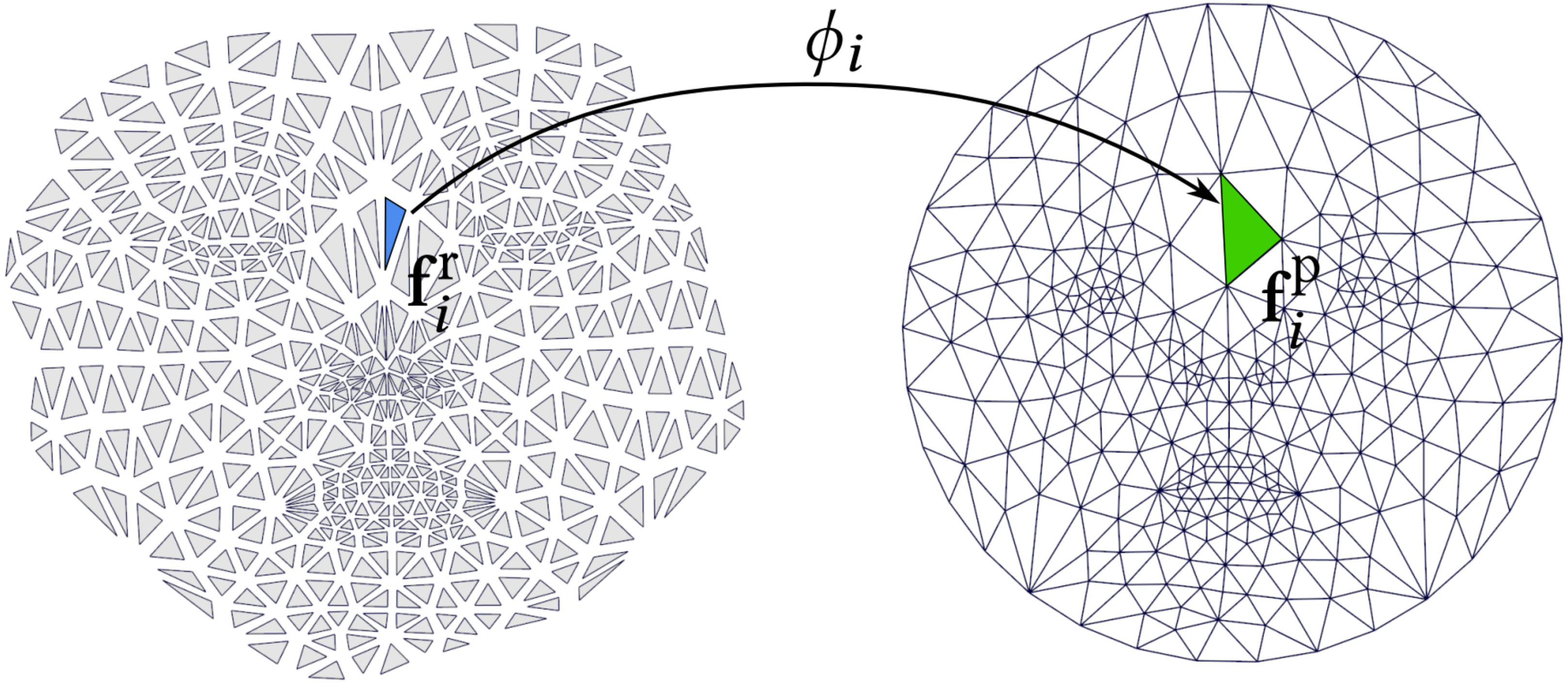
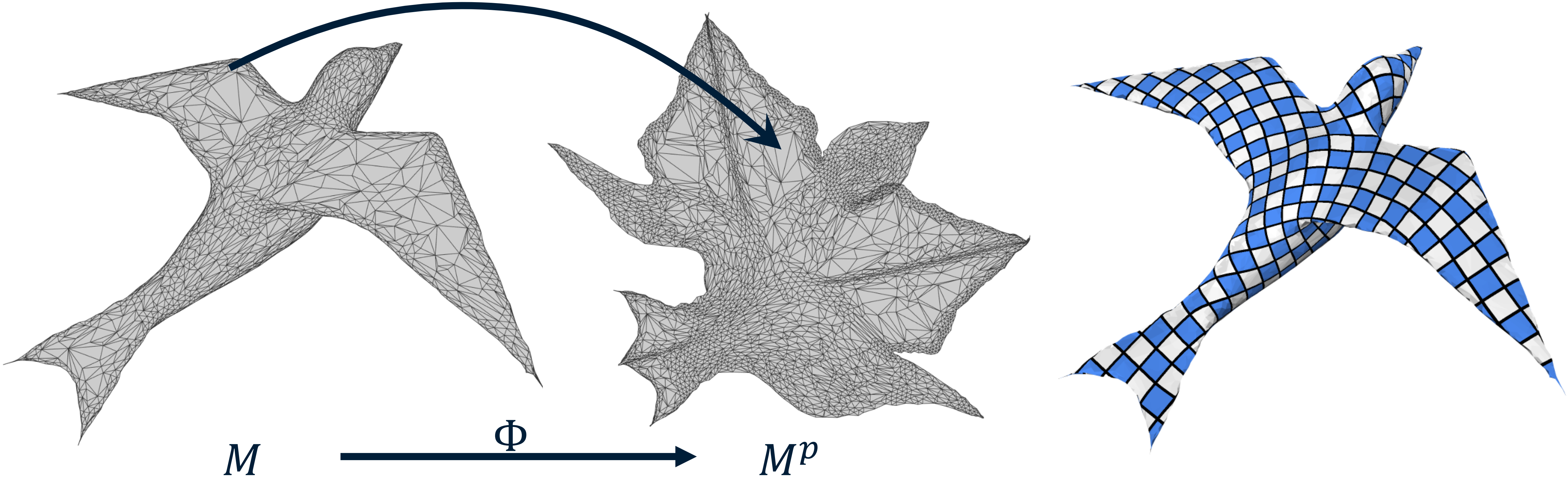


Progressive Parameterizations

Ligang Liu, Chunyang Ye, **Ruiqi Ni**, Xiao-Ming Fu

University of Science and Technology of China

Parameterizations



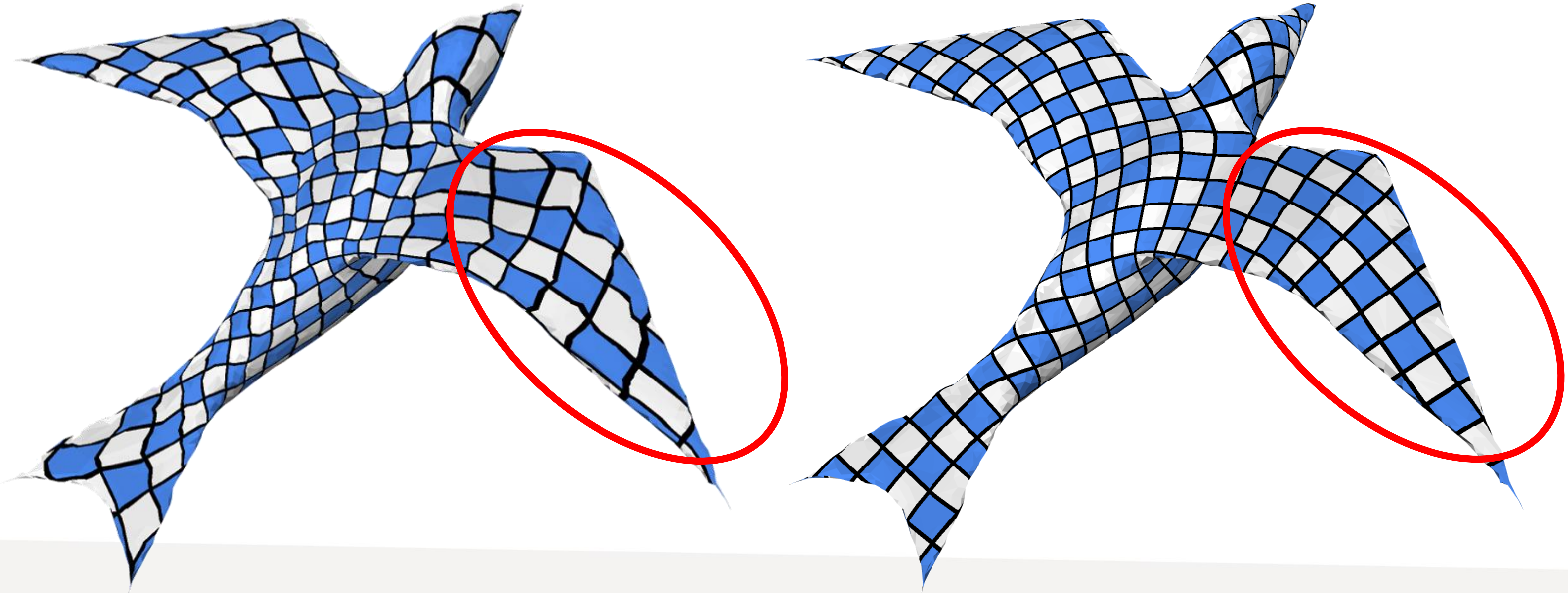
$$M \xrightarrow{\Phi} M^p$$

Applications

- Texture mapping, remeshing, inter-surface mapping, and shape analysis

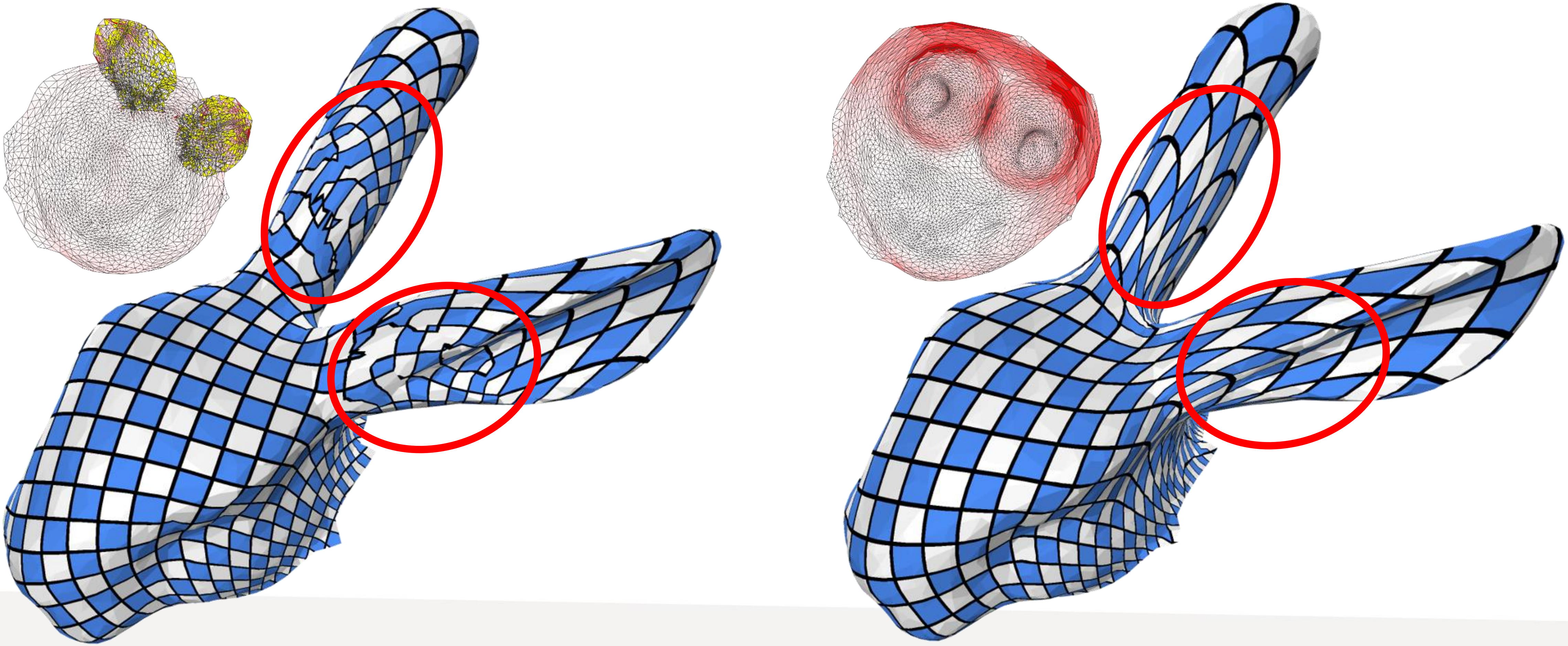
Two Basic Requirements

- Low distortion



Two Basic Requirements

- Foldover-free



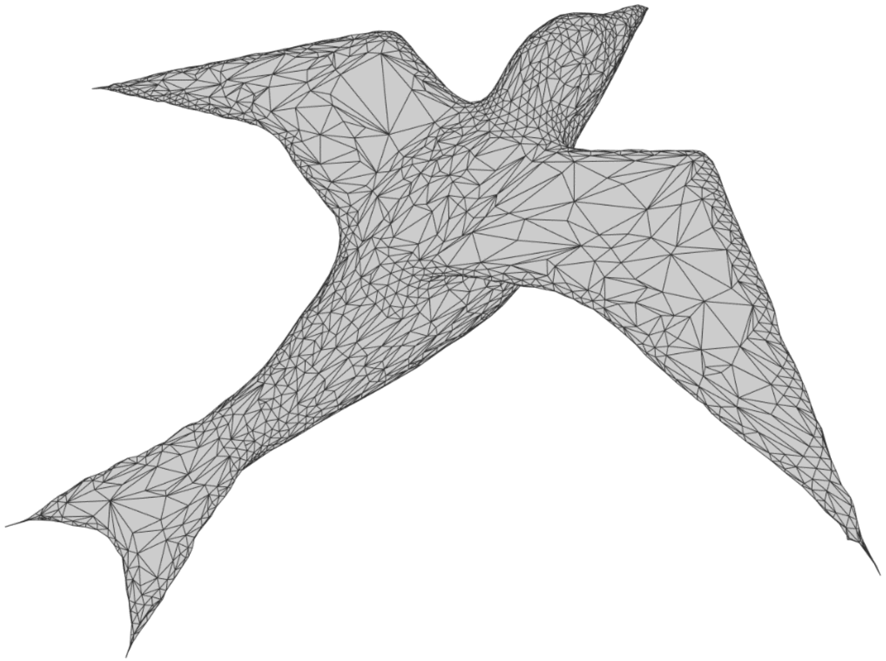
Existing Work

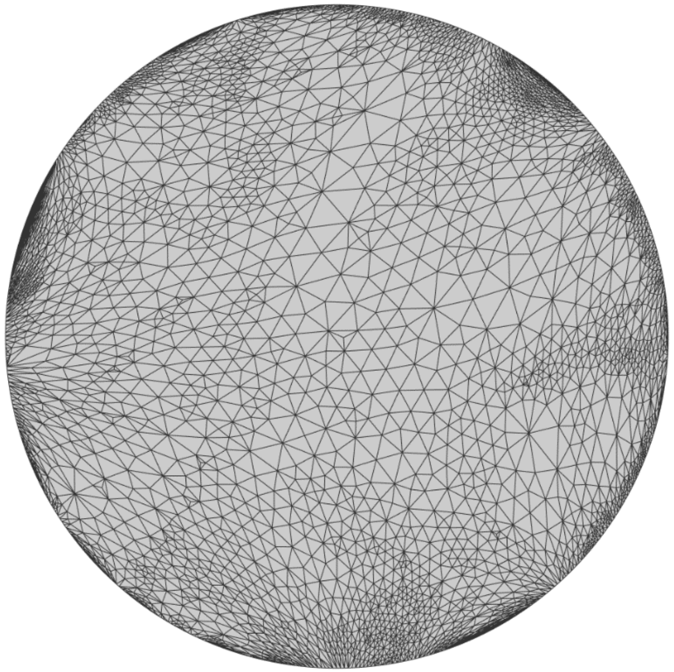
Geometric Standpoint

- Local/global methods [Liu et al. 2008; Sorkine and Alex 2007]
- Bounded distortion methods [Aigerman et al. 2014; Kovalsky et al. 2015; Lipman 2012]
- Representation based methods [Chien et al. 2016b; Fu and Liu 2016; Sheffer et al. 2005]

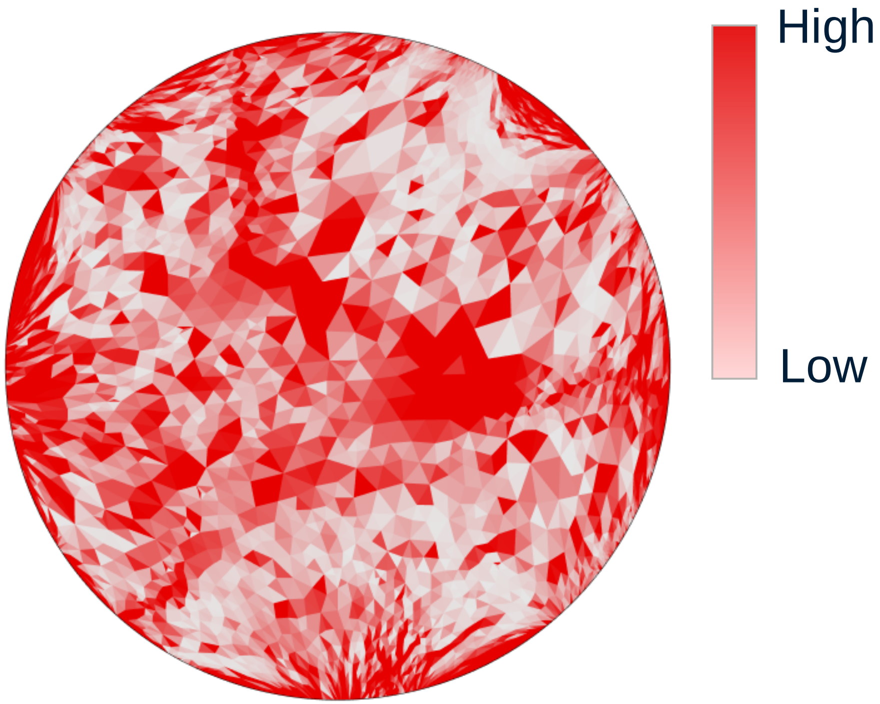
They cannot guarantee foldover-free!

Tutte’s Embedding Method



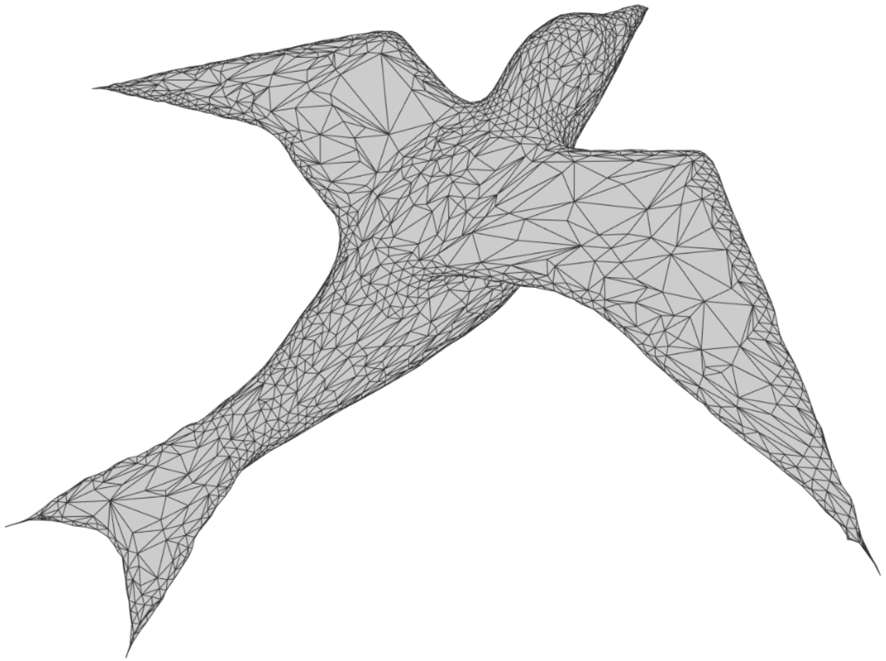


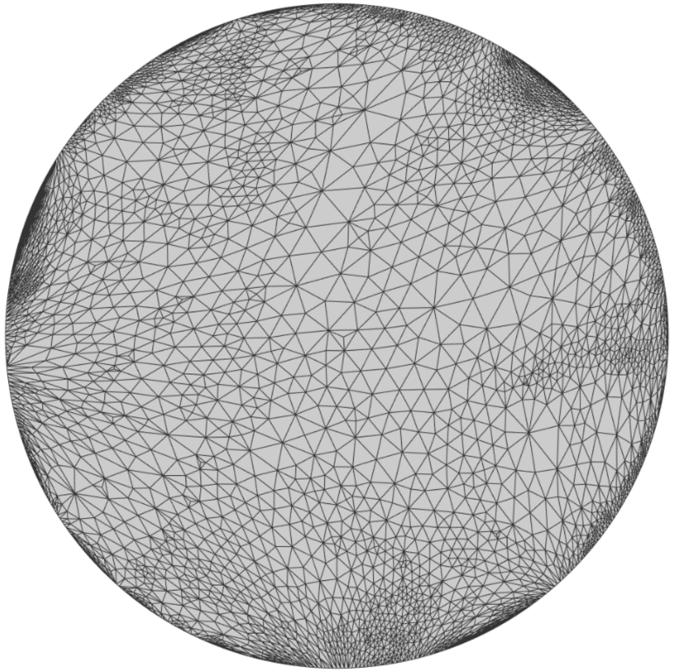
Convex boundary
Bijection guarantee



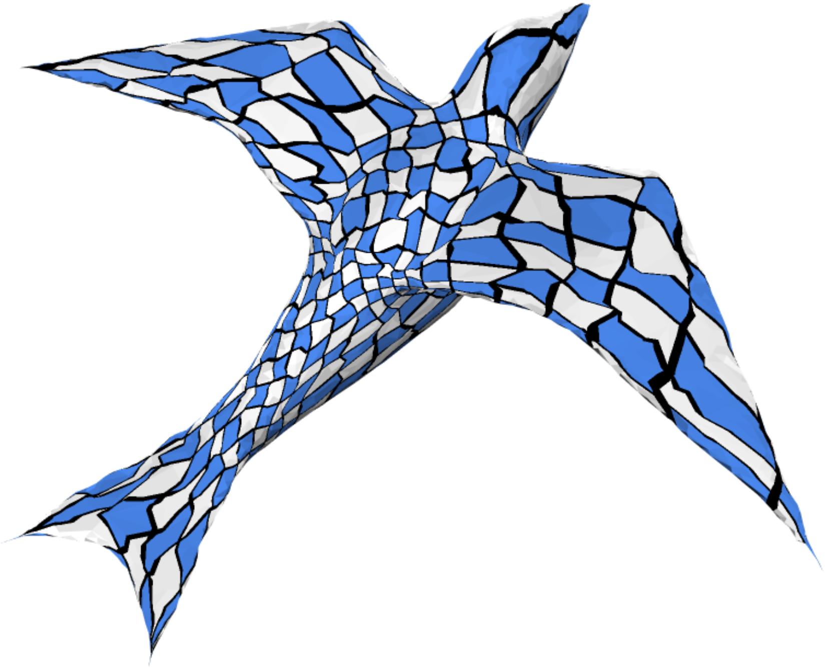
High distortion

Tutte’s Embedding Method





Convex boundary
Bijection guarantee

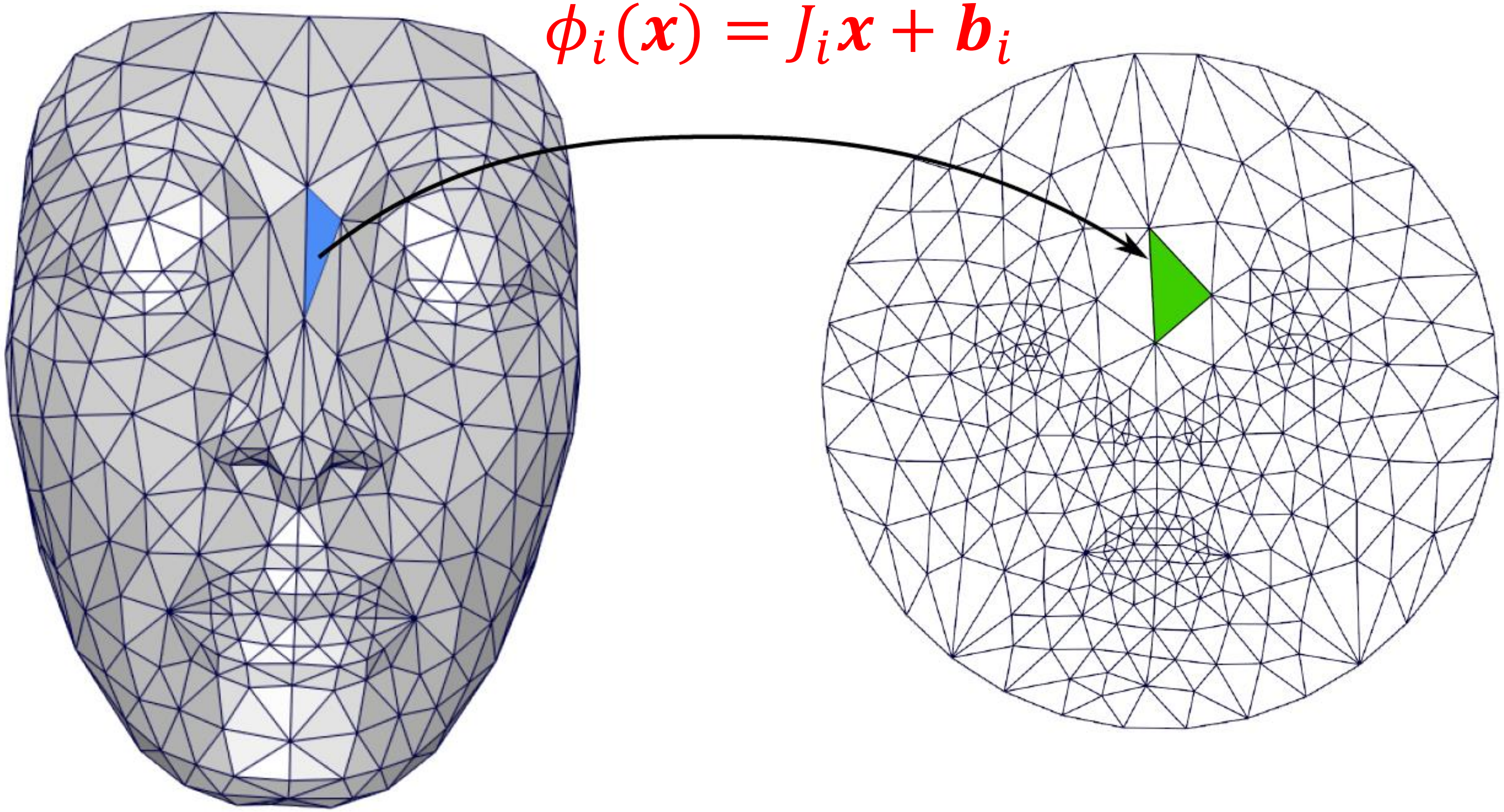


High distortion

Maintenance-based Methods

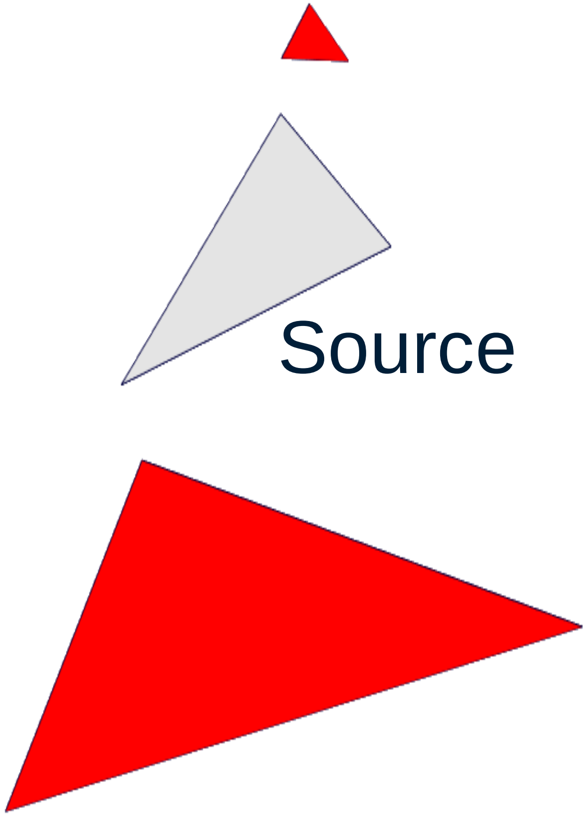
- Not violate the foldover-free constraints.

Distortion Measures



$$\phi_i(\boldsymbol{x}) = J_i\boldsymbol{x} + \boldsymbol{b}_i$$

High distortion



Source

High distortion

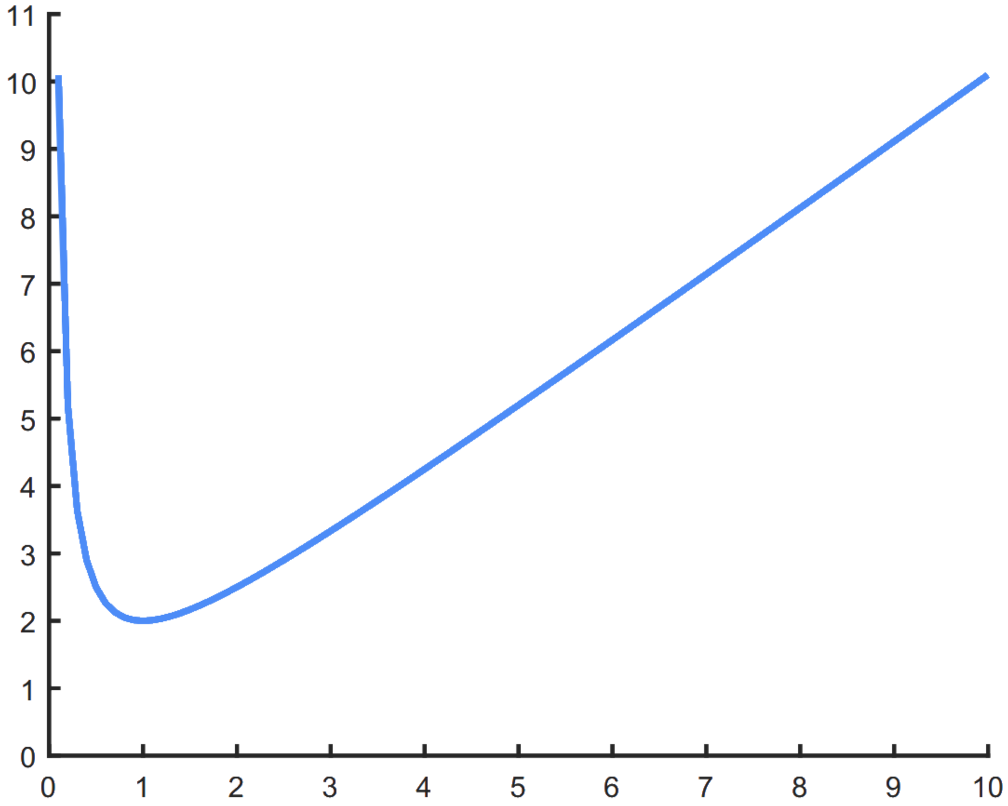
Distortion Measures

- Symmetric Dirichlet metric: [Smith and Schaefer 2015]

$$D(J_i) = \frac{1}{4}\left(\|J_i\|_F^2 + \|J_i^{-1}\|_F^2\right)$$
$$= \frac{1}{4}\left(\sigma_i^2 + \sigma_i^{-2} + \tau_i^2 + \tau_i^{-2}\right)$$

σ_i, τ_i: singular values of J_i

Opt value = 1 when $\sigma_i = \tau_i = 1$



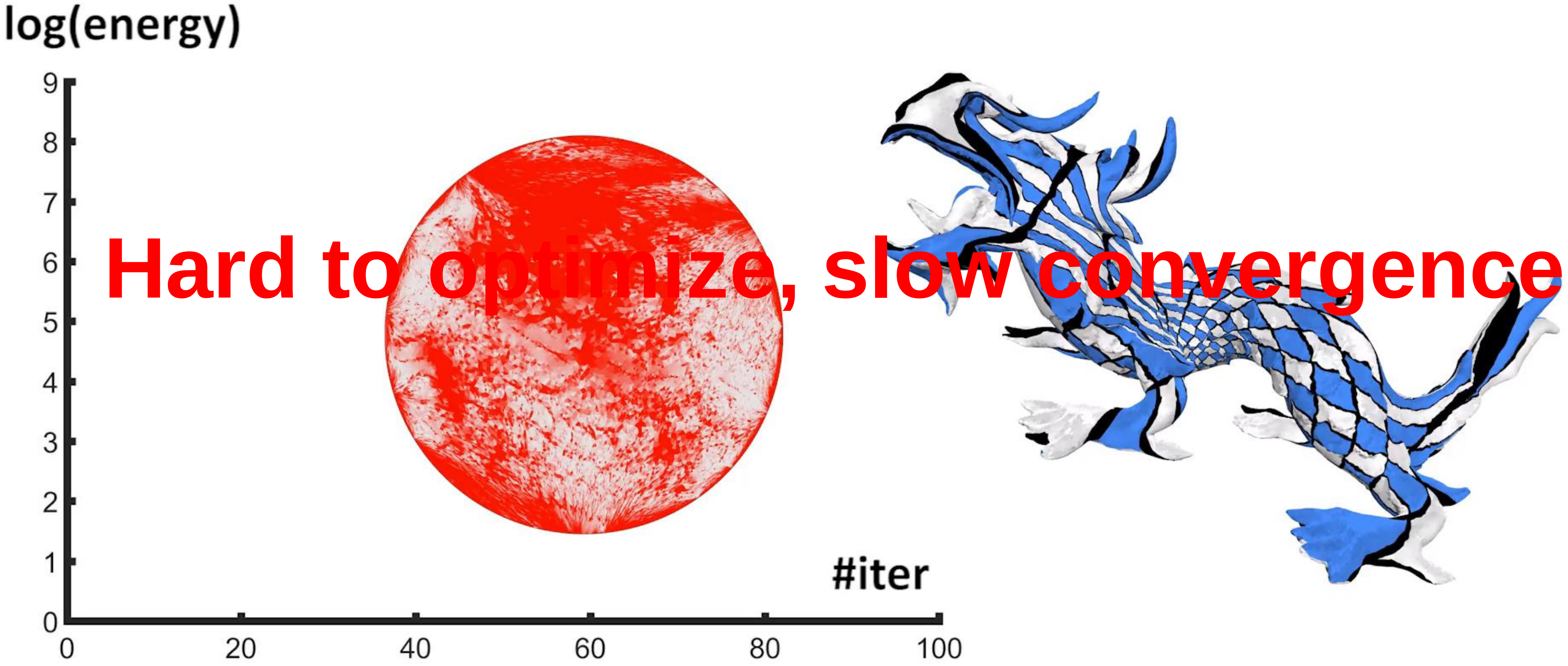
Challenge

- Highly non-convex and non-linear
- Extremely large distortion on initializations

Initializations

Bottom

Challenge



Hard to optimize, slow convergence!

Existing Work

Optimization Standpoint

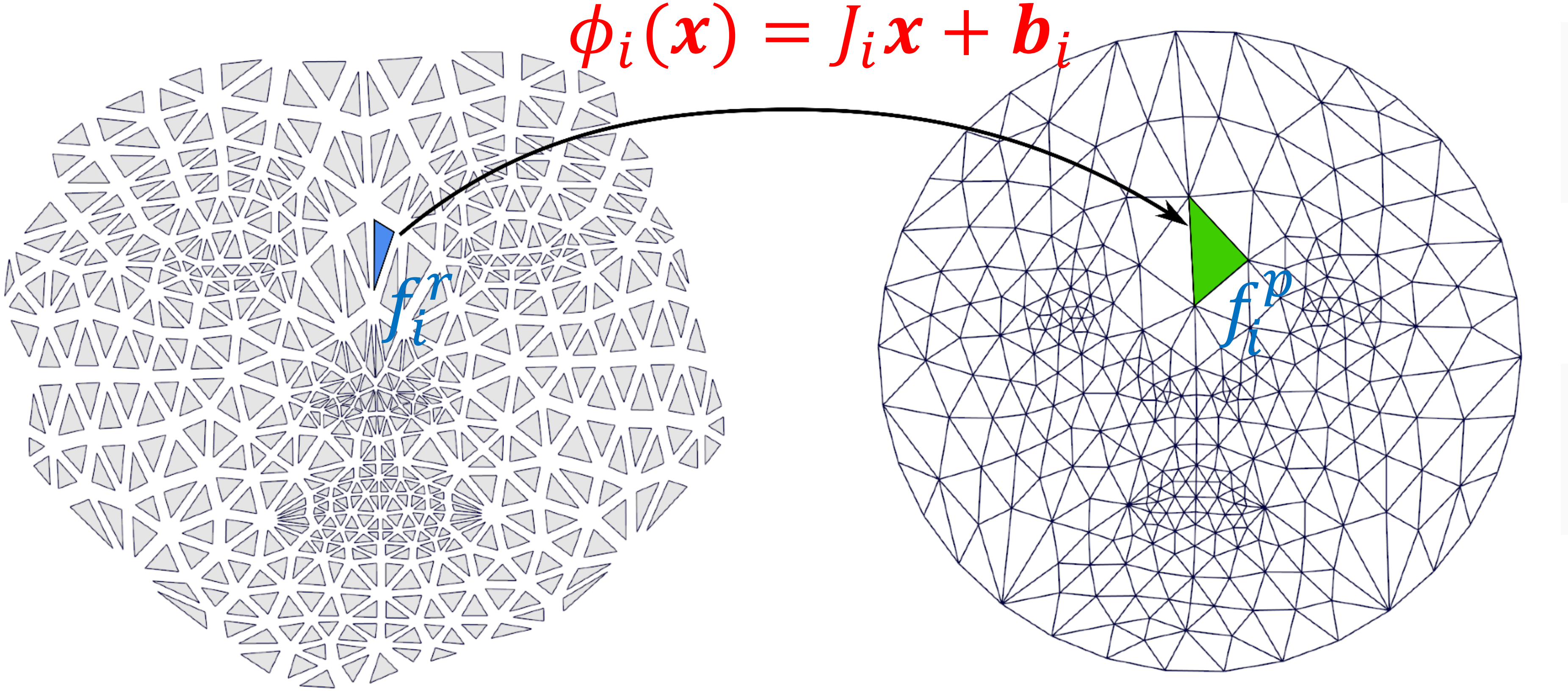
- Block coordinate descent methods [Fu et al. 2015; Hormann and Greiner 2000]
- Quasi-Newton method [Smith and Schaefer 2015]
- Preconditioning methods [Claici et al. 2017; Kovalsky et al. 2016]
- Reweighting descent method [Rabinovich et al. 2017]
- Composite majorization method [Shtengel et al. 2017]

Only thinking from the view of solver!

Our Approach

Progressive Parameterizations

Reference-guided Distortion Metric



$$\phi_i(\boldsymbol{x}) = J_i\boldsymbol{x} + \boldsymbol{b}_i$$

Reference M^r: A set of individual triangles

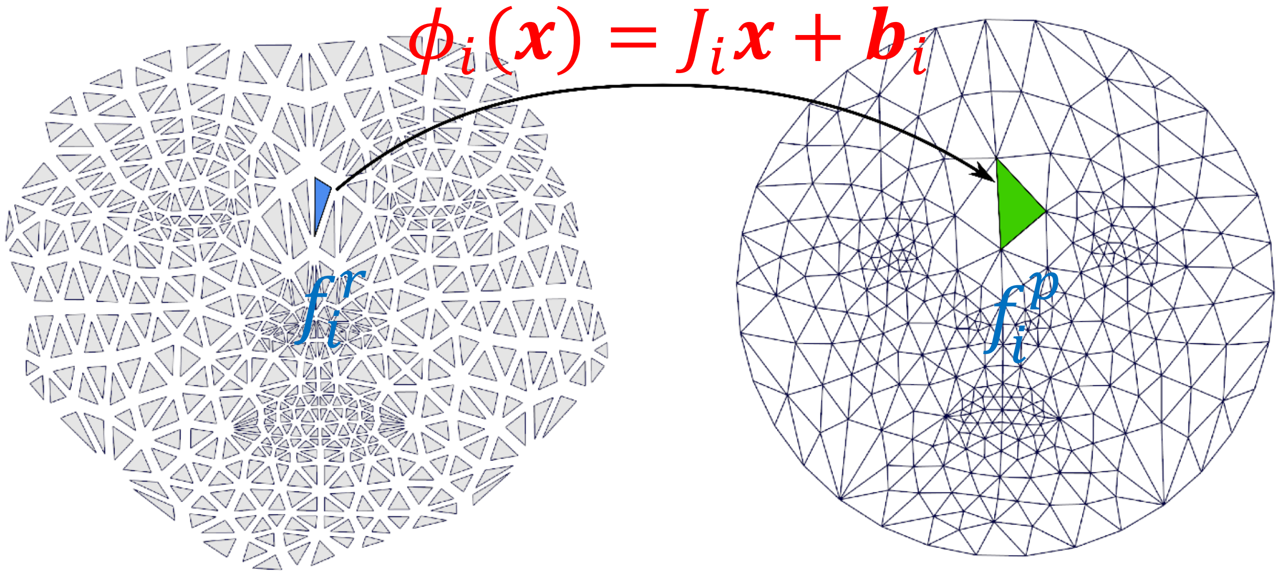
Parameterized mesh M^p

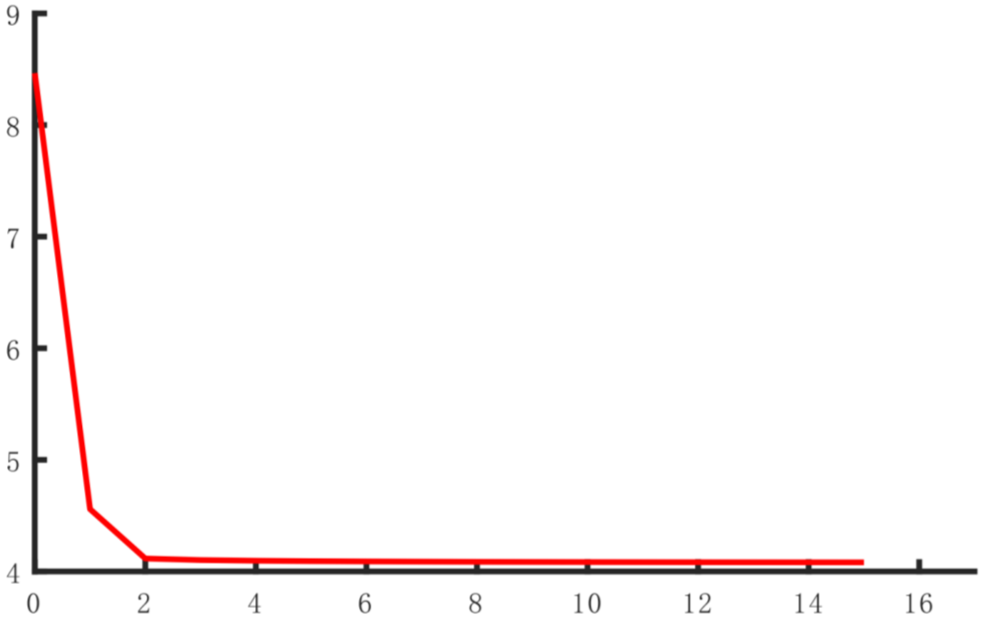
Distortion Metric:

$$D\left(f_i^r, f_i^p\right) = D(J_i)$$

Input Mesh:
Ideal Reference

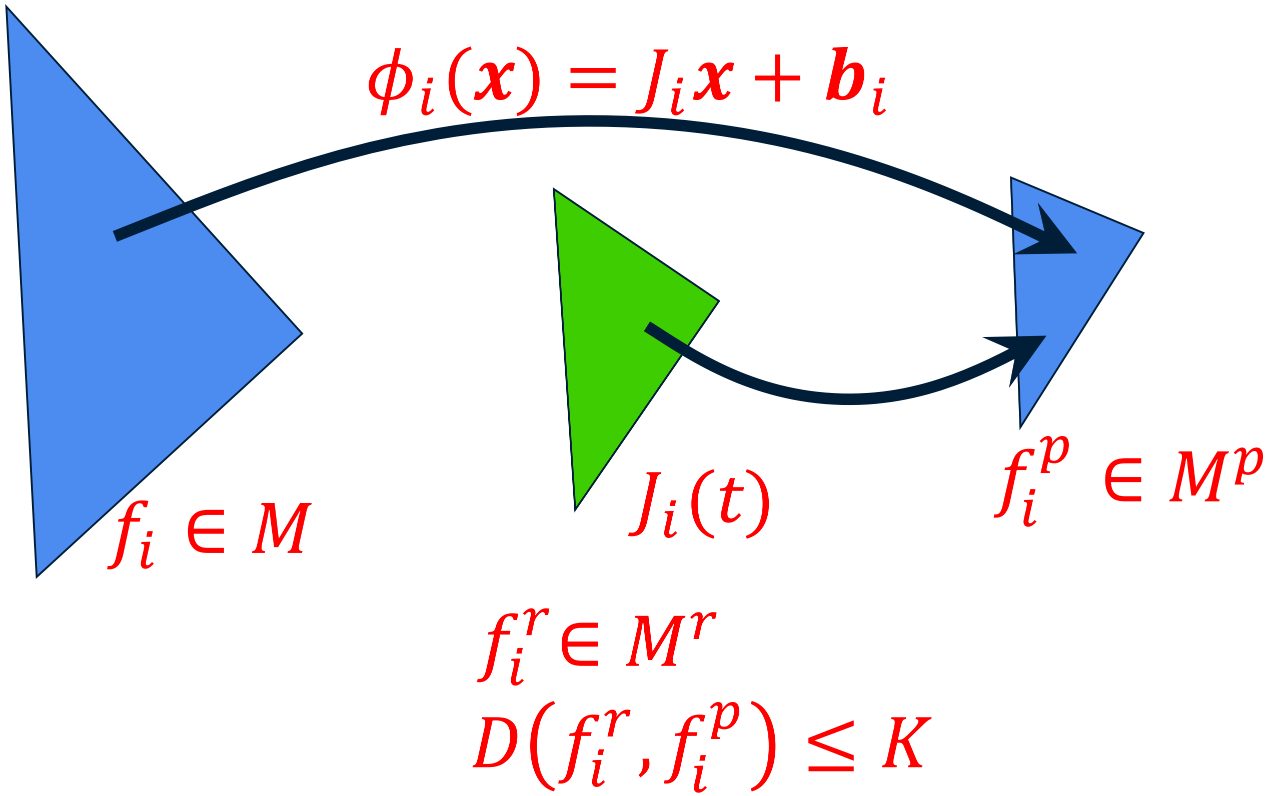
Key Observation





If all $D\left(f_i^r, f_i^p\right) \le K, \forall i$, only a few iterations in the optimization of $E(M^r, M^p)$ are necessary.

Change The Reference



$\phi_i(\boldsymbol{x}) = J_i\boldsymbol{x} + \boldsymbol{b}_i$

$f_i \in M$

$J_i(t)$

$f_i^p \in M^p$

$f_i^r \in M^r$

$D\left(f_i^r, f_i^p\right) \leq K$

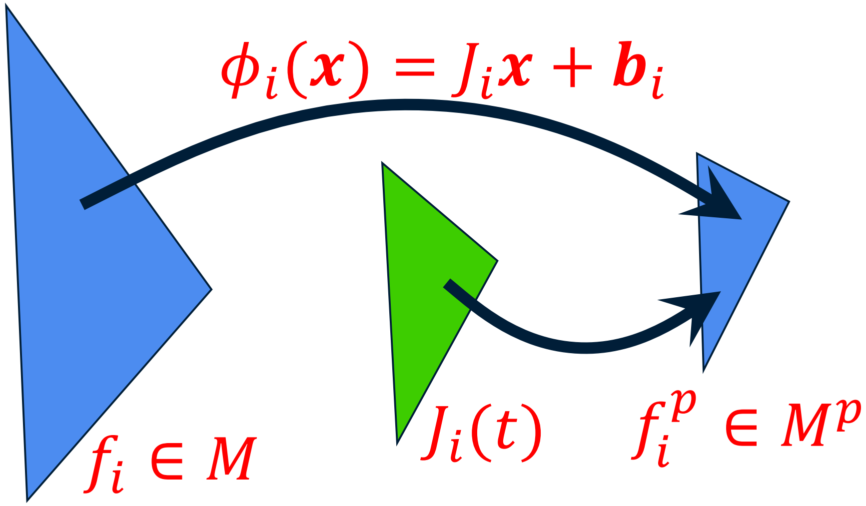
Goal: find a triangle between f_i and f_i^p as the reference f_i^r that satisfies $D\left(f_i^r, f_i^p\right) \leq K$.

Choose The Reference

- Exponential function [Alexa 2002; Grassia 1998; Rossignac and Vinacua 2011]:

$$J_i(t) = U_i \mathrm{diag}(\sigma_i^t, \tau_i^t) V_i^T$$

where $J_i = U_i \mathrm{diag}(\sigma_i, \tau_i) V_i^T$

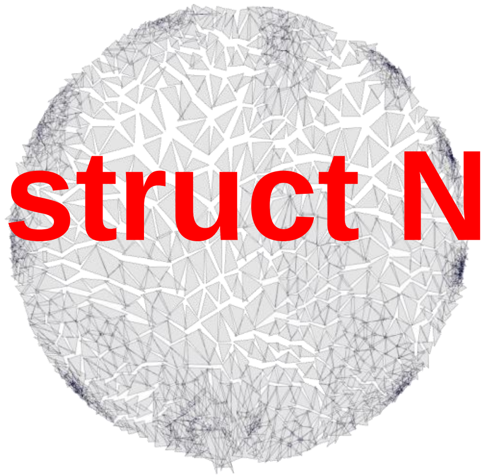


Construction of new reference

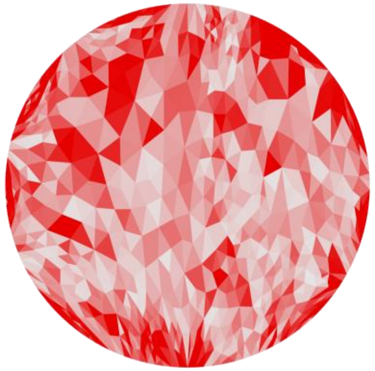
Optimization Of The New Reference

Progressively Construct New Reference!

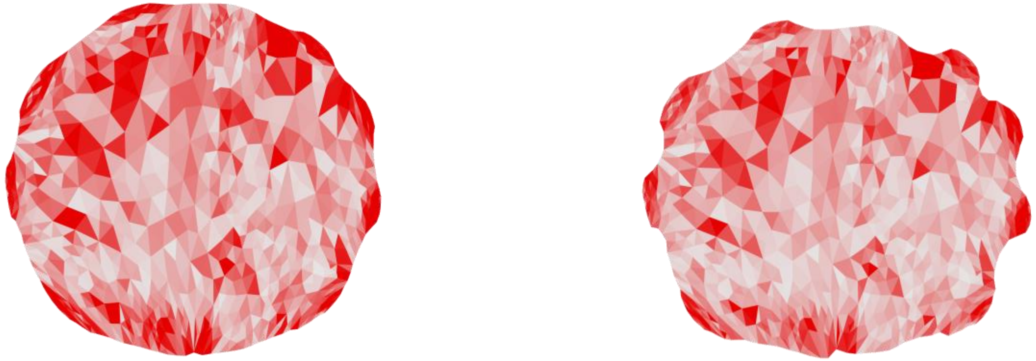
Ideal Reference



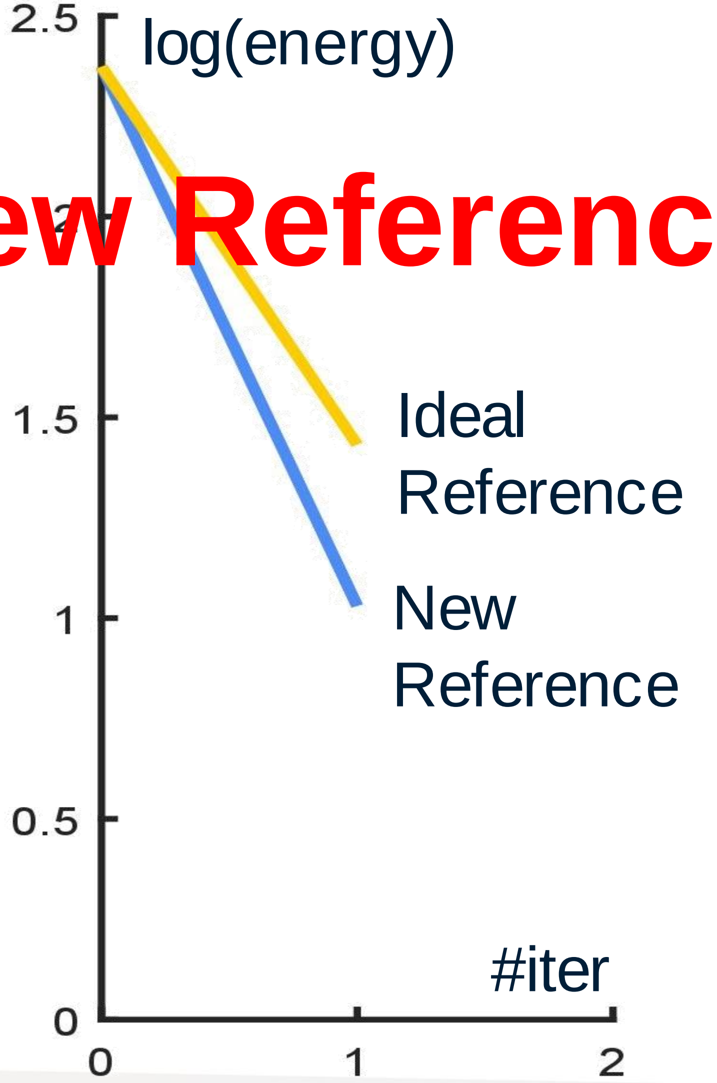
New Reference



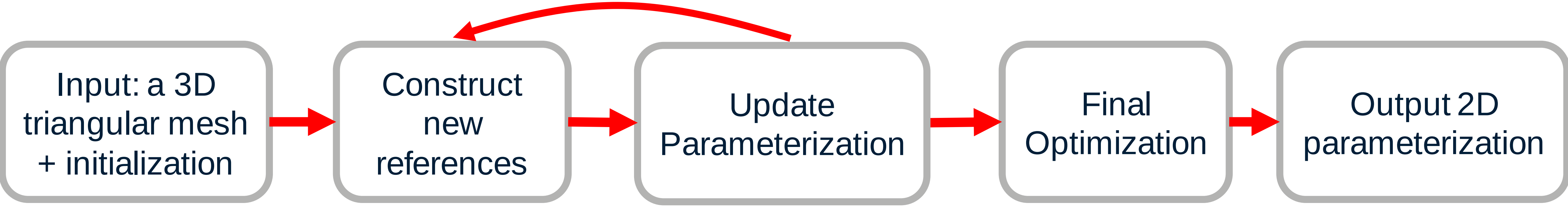
Tutte's Initialization



SLIM [Rabinovich et al. 2017]



Progressive Parameterization



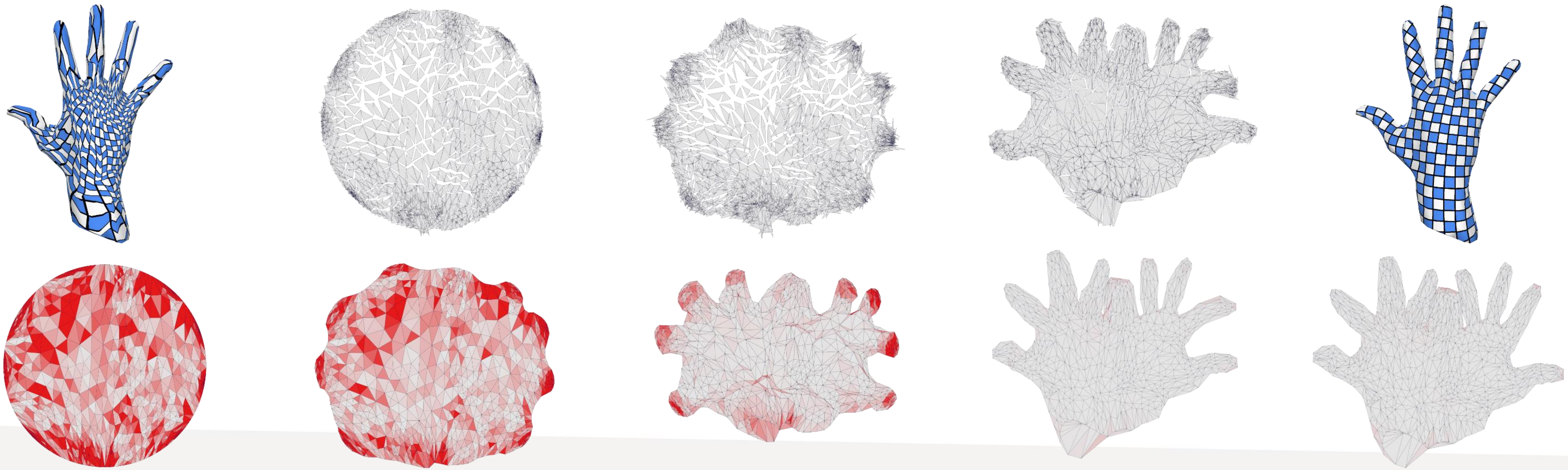
Input: a 3D triangular mesh + initialization

Construct new references

Update Parameterization

Final Optimization

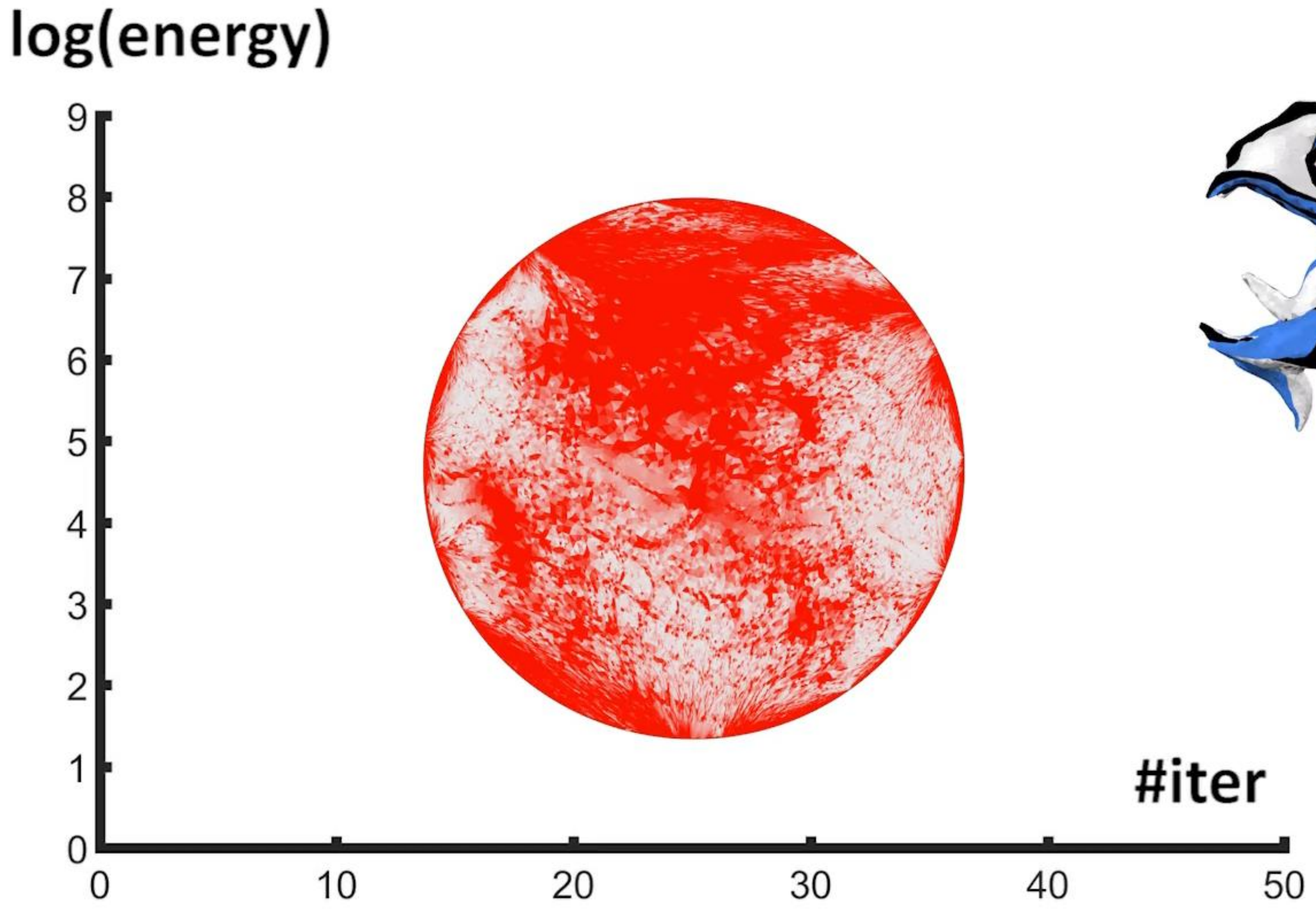
Output 2D parameterization



Hybrid Solver

- SLIM [Rabinovich et al. 2017]
  - Pros: effectively penalize the maximum distortion
  - Cons: a poor convergence rate
- CM [Shtengel et al. 2017]
  - Pros: converge quickly
  - Cons: cannot reduce large distortion quickly
- Hybrid
  - First perform SLIM solver
  - Then use the CM solver

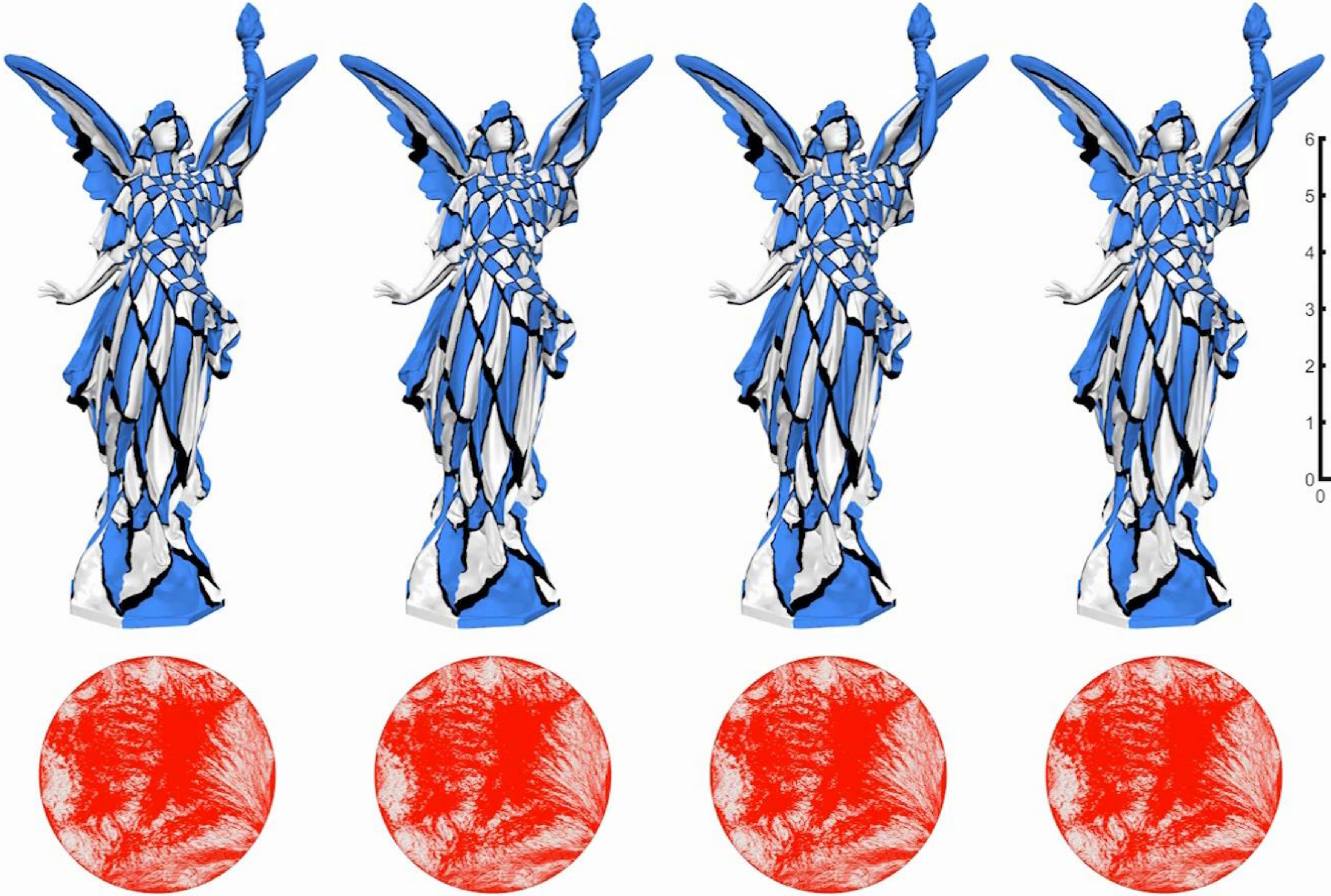
The Former Dragon Example

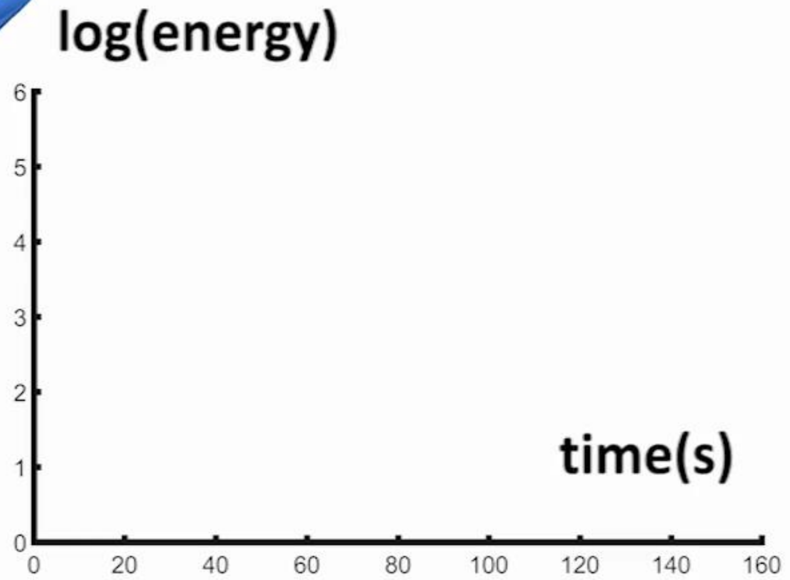


Experiments

AKVF [Claici et al. 17] **CM** [Shtengel et al. 17] **SLIM** [Rabinovich et al. 17] **Ours**

2x playback

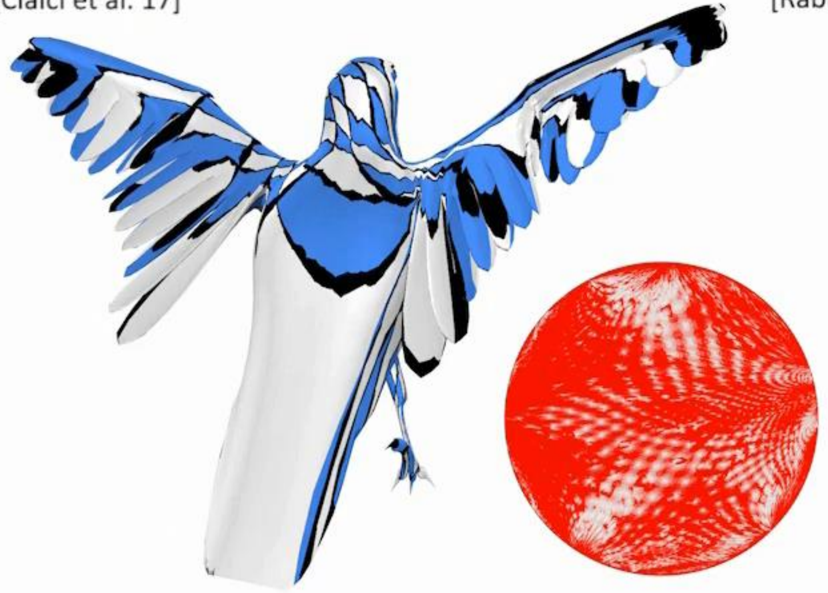




#V: 900k, #F: 1792k

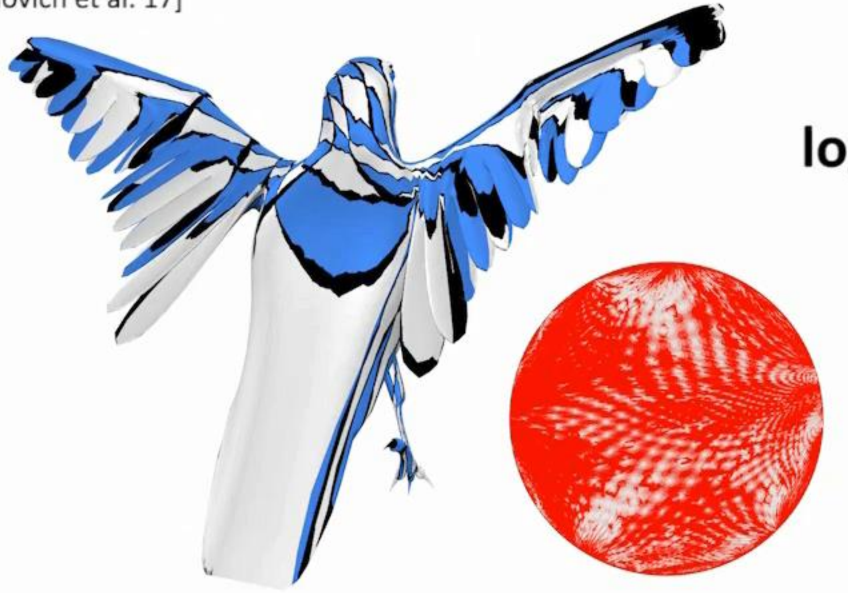
AKVF

[Claici et al. 17]

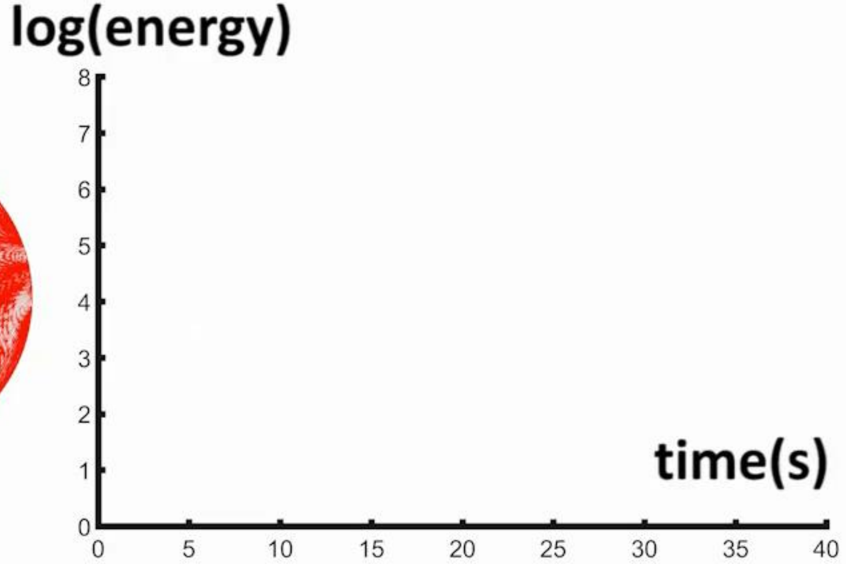


SLIM

[Rabinovich et al. 17]

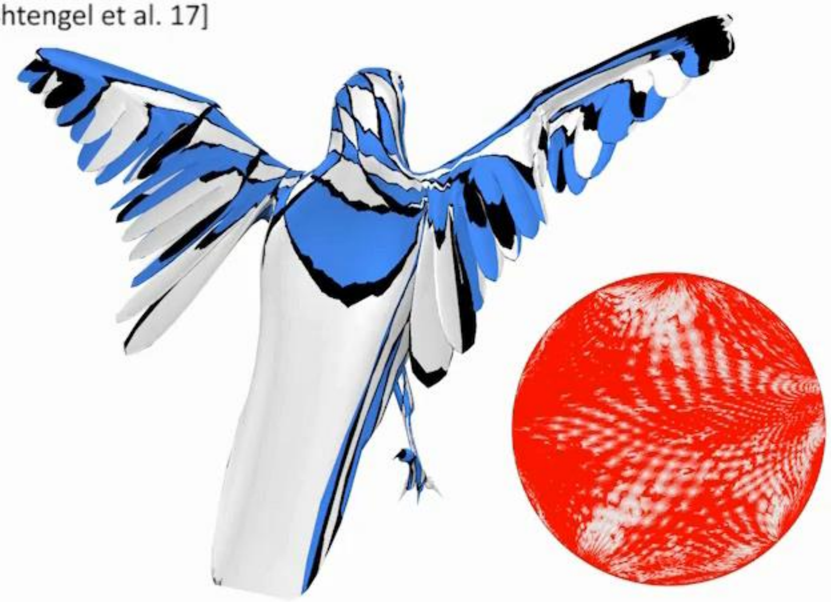


playback

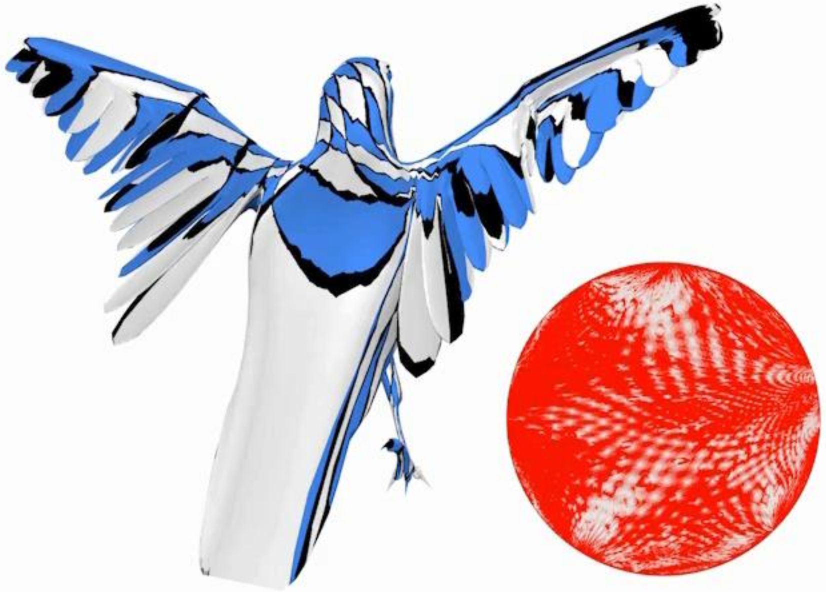


CM

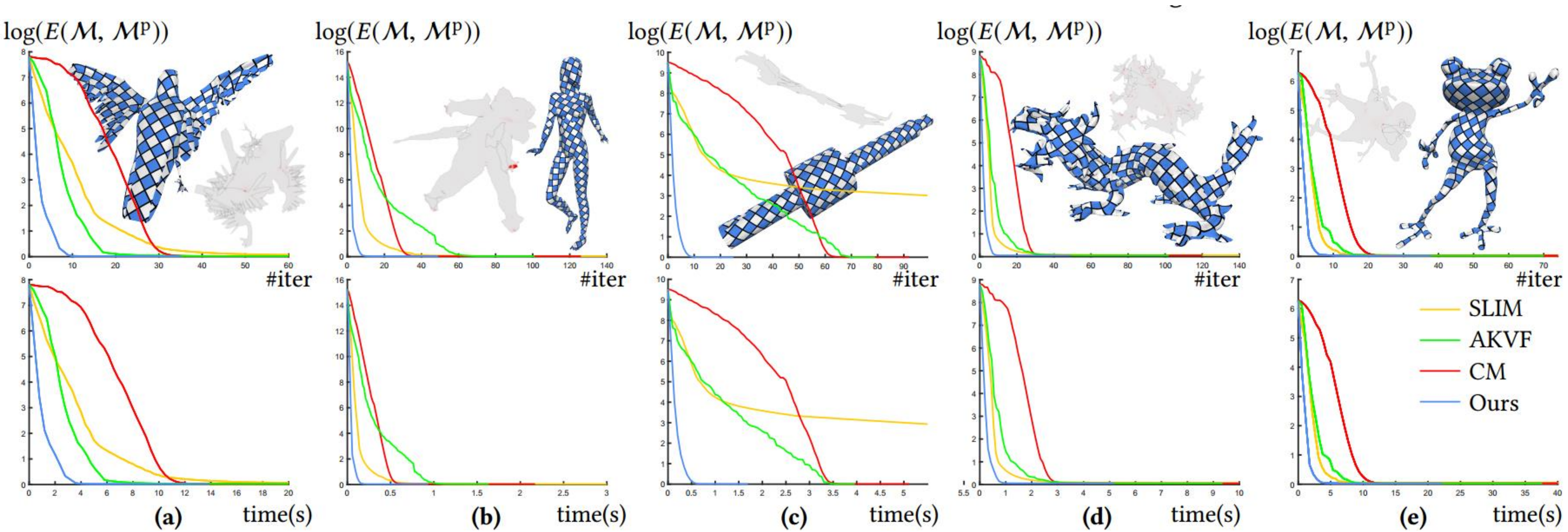
[Shtengel et al. 17]



Ours



#V: 195k, #F: 382k

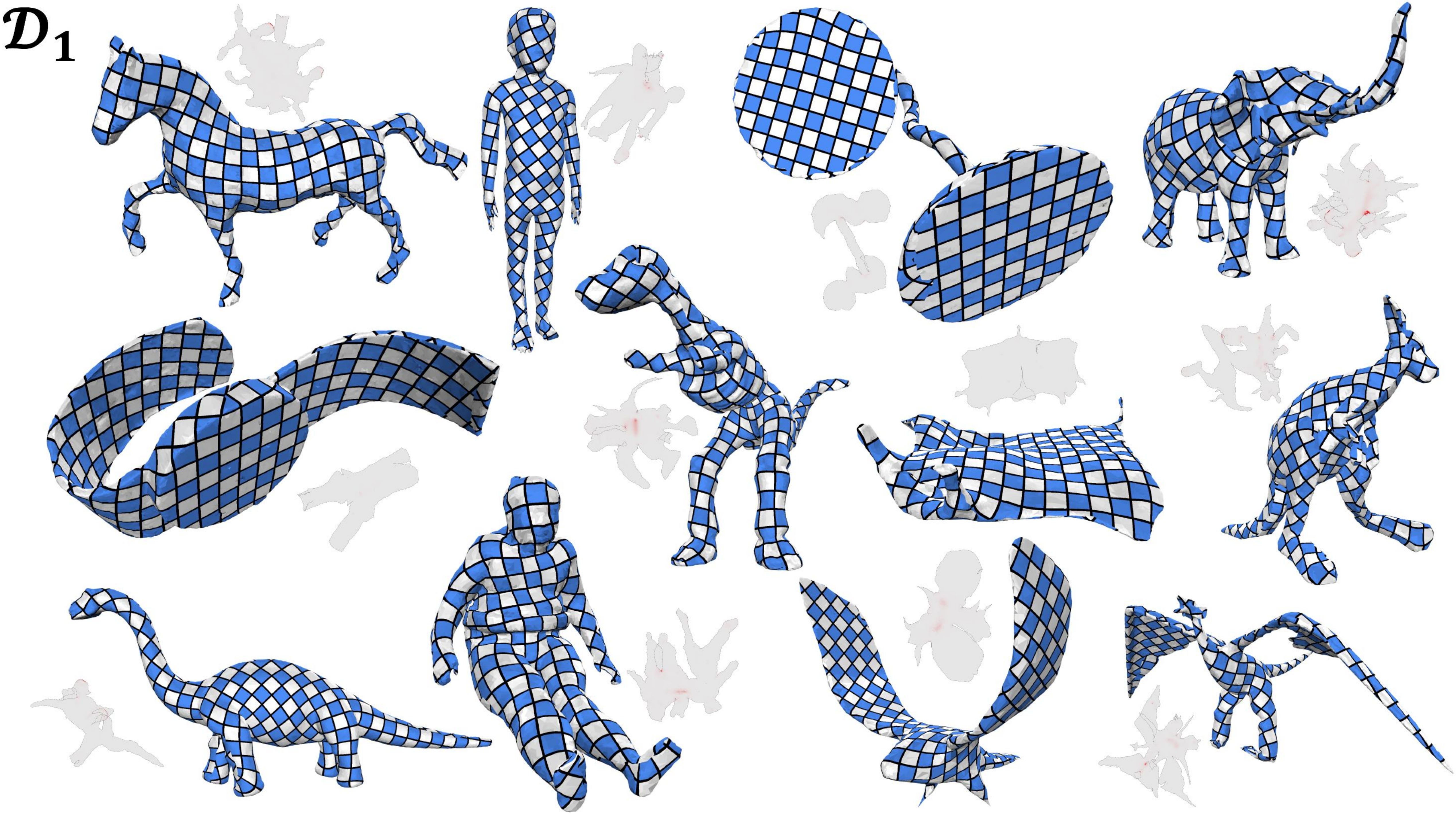

log(E(M, M^p))
#iter
time(s)
0 10 20 30 40 50 60
0 2 4 6 8 10 12 14 16 18 20
0 20 40 60 80 100 120 140
0 0.5 1 1.5 2 2.5 3
0 10 20 30 40 50 60 70 80 90
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5
0 1 2 3 4 5 6 7 8 9 10
0 10 20 30 40 50 60 70
0 5 10 15 20 25 30 35 40
SLIM
AKVF
CM
Ours
(a)
(b)
(c)
(d)
(e)

Benchmark

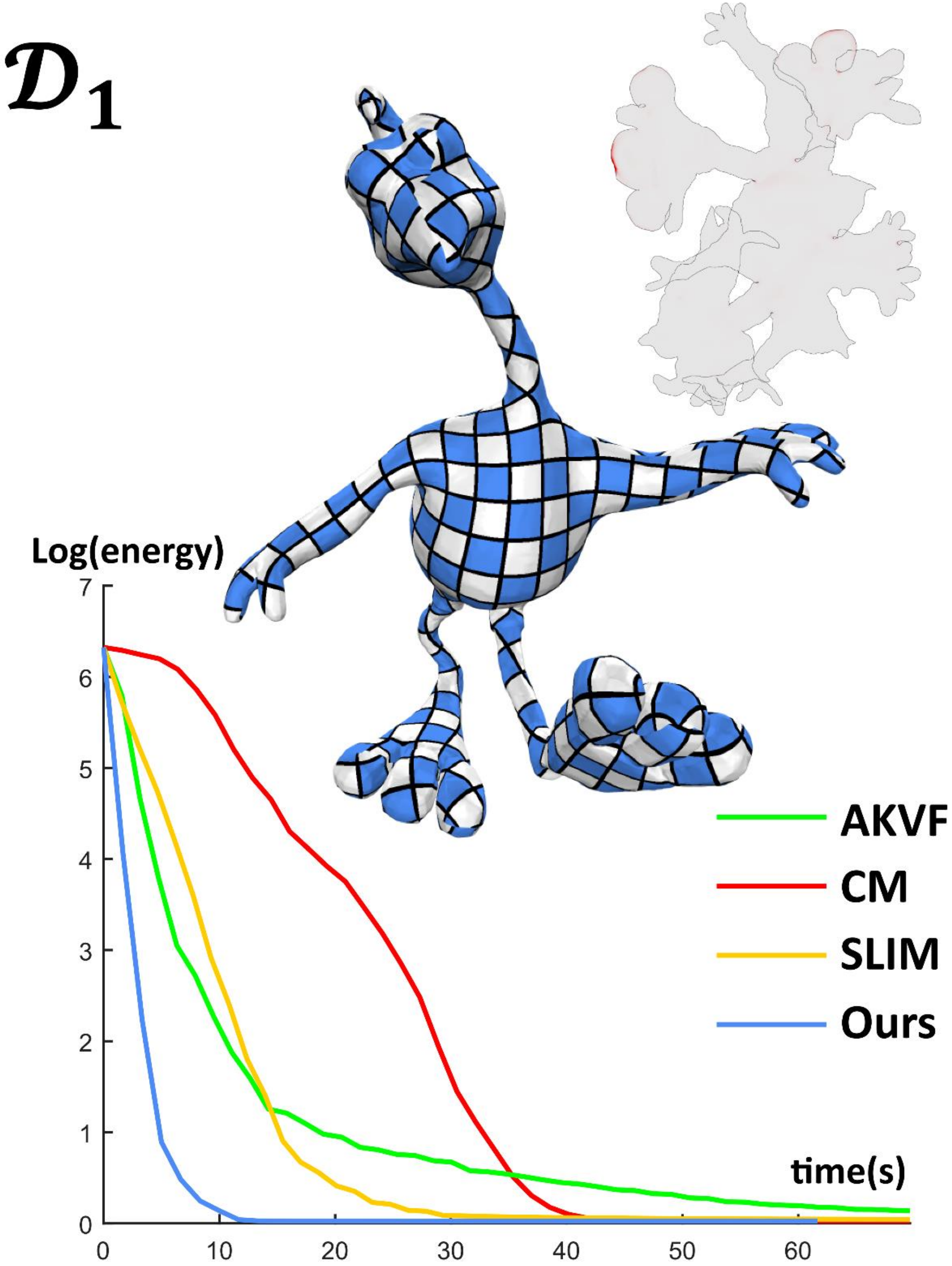
$\mathcal{D}_1$: ***10273*** well cut meshes

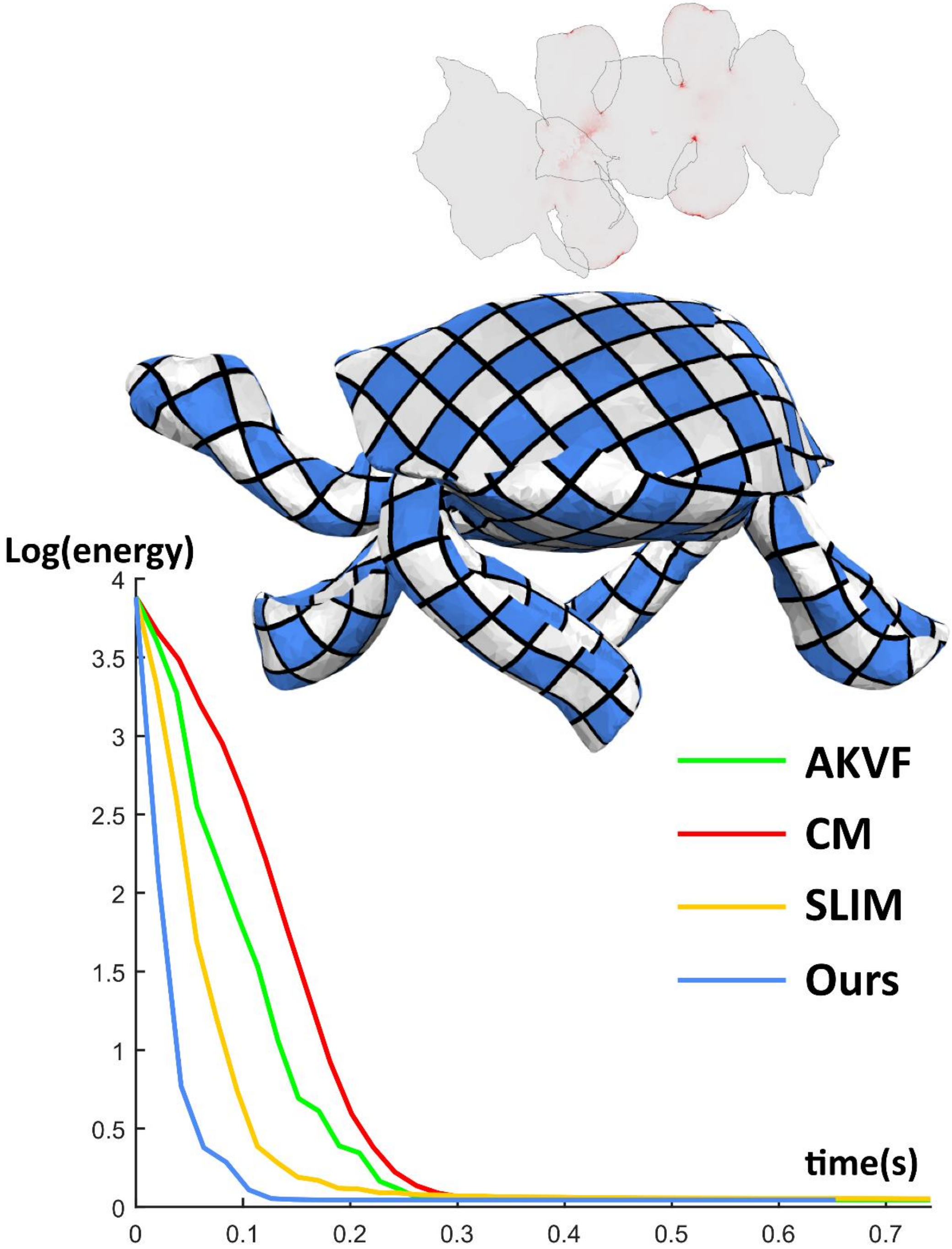
$\mathcal{D}_2$: ***6189*** moderately bad cut meshes

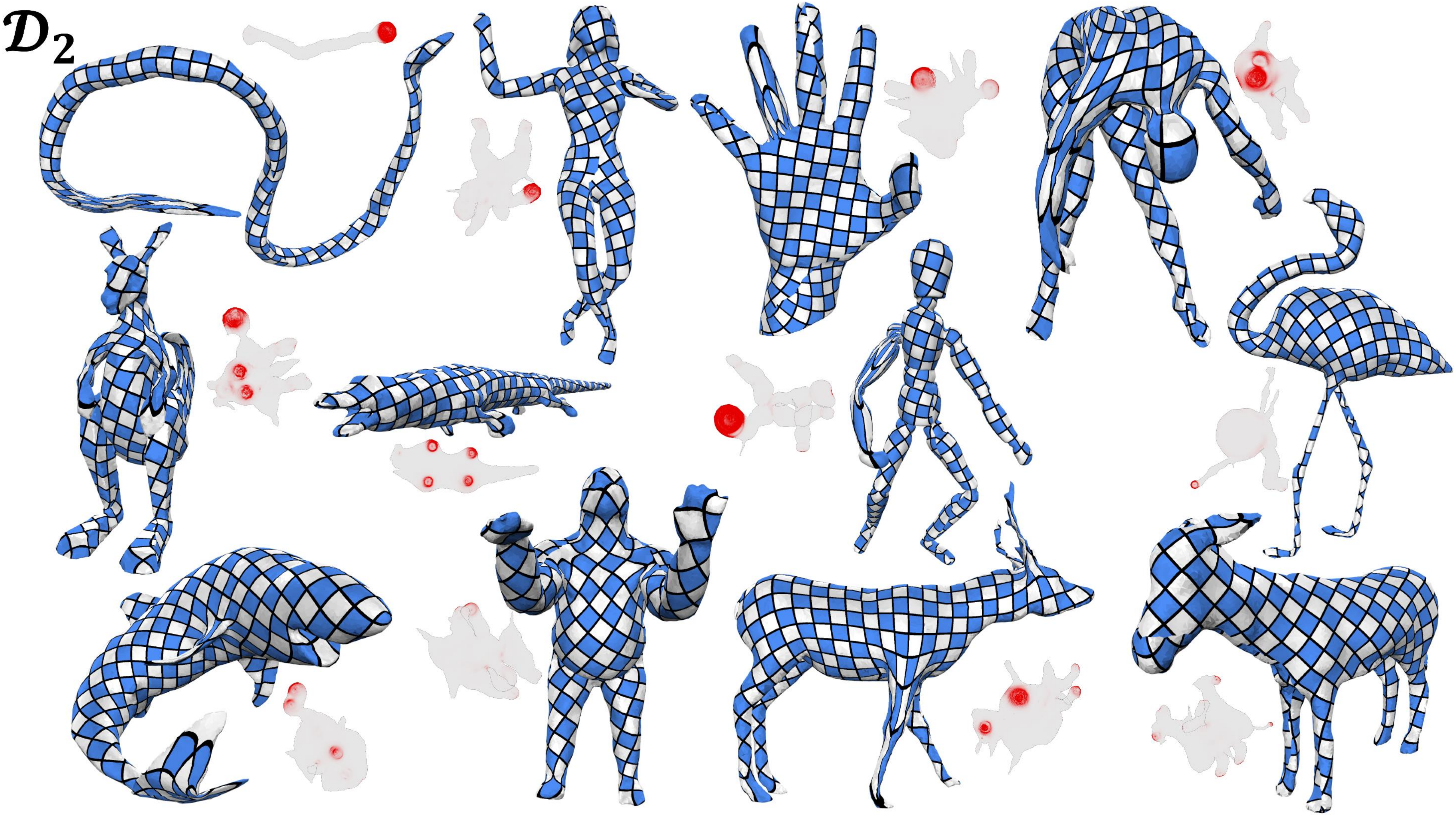
$\mathcal{D}_3$: ***4250*** extremely challenging examples


$\mathcal{D}_1$

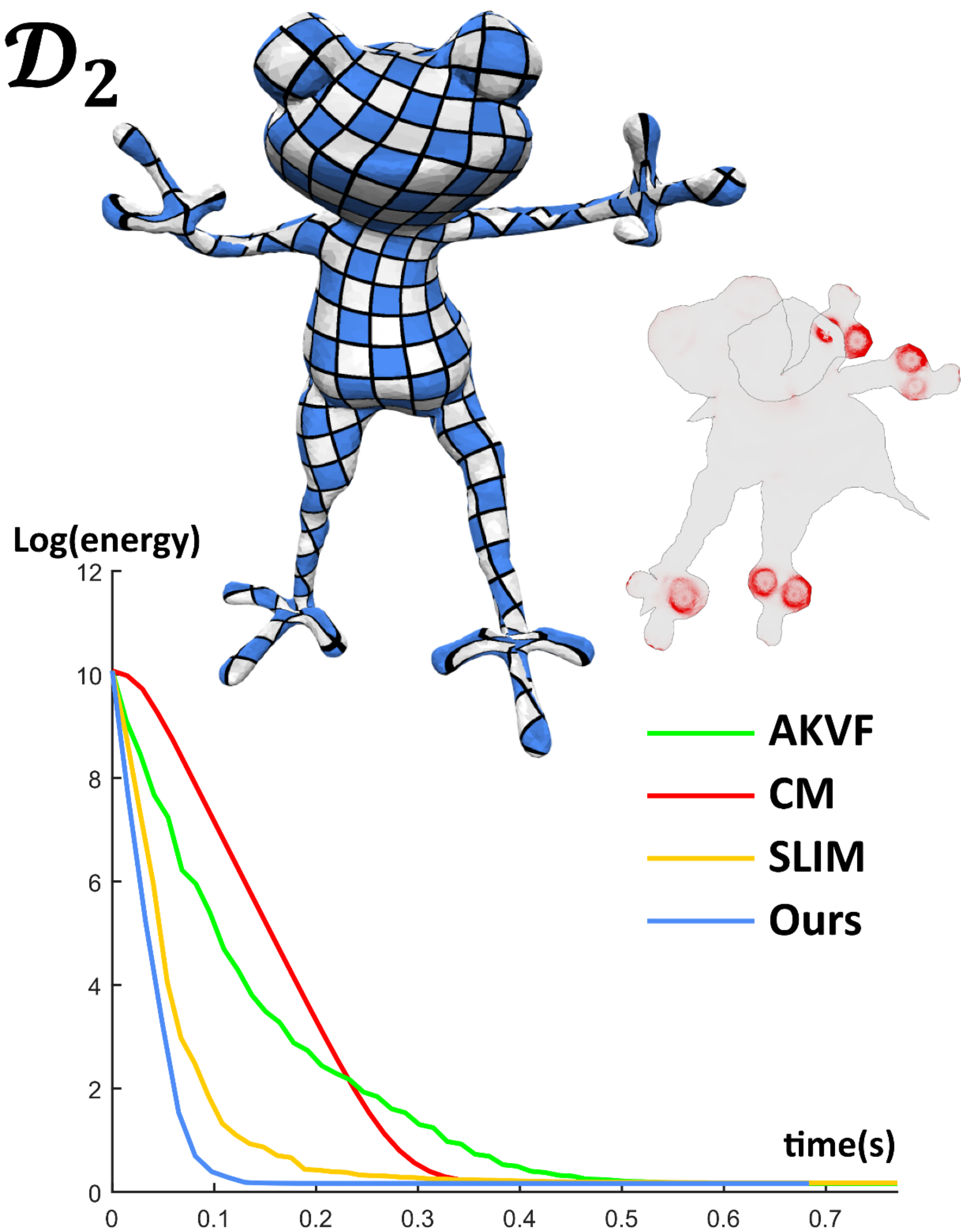
$\mathcal{D}_1$

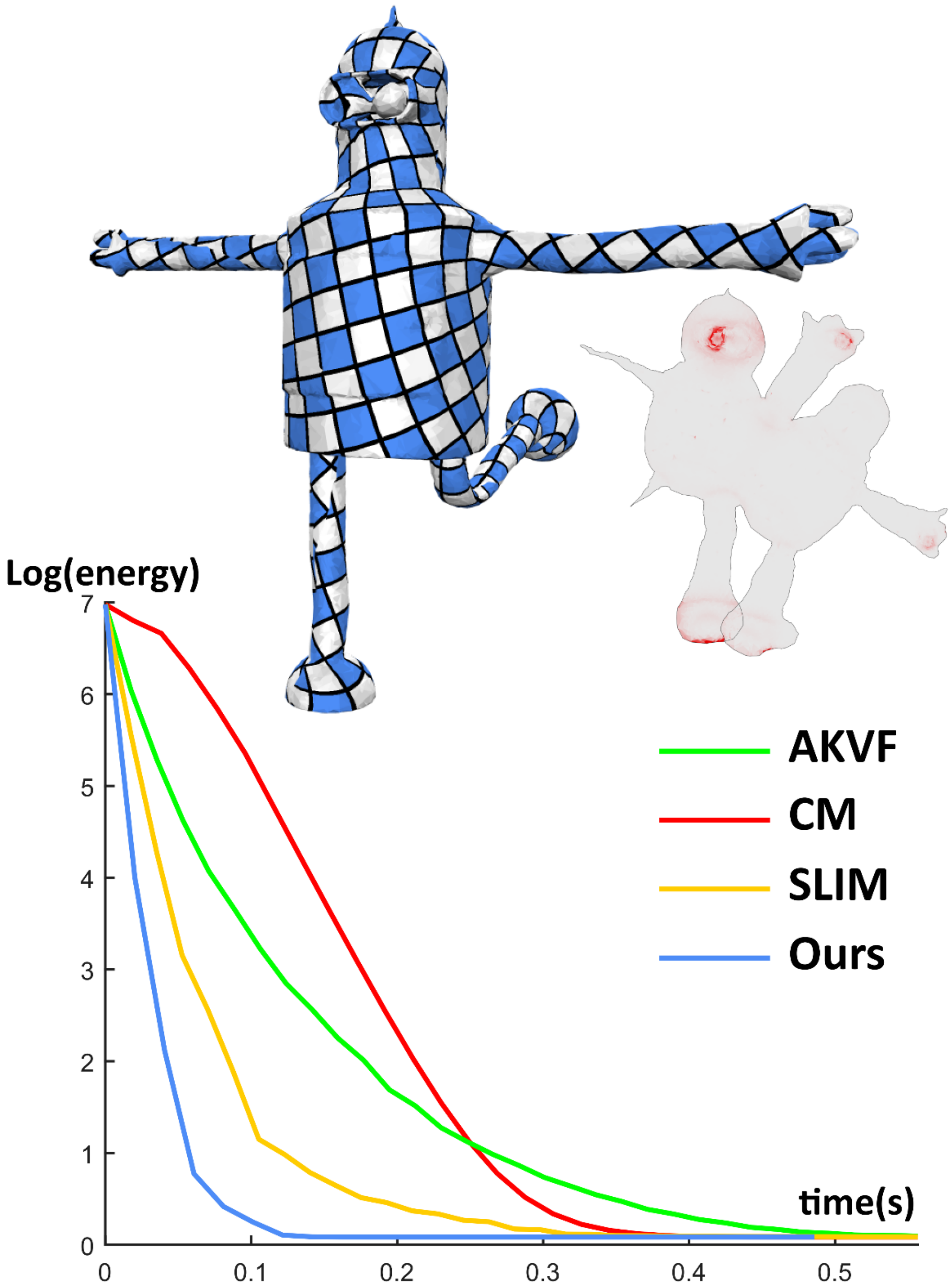

Log(energy)
7
6
5
4
3
2
1
0
0
10
20
30
40
50
60
time(s)
AKVF
CM
SLIM
Ours

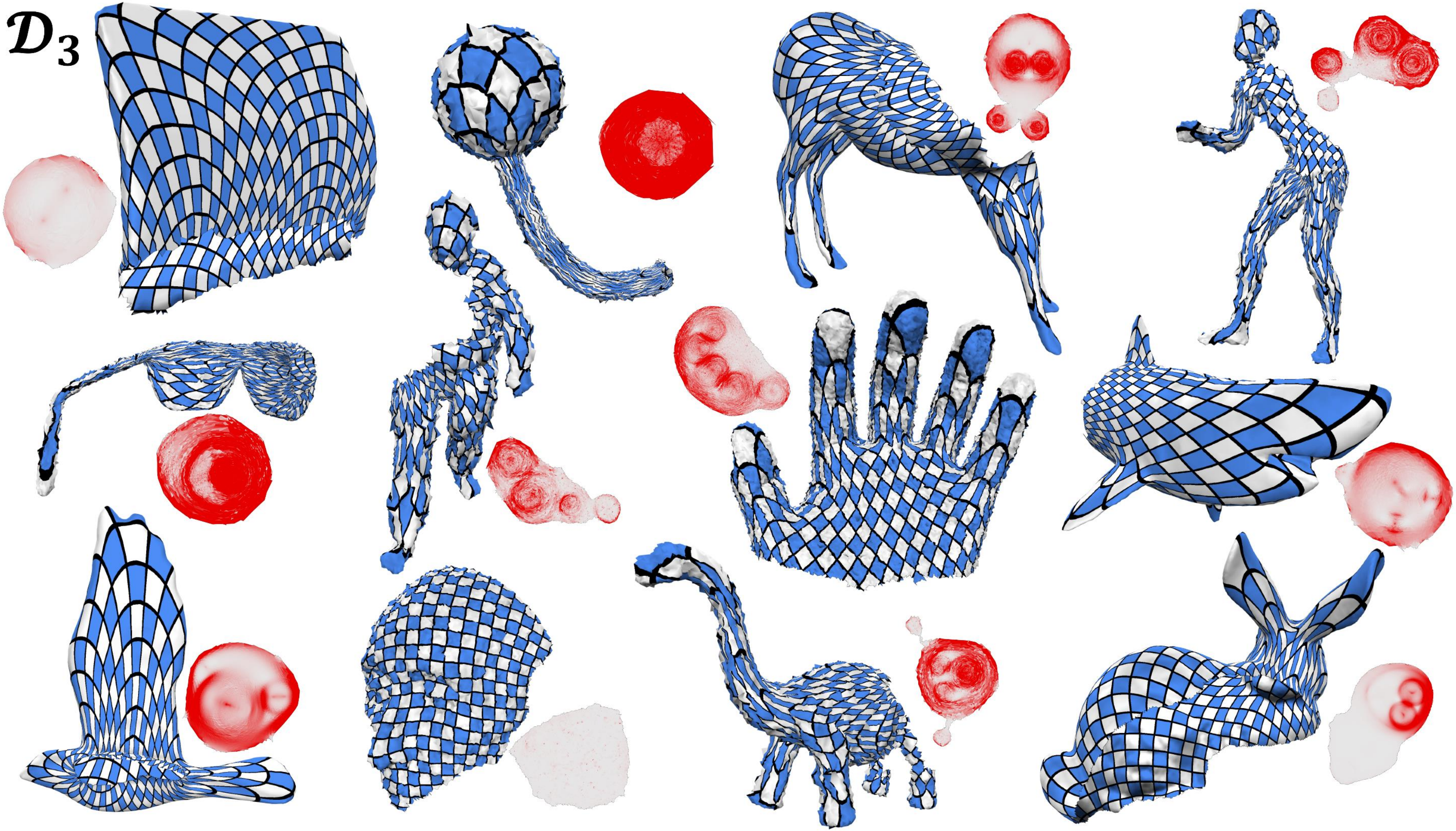

Log(energy)
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
time(s)
AKVF
CM
SLIM
Ours

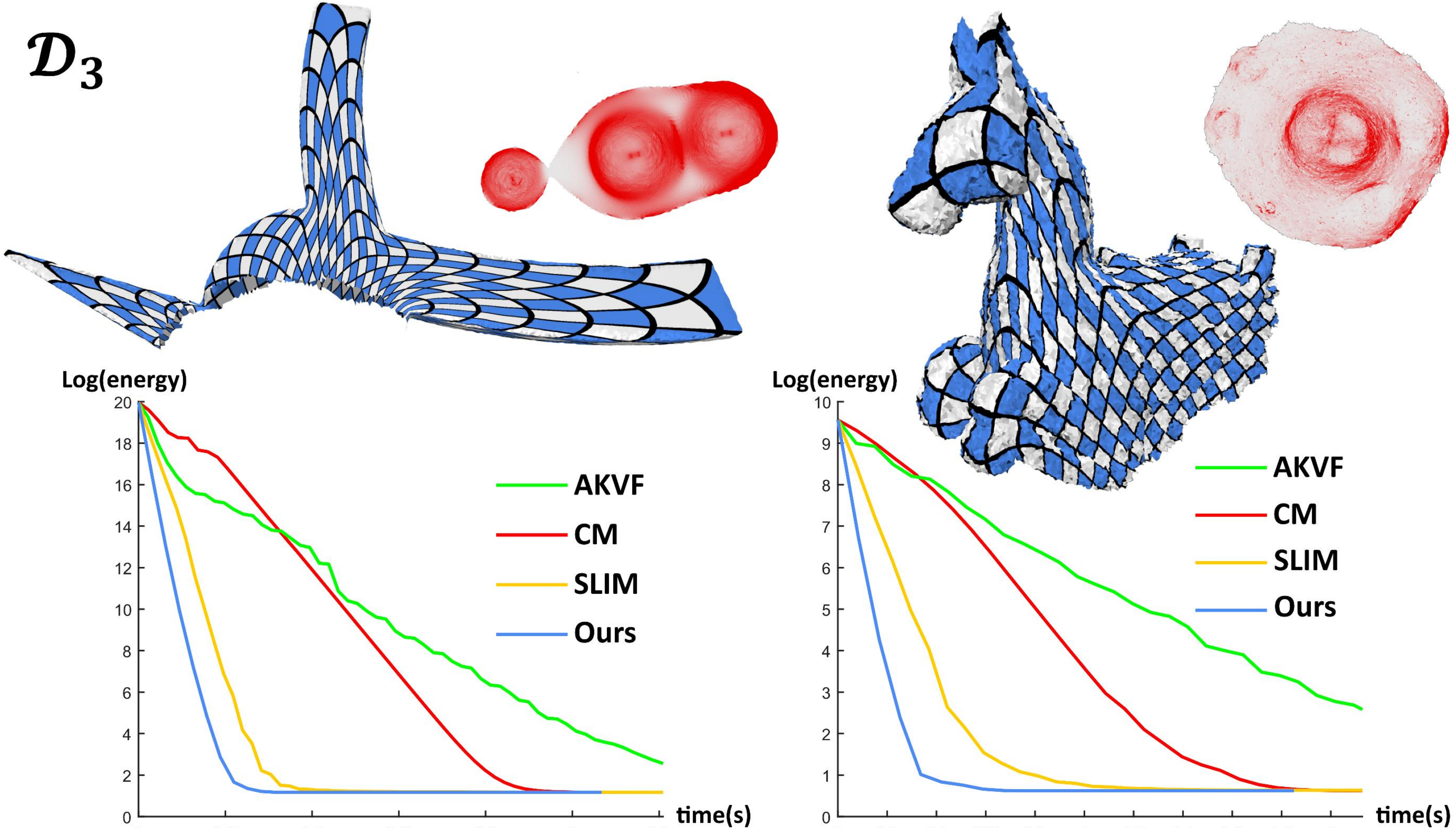

$\mathcal{D}_2$

$\mathcal{D}_2$


Log(energy)
12
10
8
6
4
2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
time(s)
AKVF
CM
SLIM
Ours

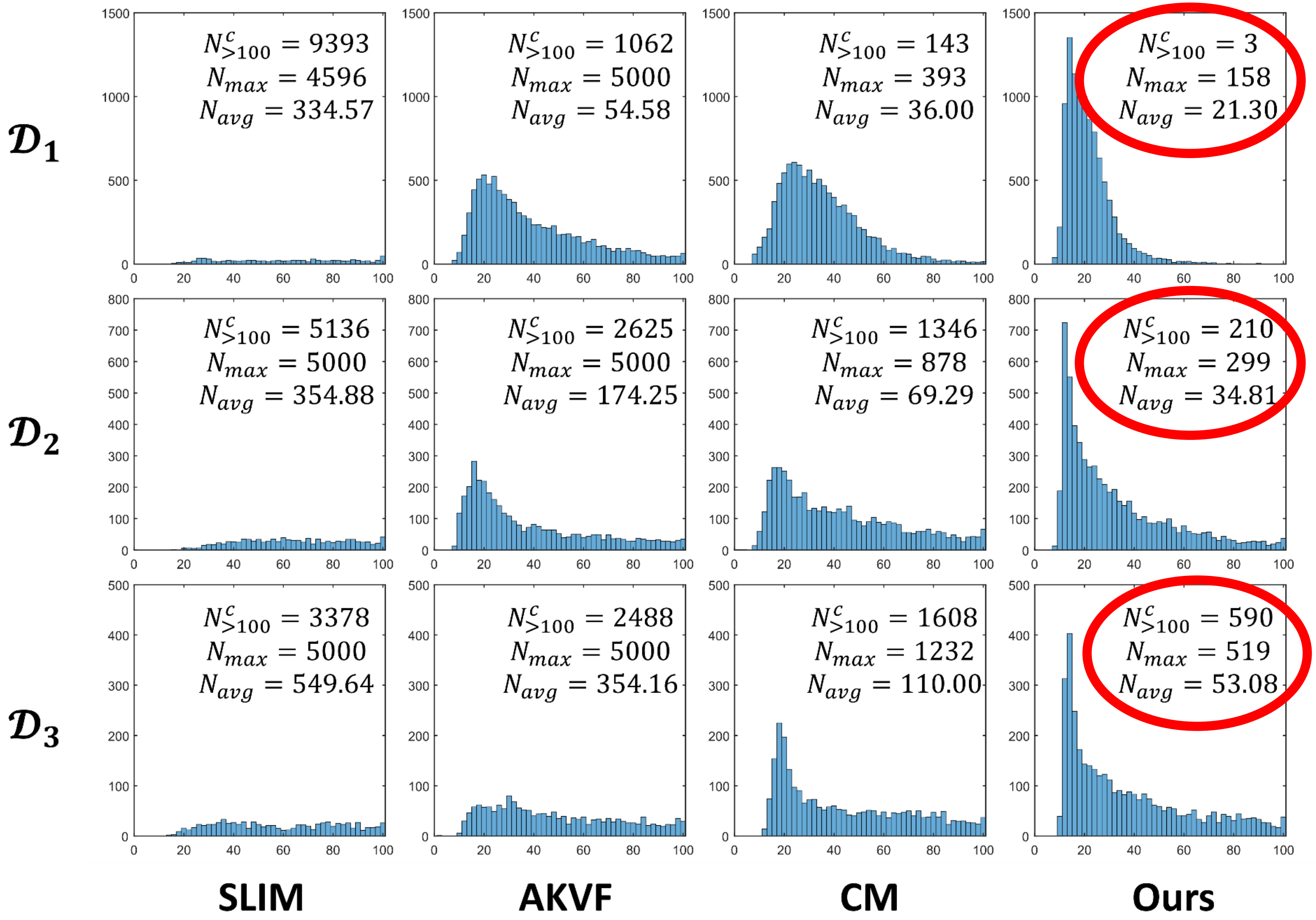

Log(energy)
7
6
5
4
3
2
1
0
0
0.1
0.2
0.3
0.4
0.5
time(s)
AKVF
CM
SLIM
Ours


$\mathcal{D}_3$



$\mathcal{D}_3$
Log(energy)
20
18
16
14
12
10
8
6
4
2
0
0
0.2
0.4
0.6
0.8
1
1.2
time(s)
AKVF
CM
SLIM
Ours
Log(energy)
10
9
8
7
6
5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
time(s)
AKVF
CM
SLIM
Ours

Distributions of iteration number



N_c: iteration number of convergence

N_{max}: maximum of N_c

N_{avg}: average of N_c

$N^c_{>100}$: the number of models whose N_c >100

Conclusions

- Progressive parameterizations: a novel and simple method to generate low isometric distortion parameterizations with no foldovers.

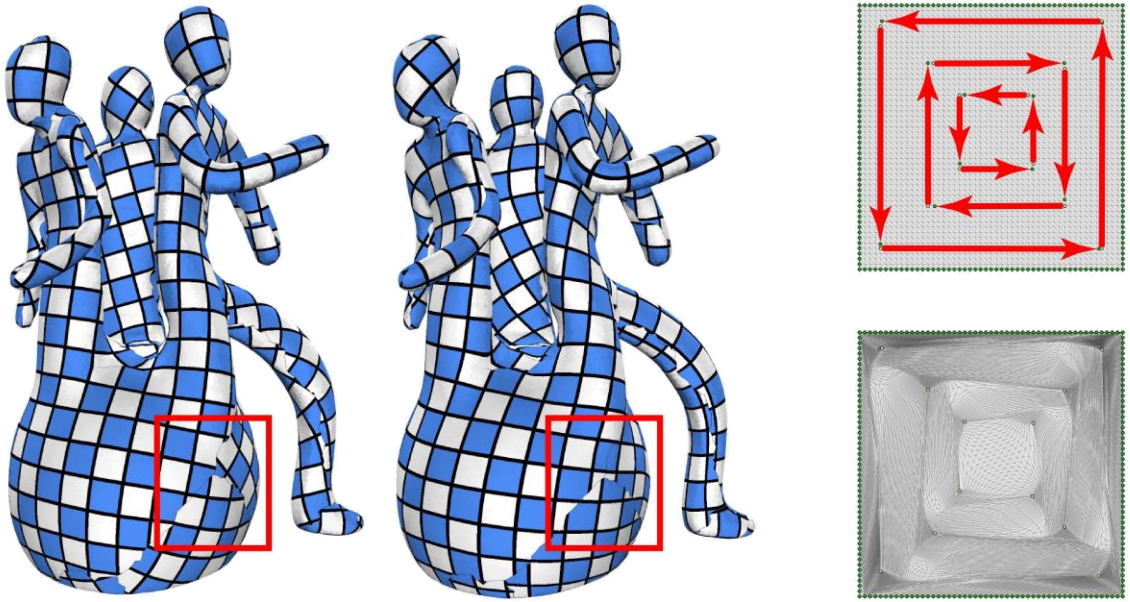
✓Thinks from the view of reference triangle.

✓Exhibits strong practical reliability and high efficiency.

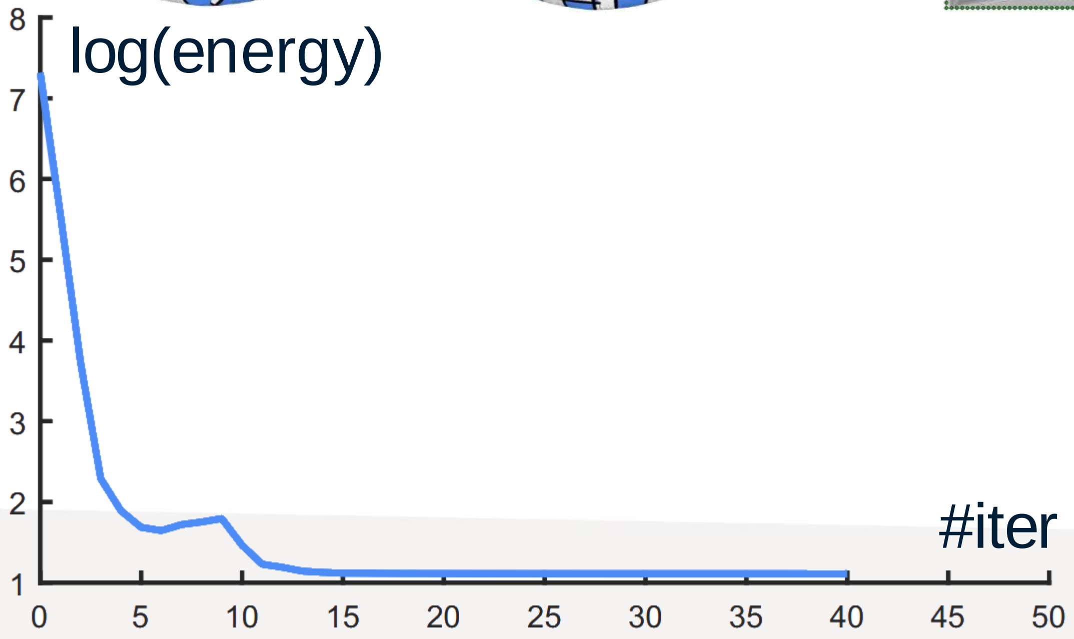
✓Demonstrates the practical robustness on a large data set containing 20712 models

Limitations

- Cannot fit constraint condition well.



- No theoretical guarantee to reduce $E(M, M^p)$ monotonously.



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Thank you!

http://staff.ustc.edu.cn/~fuxm/projects/ProgressivePara/