Local Barycentric Coordinates

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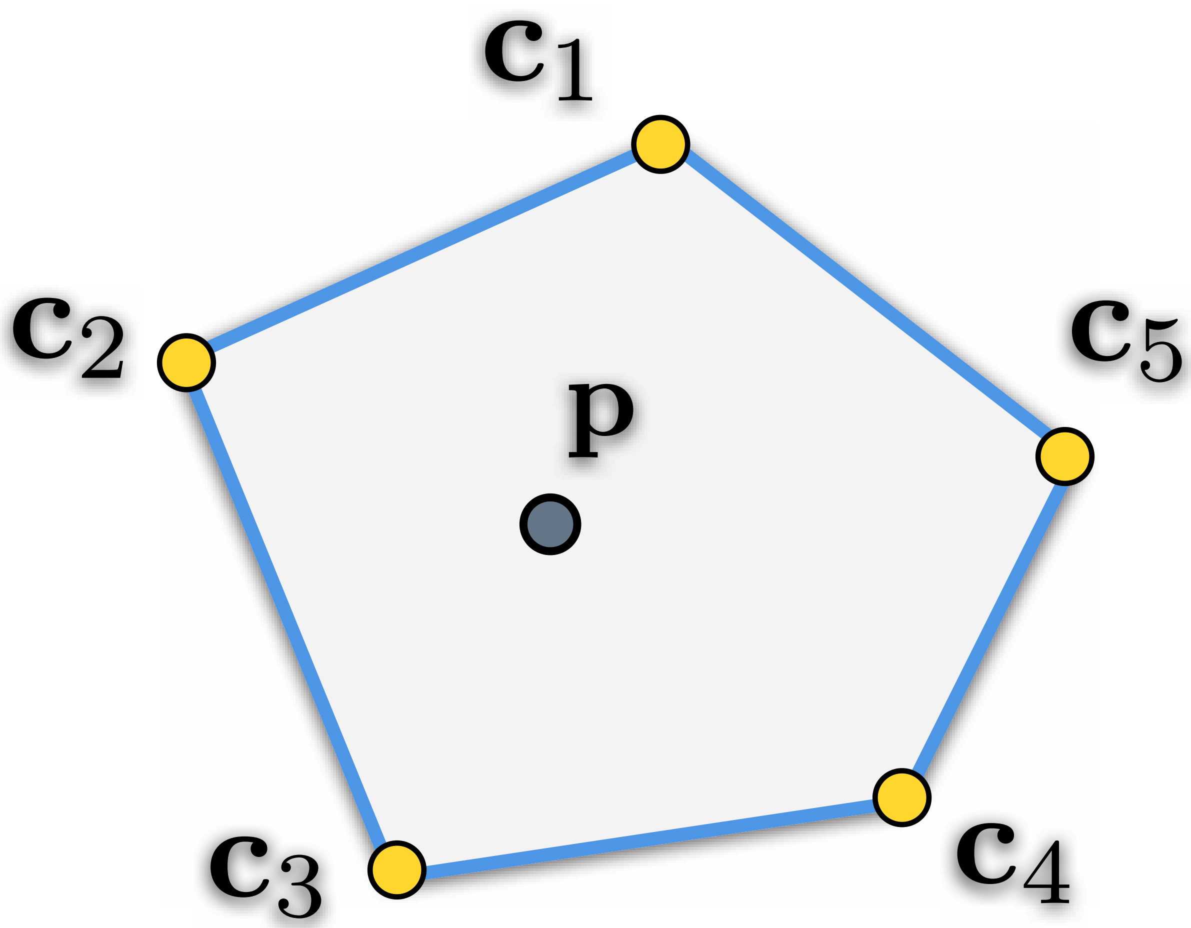
Introduction

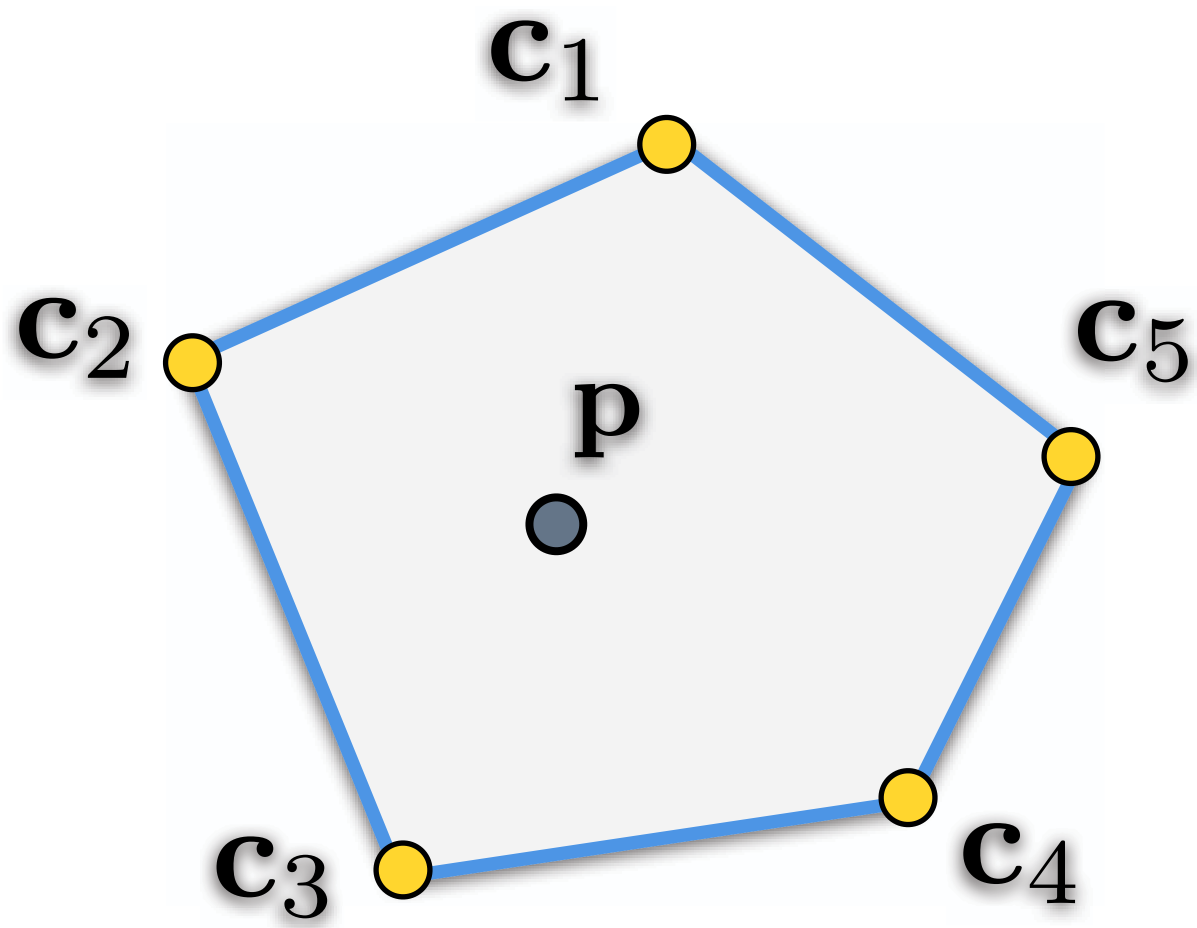
- Given a point $\mathbf{p}$ inside a polygon with vertices $\{\mathbf{c}_i\}$



Introduction

- Given a point $\mathbf{p}$ inside a polygon with vertices $\{\mathbf{c}_i\}$
- Barycentric coordinates $\{w_i\}$ of $\mathbf{p}$:

$$\mathbf{p} = \sum_i w_i \mathbf{c}_i, \quad \sum_i w_i = 1$$



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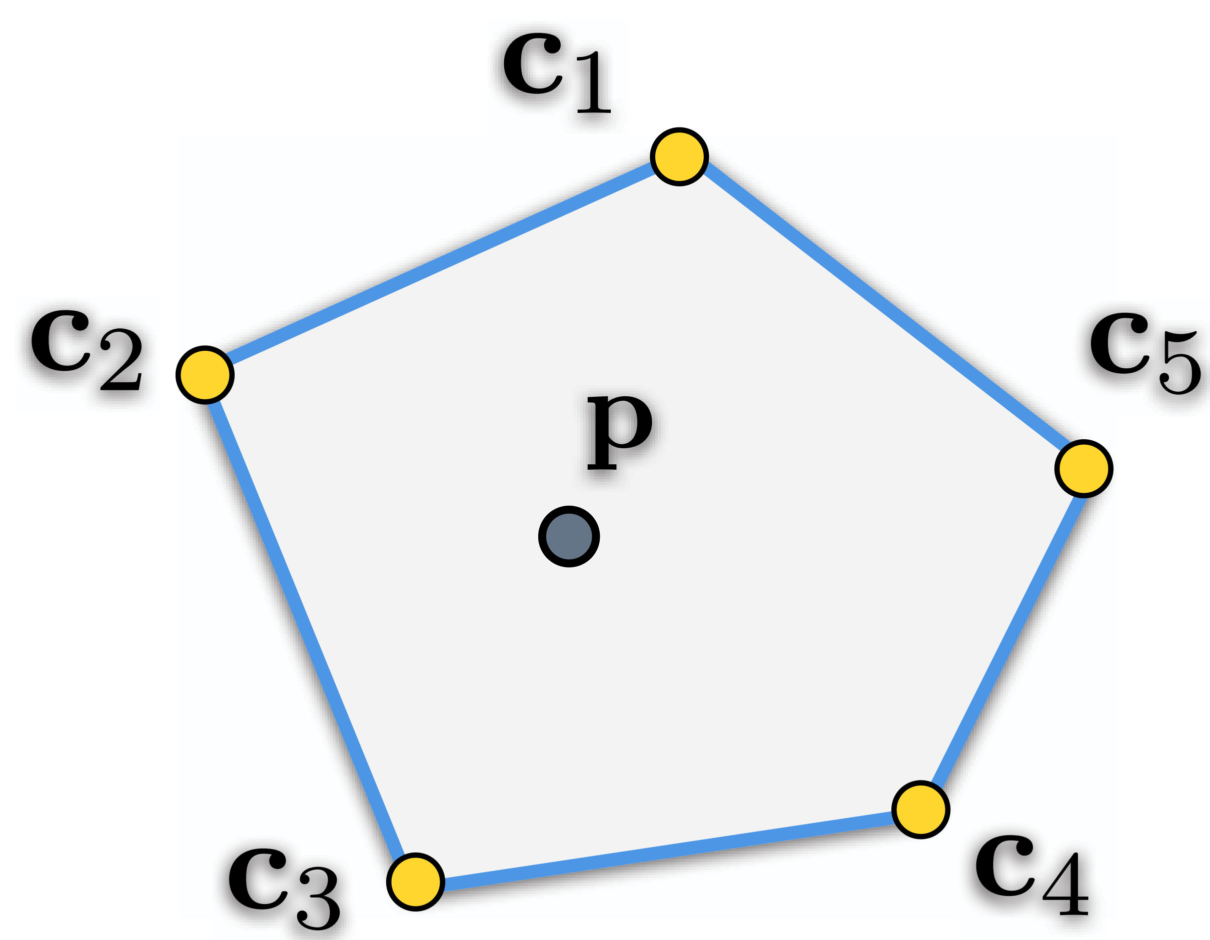
functions inside the polygon



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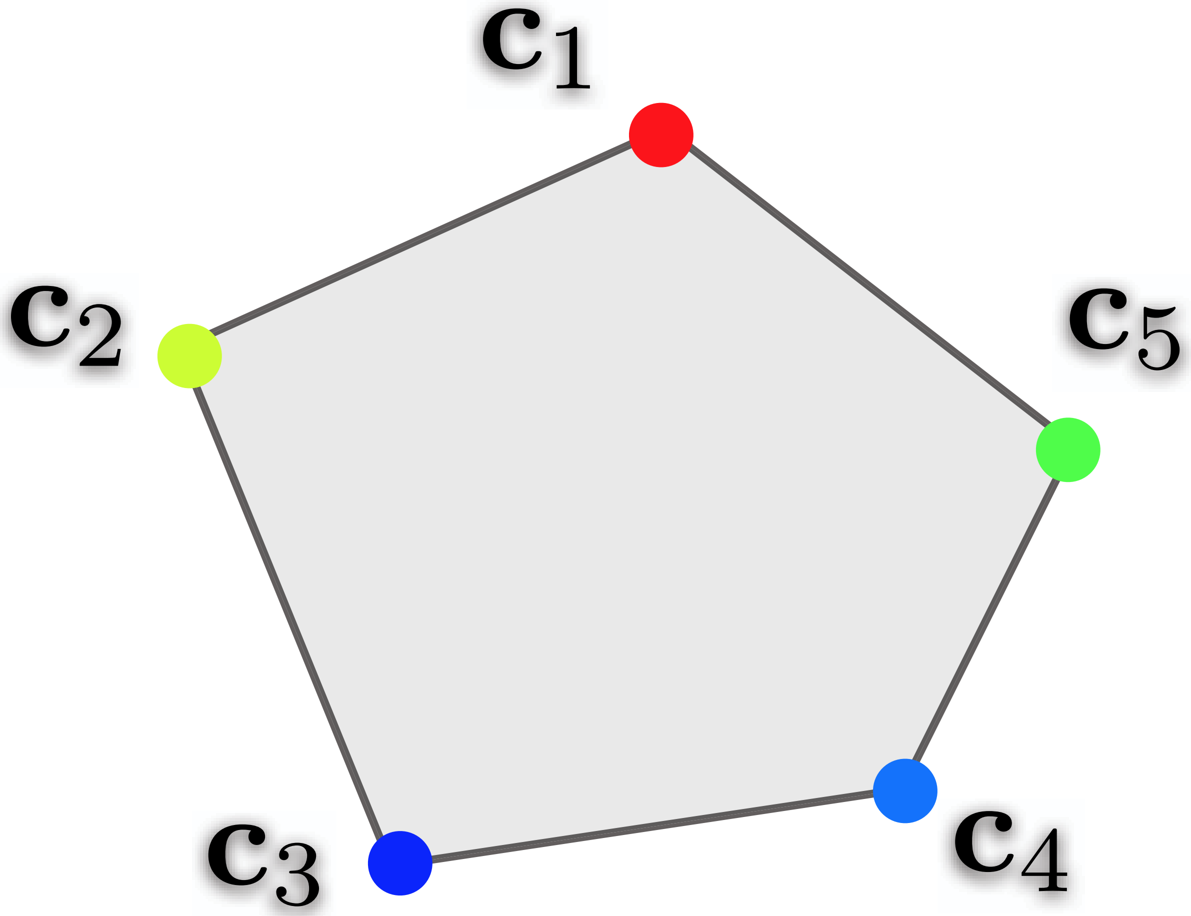
Introduction

- Application: interpolation



Introduction

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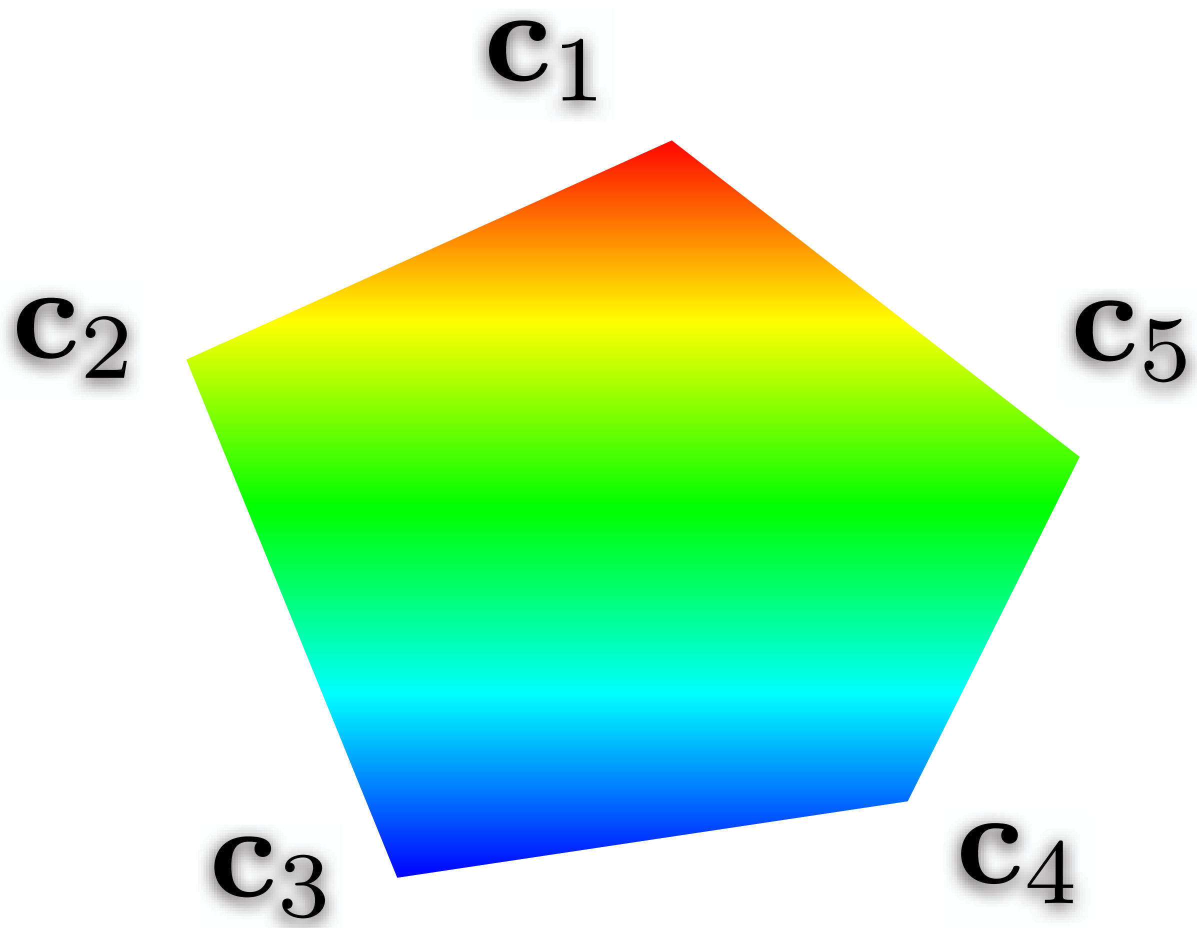


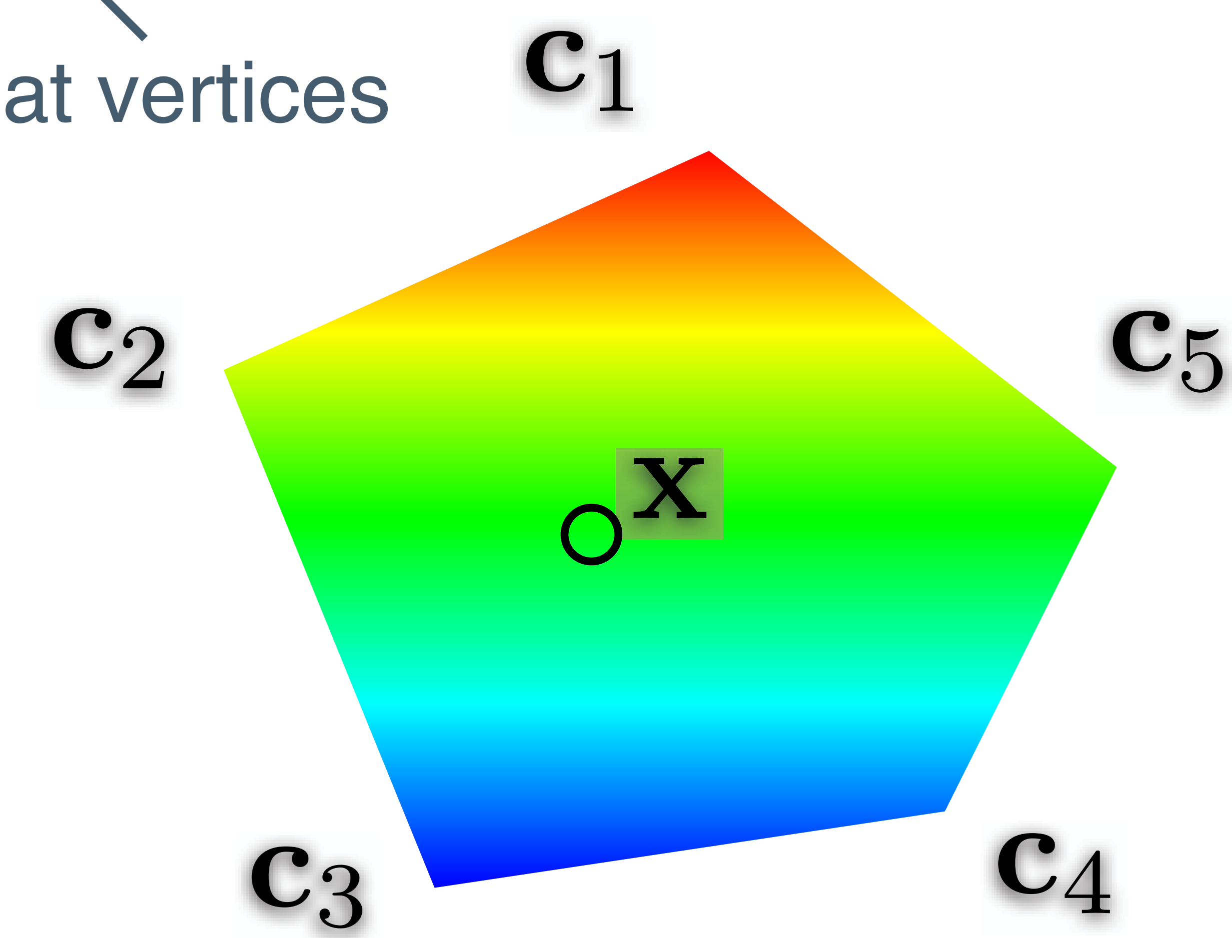
Introduction

- Application: interpolation

$$f(\mathbf{x}) = \sum_i w_i(\mathbf{x})\ f(\mathbf{c}_i)$$

interpolated value

values at vertices



Introduction

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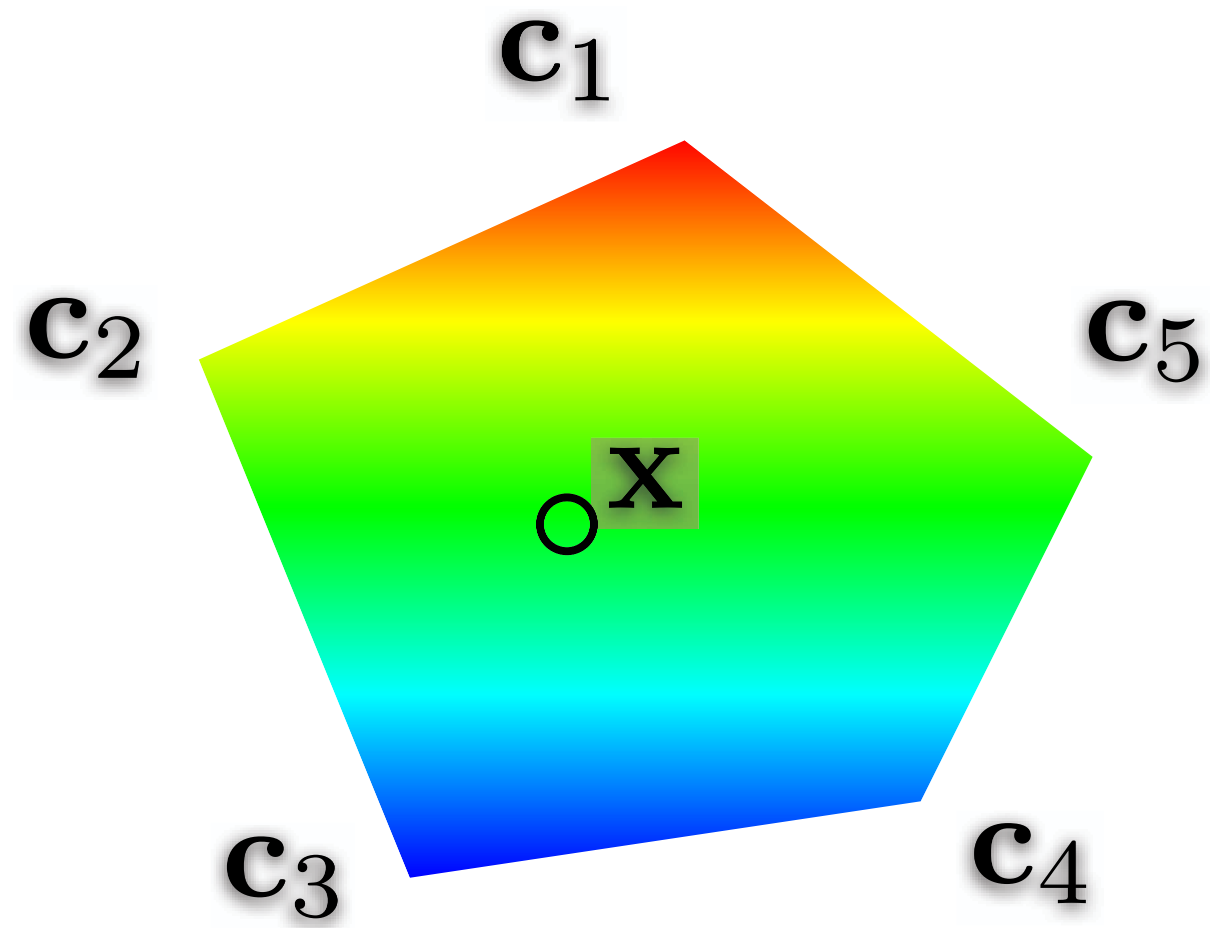
barycentric coordinates



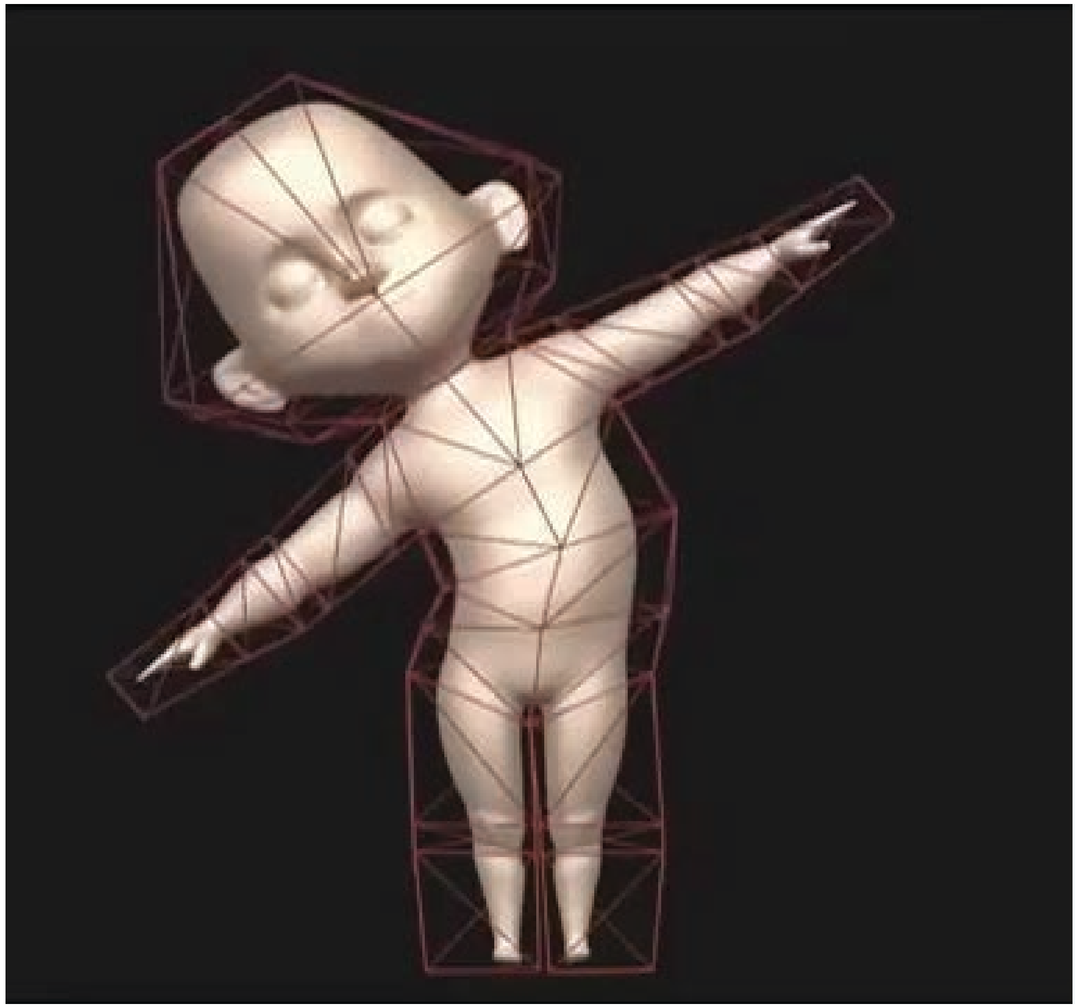
Introduction

- Cage based deformation

cage vertex deformation

$$\mathbf{d}(\mathbf{x}) = \sum_i w_i(\mathbf{x})\, \mathbf{d}(\mathbf{c}_i)$$

deformation field inside the cage



Global Deformation

- Global influence

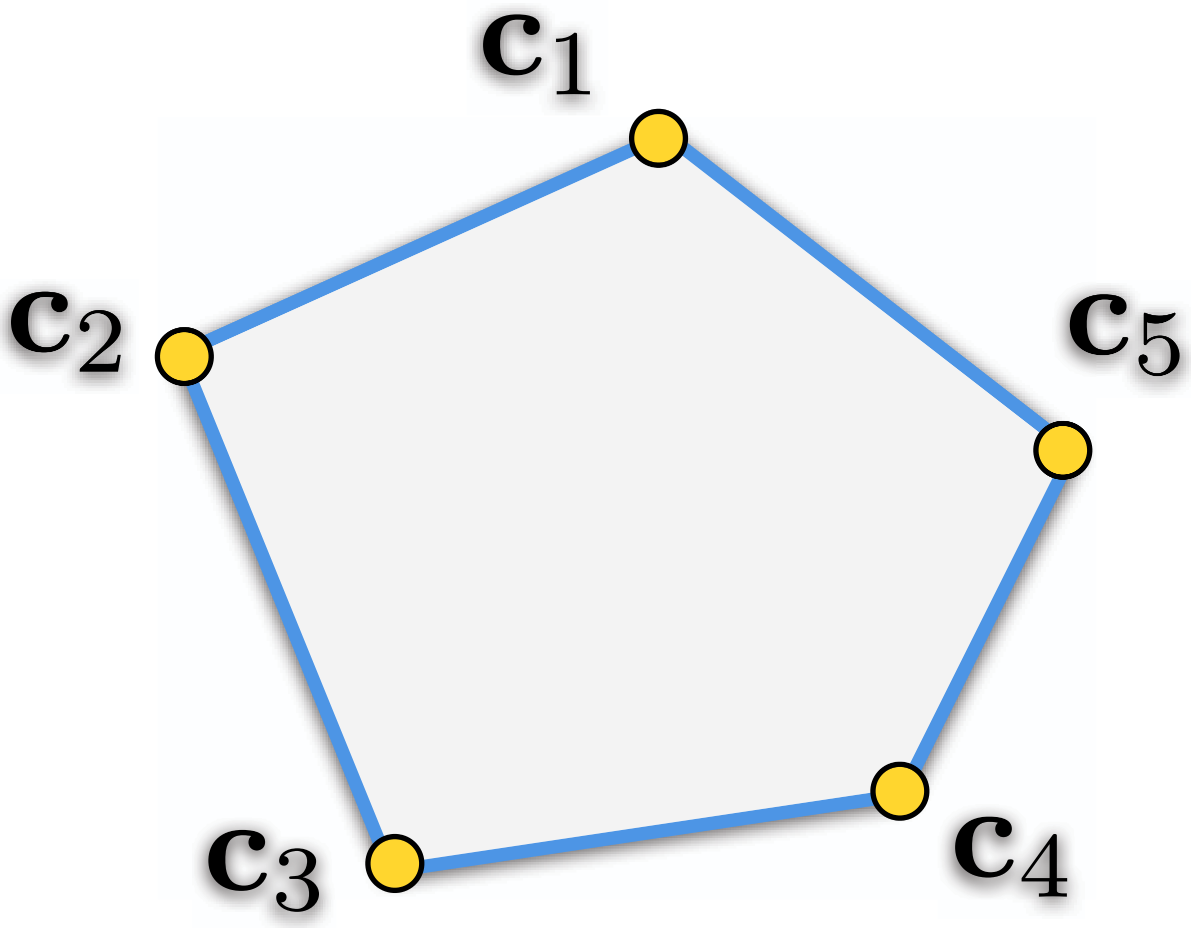
Mean Value Coordinates (MVC)

Our Goal: Local Control

- Control points influence nearby regions only

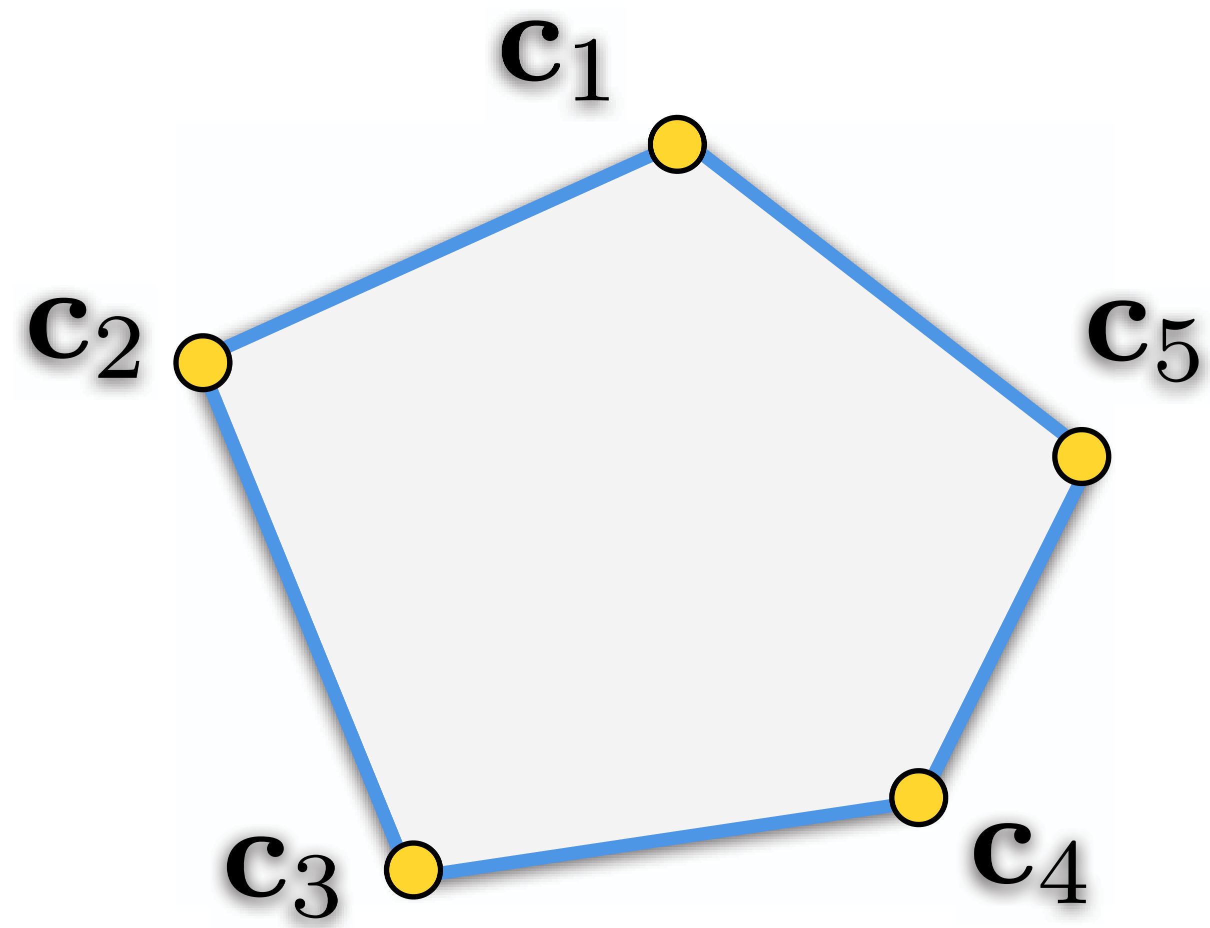
Problem Formulation

- Input: control cage with vertices $\{\mathbf{c}_i\}$

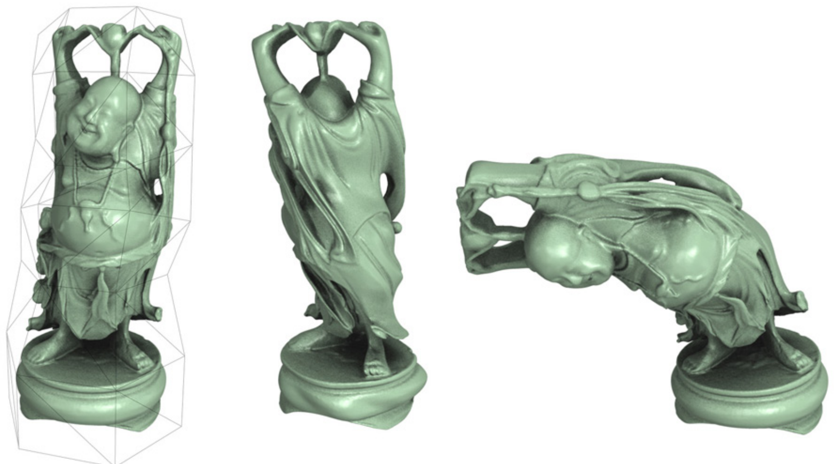


Problem Formulation

- Input: control cage with vertices $\{\mathbf{c}_i\}$
- Output: barycentric coordinate functions $\{w_i(\mathbf{x})\}$ with local influence



Previous Work



Poisson-based Weight Reduction
[Landreneau & Schaefer 2009]

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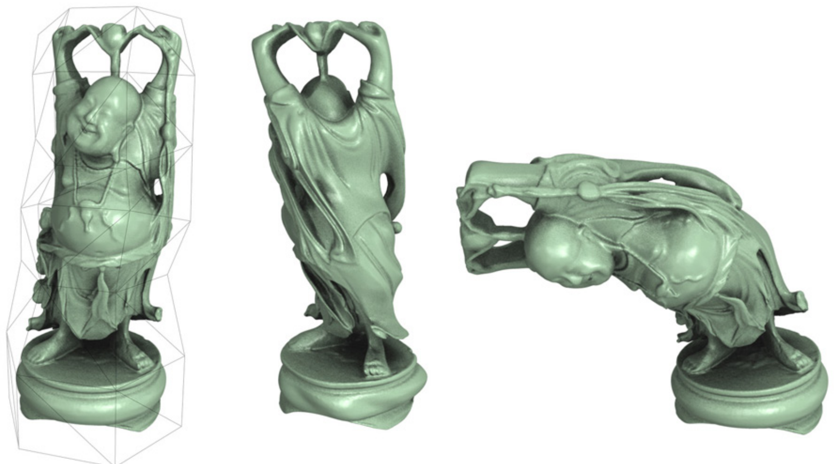

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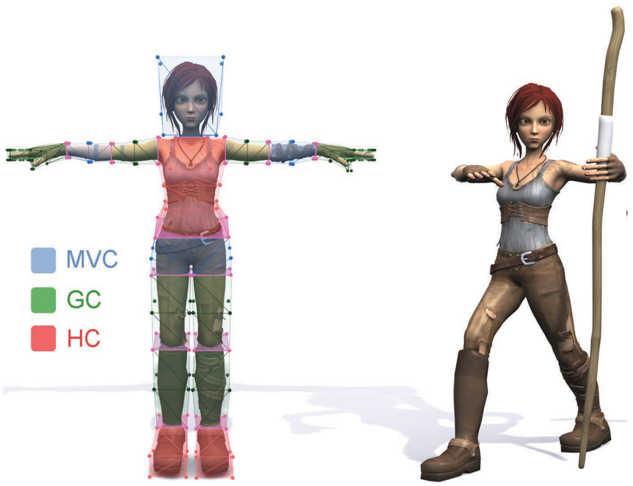

*Cages
[García et al. 2013]

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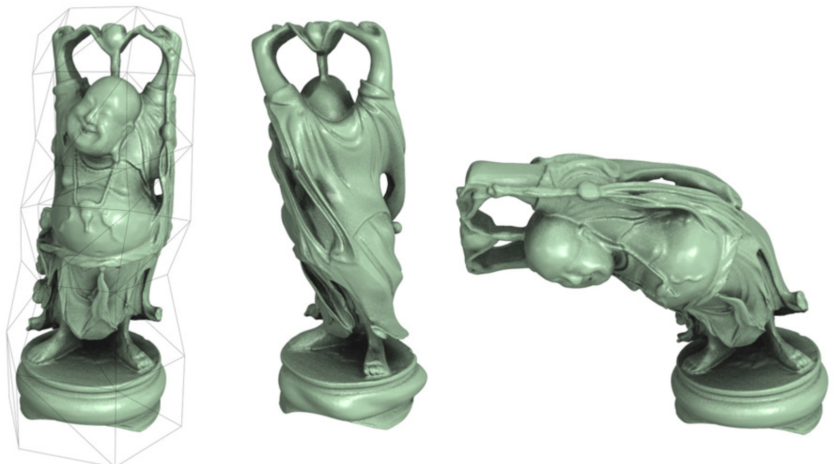

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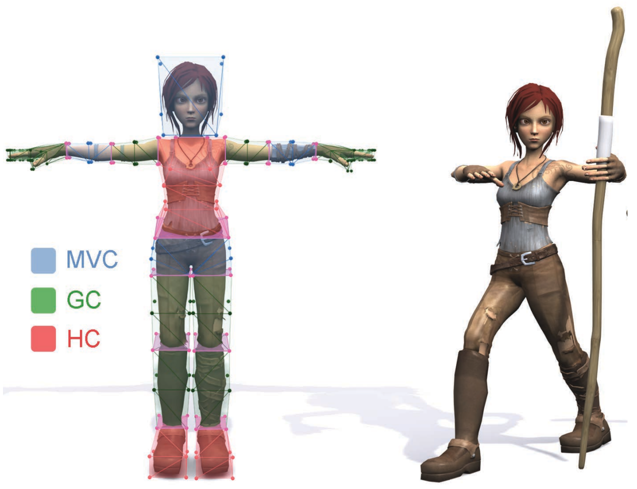

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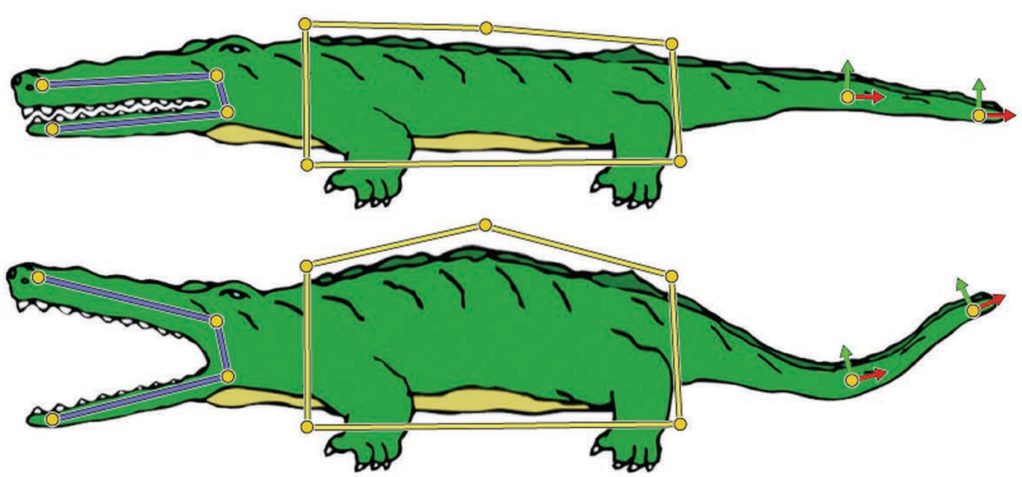
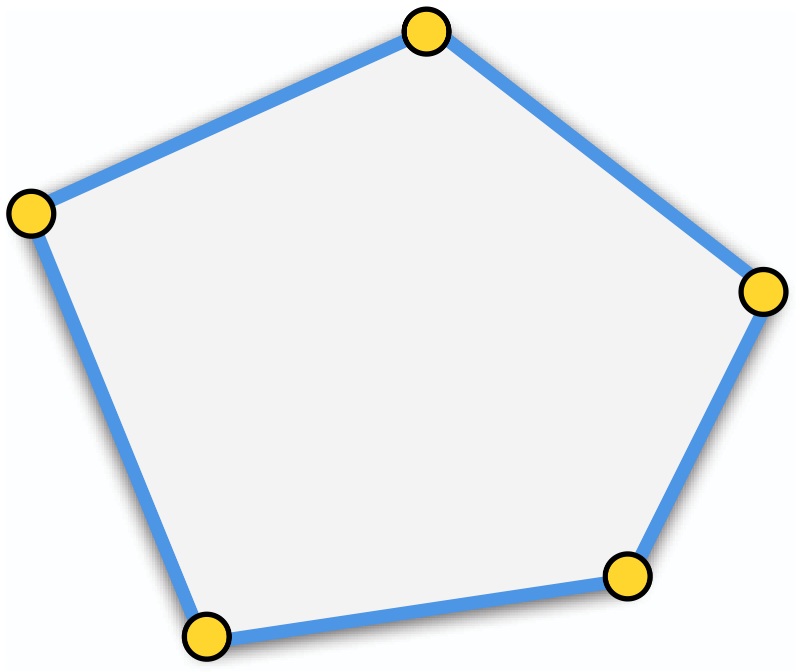


Bounded Biharmonic Weights (BBW)
[Jacobson et al. 2011]

Optimization Approach

- $\min_{w_1,\ldots,w_n} F(w_1,\ldots,w_n)$ subject to some constraints:



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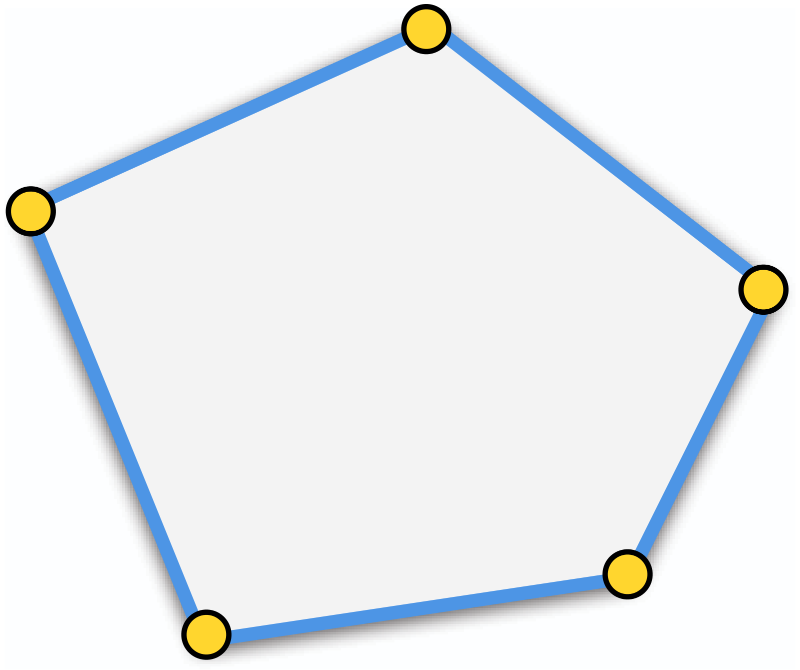

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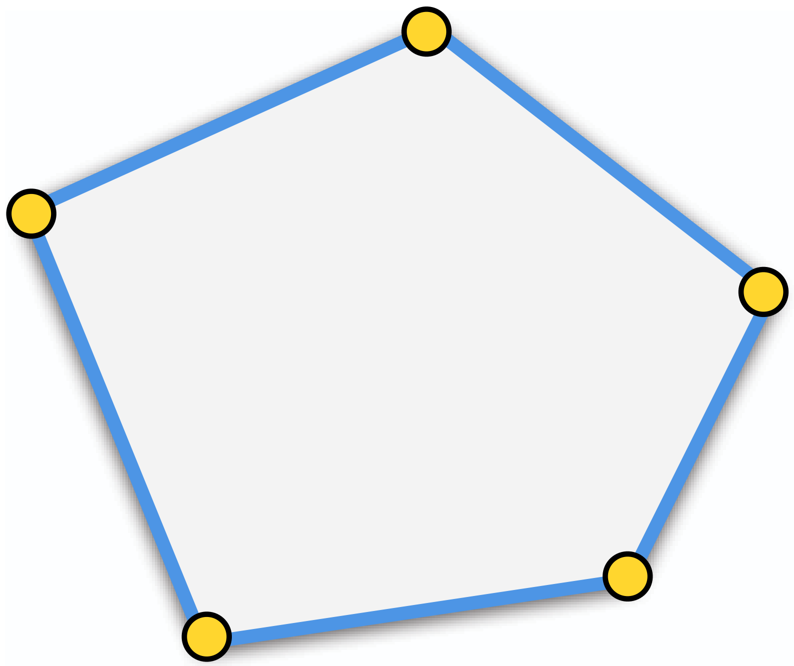
  - $w_i \geq 0$



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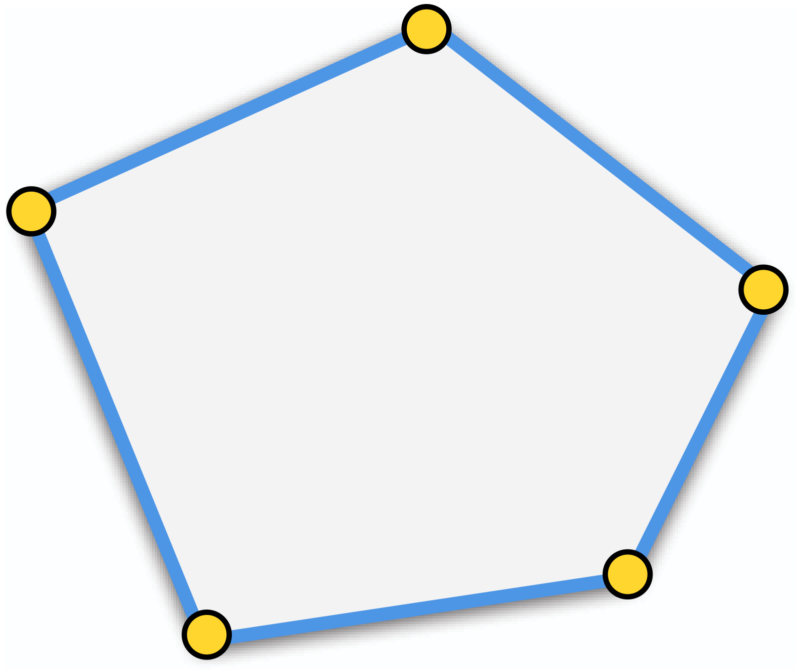
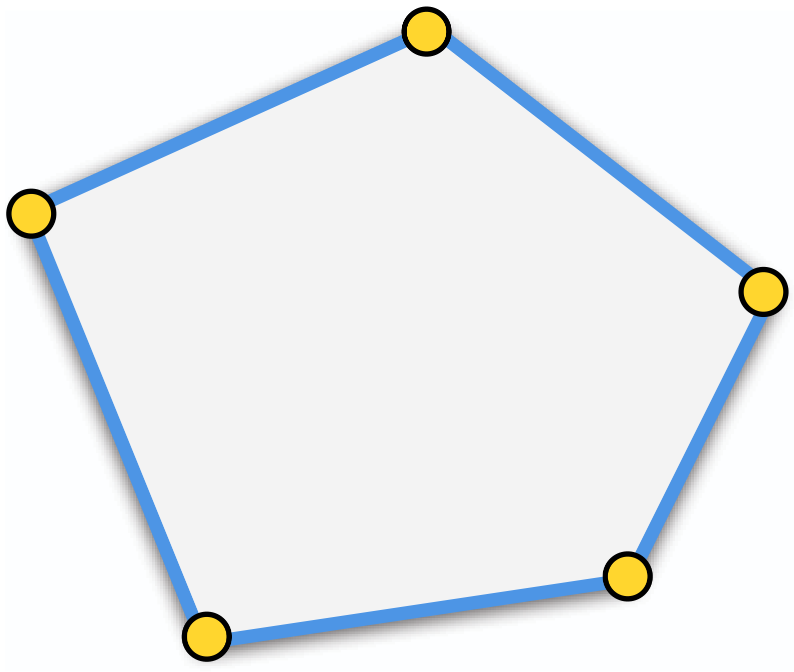
  - $w_i \geq 0$
  - $w_i(\mathbf{c}_j) = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$
  - w_i linear on cage edges



Optimization Approach

- $\min_{w_1,\ldots,w_n} F(w_1,\ldots,w_n)$ subject to some constraints:

Convex functional inducing locality



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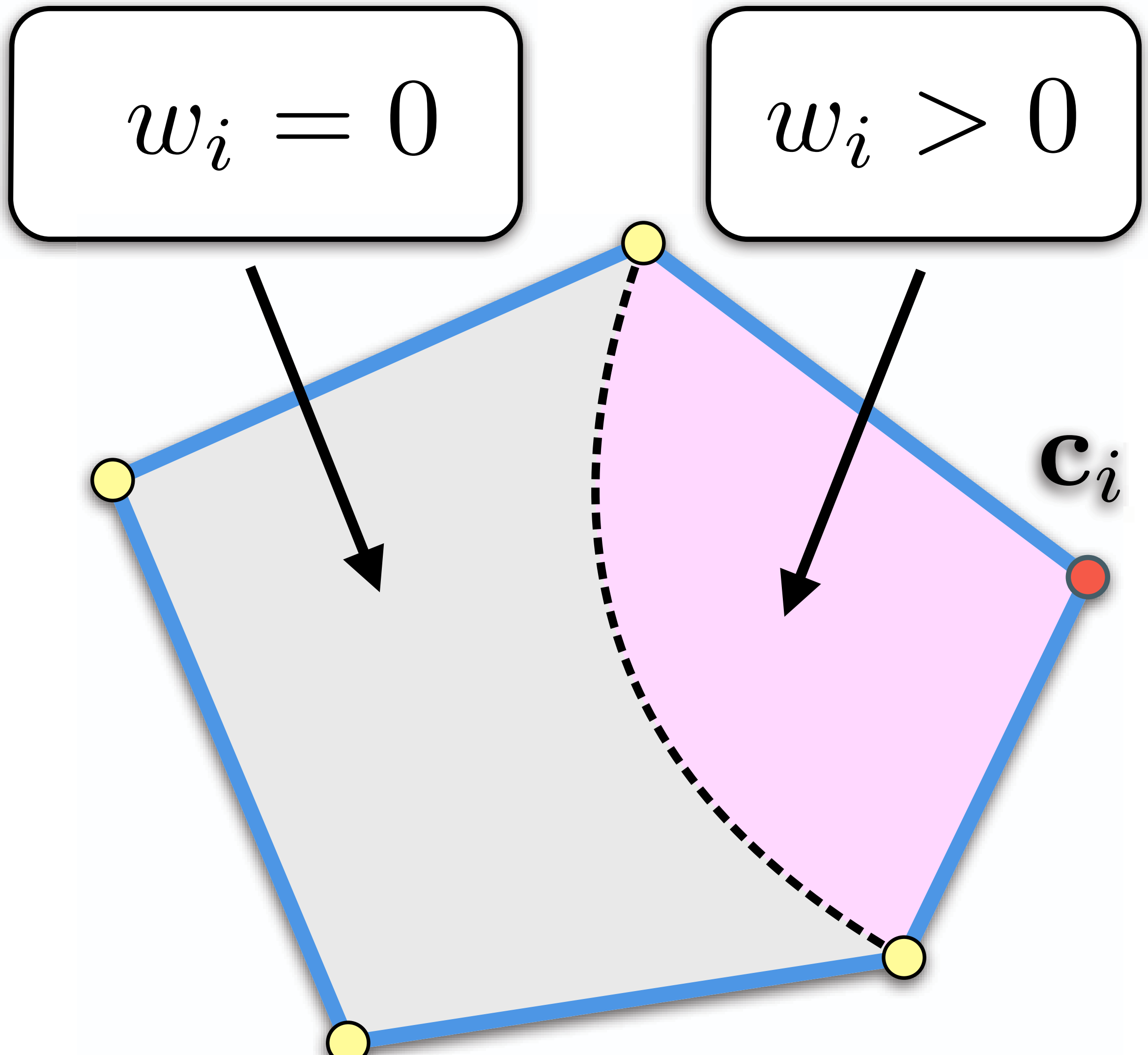
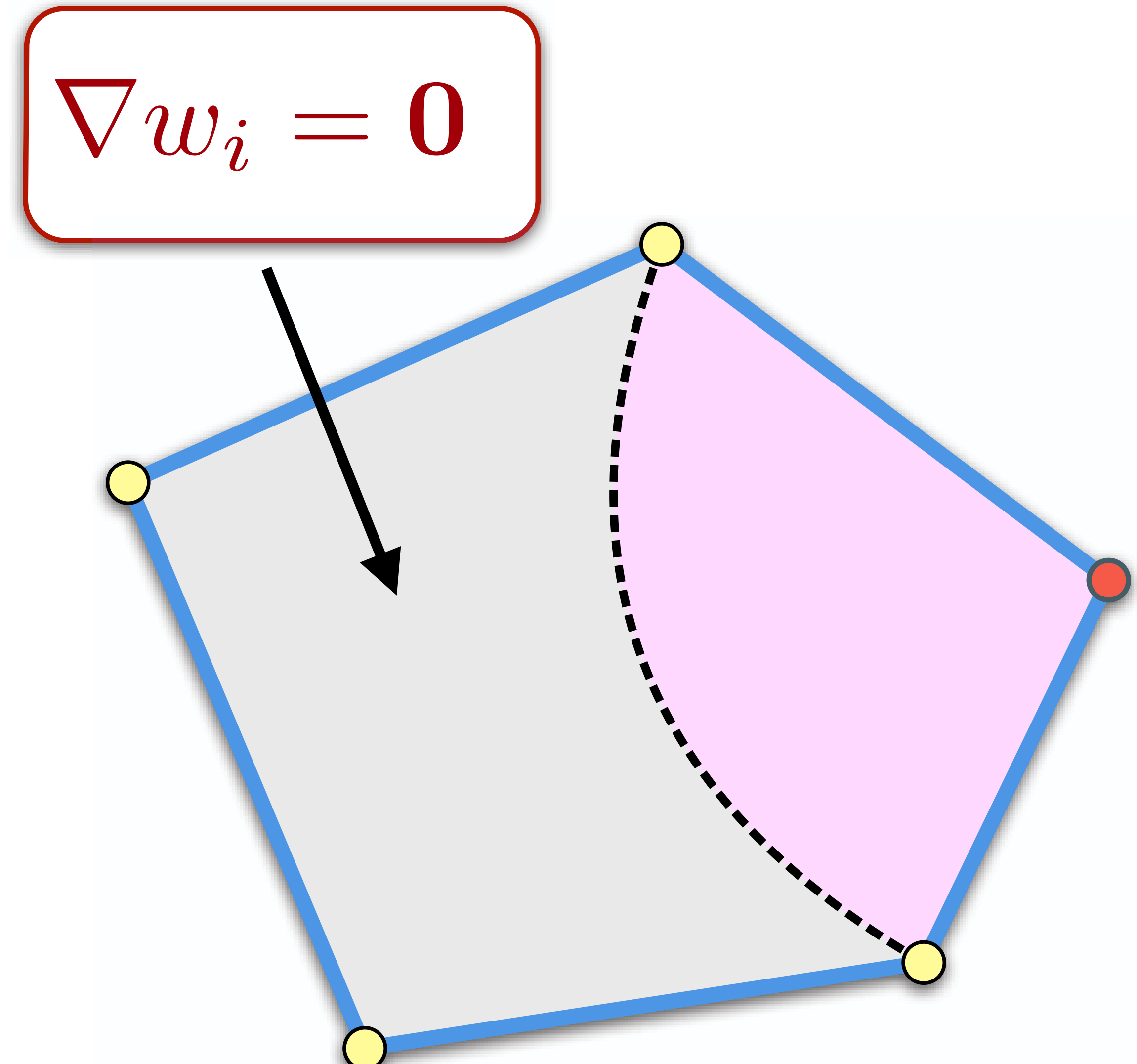
Local Influence

- Function w_i for control vertex $\mathbf{c}_i$



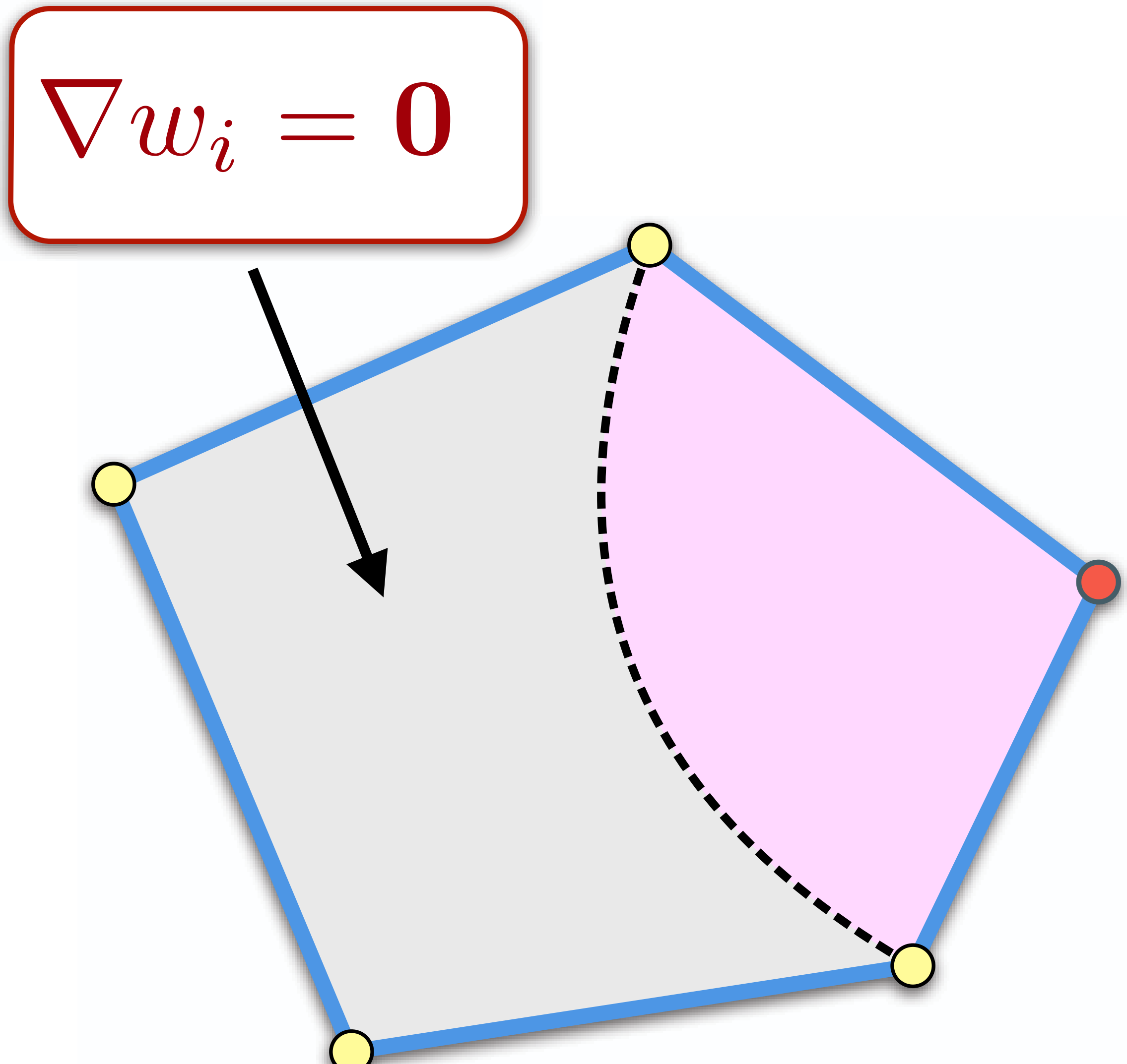
Condition for the Gradient

- Function w_i for control vertex $\mathbf{c}_i$



Condition for the Gradient

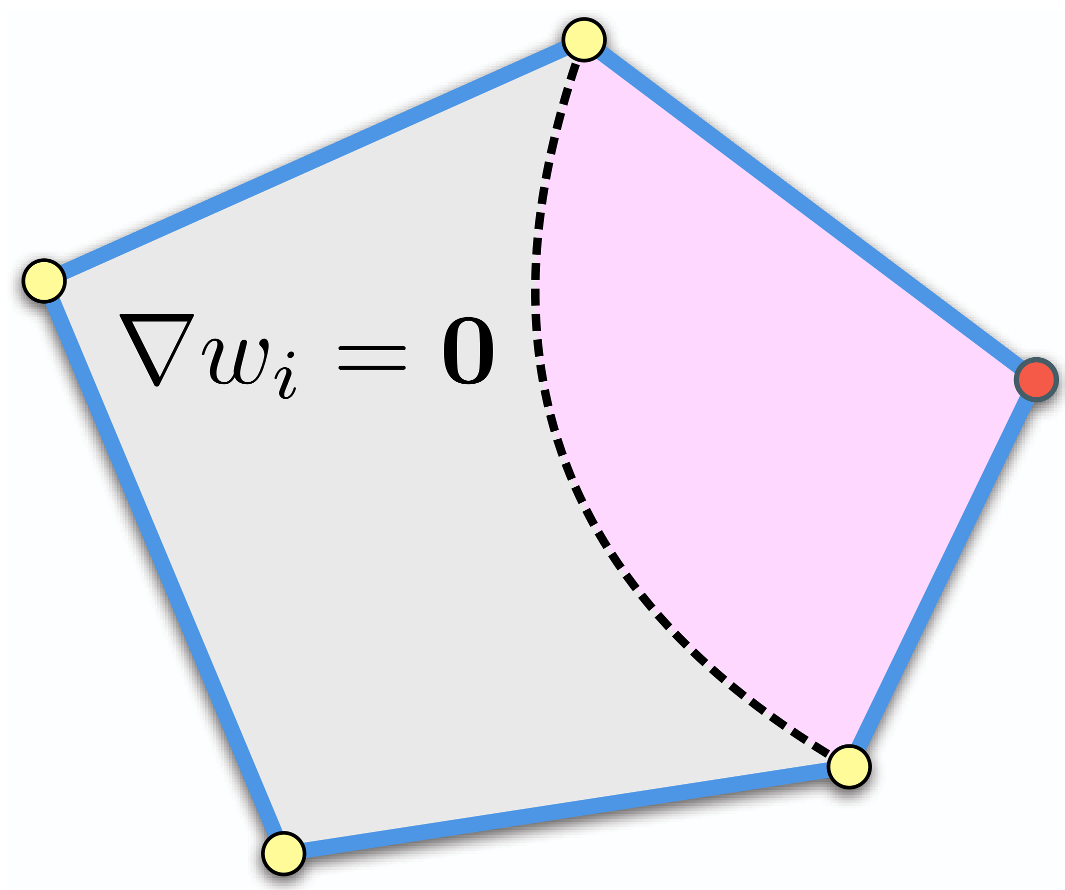
Necessary condition: large region with zero gradient



Condition for the Gradient

Necessary condition: large region with zero gradient

$$\min \int |\nabla w_i(\mathbf{x})| \; d\mathbf{x}$$

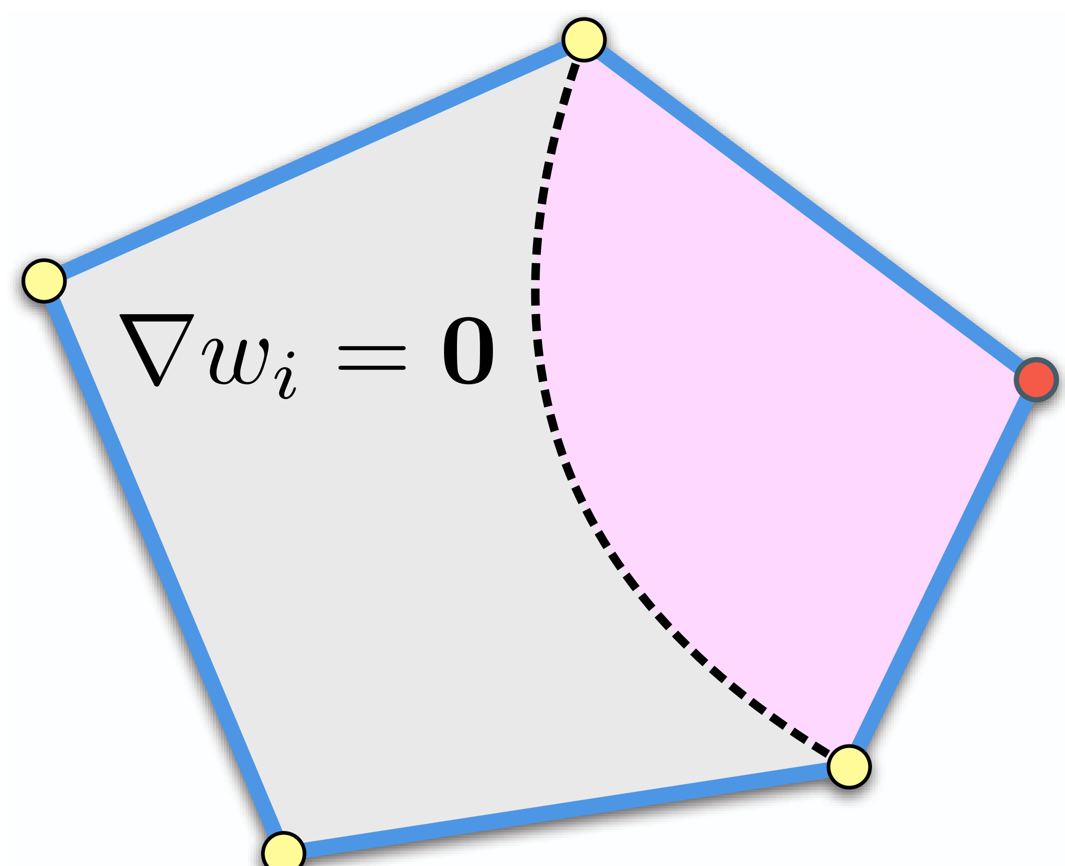


Condition for the Gradient

Necessary condition: large region with zero gradient

$$\min \int |\nabla w_i(\mathbf{x})| \, d\mathbf{x}$$

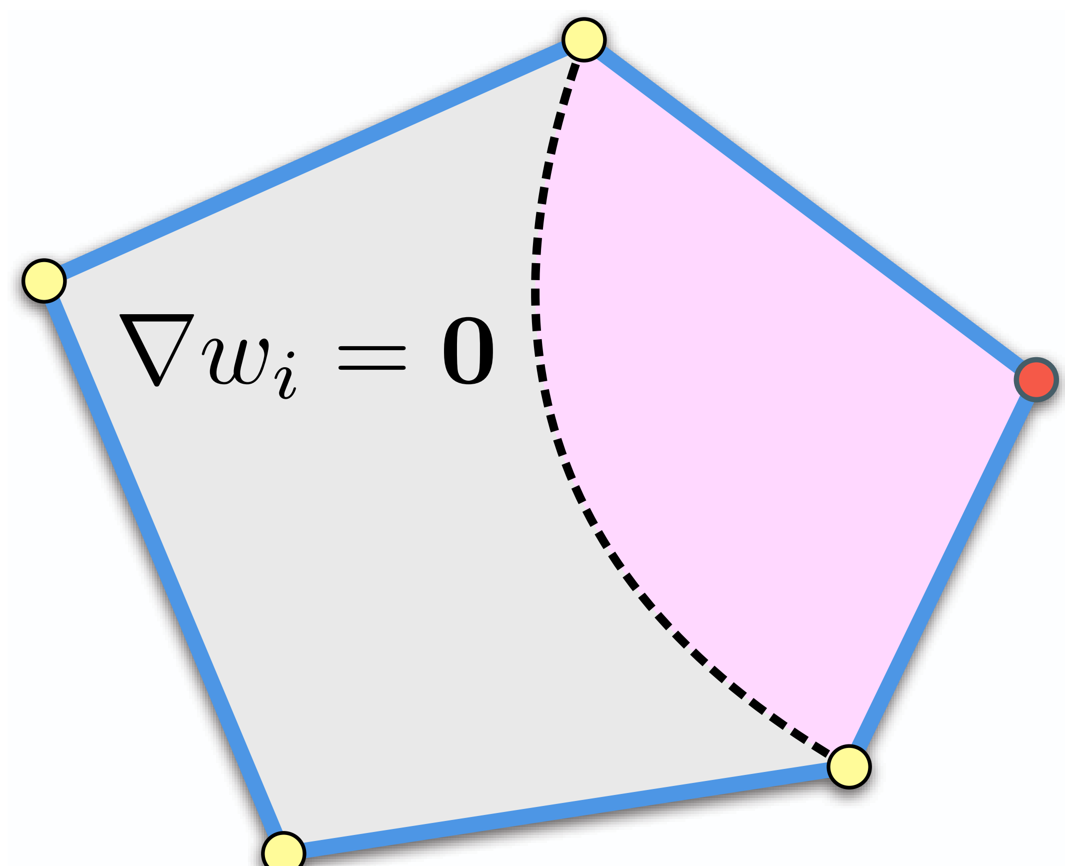
Total variation of w_i:
convex functional



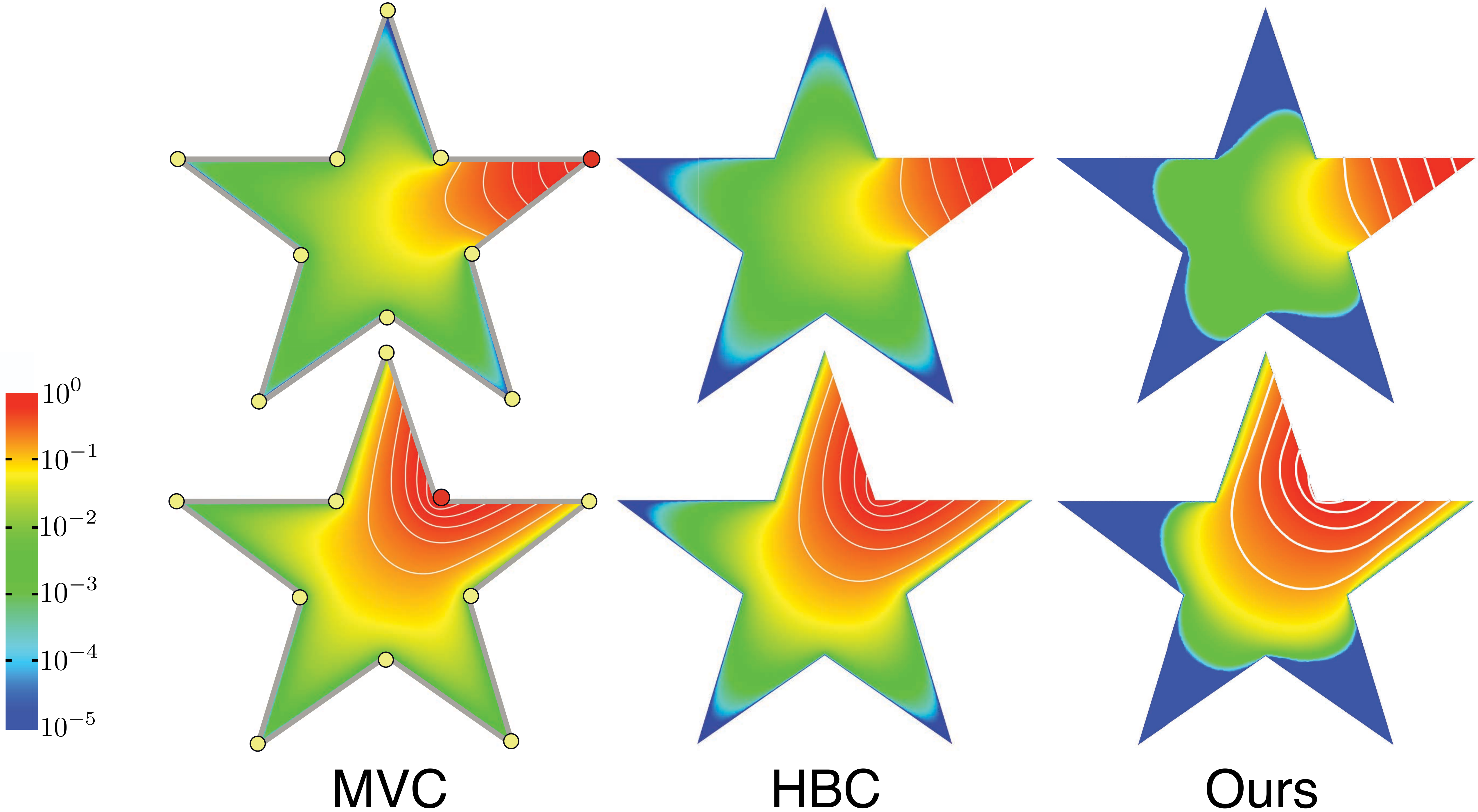
Condition for the Gradient

Target functional:

$$F = \sum_{i=1}^{n} \int |\nabla w_i(\mathbf{x})| \, d\mathbf{x}$$

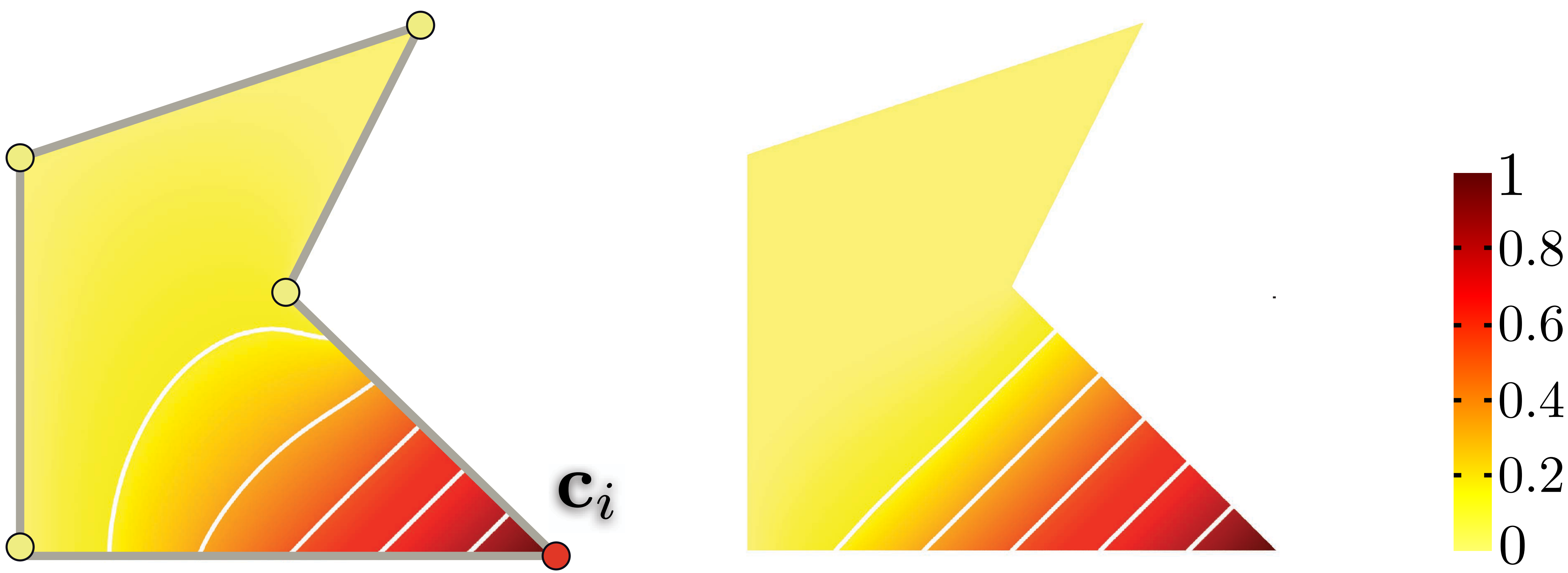


Comparison



Weighted Total Variation

- Extension: more locality using weighted total variation

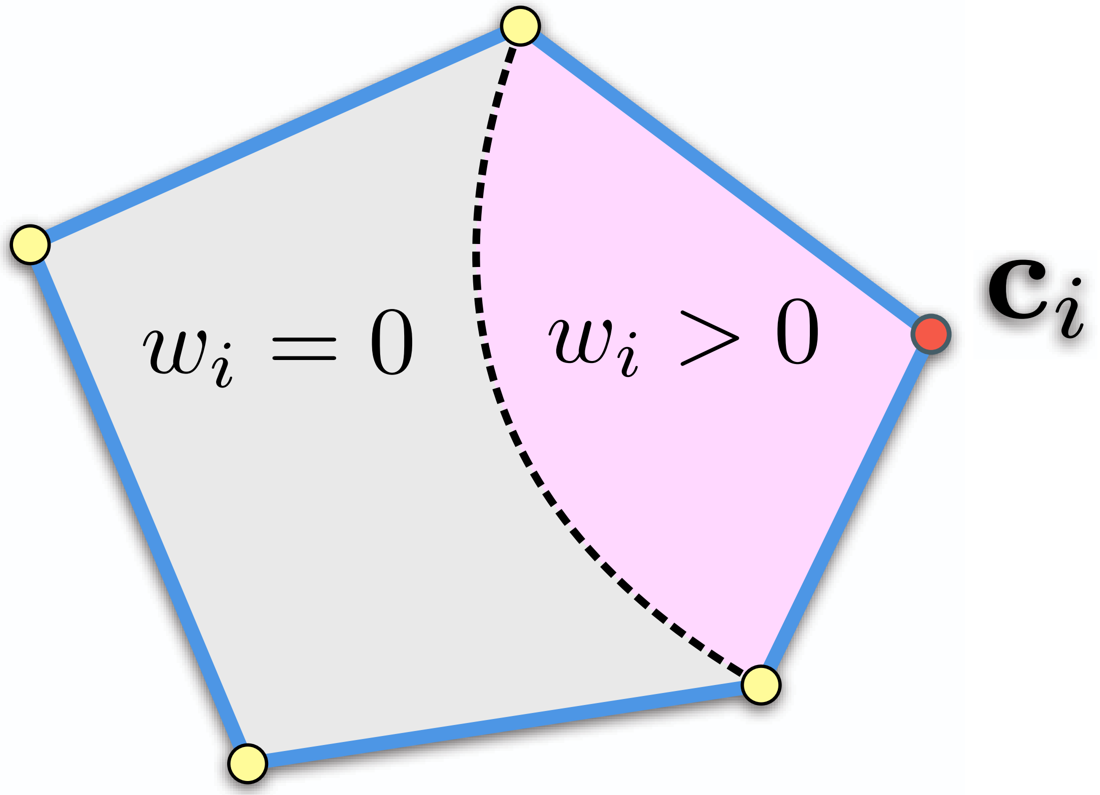


unweighted

weighted

Controlling Locality

- Local influence: w_i decreases to zero quickly



Controlling Locality

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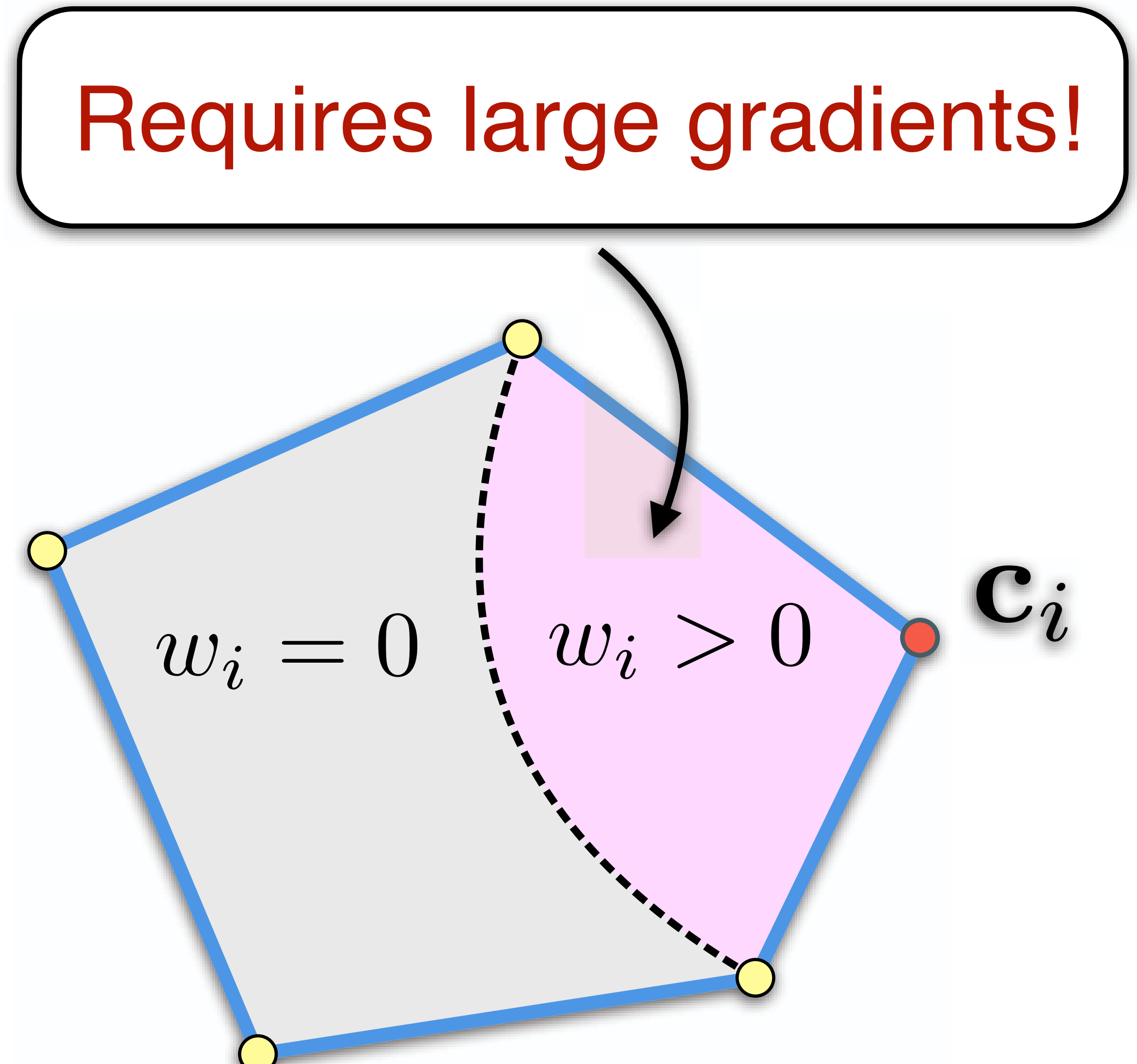
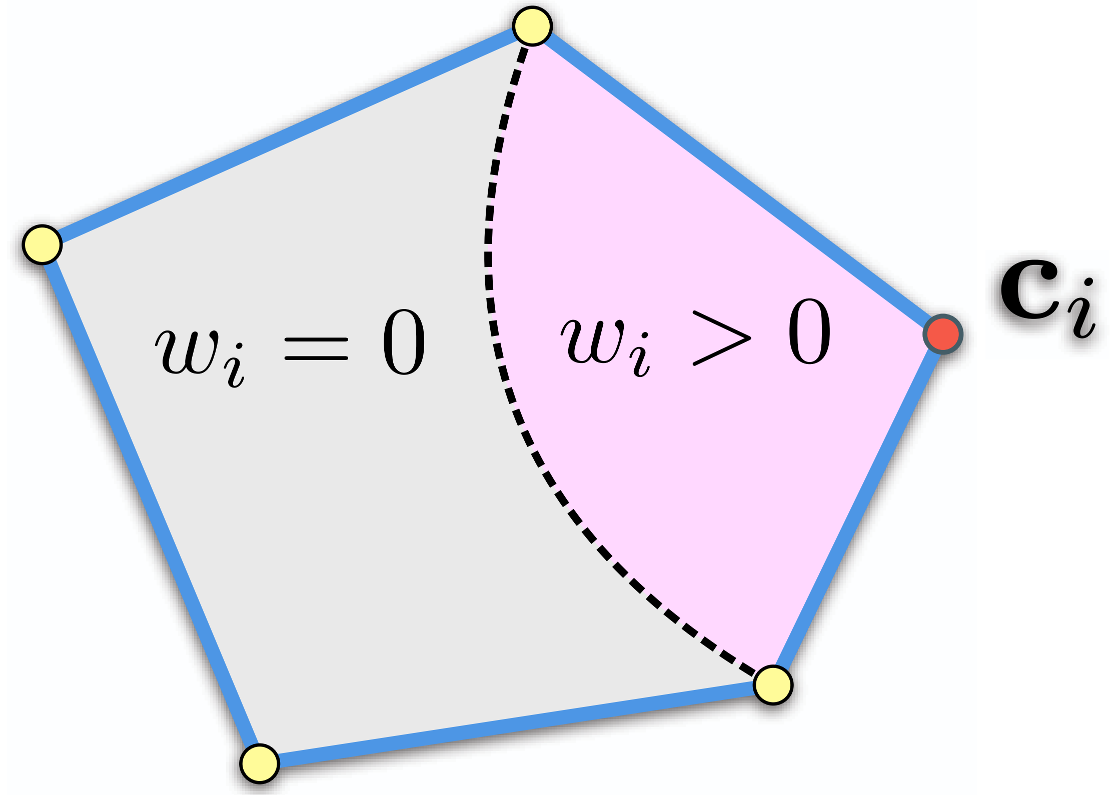


Controlling Locality

Total variation:

$$\int |\nabla w_i(\mathbf{x})| \, d\mathbf{x}$$

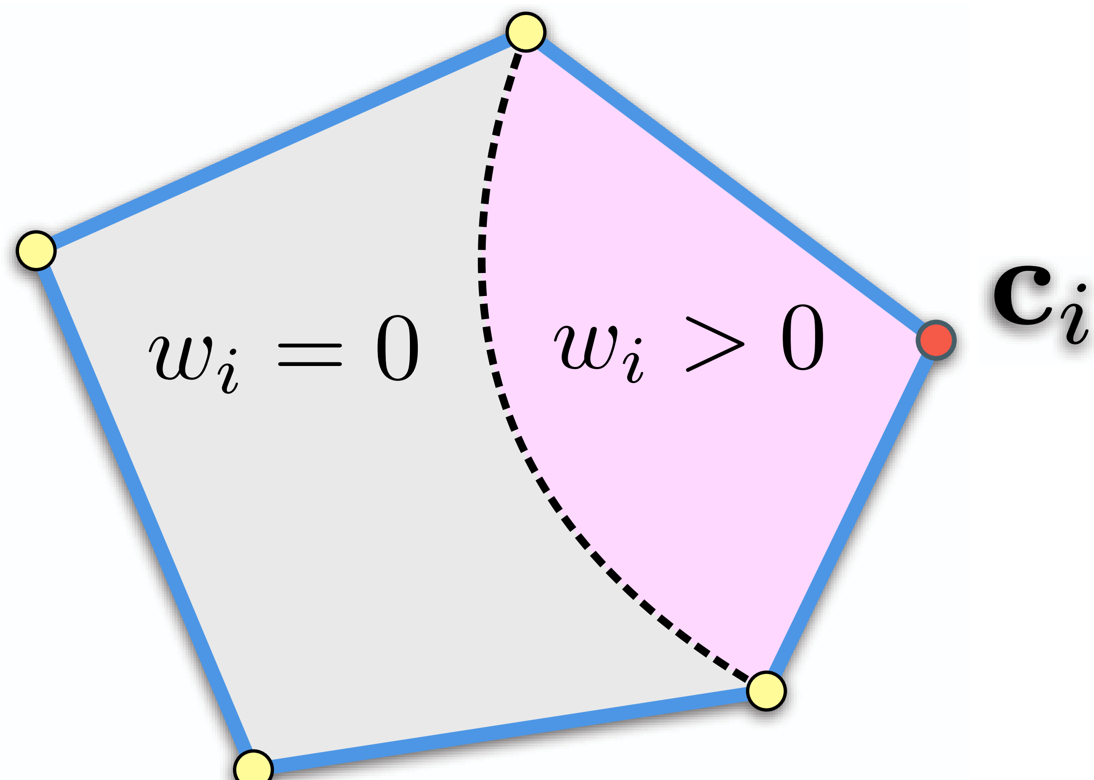


Controlling Locality

Total variation:

$$\int \boxed{|\nabla w_i(\mathbf{x})|}\, d\mathbf{x}$$

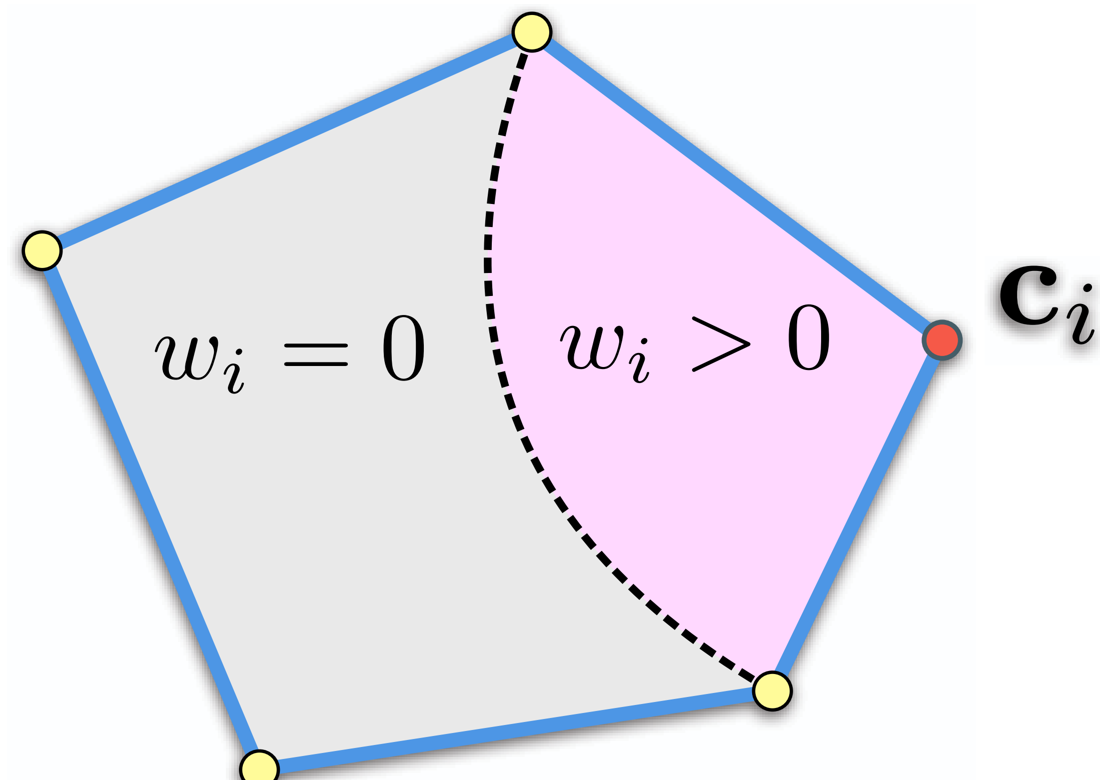
Same penalty everywhere



Controlling Locality

Weighted total variation:

$$\int \phi_i(\mathbf{x}) \; |\nabla w_i(\mathbf{x})| \; d\mathbf{x}$$

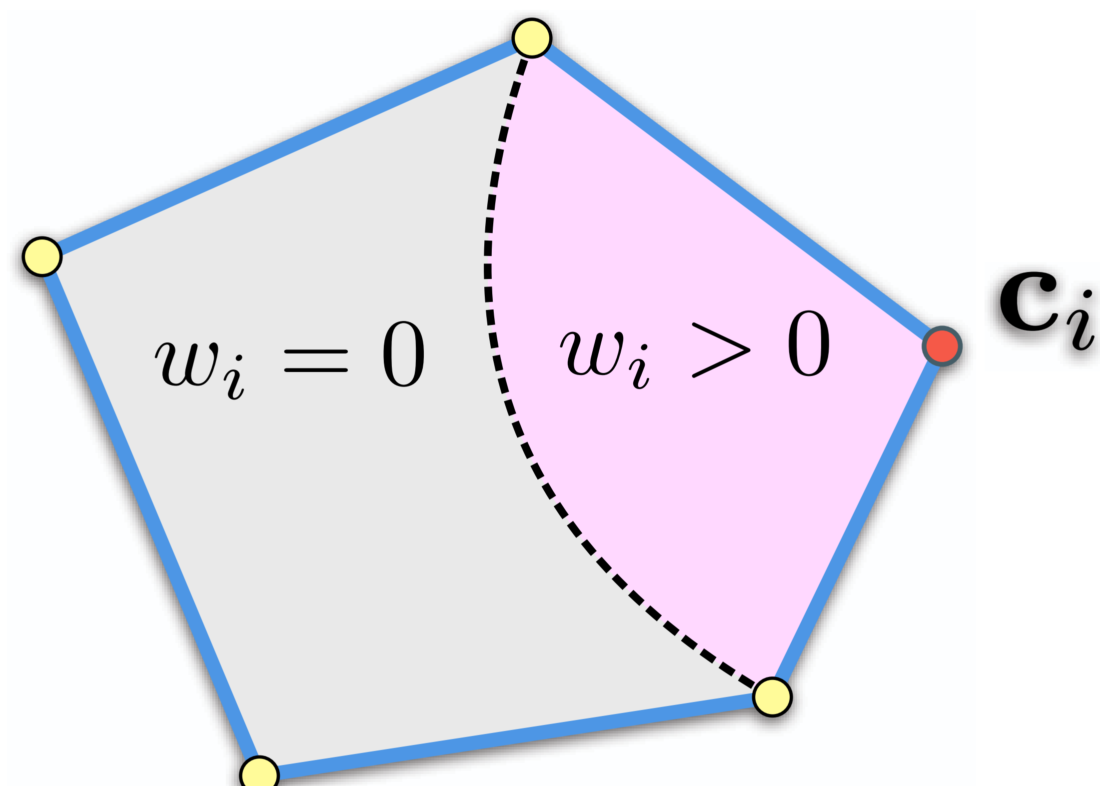


Controlling Locality

Weighted total variation:

$$\int \phi_i(\mathbf{x}) \, |\nabla w_i(\mathbf{x})| \; d\mathbf{x}$$

Monotonically increasing w.r.t. geodesic distance to cage vertex



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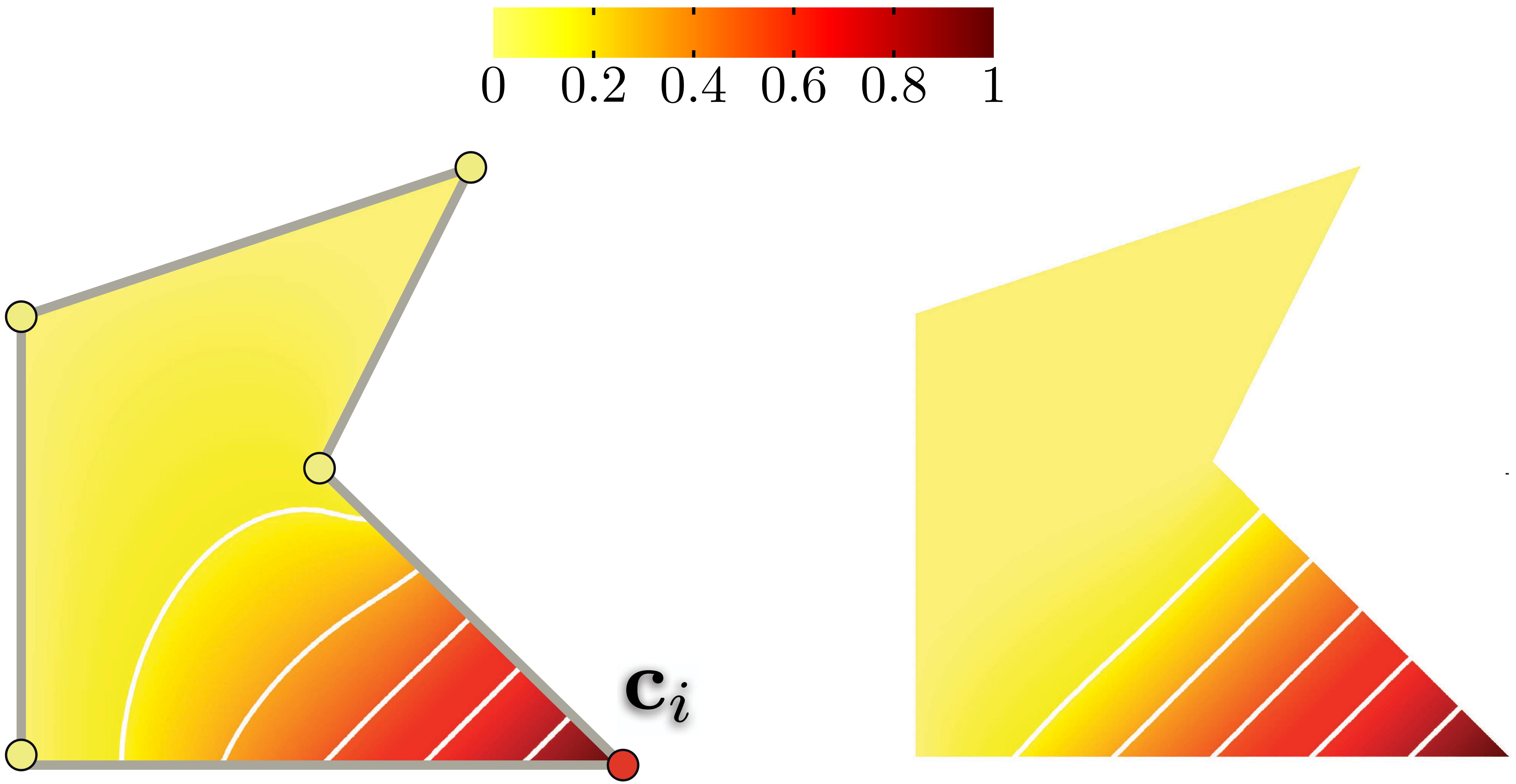
Comparison



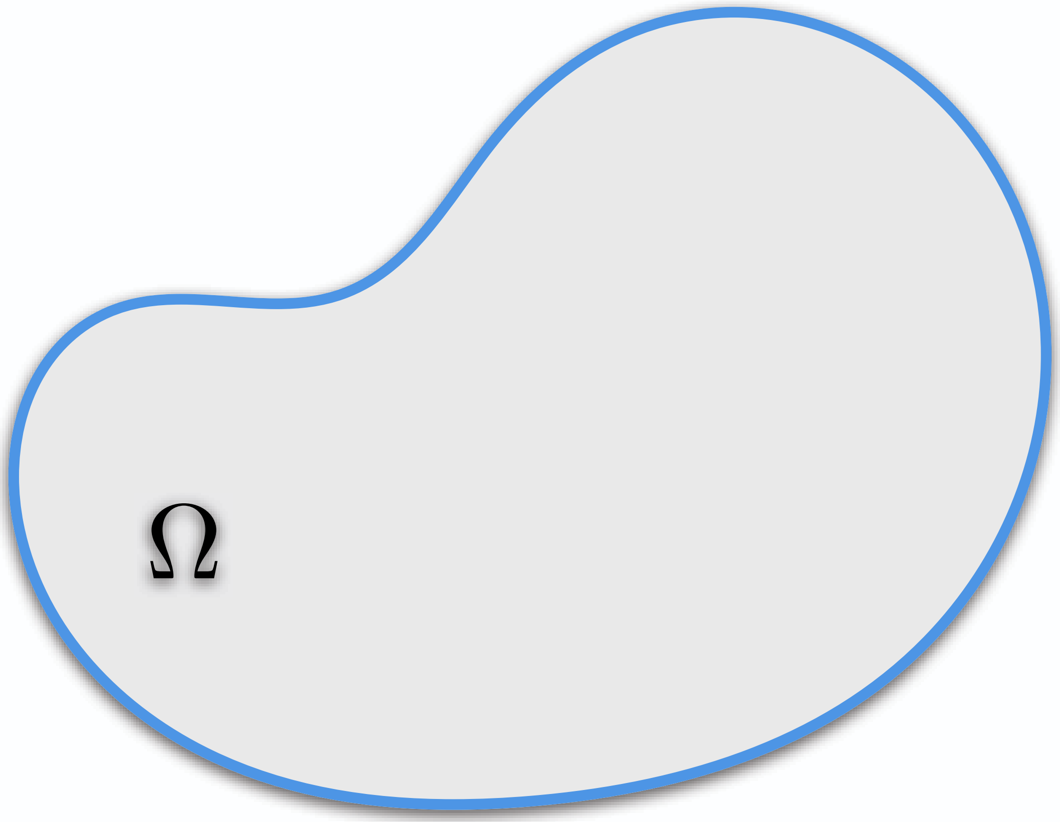
$\phi_i(\mathbf{x}) = 1$

$\phi_i(\mathbf{x}) = [d_i(\mathbf{x})]^2$

Geometry of Total Variation

Geometry of Total Variation

- Scalar function w defined on domain Ω



Geometry of Total Variation

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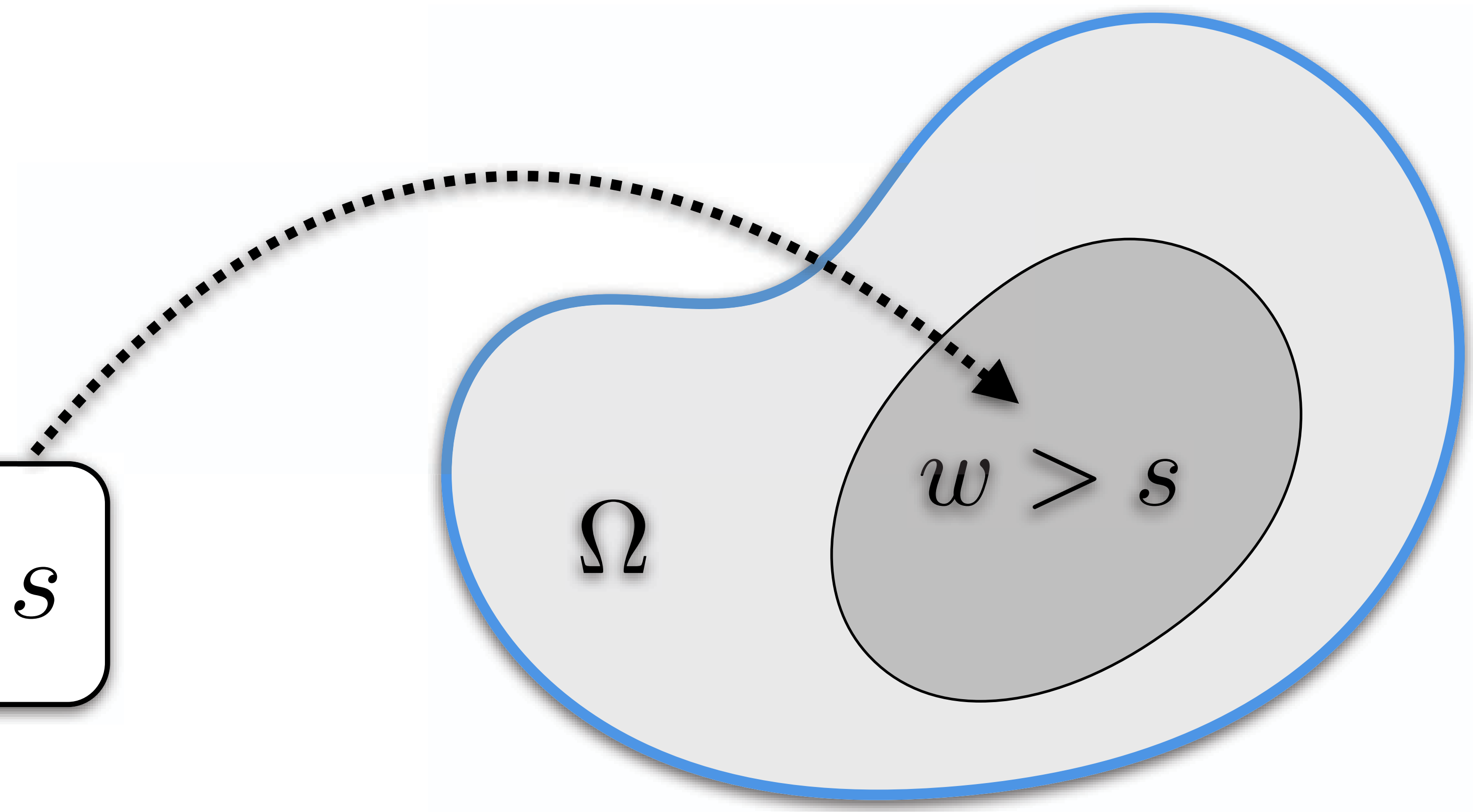
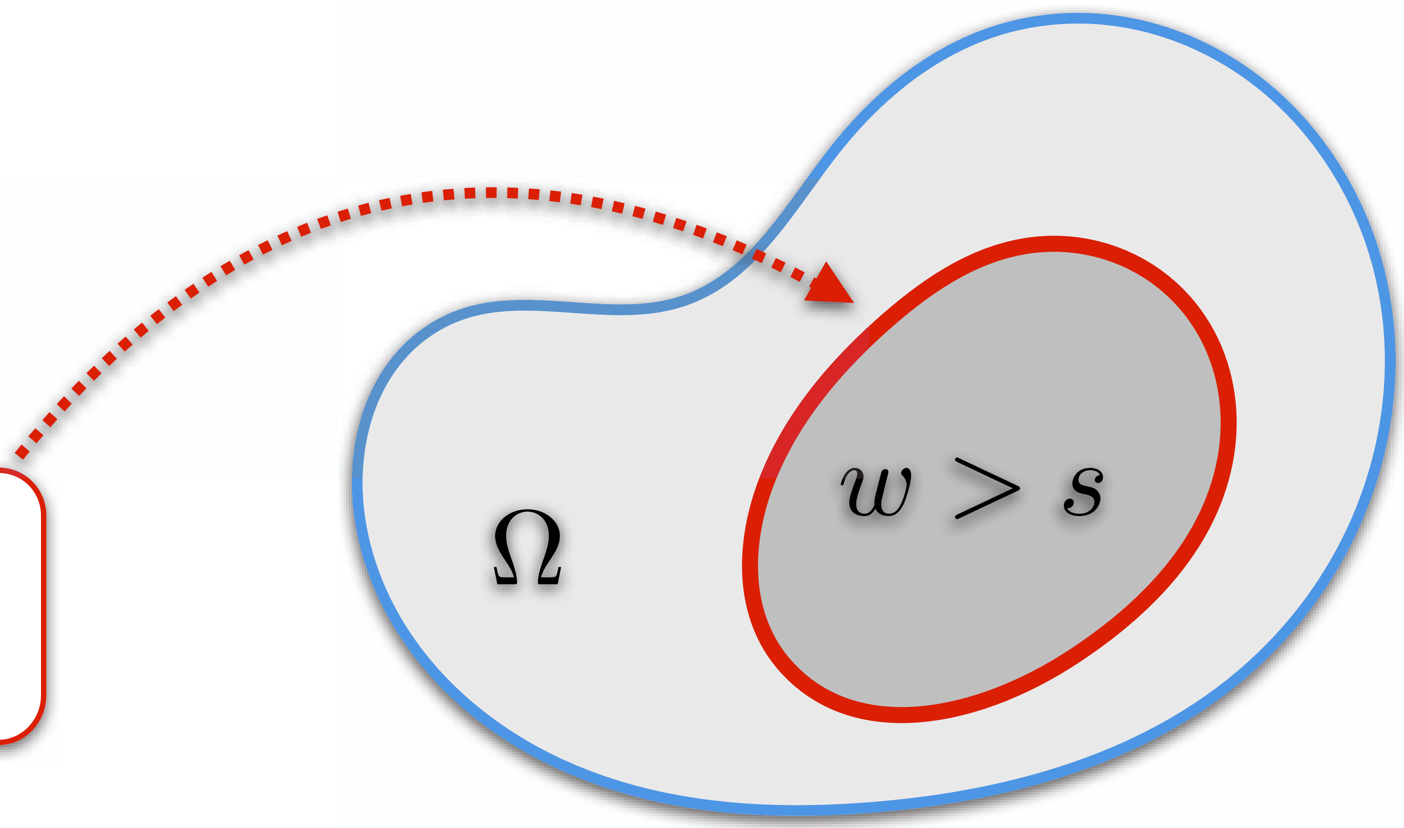


Superlevel set of s

Geometry of Total Variation

- Scalar function w defined on domain Ω



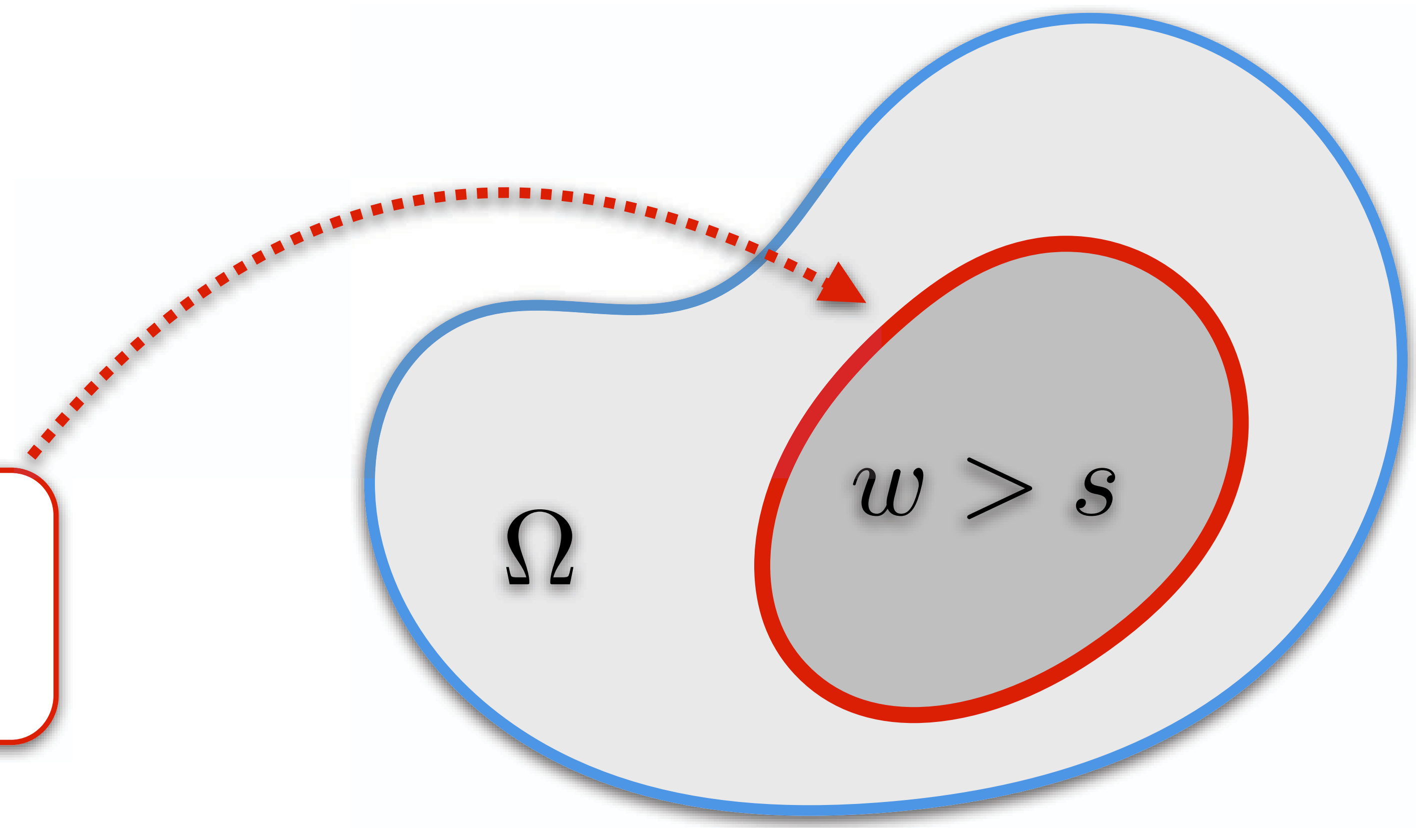
Perimeter: $P(w > s)$

Geometry of Total Variation

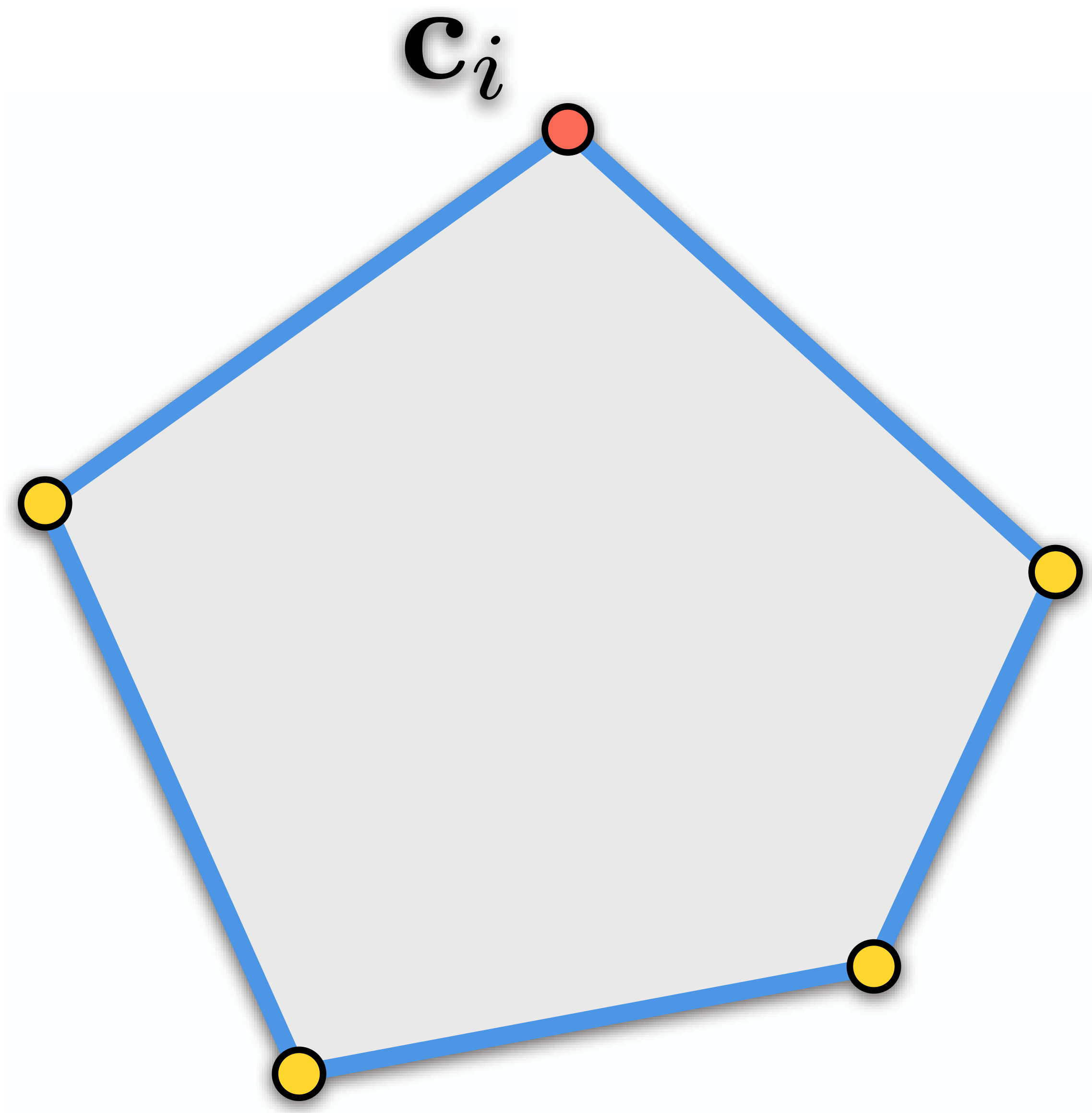
- Coarea formula:

$$\int_{\Omega} |\nabla w_i(\mathbf{x})| \; d\mathbf{x} = \int_{-\infty}^{+\infty} P(w > s) \; ds$$

Perimeter: $P(w > s)$

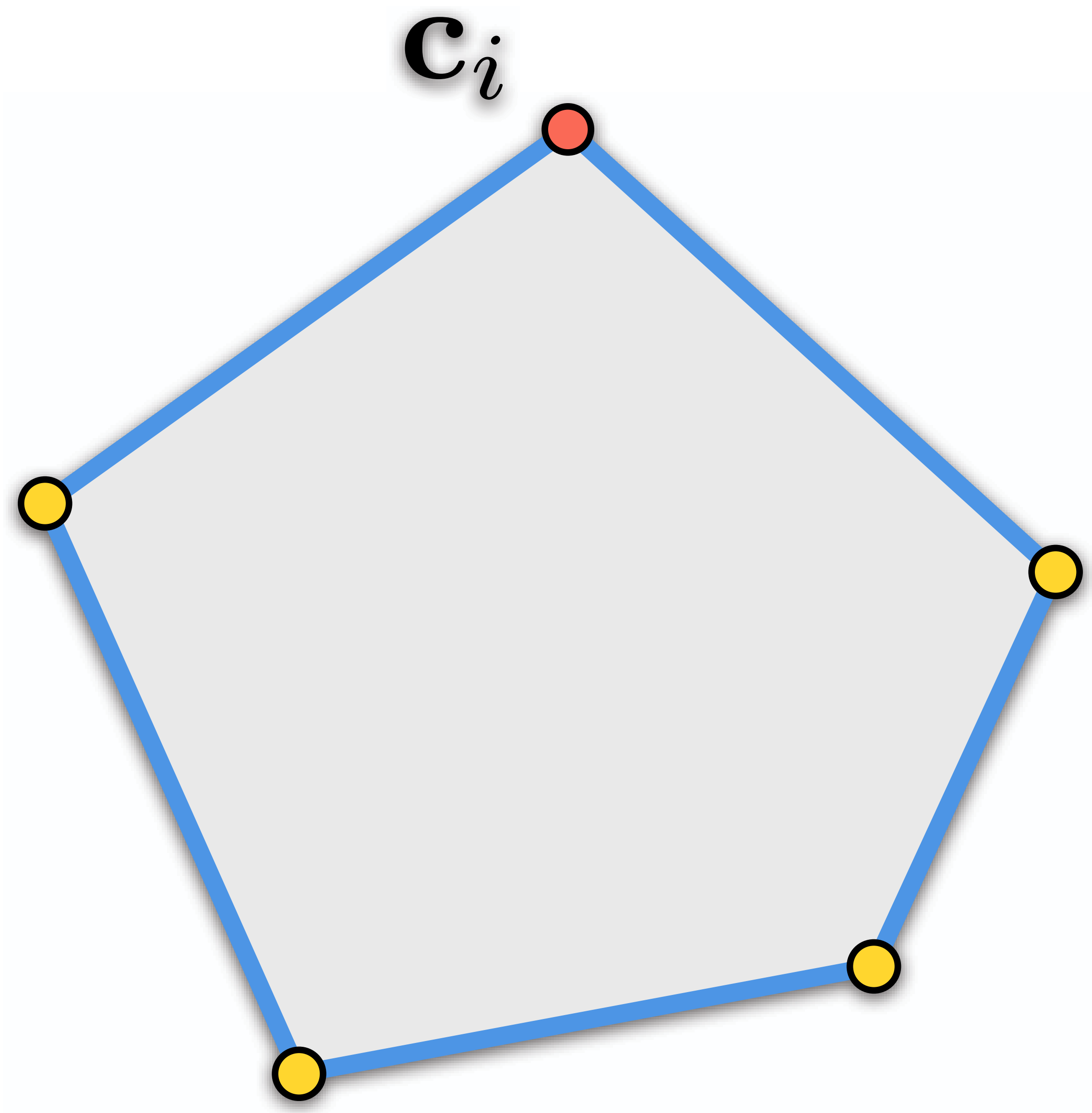


Geometry of Total Variation



Geometry of Total Variation

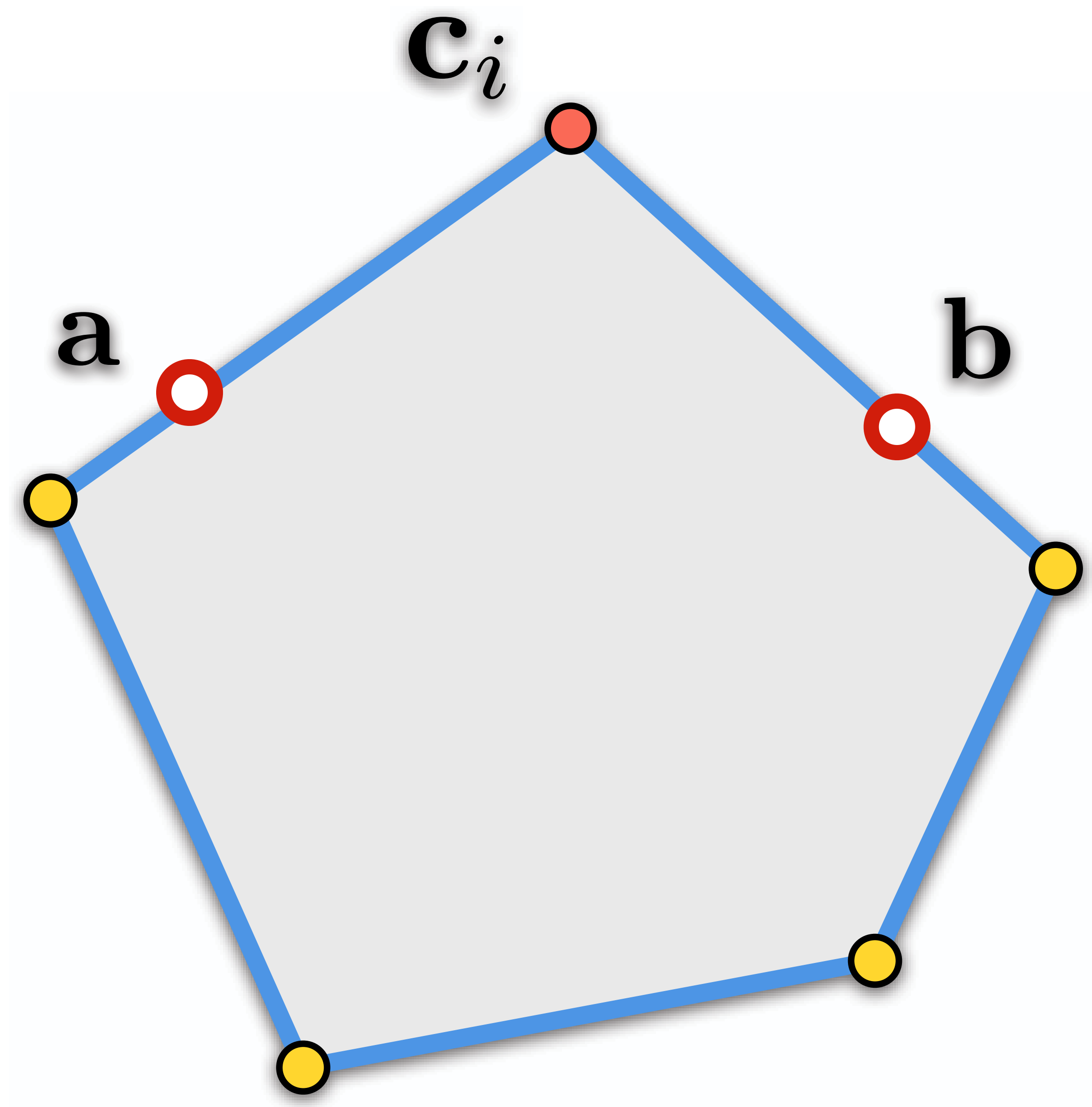
Superlevel set of w_i for $s \in [0, 1)$:



Geometry of Total Variation

Superlevel set of w_i for $s \in [0, 1)$:

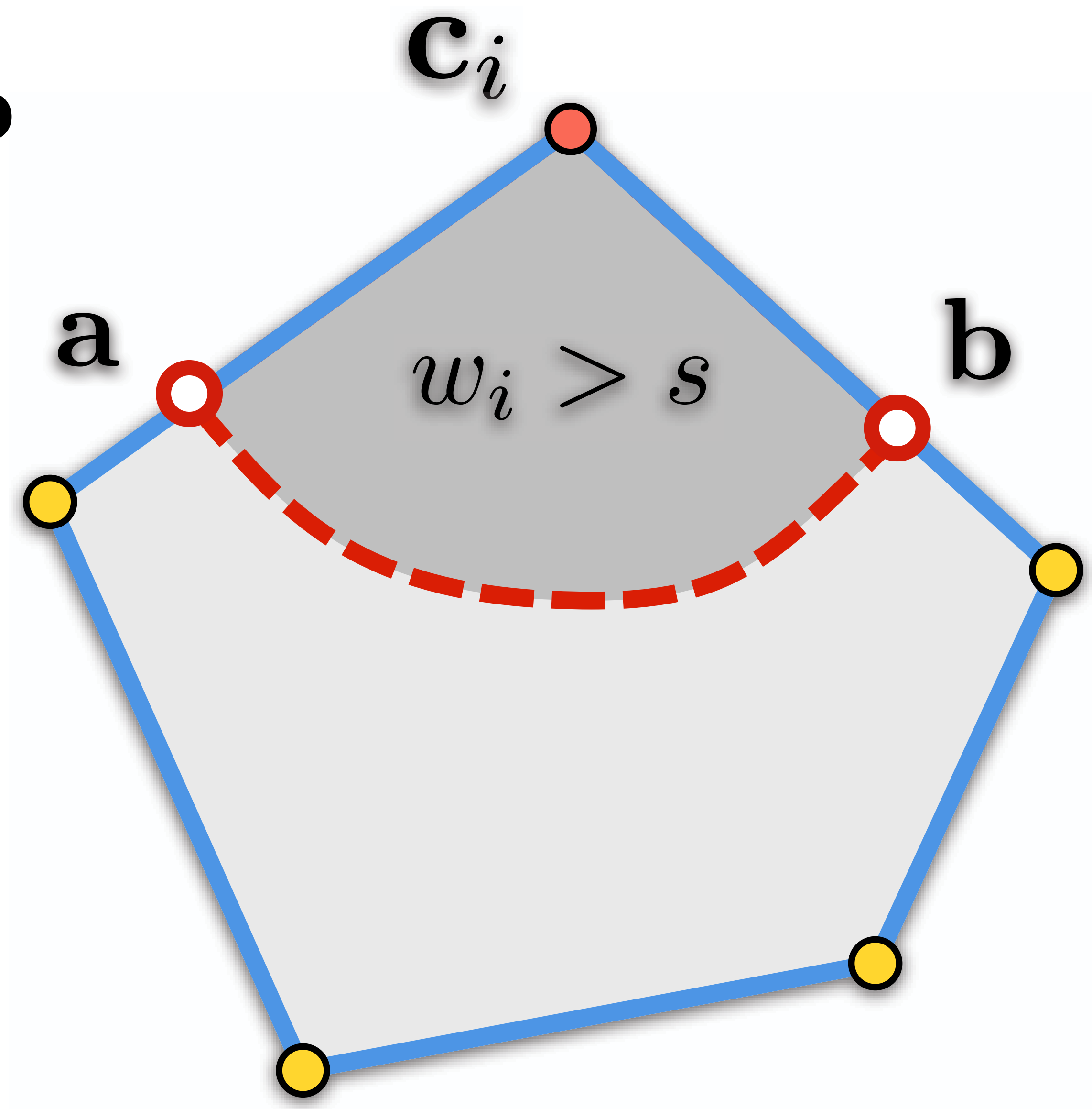
- $w_i(\mathbf{a}) = w_i(\mathbf{b}) = s$



Geometry of Total Variation

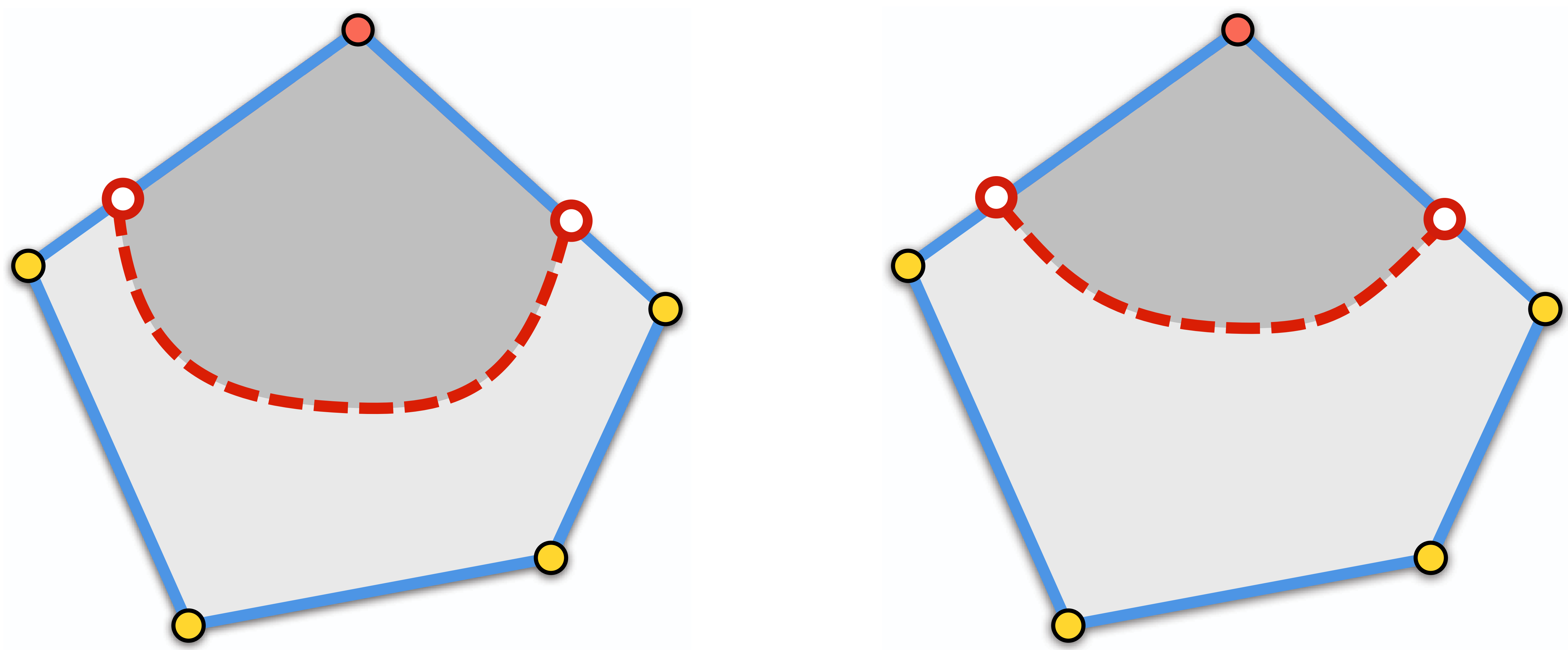
Superlevel set of w_i for $s \in [0, 1)$:

- $w_i(\mathbf{a}) = w_i(\mathbf{b}) = s$
- boundary curve connects $\mathbf{a}, \mathbf{b}$



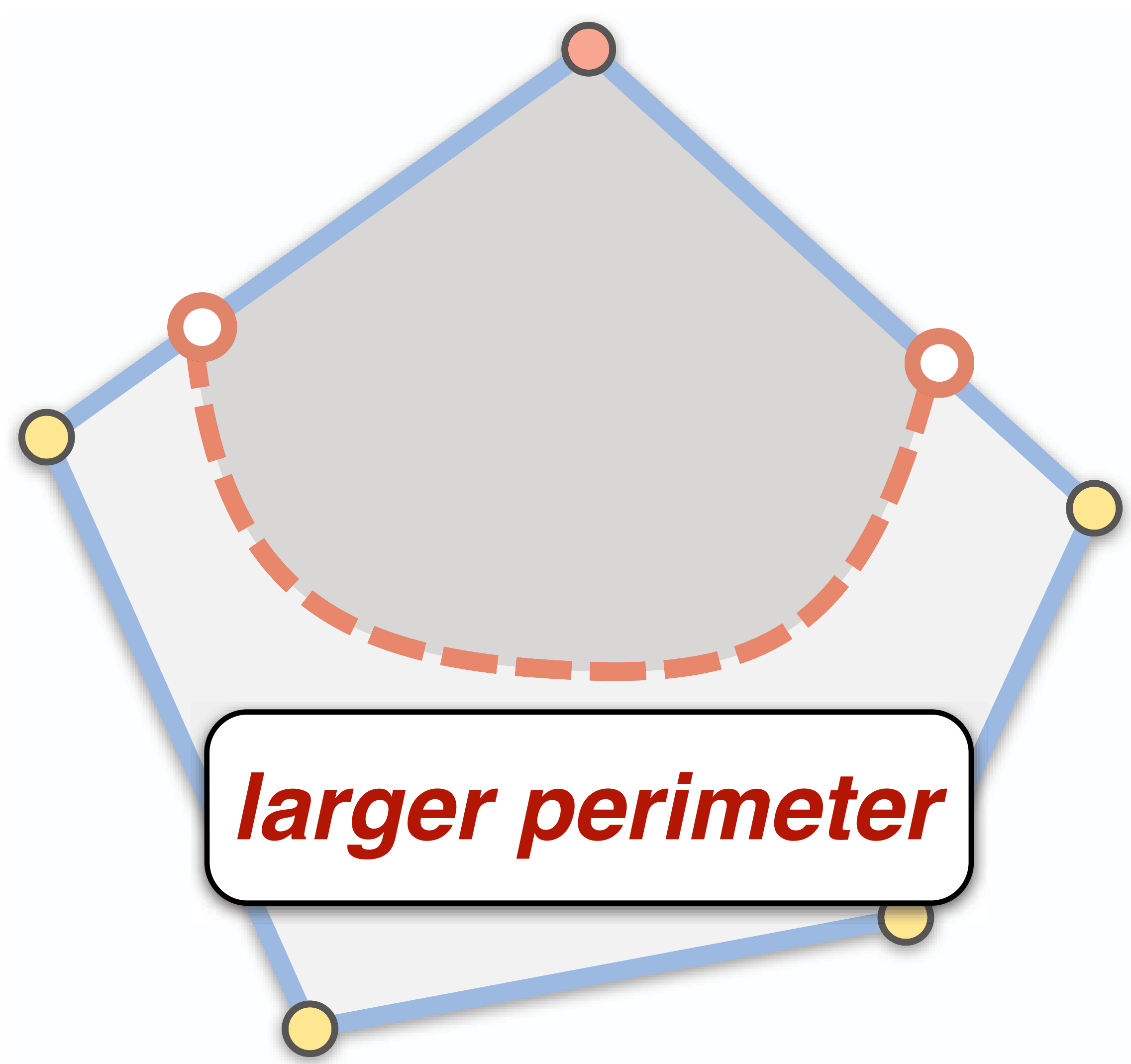
Geometry of Total Variation

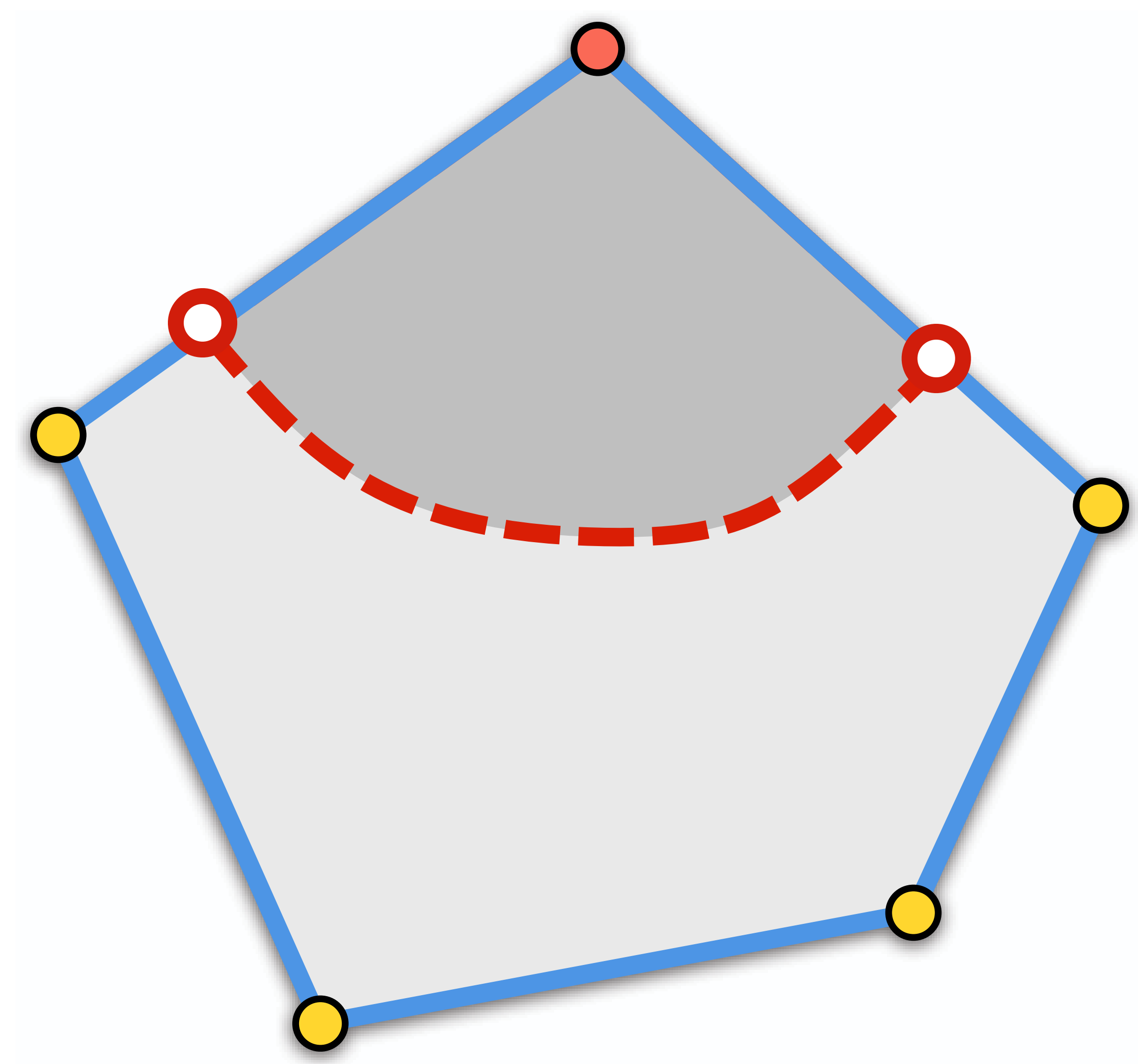
- Penalizing the superlevel set area



Geometry of Total Variation

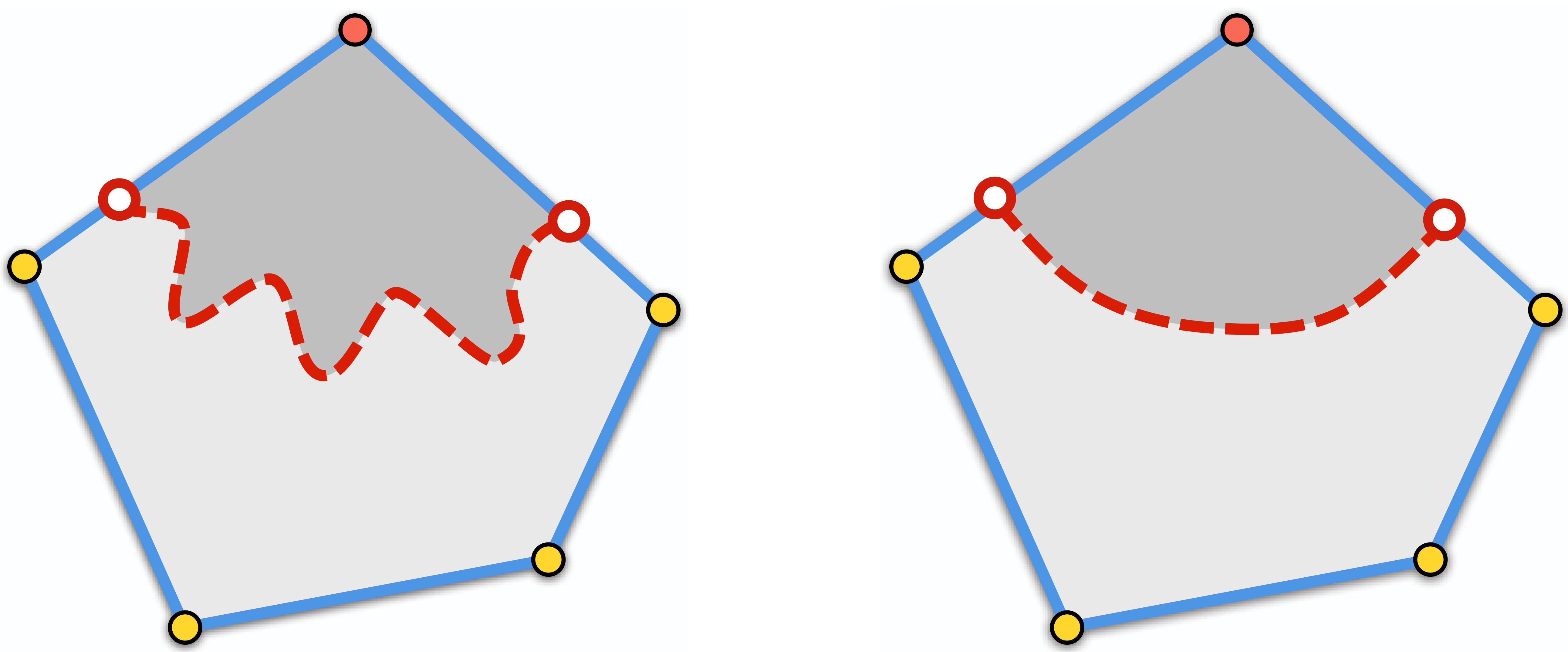
- Penalizing the superlevel set area





Geometry of Total Variation

- Regularizing the boundary curve



Geometry of Total Variation

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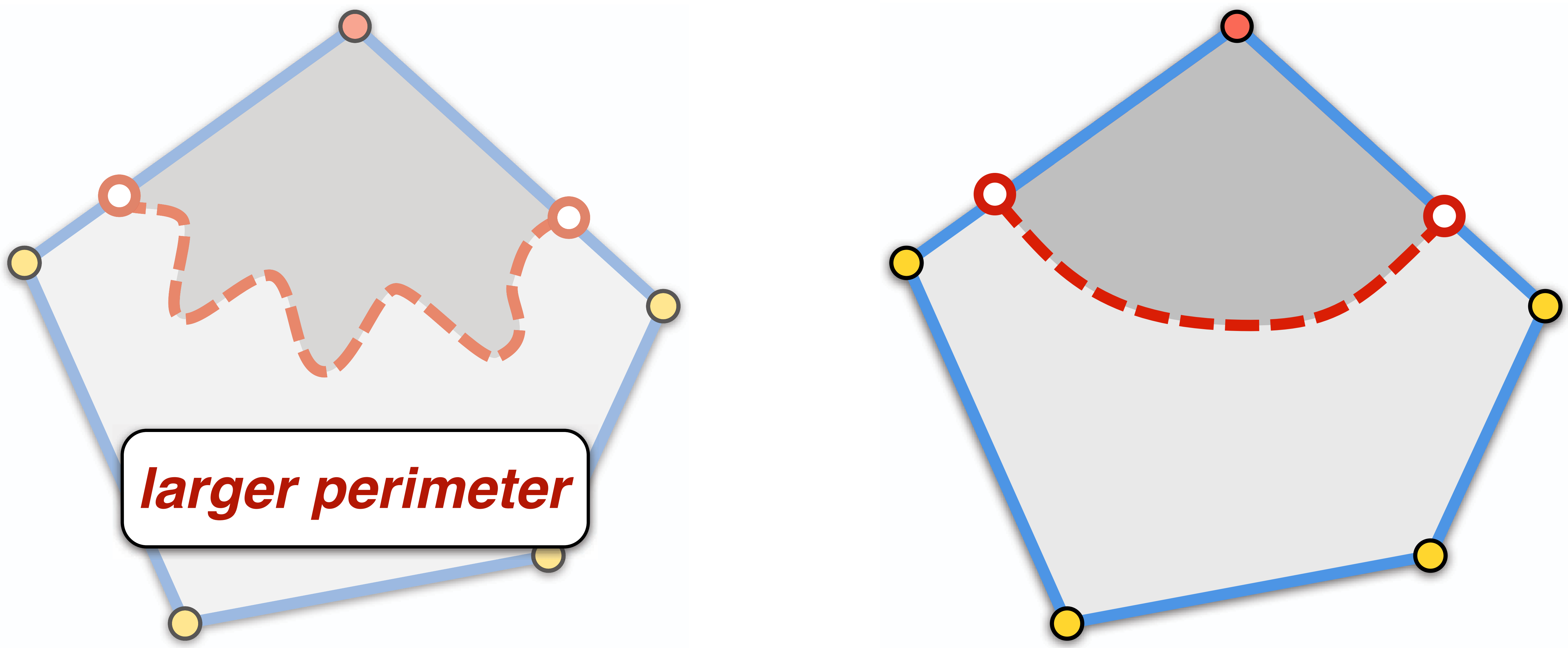
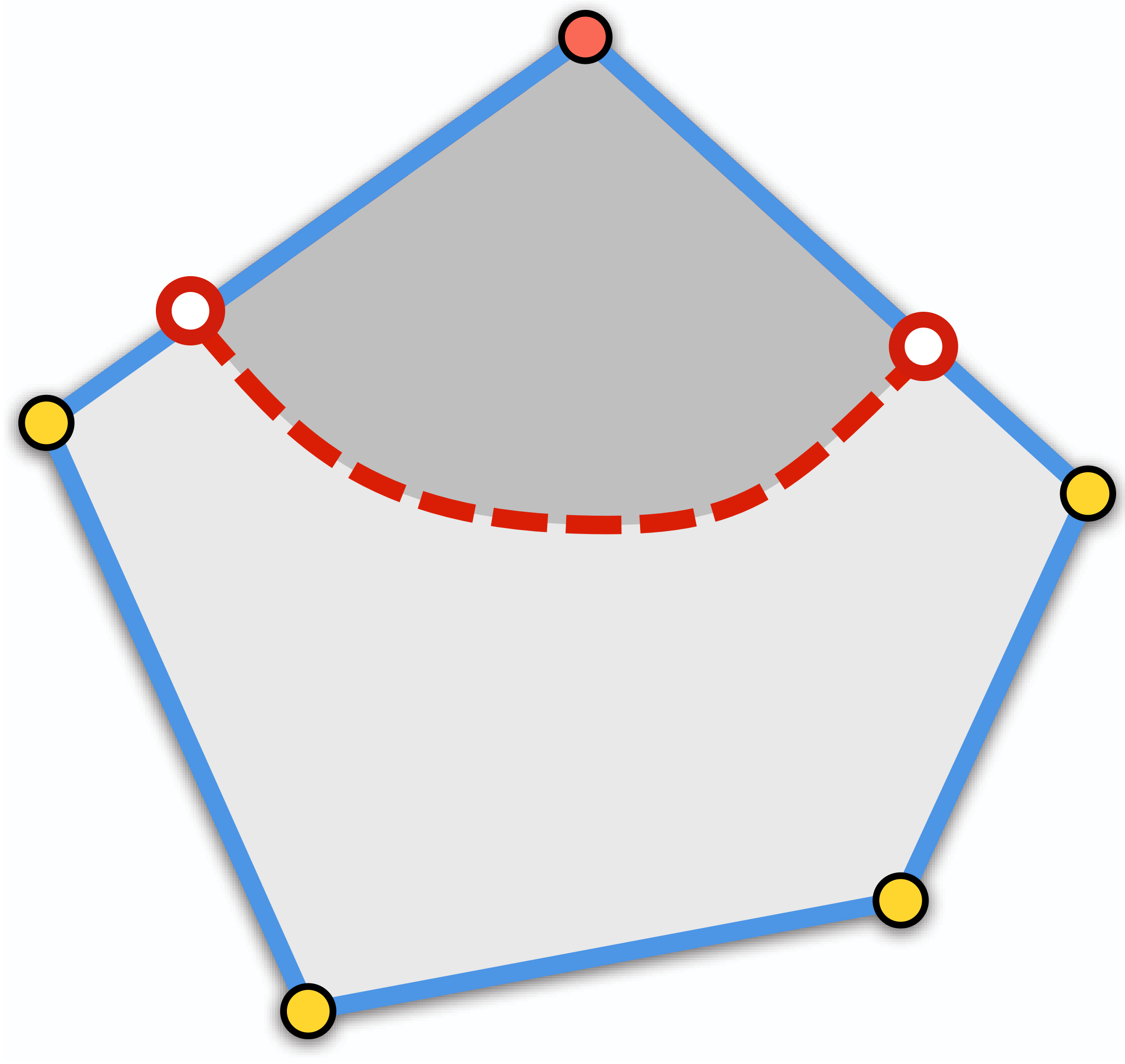
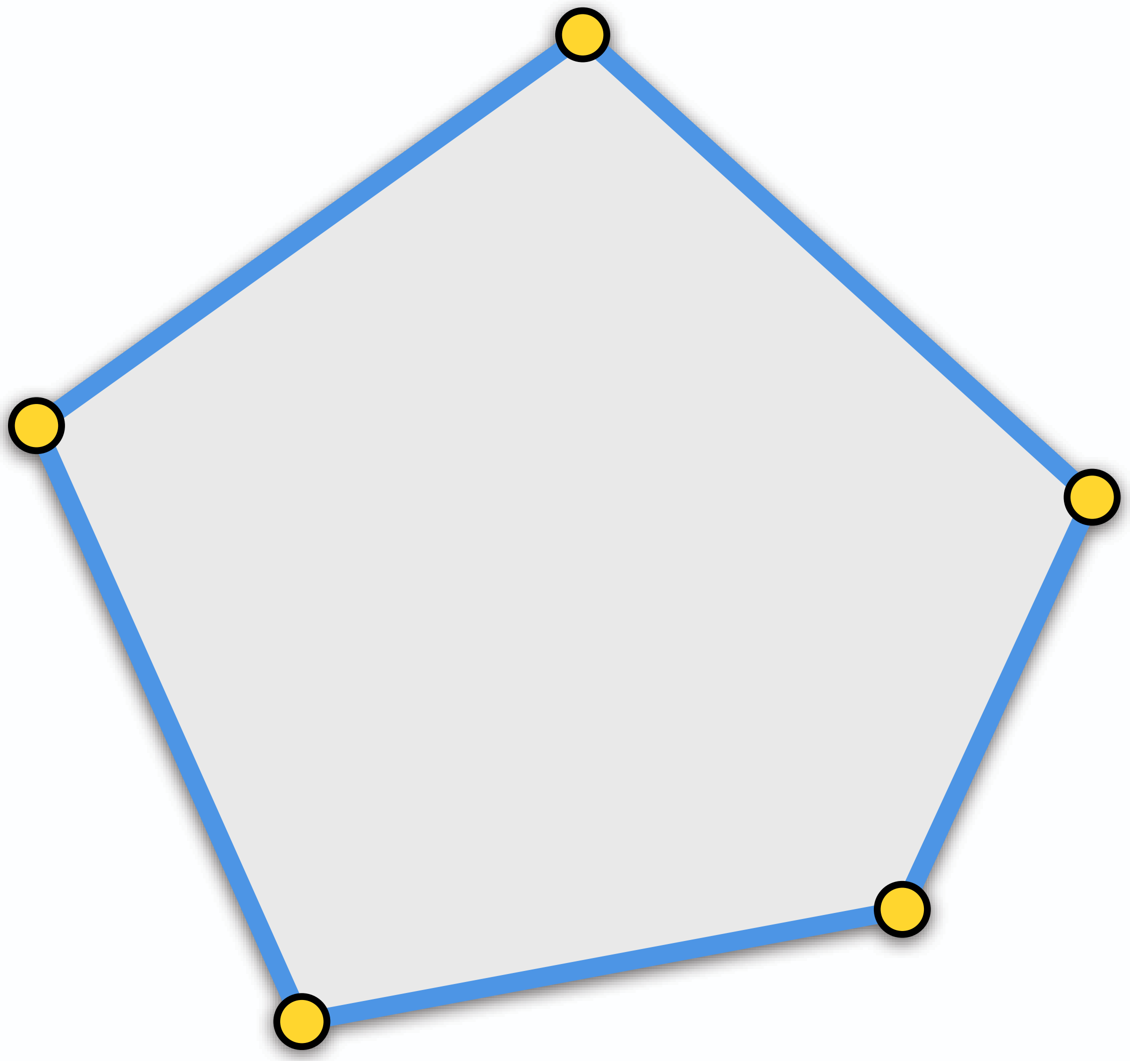


Geometry of Total Variation

- Total variation
  - penalize superlevel set size
  - regularize level set curves

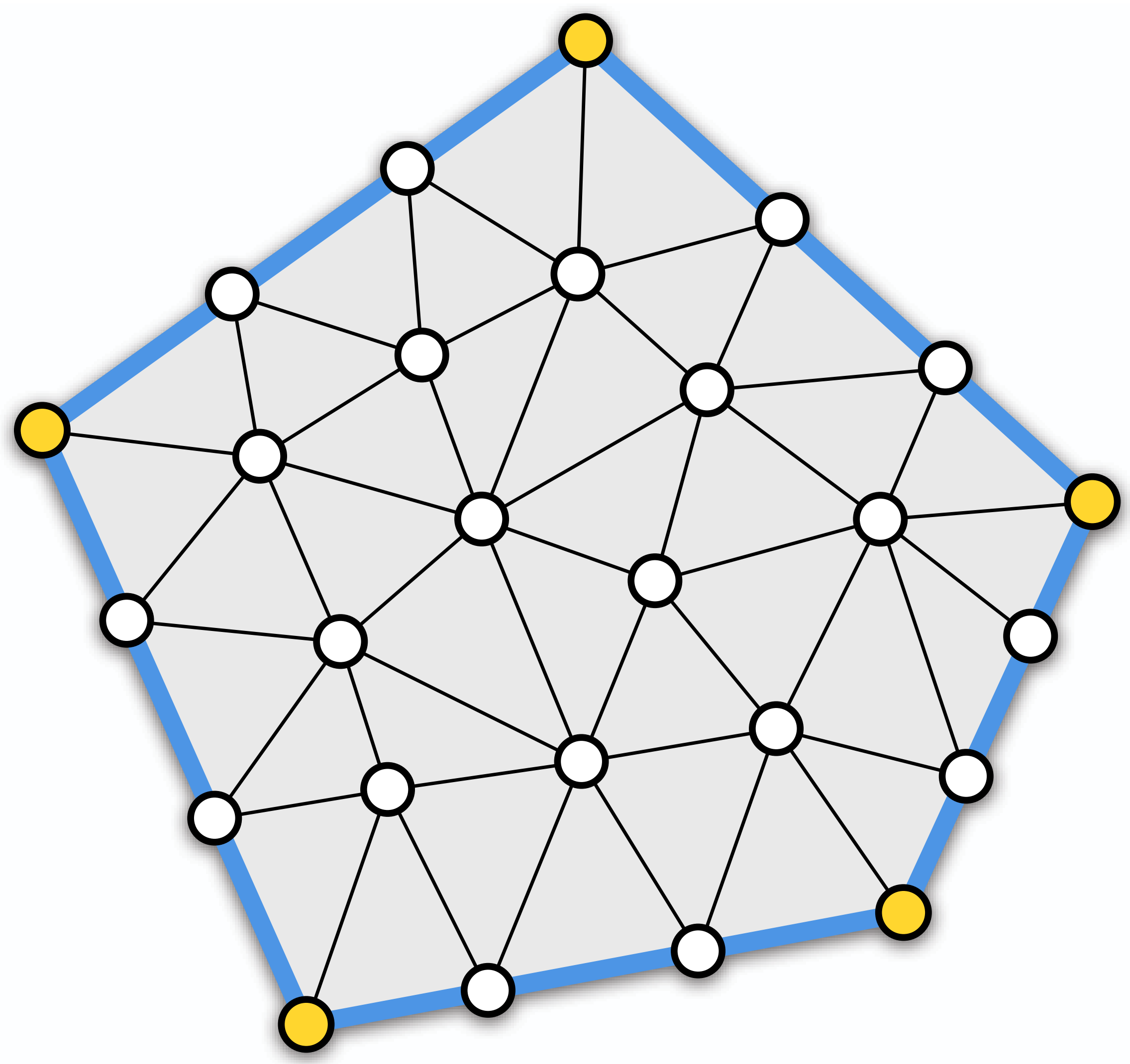


Numerical Optimization



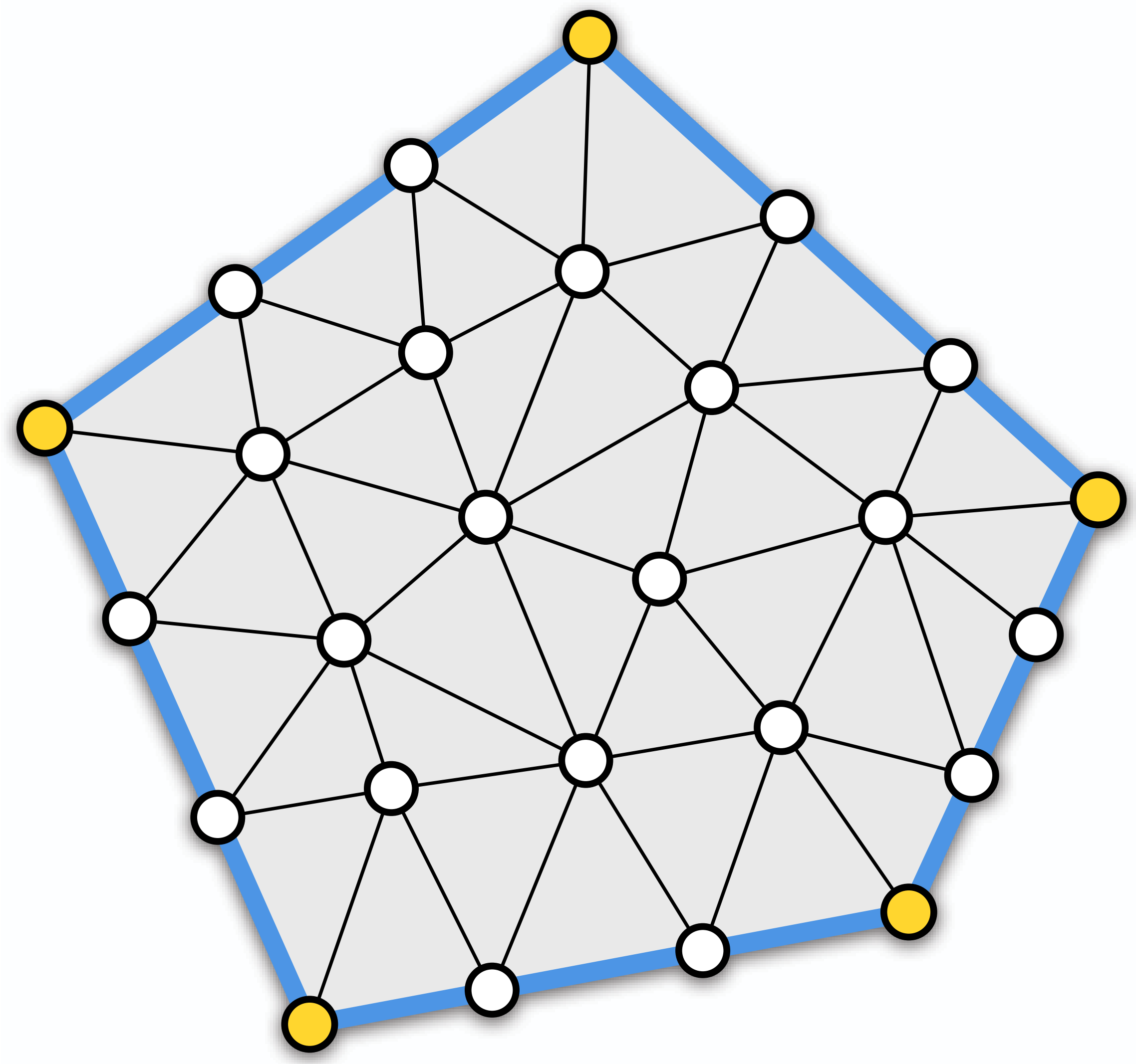
Numerical Optimization

- Discretization: triangulated domain



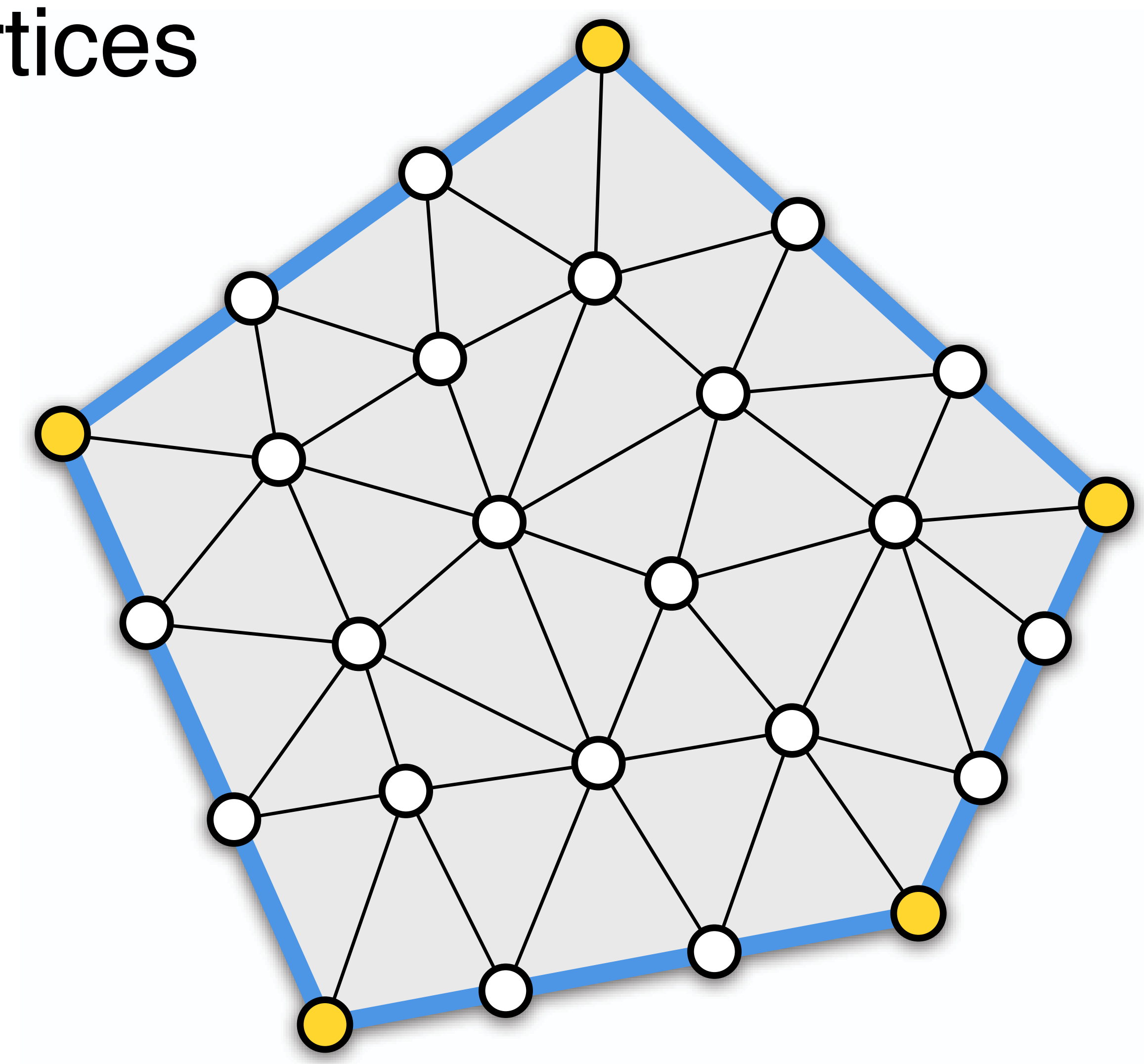
Numerical Optimization

- Discretization: triangulated domain
- Piecewise linear functions



Numerical Optimization

- Discretization: triangulated domain
- Piecewise linear functions
  - determined by values at vertices



Numerical Optimization

- Discretization: triangulated domain
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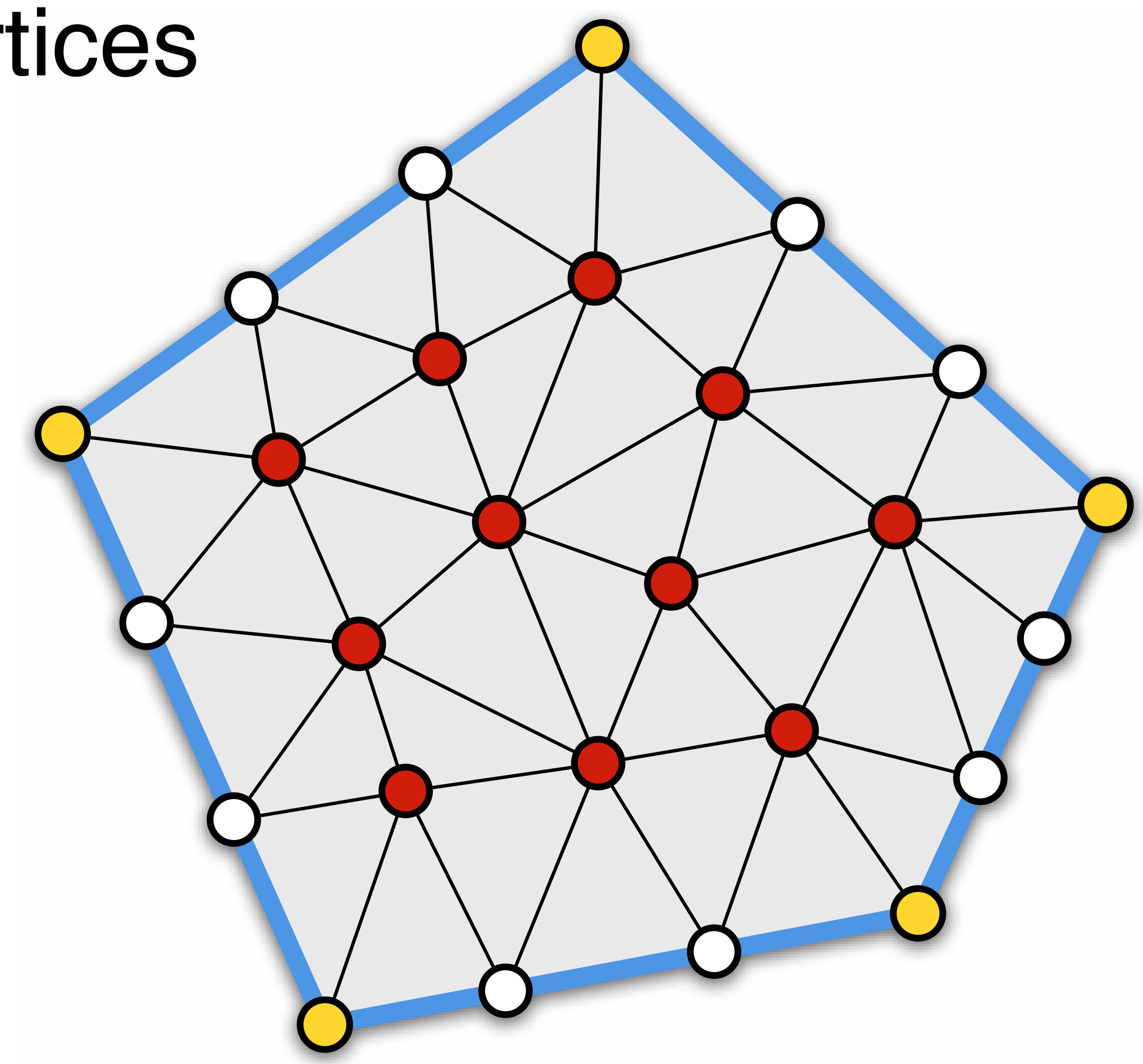
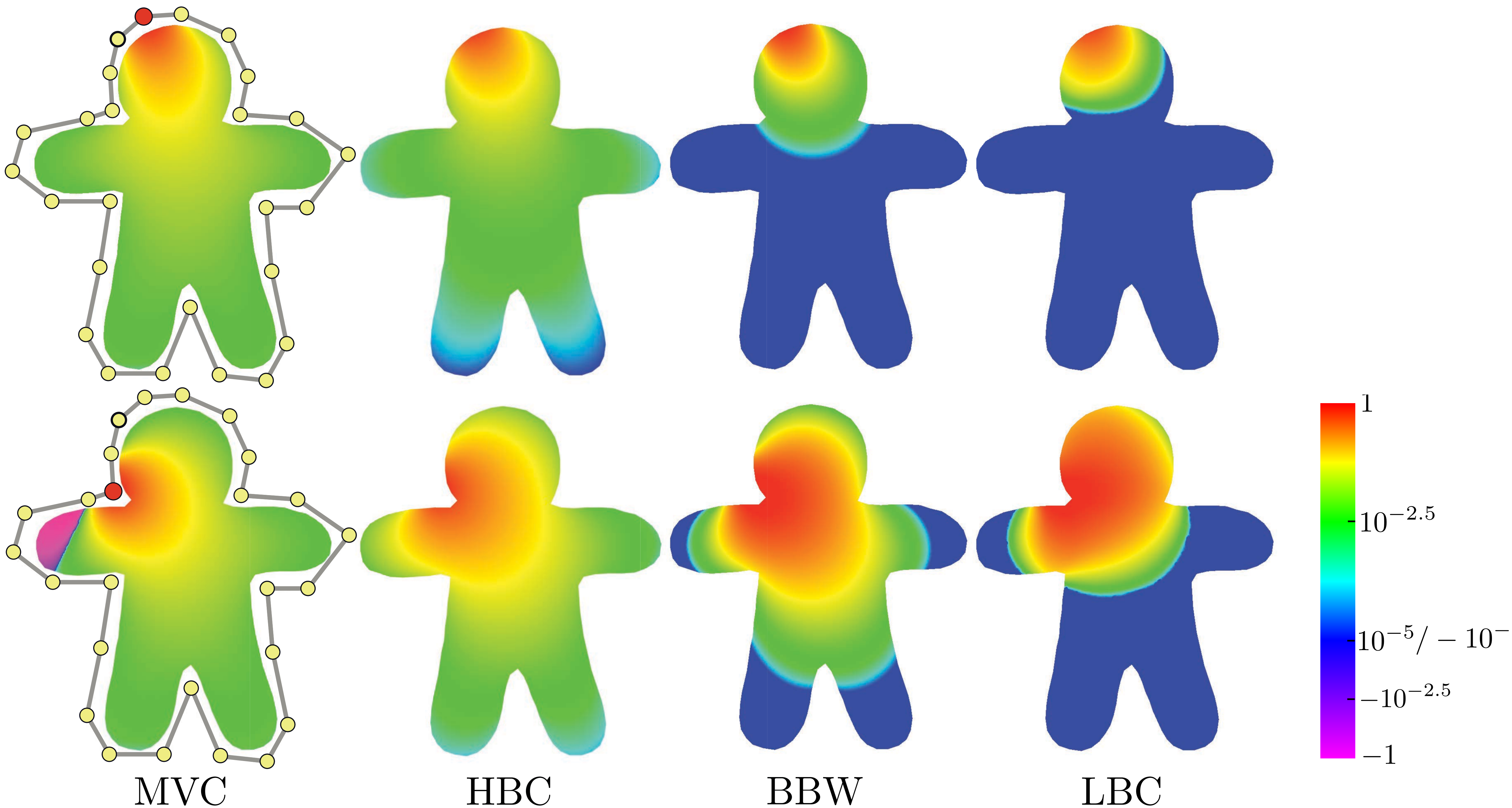
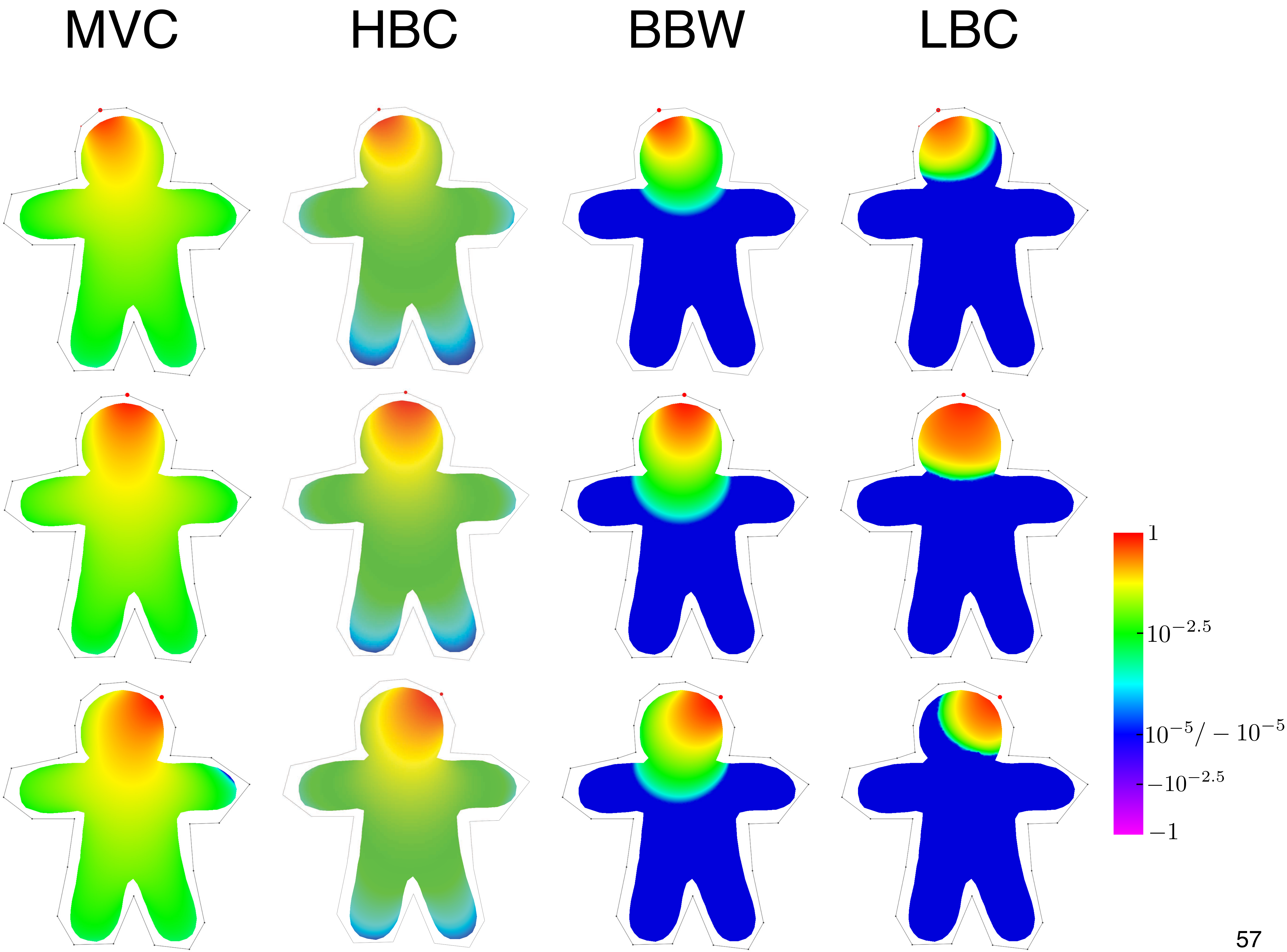


Convex optimization for values at interior vertices

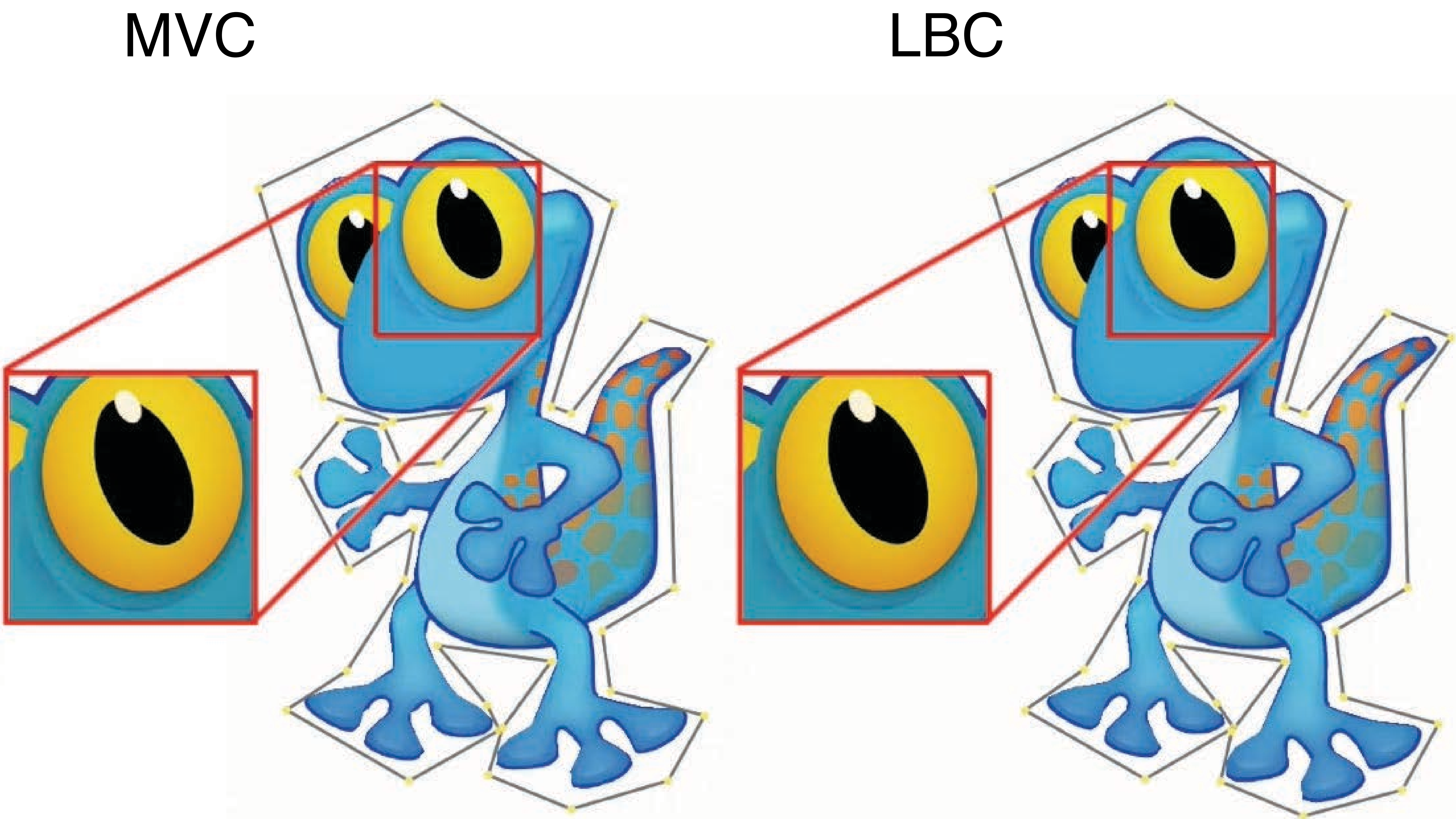
Comparison



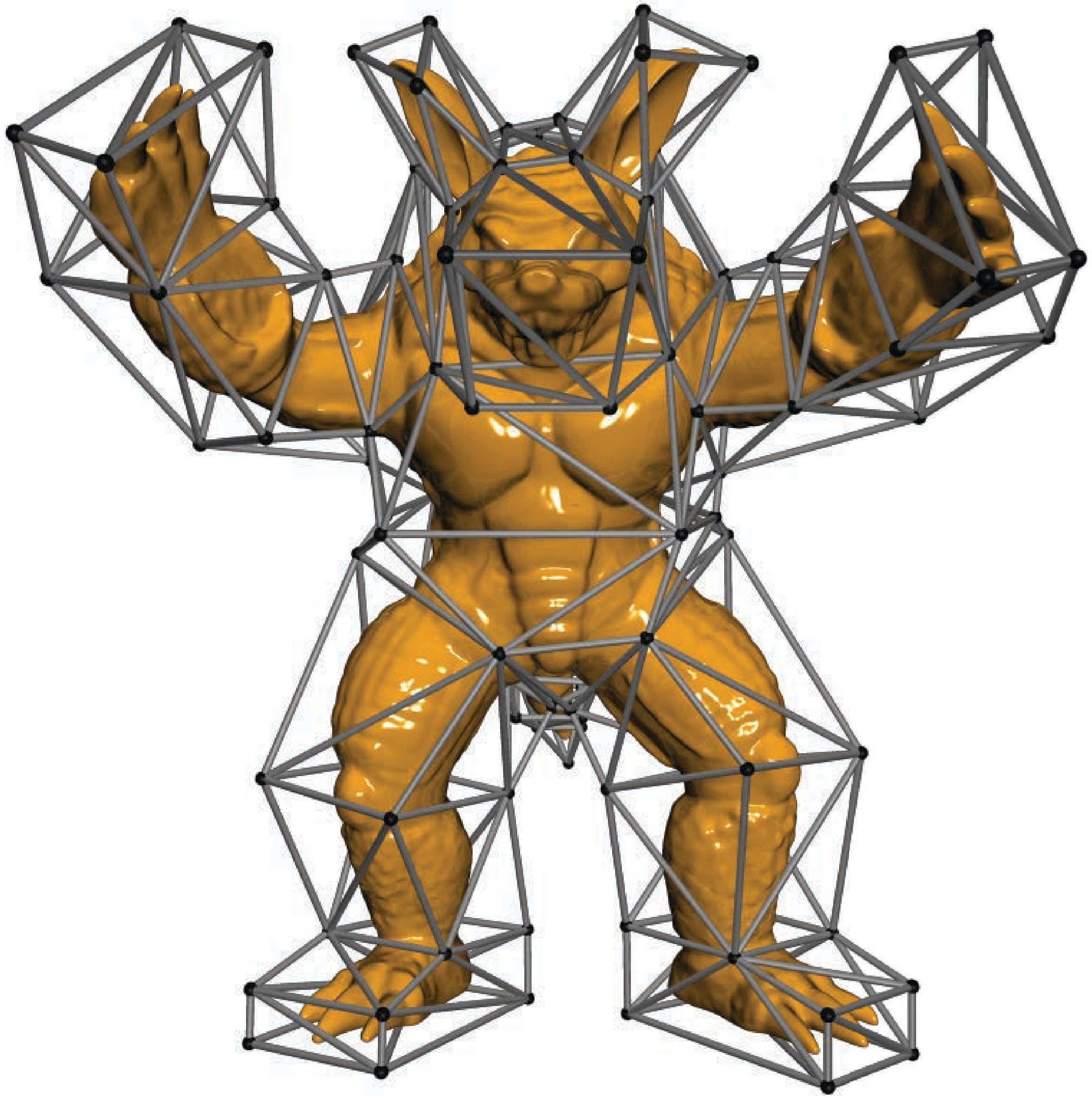

MVC
HBC
BBW
LBC
1
$10^{-2.5}$
$10^{-5}/-10^{-5}$
$-10^{-2.5}$
-1

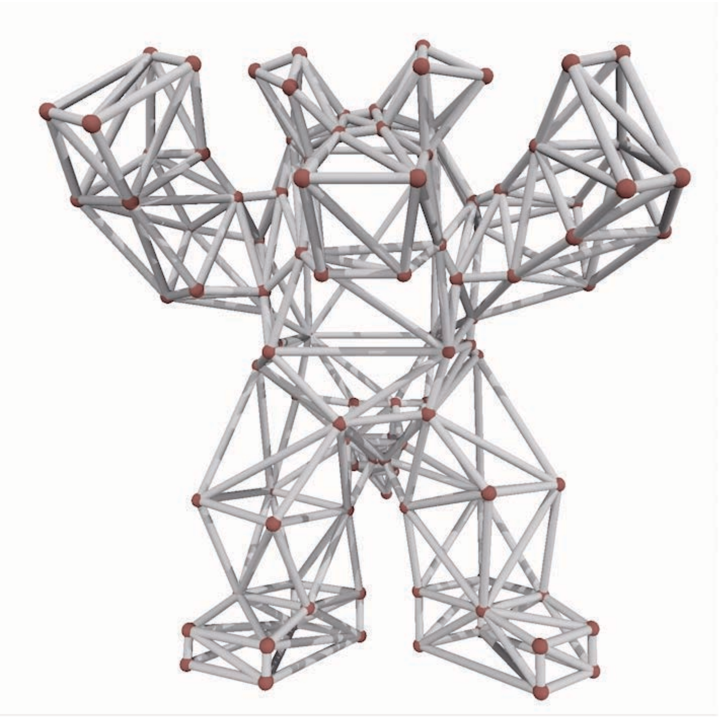
MVC

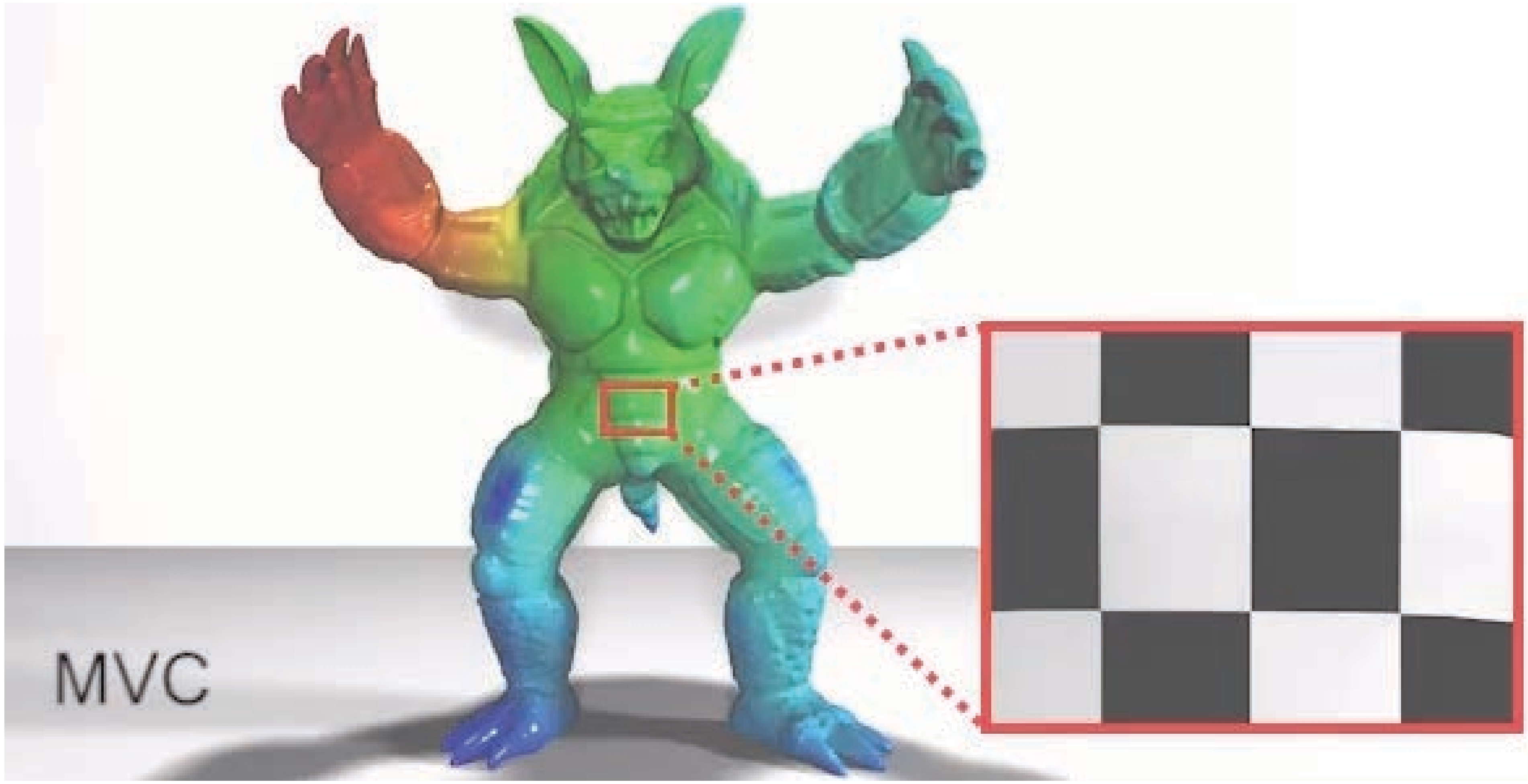
LBC

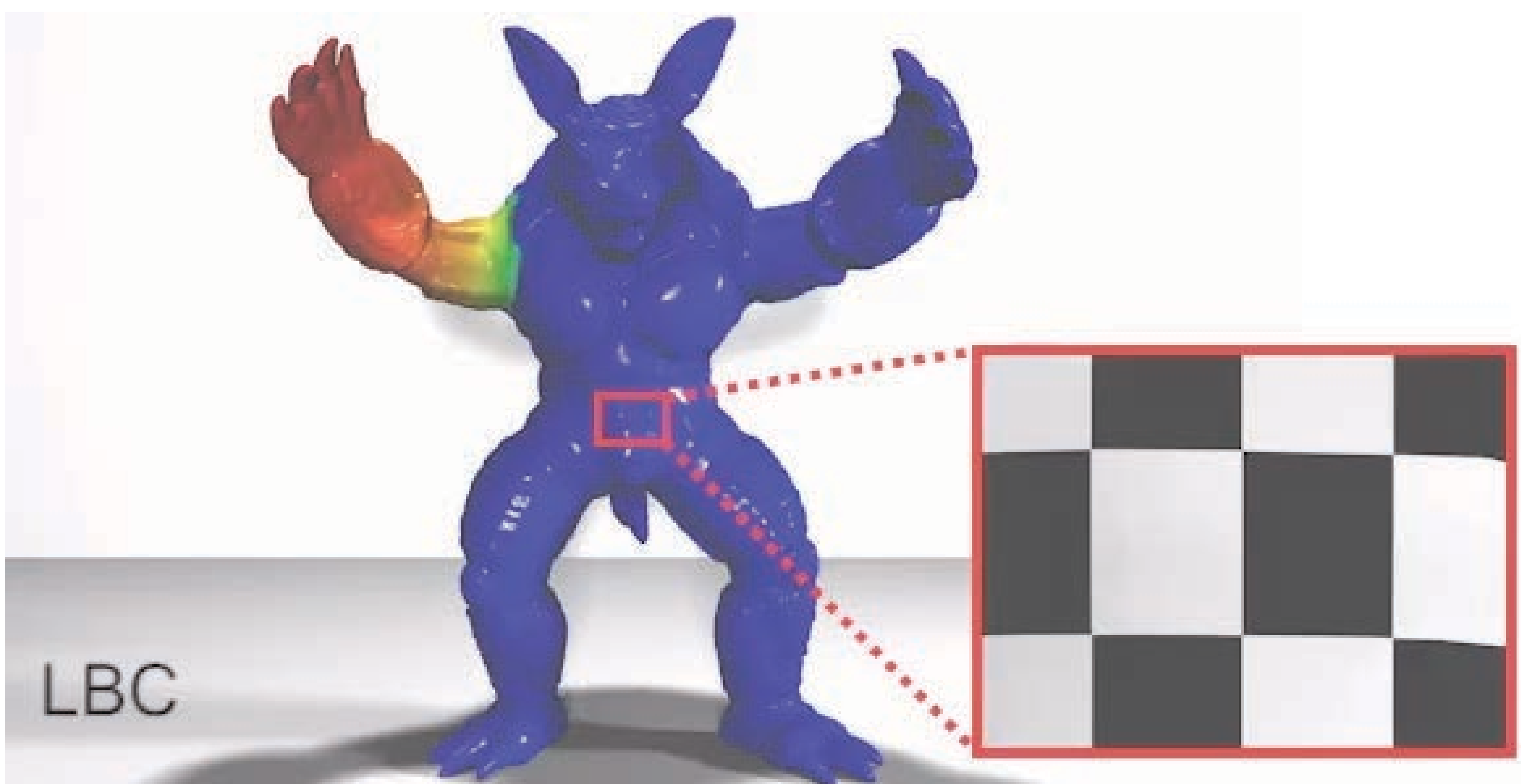

MVC
LBC

3D Example

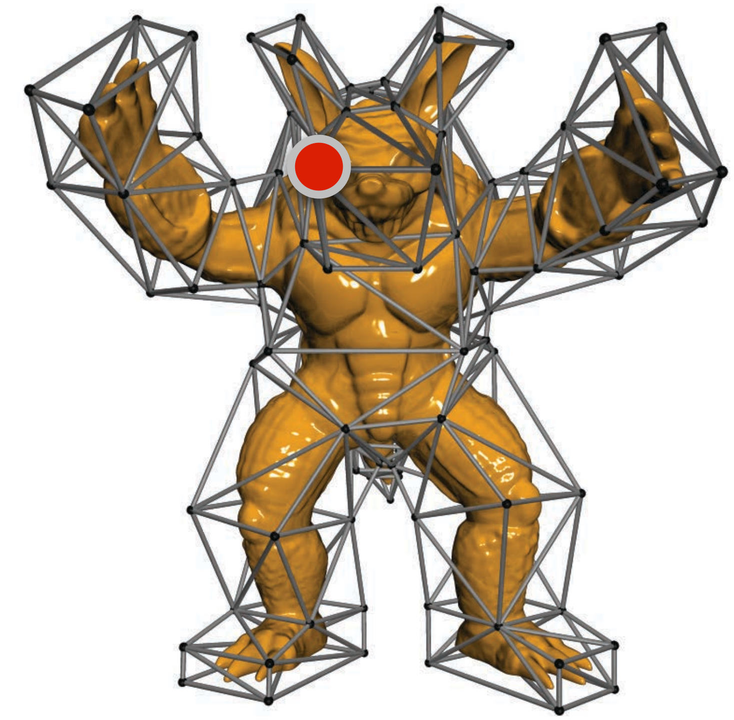


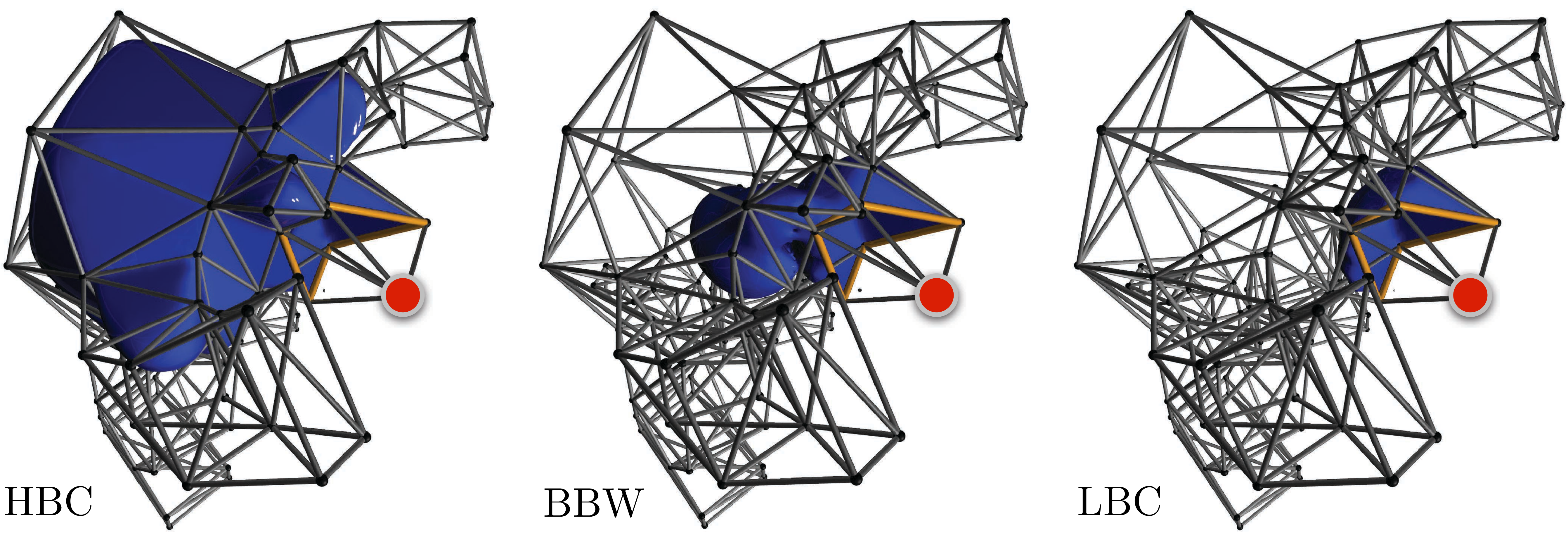



MVC


LBC

Superlevel set of $10^{-3}/n$





Weight Reduction

- Cage based deformation: matrix multiplication

$$\mathbf{W}\ \mathbf{C} = \mathbf{P}$$

Weight Reduction

new positions of
control points

$$\mathbf{W}\ \mathbf{C} = \mathbf{P}$$

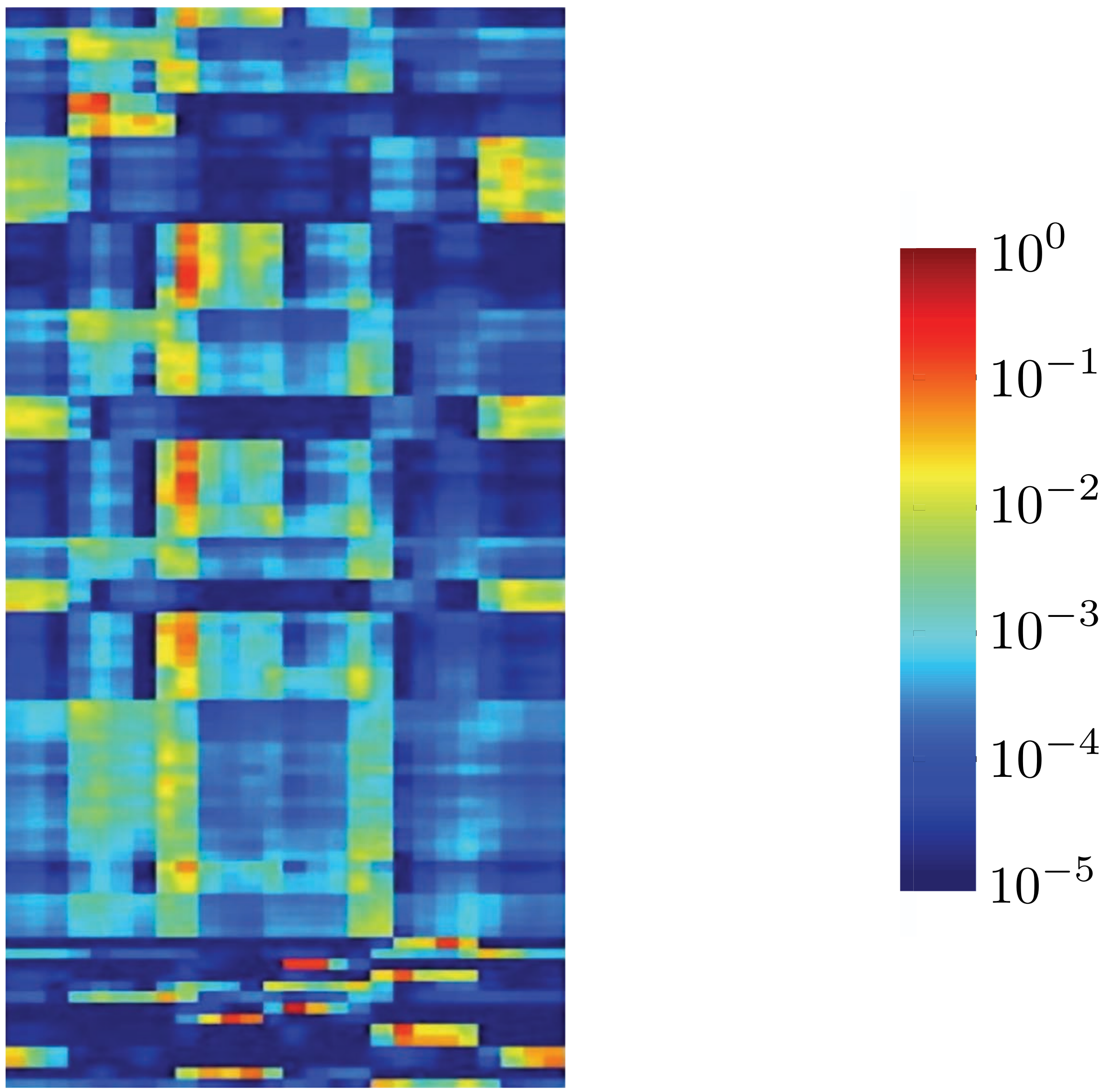
barycentric coordinates of
sample points

new positions of
sample points

Weight Reduction

- Global influence: dense matrix

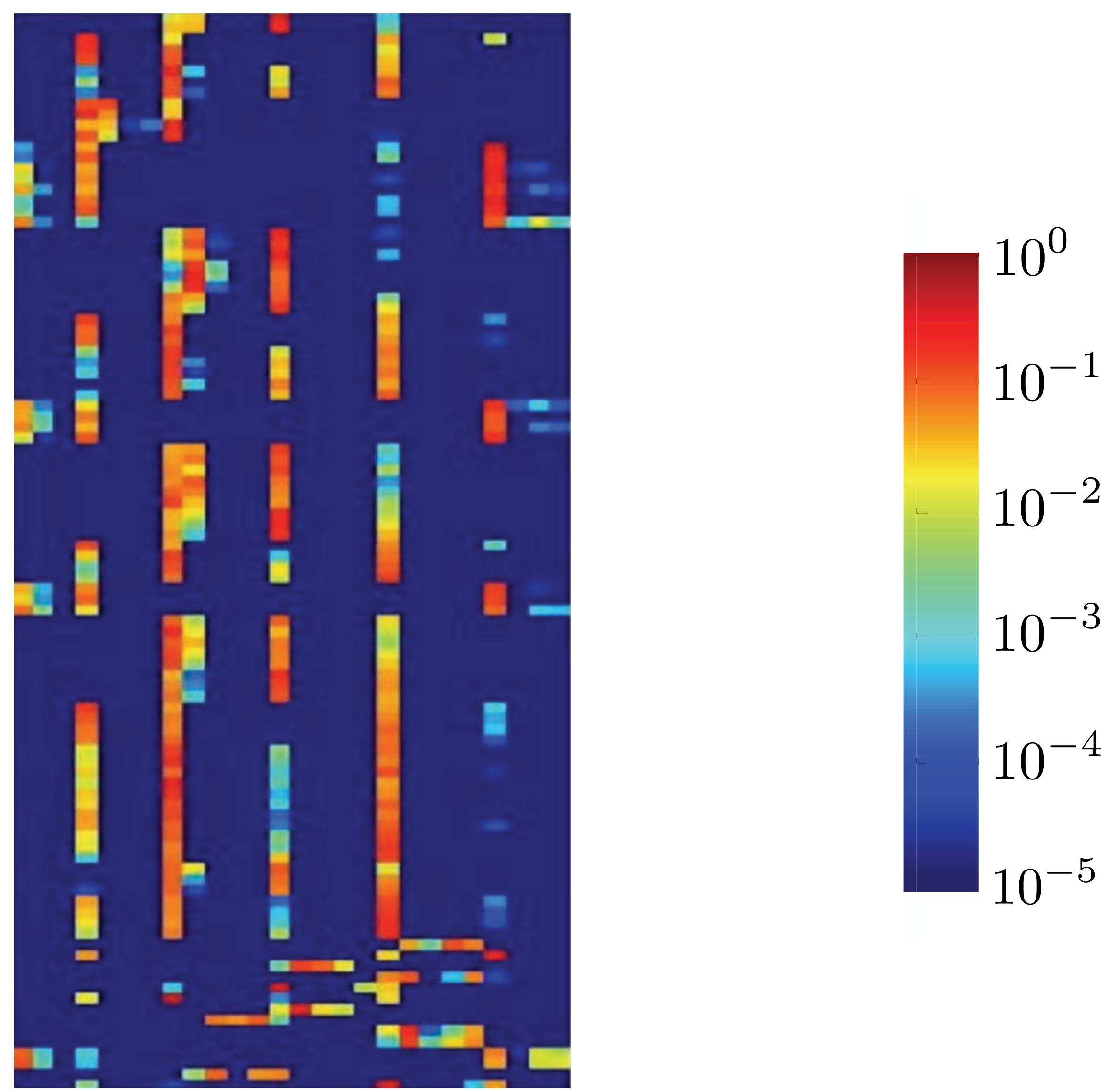
$\mathbf{W}$:



Weight Reduction

- Local influence: sparse matrix

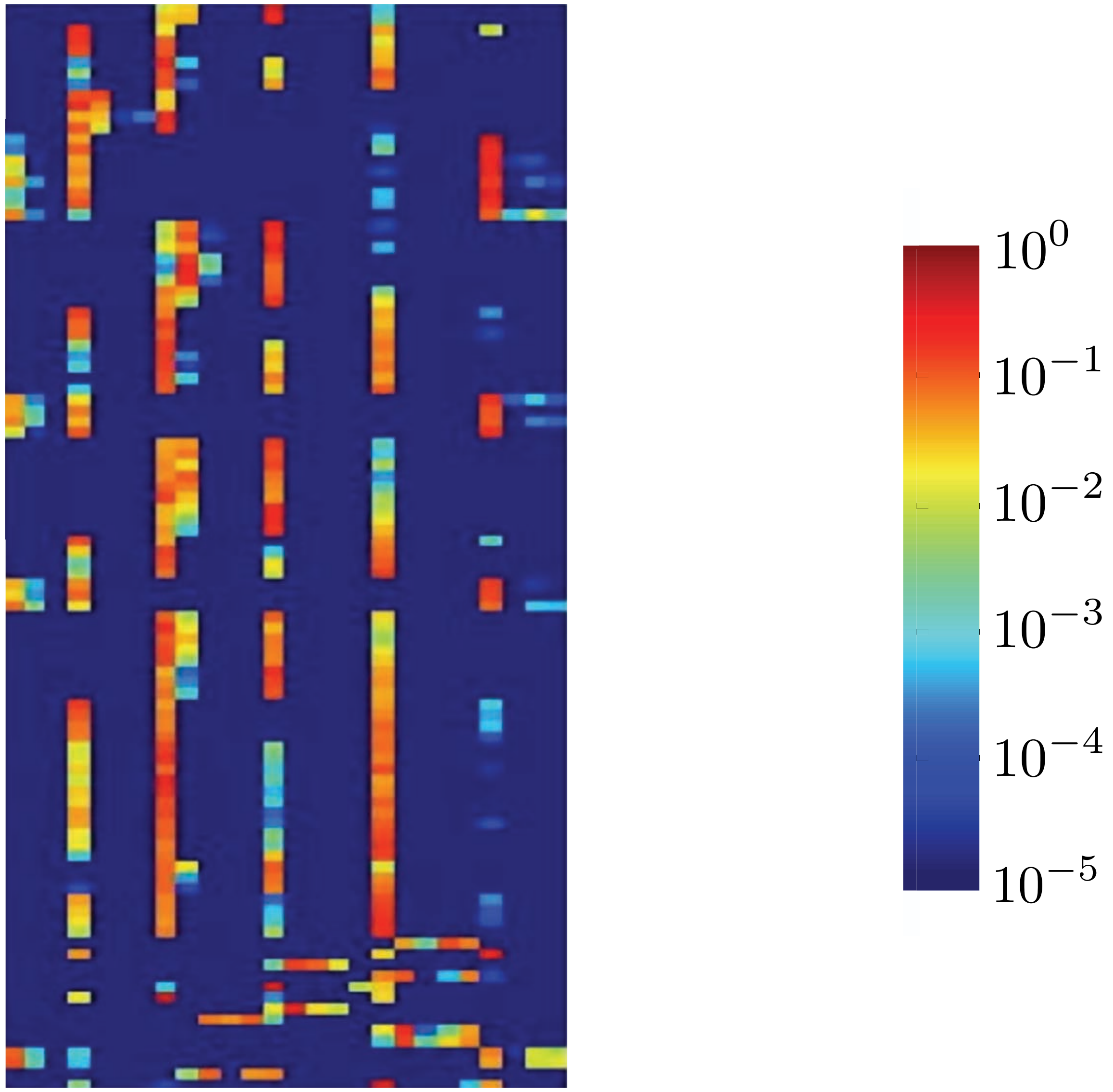
$\mathbf{W}$:



Weight Reduction

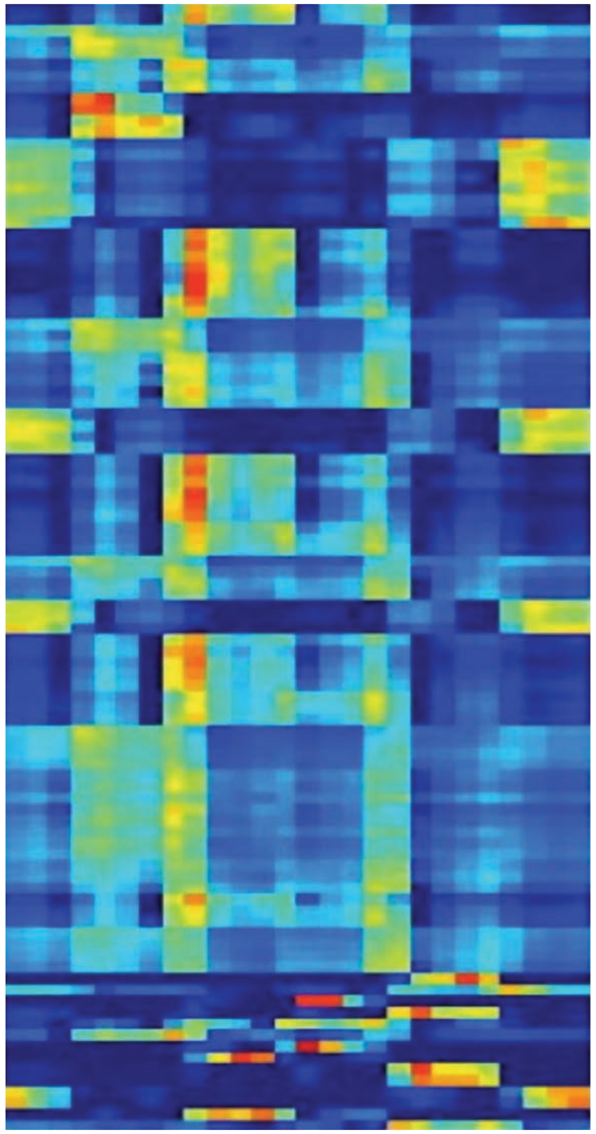
- Local influence: sparse matrix
  - lower memory footprint
  - faster multiplication

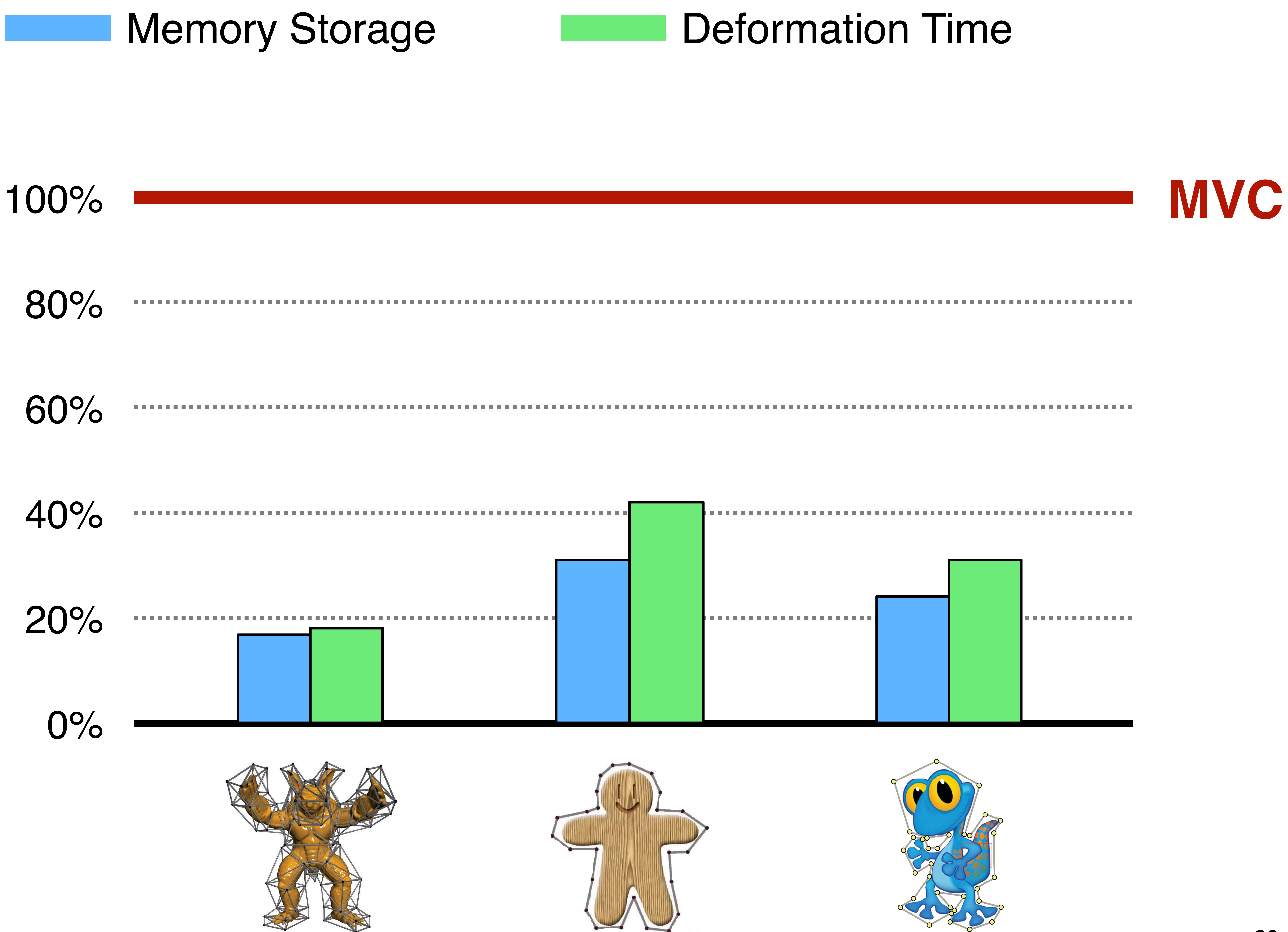
$\mathbf{W}$:



Weight Reduction

- Store LBC values





Memory Storage
Deformation Time
100%
80%
60%
40%
20%
0%
MVC

Limitation

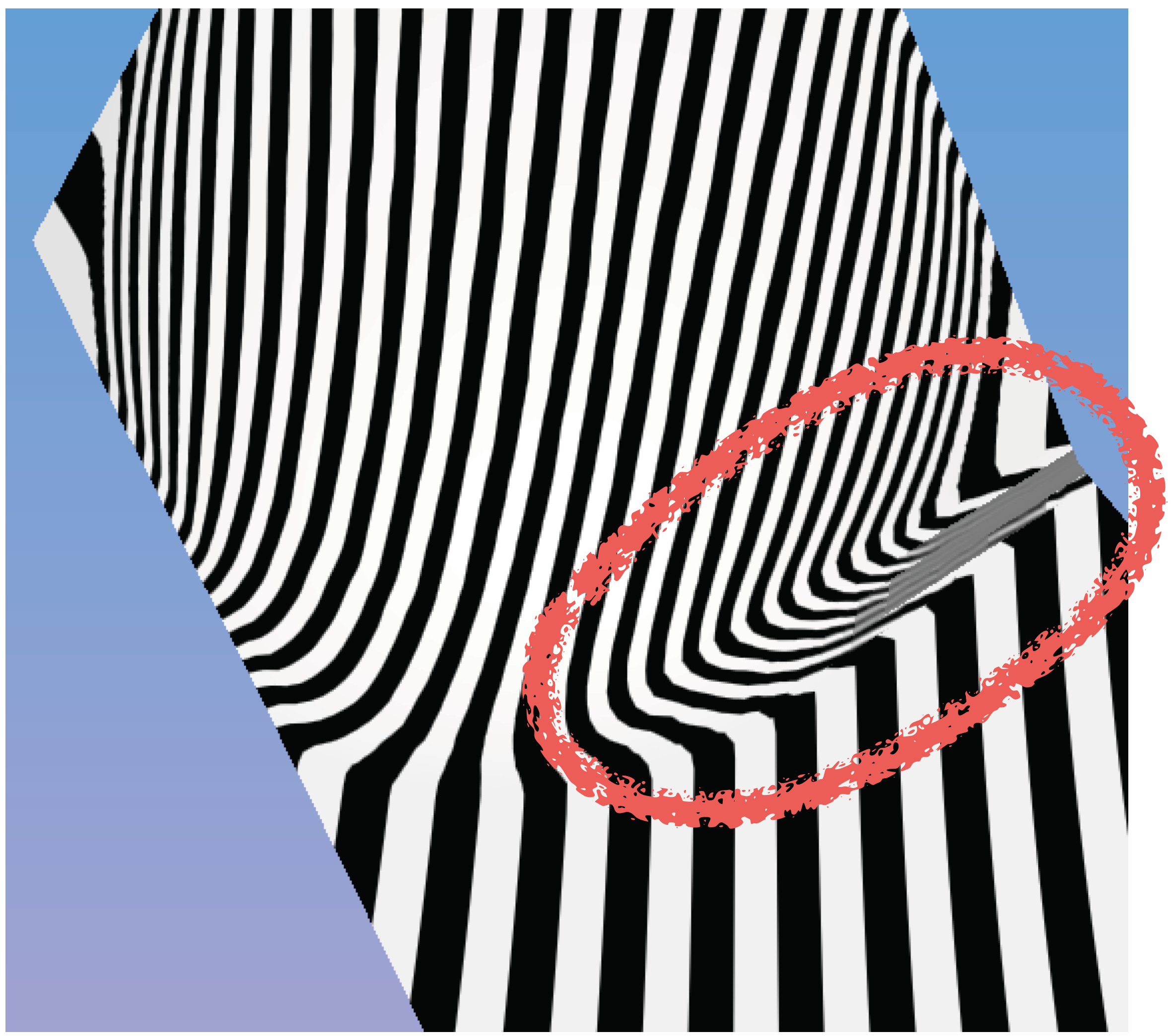
- Less smoothness: C^1 almost everywhere

LBC

BBW

Limitation

- Less smoothness: C^1 almost everywhere



LBC

Conclusion

- Local barycentric coordinates by convex optimization
- Total variation induces locality via superlevel set perimeters

Future Work

- Higher order continuity
- Fundamental question: how local can smooth barycentric coordinates become?

Acknowledgements

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Thank You!

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