# Surface Reconstruction

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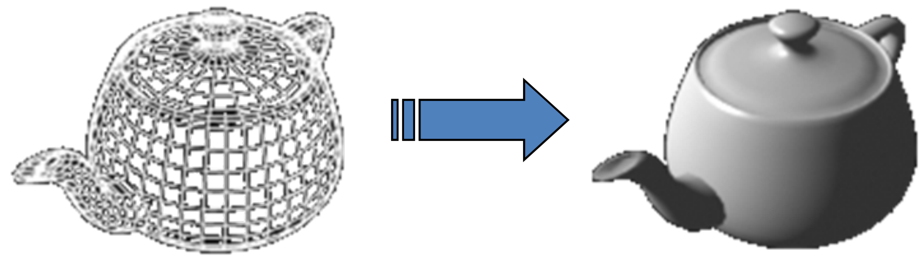
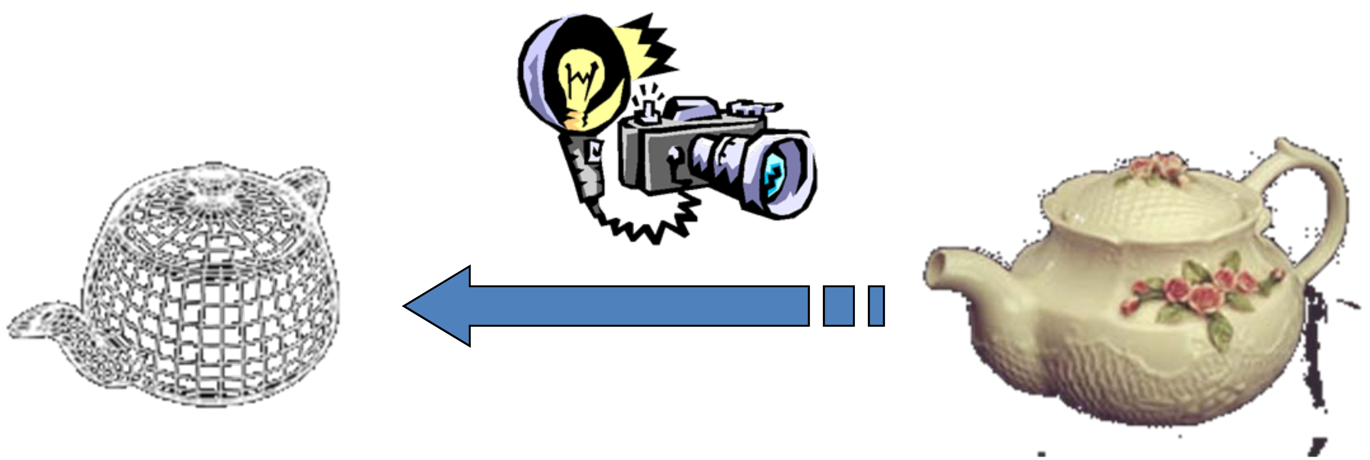
# Introduction

- Rendering:

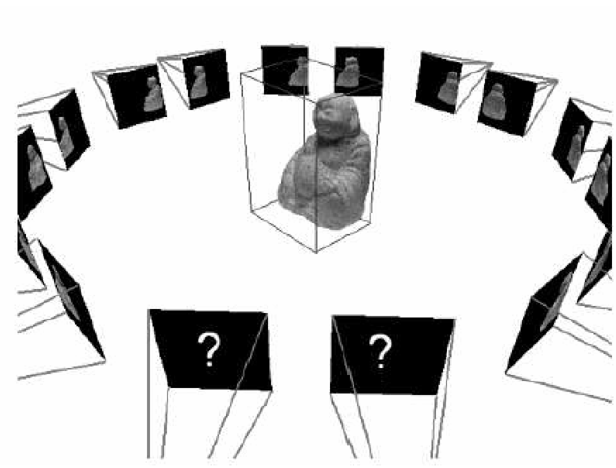


- Reconstruction:



# Shape from "X"

- Shape from Structured Light
- Shape from Illumination
- Shape from Silhouette
- Shape from Stereo
- Shape from Data

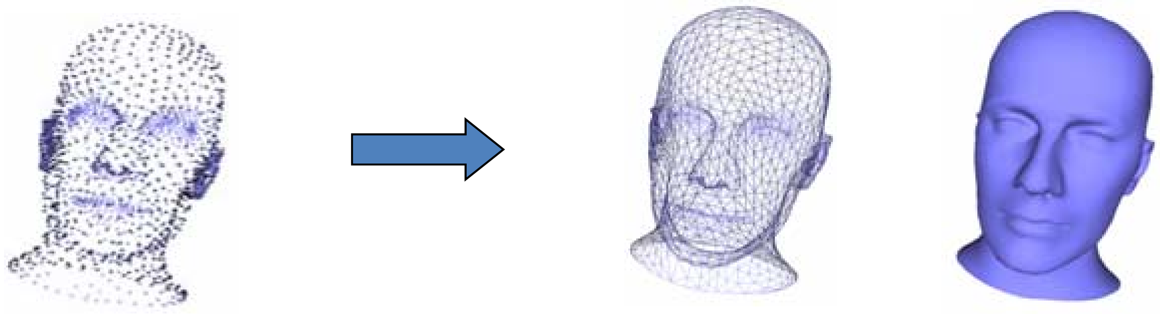


# Applications

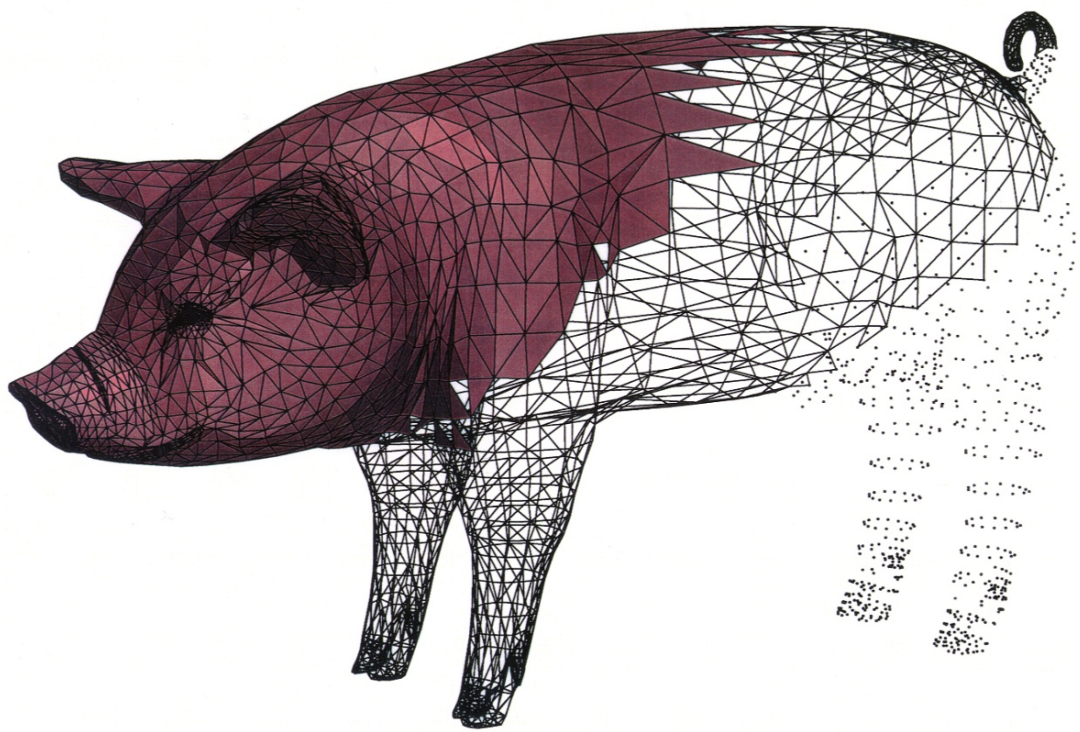
- Reverse engineering
  - Industrial design
- Augmented reality
- Medical Imaging
- Human computer interaction
  - Realistic virtual environments
- Animation
- …

# Problem

- Input
  - A set of points in 3D that sampled from a model surface
- Output
  - A 2D manifold mesh surface that closely approximates the surface of the original model

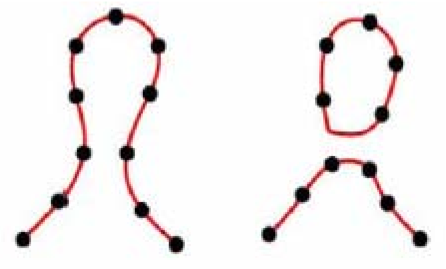


# Problem



# Desirable Properties

- No restriction on topological type

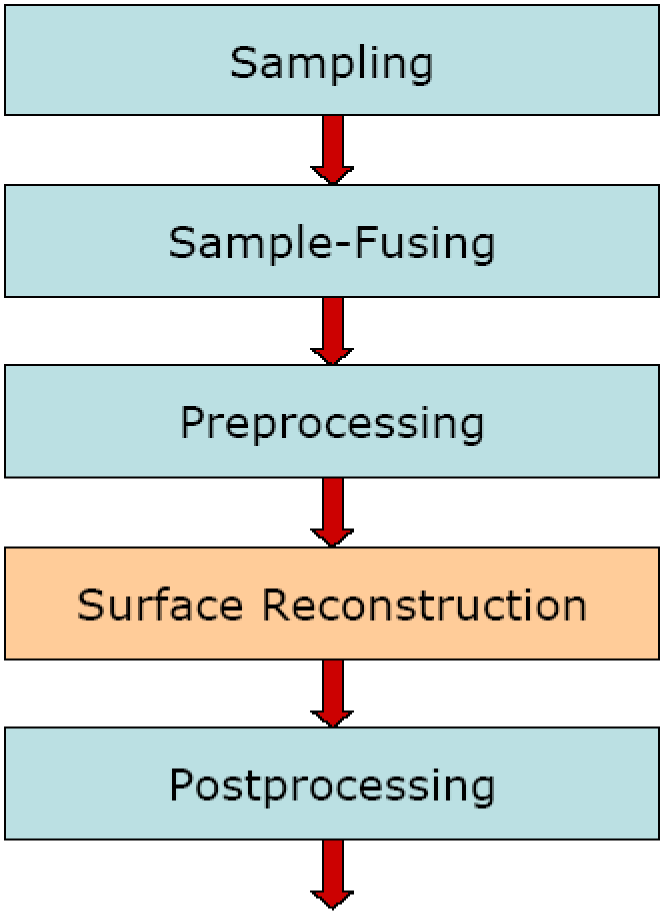


- Representation of range uncertainty
- Utilization of all range data
- Incremental and order independent updating
- Time and space efficiency
- Robustness
- Ability to fill holes in the reconstruction

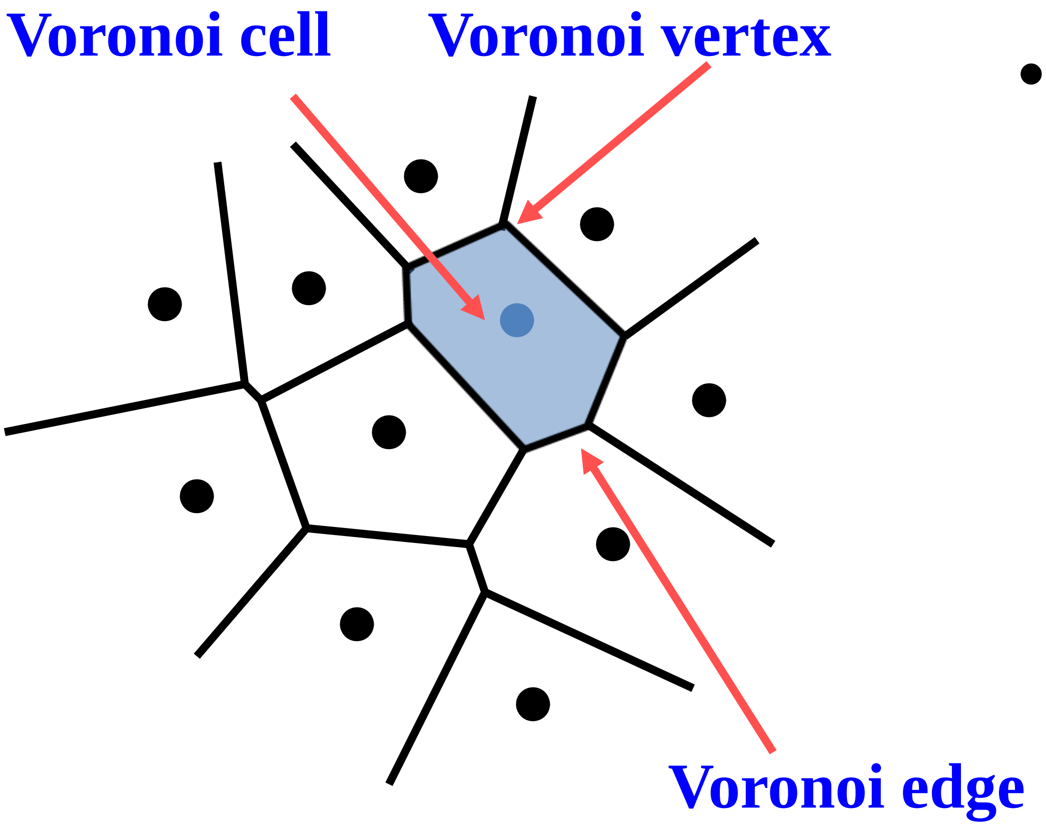
# Solutions

- Methods that construct triangle meshes directly — *Explicit methods*
  - Voronoi diagram and Delaunay triangulation
  - Zippering in 3D
- Methods that construct volumetric implicit functions — *Implicit methods*
  - Signed distances
  - Radial basis function reconstruction
  - Poisson reconstruction

# General Pipeline

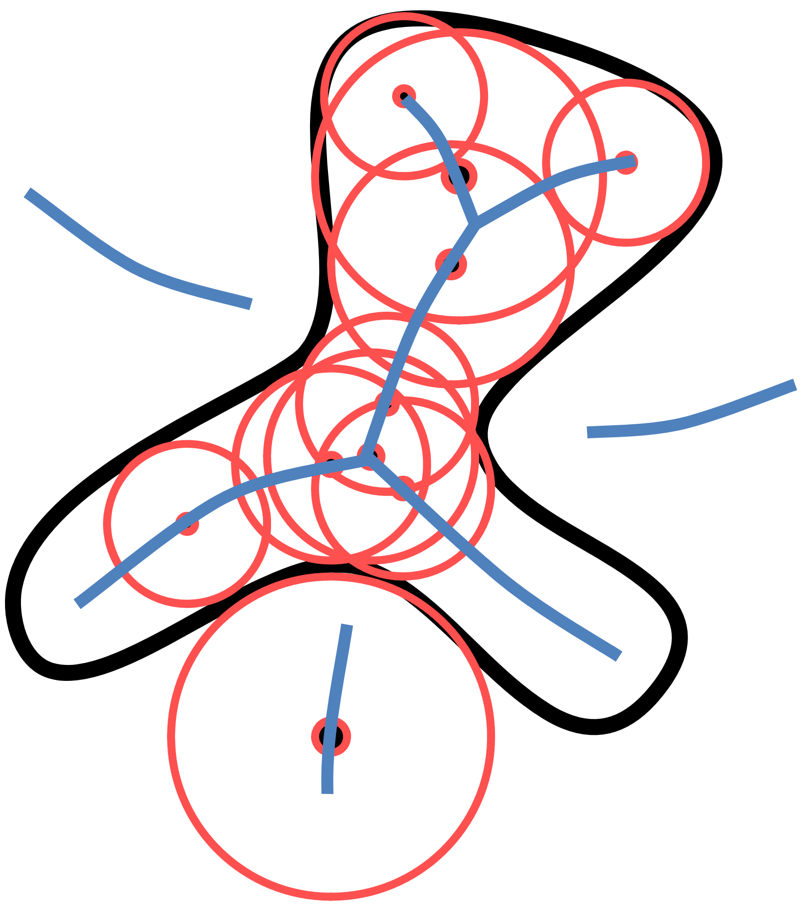


# Voronoi Diagram



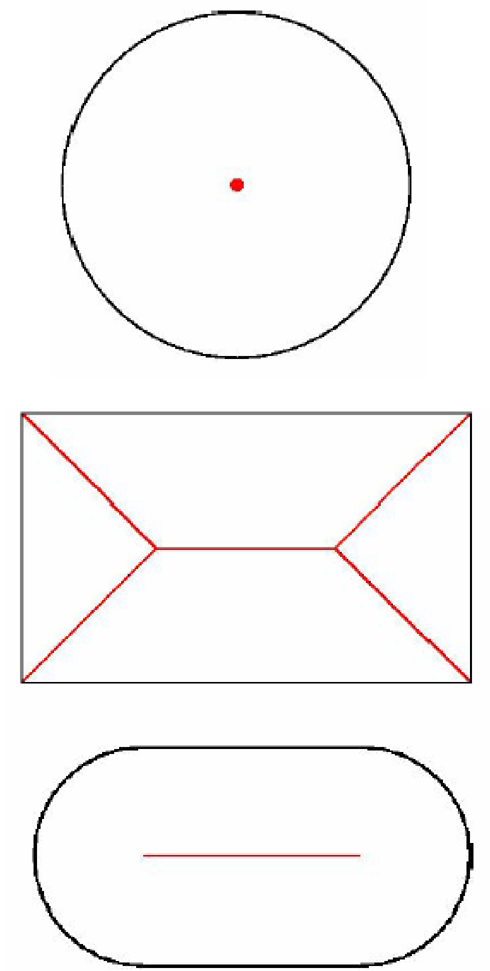

- Voronoi Cell of ***x***
  - The set of points that are closer to ***x*** than to any other sample point

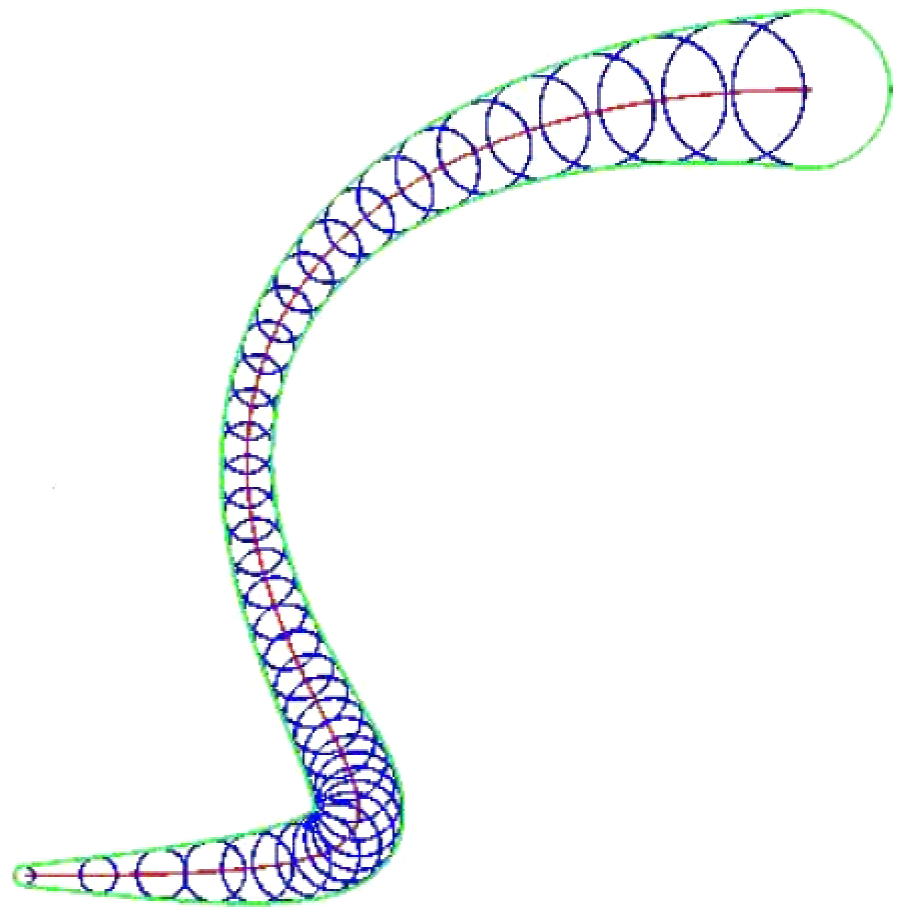
# Medial Axis



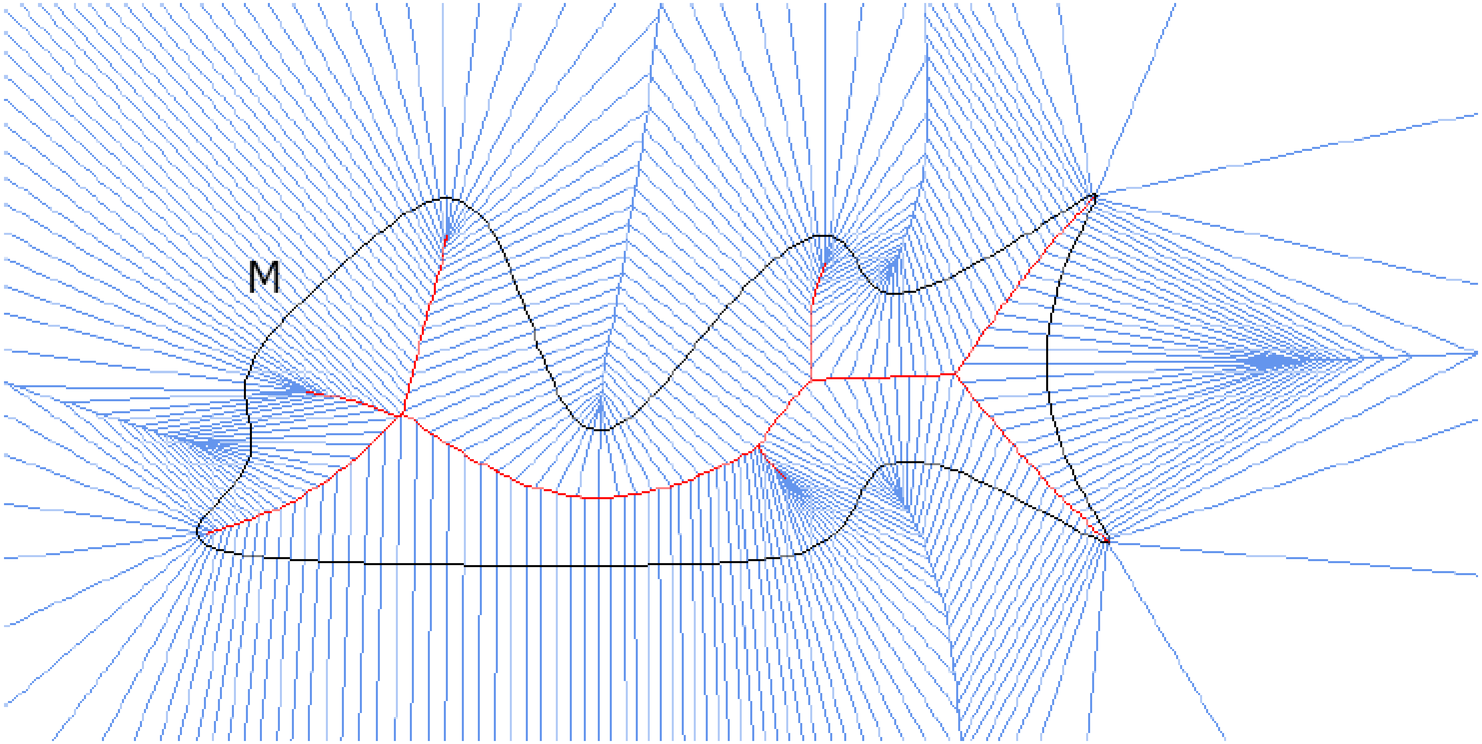
- Medial Axis:
  - Find all circles that tangentially touch the curve in at least 2 points
  - Medial axis = centers of all those circles

# Simple Examples



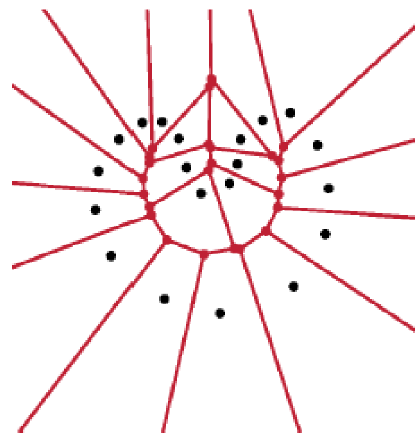


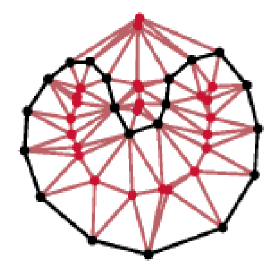
# Medial Axis vs. Voronoi Diagram



# Definitions (1)

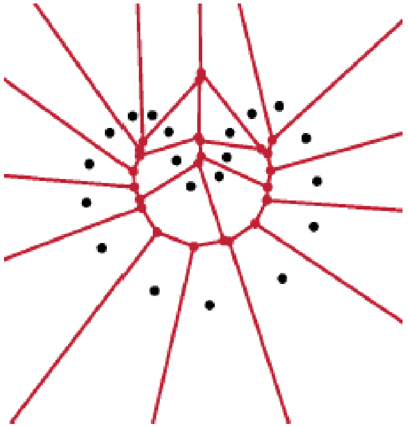
- Voronoi cell
  - A cell where all points in the cell are closer to a given sample point than any other point
- Voronoi diagram
  - A space partitioned into Voronoi cells
- Voronoi vertex
  - A point equidistant to d+1 sample points in $R^d$

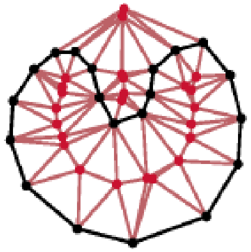


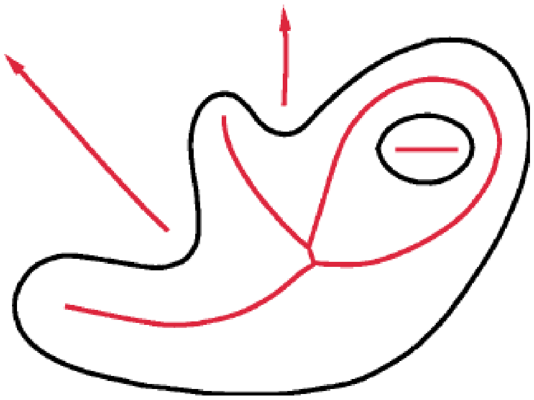


# Definitions (2)

- Delaunay triangulation
  - Dual of Voronoi diagram
  - Each triangle's circumcircle contains no other vertices

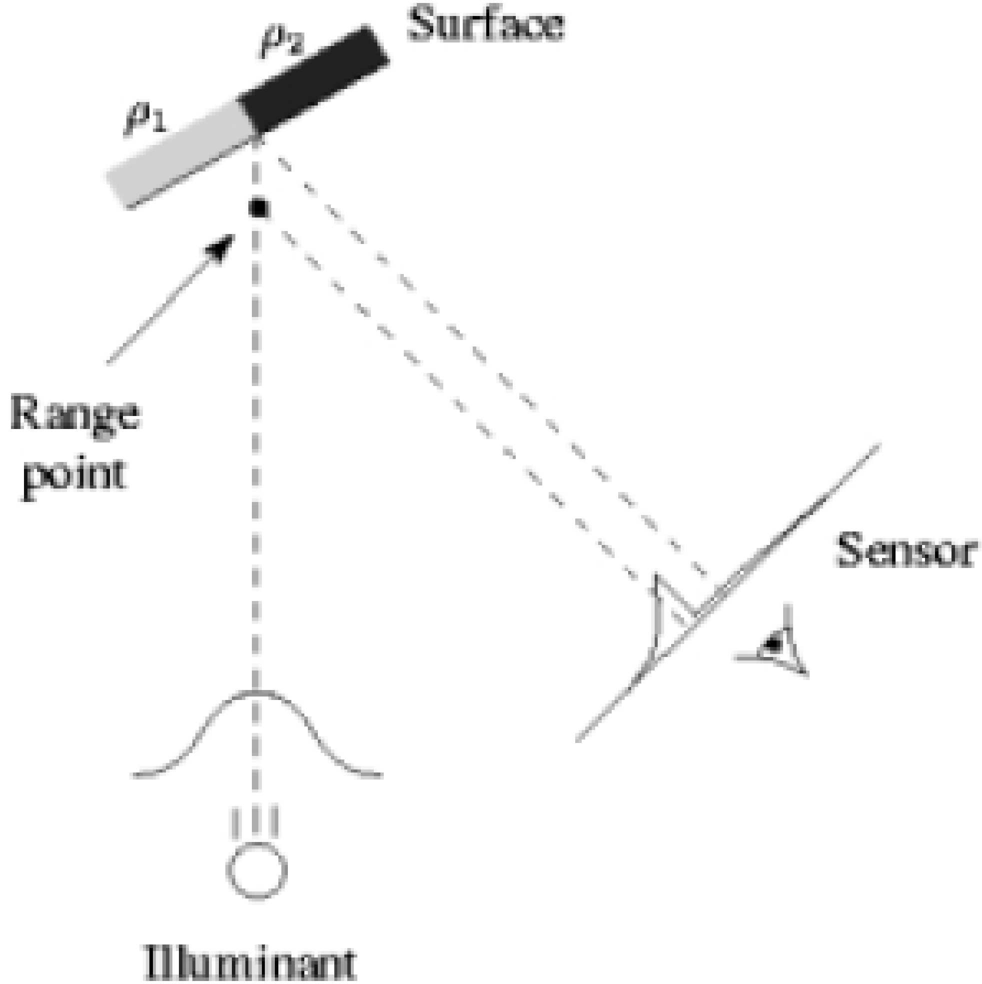






- Medial axis
  - Set of points with more than one closest point

# Range Data

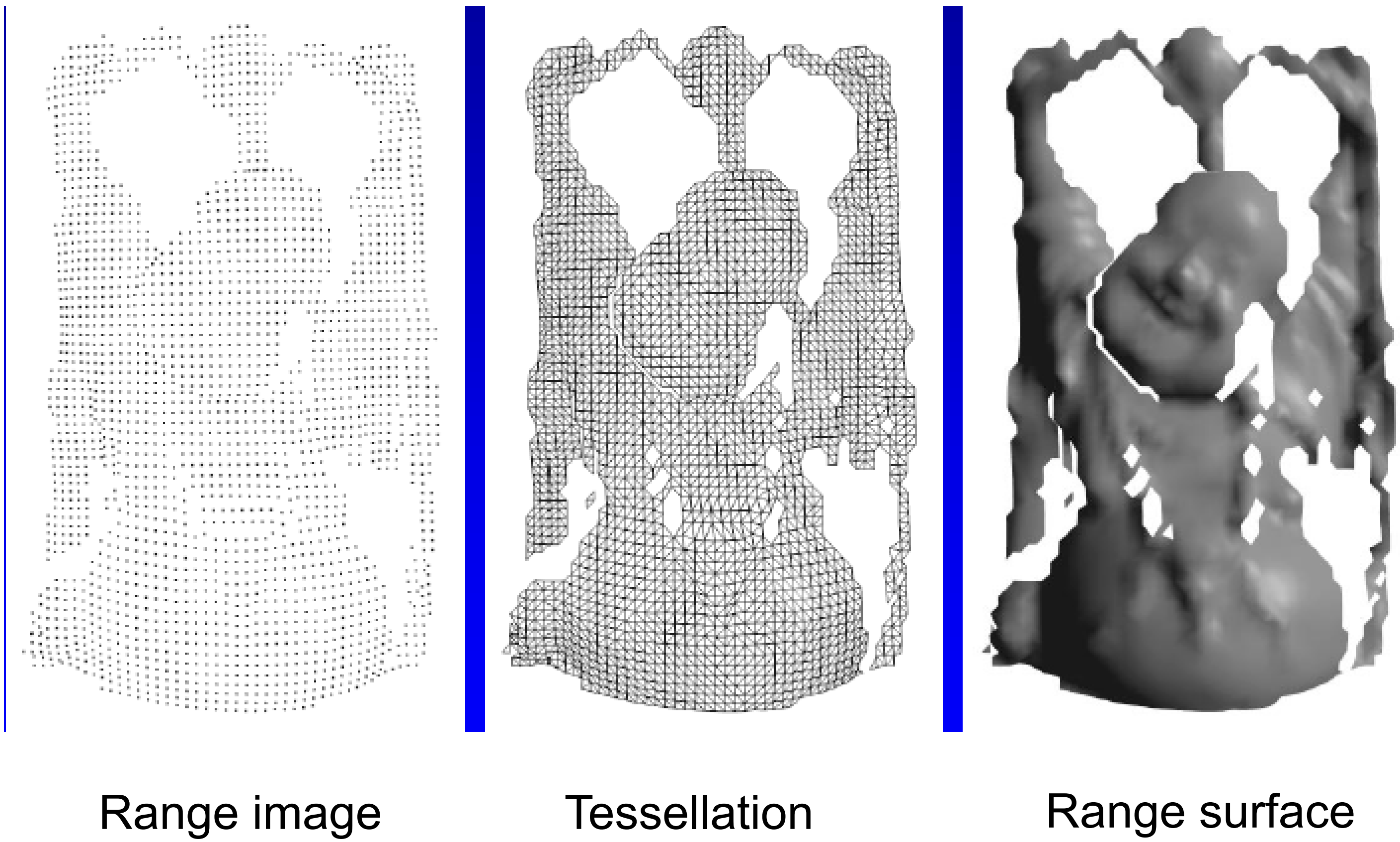


# Optical Range Acquisition

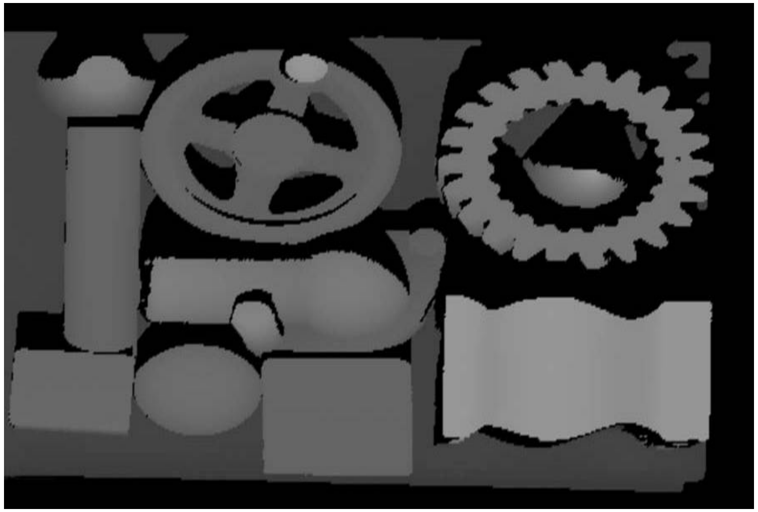
- Strengths
  - Non-contact
  - Safe
  - Inexpensive (?)
  - Fast
- Limitations
  - Can only acquire visible portions of the surface
  - Sensitivity to surface properties
    - transparency, shininess, rapid color variations, darkness
  - Confused by interreflections

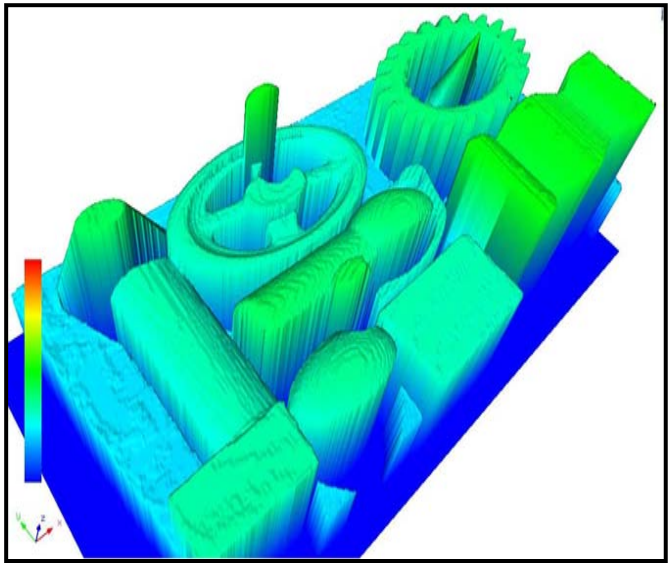
# Range Images

- Construct a range surface



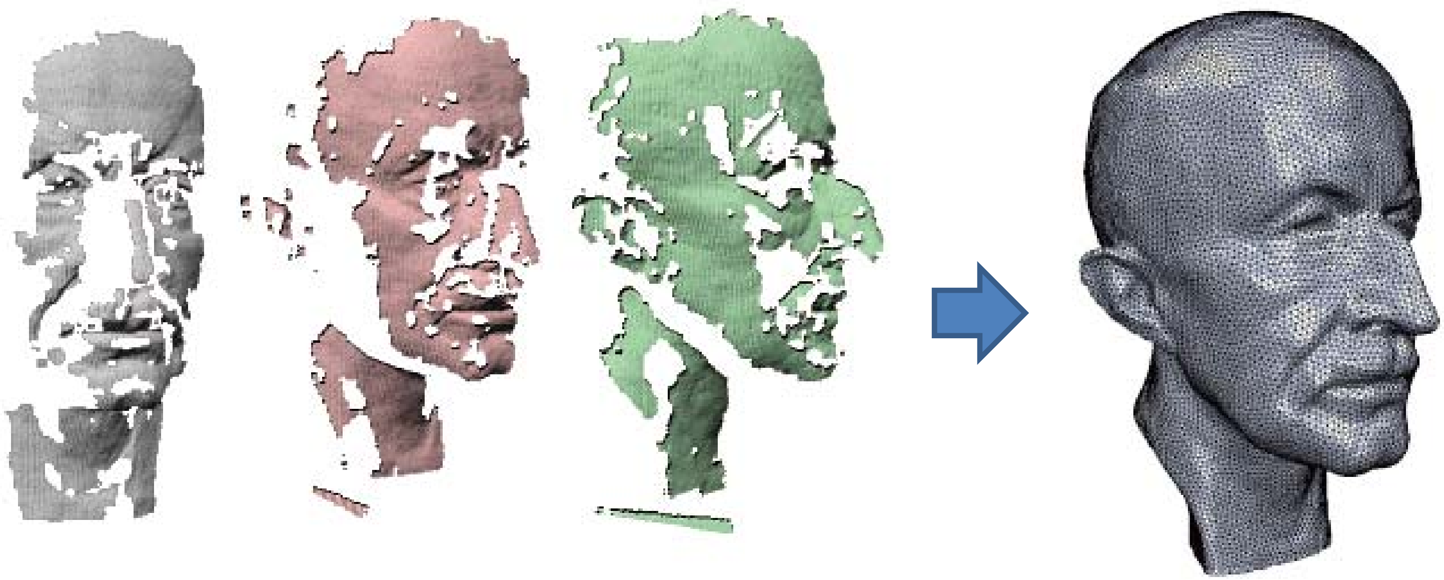
# Range Images



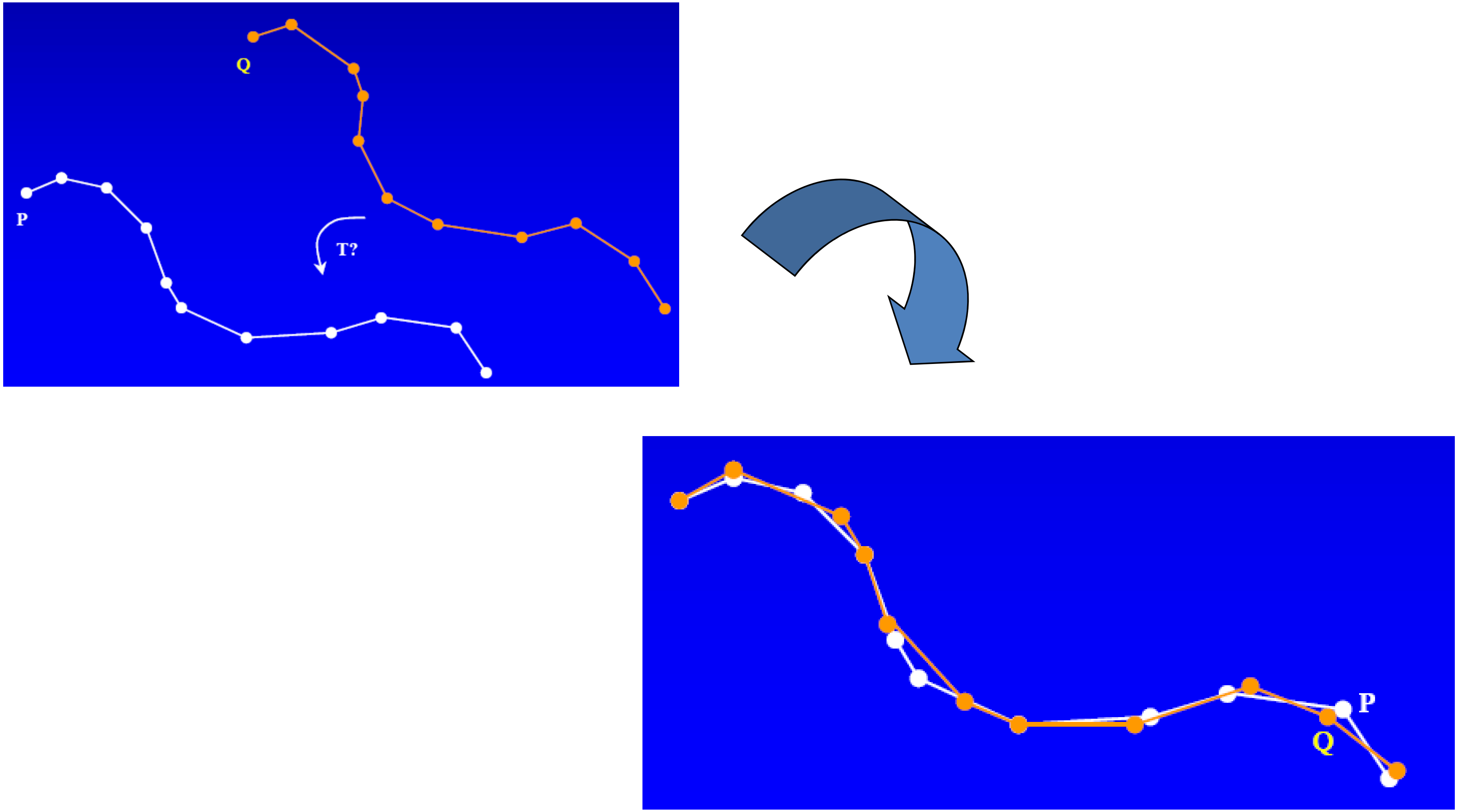


# Registration

- Any surface reconstruction algorithm should strive to use all of the detail in all the available range data.
- Accurate registration may require:
  - Calibrated scanner/object positioning
  - Software-based optimization
  - Both

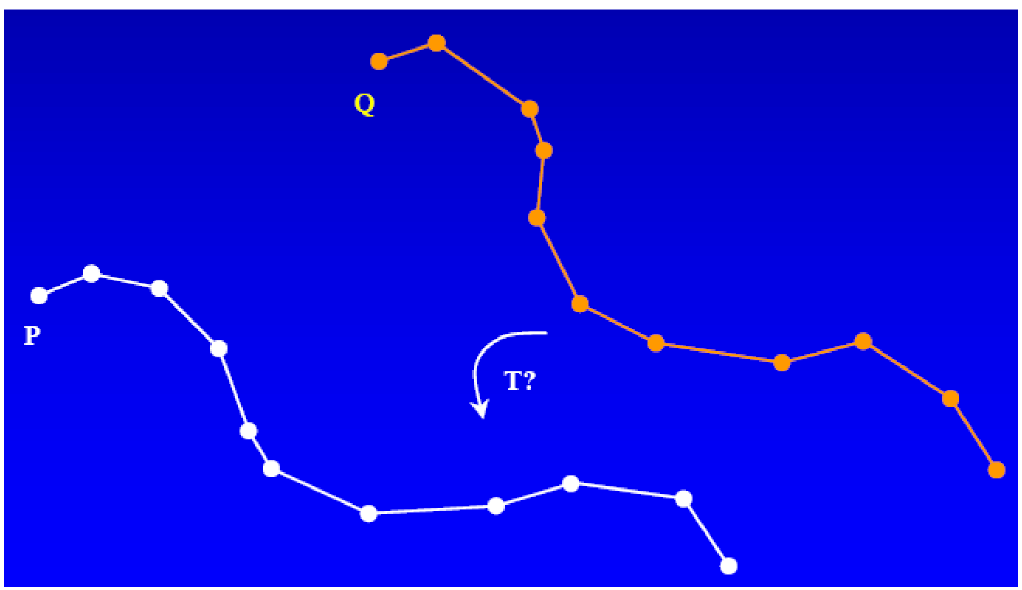


# Registration



# Registration as Optimization (1)

- Given two overlapping range scans, we wish to solve for the rigid transformation, T, that minimizes the distance between them.



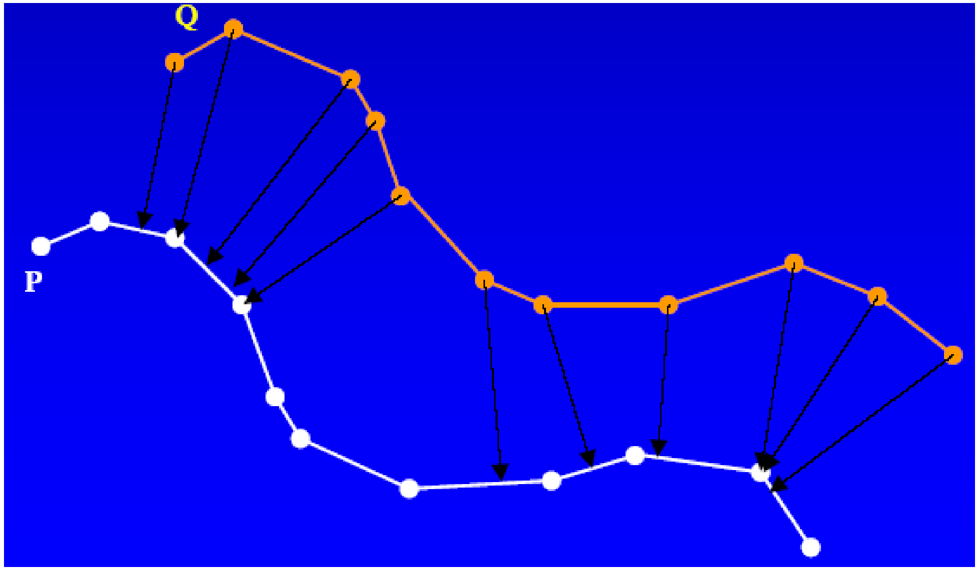

$$E = \sum_{i}^{N_P} \left\| Tq_i - p_i \right\|^2$$

# Registration as Optimization (2)

- If the correspondences are known a priori, then there is a closed form solution for T.
- This is not the case.

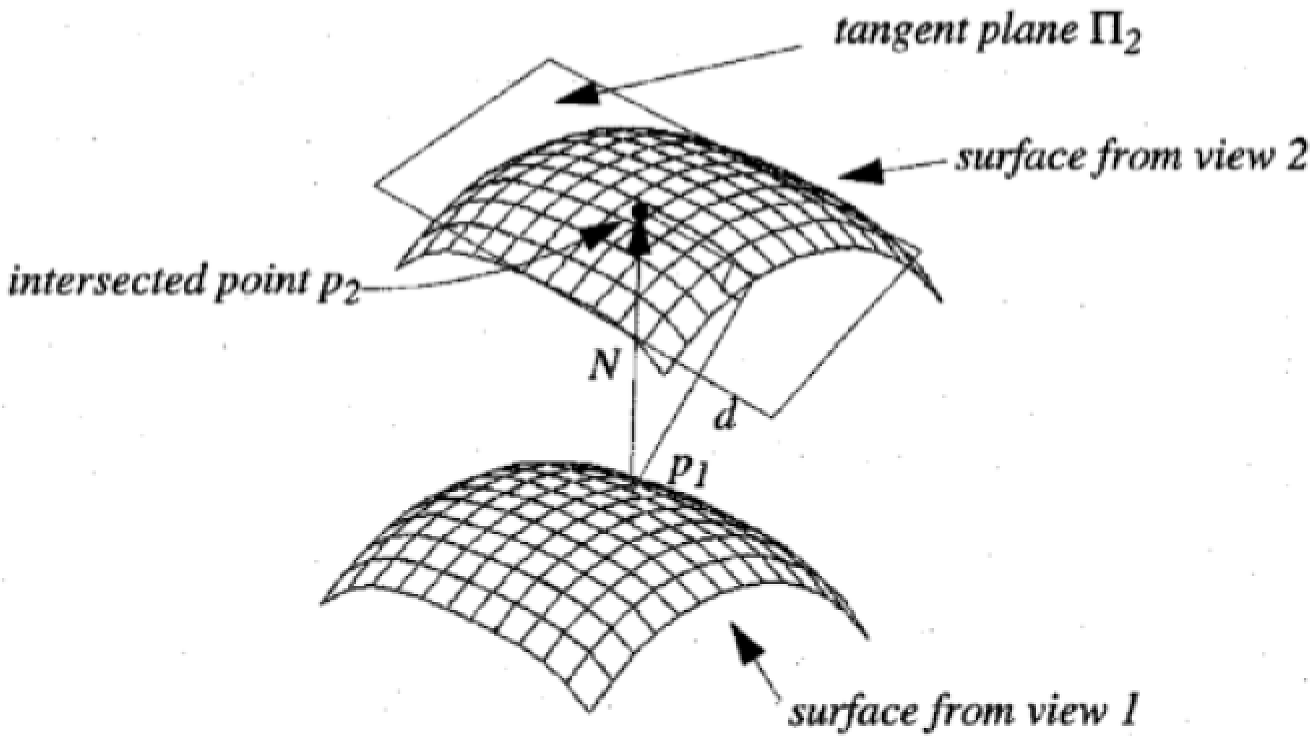
# Registration as Optimization (3)

- Iterative close-point (ICP) solutions
  - Identify nearest points
  - Compute the optimal T
  - Repeat until E is small



# Registration as Optimization (4)

- In 3D:



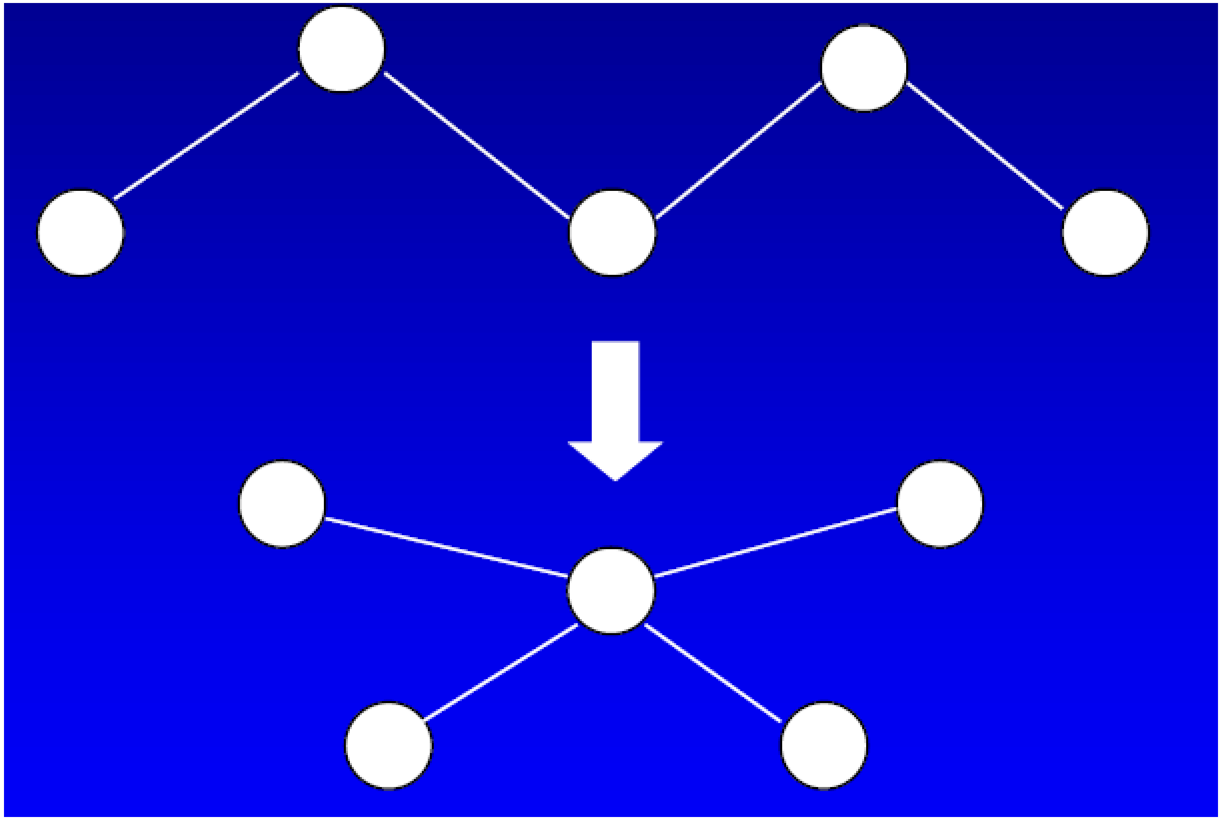
# Registration as Optimization (5)

- Sequential registration and integration is not optimal.

- Multiple range scans could be simultaneously registered. This provides greater information to assist in generating a more accurate registration.

# Global Registration

- Network of range views



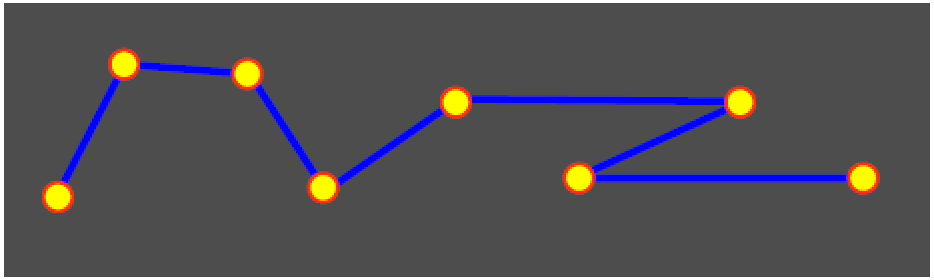
# Registration Result

- Registration Error < Measurement Uncertainty

# Curve from Points

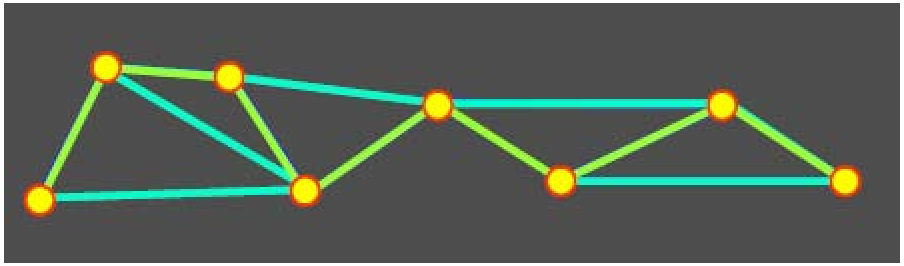
# Curve from Points - Connect the Dots (1)

- Given unordered set of points P
  - connect them by linear segments



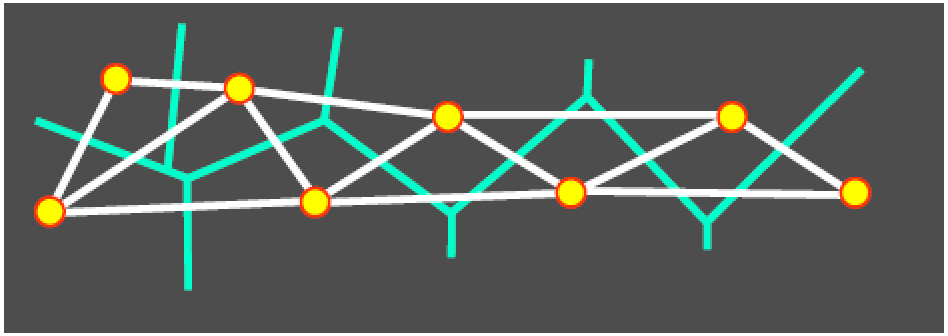
# Curve from Points
# - Connect the Dots (2)

- Can be ambiguous
- Harder when topology not known



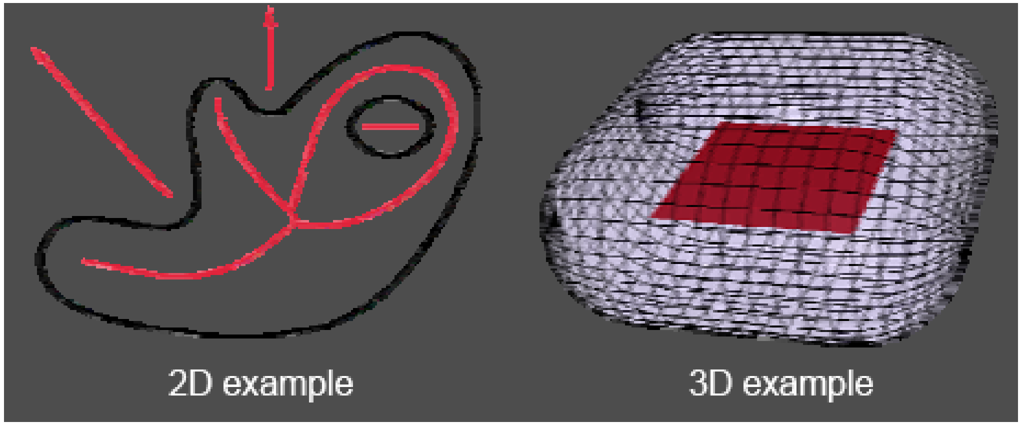
# Curve from Points
# - Connect the Dots (3)

- Use Voronoi Diagram
- Construct Delaunay triangulation
- Which edges to choose?



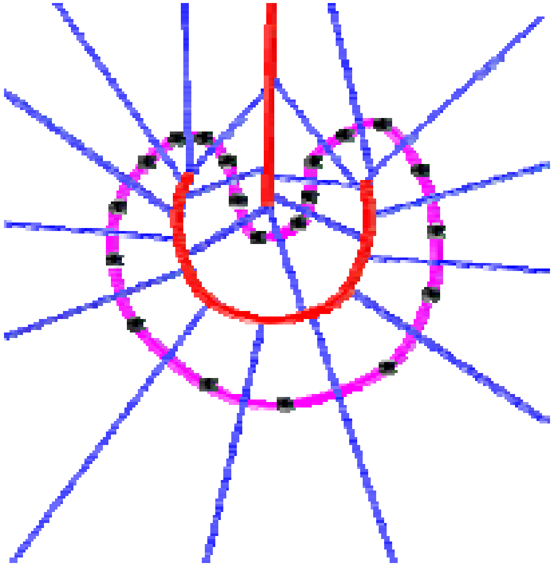
# Medial Axis

- Medial axis of ($d$-1)-dimensional surface in $R^d$ - set of points with more than one closest point on the surface
- Alternative definition: locus of centers of maximal inscribed spheres



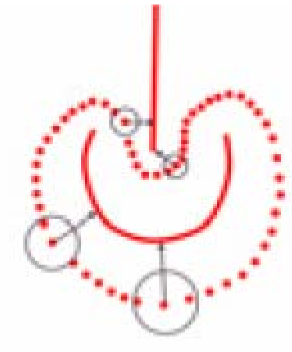
# Medial Axis and VD

- Voronoi diagram of set of points on curve approximates Medial Axis
  - if points sampled densely enough



# Sampling Criterion (1)

- Good sample - sampling density (at least) inversely proportional to distance from medial axis
- r-sample : distance from any point on surface to nearest sample point ≤ r * distance from point to medial axis
- In general, $r \in (0,1]$
- r=0.5 good enough



r = 0.5

# Sampling Criterion (2)

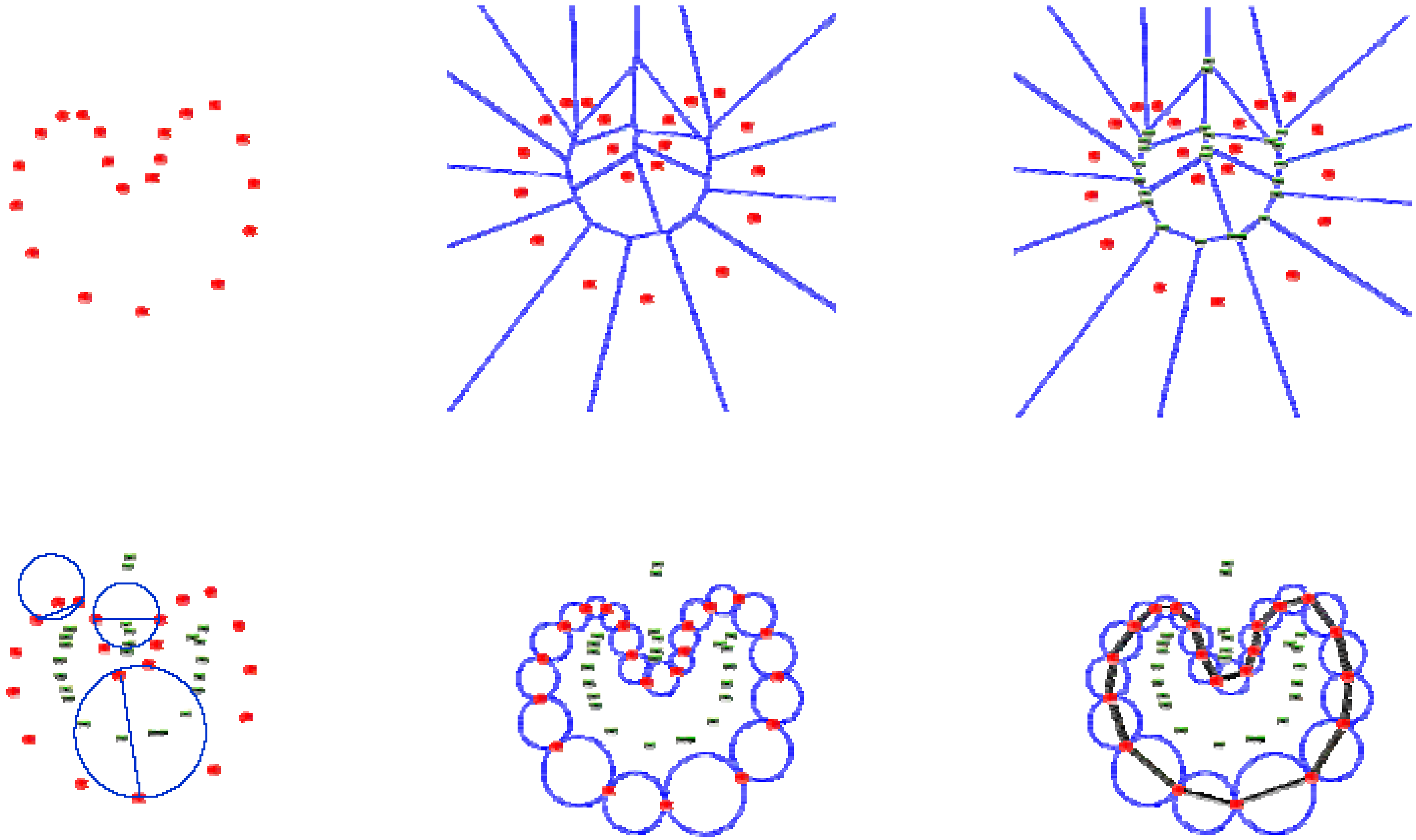
- Inherently takes into account
  - curvature of the surface
  - proximity of other parts of the surface

# 2D Crust Algorithm

# Idea

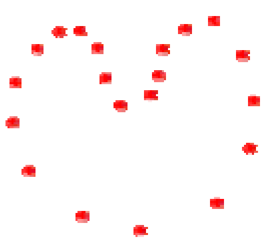
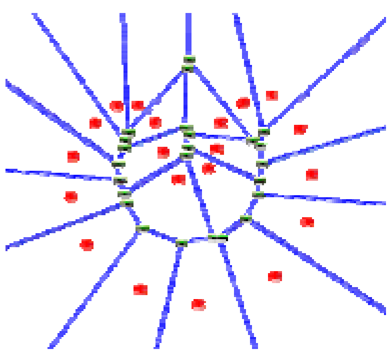
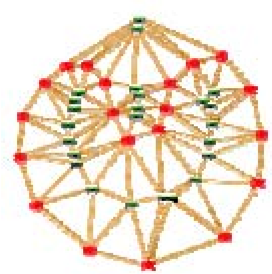
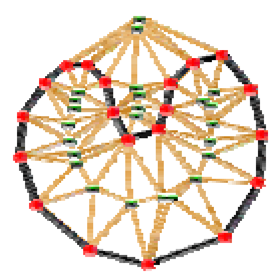
- Adopt Delaunay edges which are “far” from MA
- To represent MA use Voronoi vertices
- Edge e in crust <=> circumcircle of e contains no other sample points or Voronoi vertices of S

# Algorithm Process



# Crust Algorithm

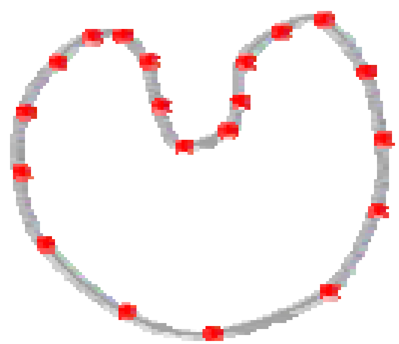
- Compute Voronoi diagram of S
  - – let V be set of Voronoi vertices
- Compute Delaunay triangulation of SUV
- Return all Delaunay edges between points of S

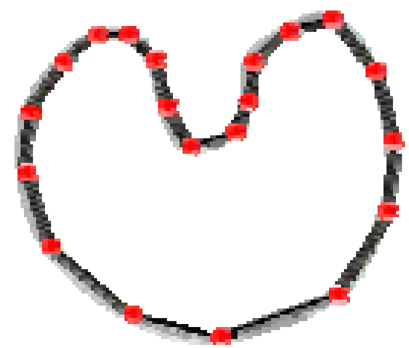
# Theory

- **Theorem**: The crust of an *r*-sample from a smooth curve *F*, for $r \leq 0.25$ connects only adjacent samples of *F*

Smooth Curve *F*



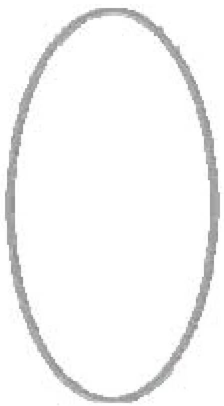
*r*-sample (r ≤ 0.25)

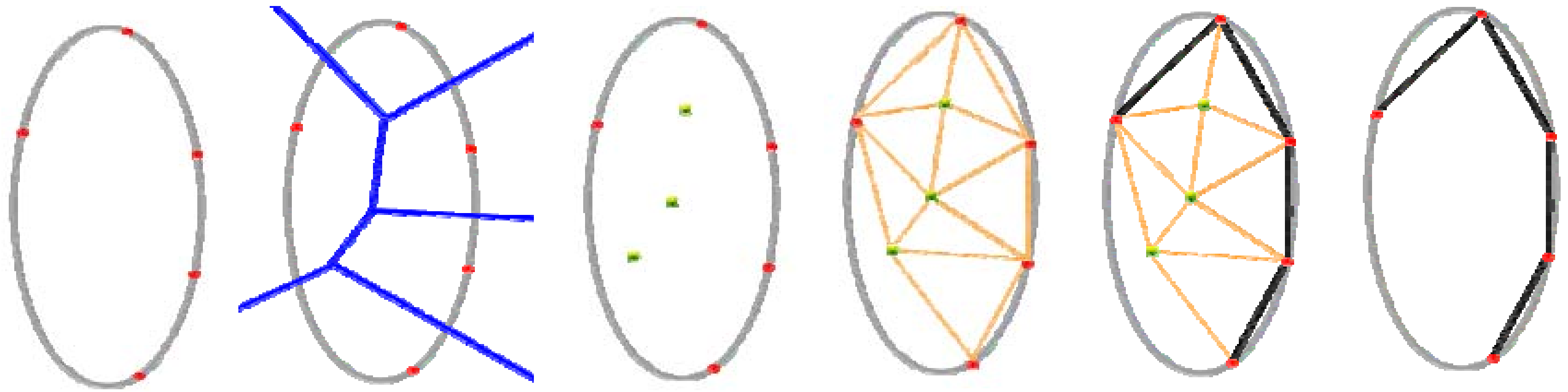


The crust

# Theory

- The algorithm may fail when r is too large





# 3D Crust Algorithm

A New Voronoi-Based Surface Reconstruction Algorithm

N. Amenta, M. Bern, and M. Kamvysselis

Siggraph 1998

# Idea

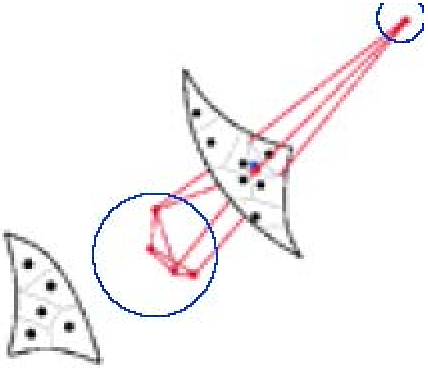
- Extend 2D approach
- Voronoi cells are polyhedra
- Voronoi vertex is equidistant from 4 sample points
- BUT in 3D not all Voronoi vertices are near medial axis (regardless of sampling density)

# Concepts

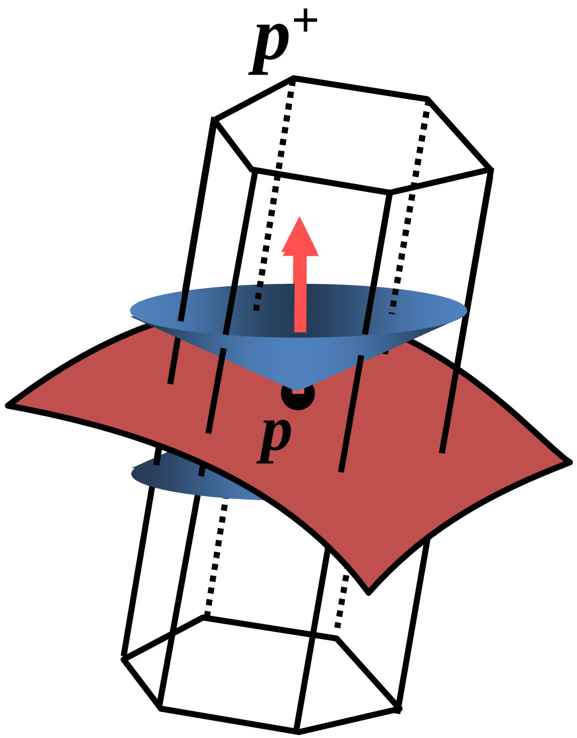
- Poles
  - Farthest Voronoi vertices for a sample point that are on opposite sides
- Crust
  - Shell created to represent the surface

# Problem

- But some vertices of the Voronoi cell are near medial axis
- Intuitively – cell is closed not just from the sides but also from both sides of the surface



# Observation

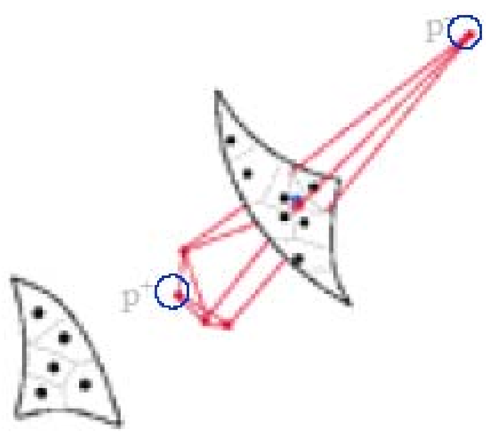


Voronoi cell of ***p***

- ***p+*** ≡ pole of ***p*** = point in the Voronoi cell farthest from ***p***
- ε < *0.1* →
  - the vector from ***p*** to ***p+*** is within π/8 of the true surface normal
  - The surface is nearly flat within the cell

# Solution

- Solution
  - use only two farthest vertices of Vs - one on each side of the surface
- Call vertices poles of s: $p^+(s)$, $p^-(s)$

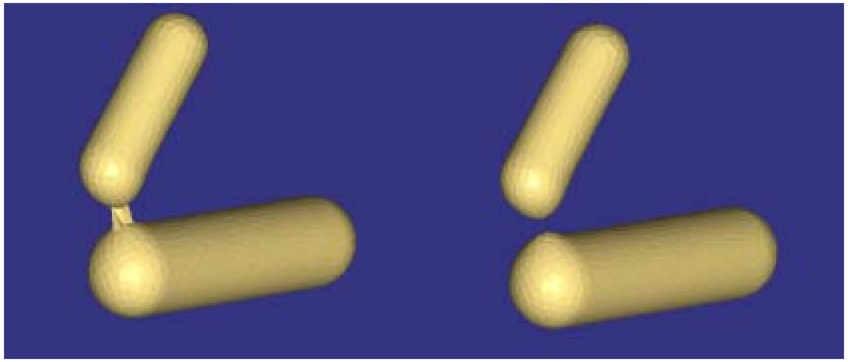


# 3D Algorithm

- Compute Voronoi diagram of S
- For each s $\in$ S, identify the poles $p^+(s)$ and $p^-(s)$
  - $p^+(s)$ is the vertex of Vs most distant from s
  - $p^-(s)$ is the vertex of Vs most distant from s in the opposite direction
- Let P be the set of all poles
- Compute Delaunay triangulation T of S U P
- Add to crust all triangles in T with vertices only in S

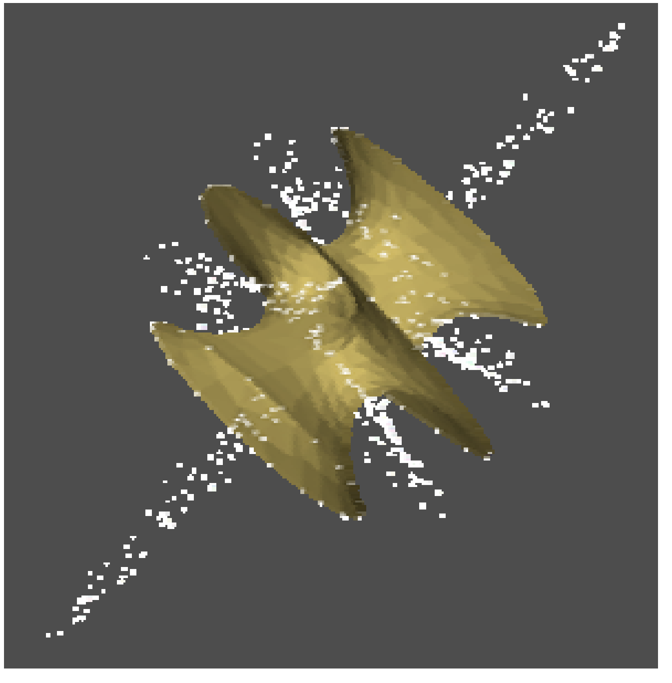
# Post-processing

- Delete triangles whose normals differ too much from the direction vectors from the triangle vertices to their poles
- Orient triangles consistently with its neighbors and remove sharp dihedral edges to create a manifold



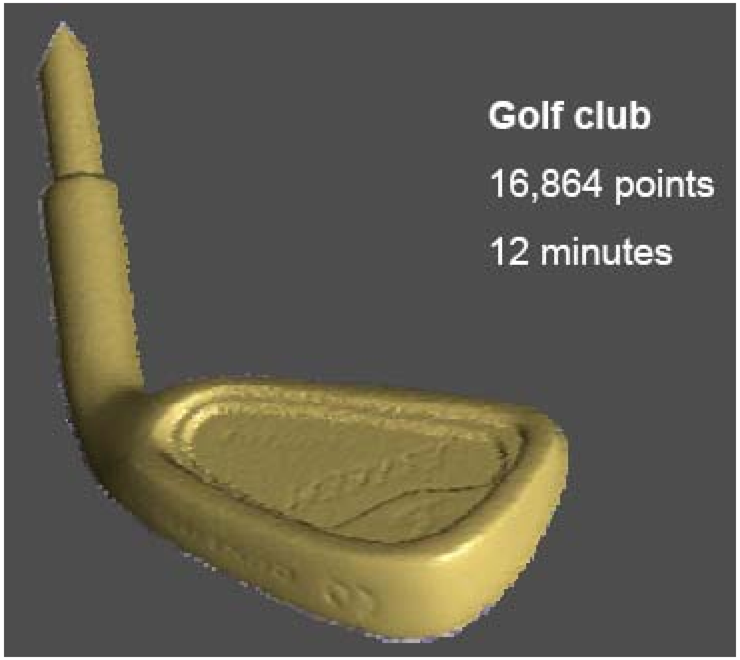
# Example

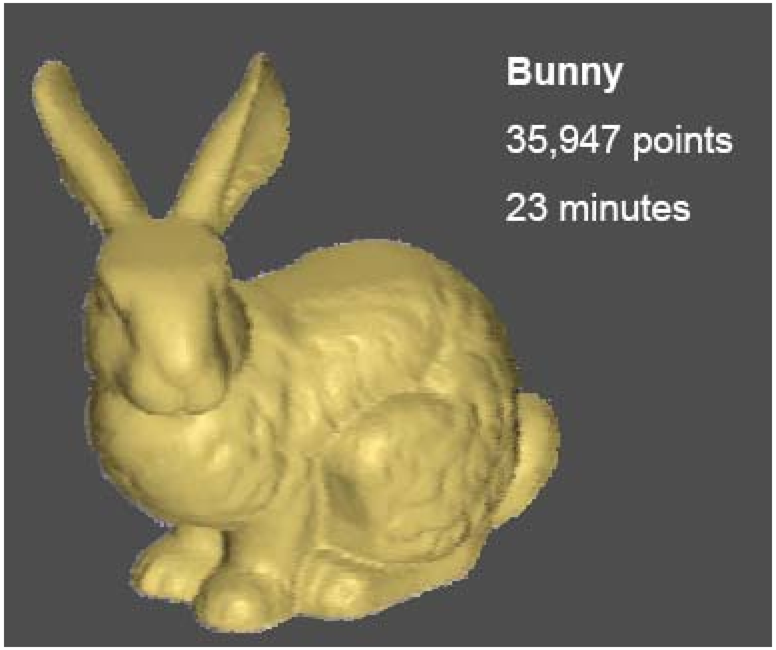
- Crust of set of points and poles used in its reconstruction



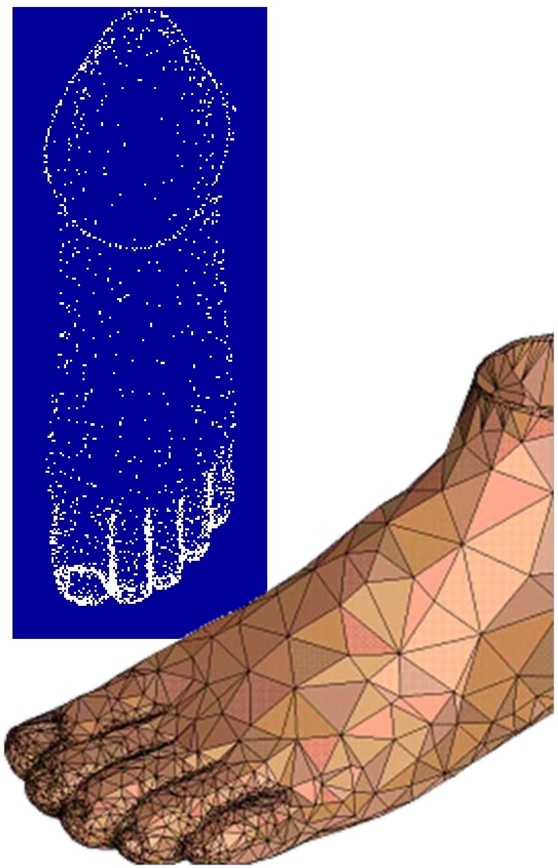
# Examp


Femur
939 points
2 minutes

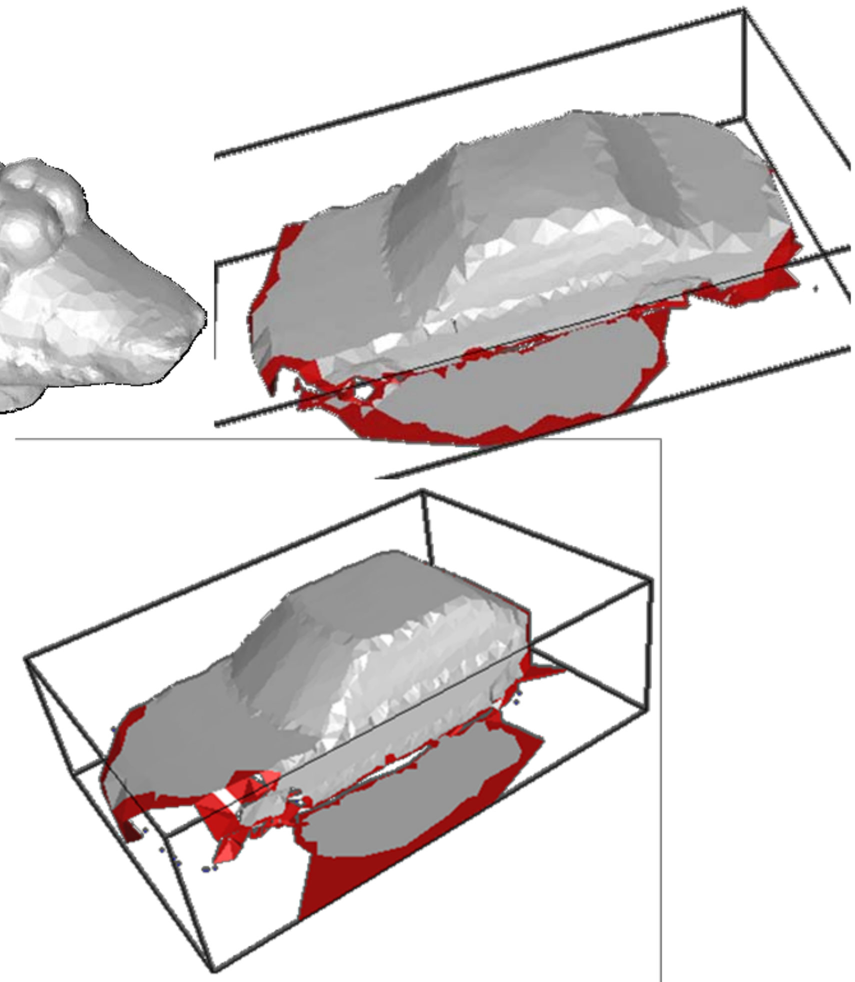

Golf club
16,864 points
12 minutes


Bunny
35,947 points
23 minutes

# Examples



Foot images from Prof. Dey



# Advantages

- No need for experimental parameters in basic algorithm
- Not sensitive to distribution of points

# Problems

- Sampling of points needs to be dense
  - Undersampling causes holes
- Problems at sharp corners
- Another way to choose poles gives better reconstruction
  - choosing the farthest and the second farthest Voronoi vertices, regardless of direction
- Correct, BUT slow

# Q&A

# Zippered Polygon Meshes From Range Images

Greg Turk and Mark Levoy

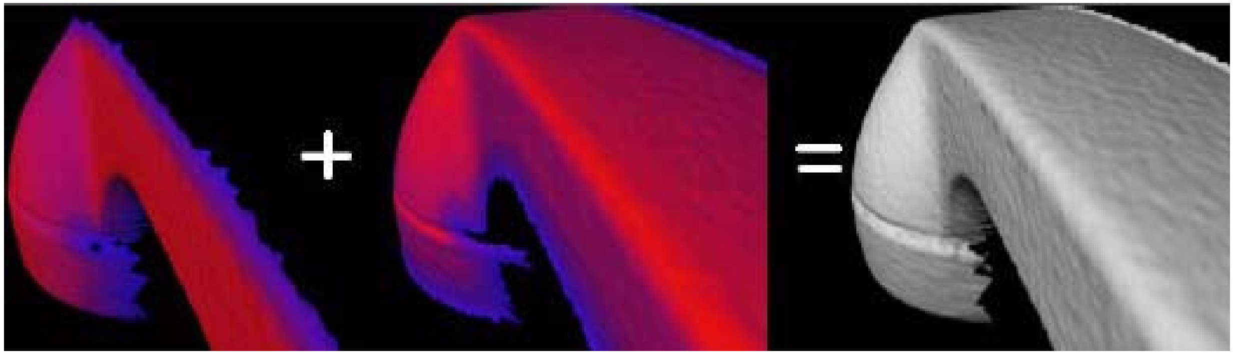
Siggraph'1994

# Idea

- Use range scanner properties for reconstruction
- Single scan from given direction produces regular lattice of points in X and Y with changing depth (Z)
- Take multiple scans to create complete model

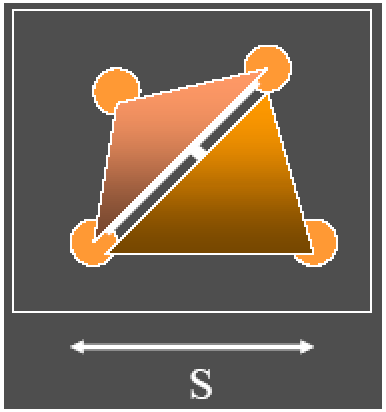
# Algorithm

- Generate separate mesh from each scan (range image)
  - Use X &Y adjacency info
- Combine
  - Register positions
  - Merge meshes



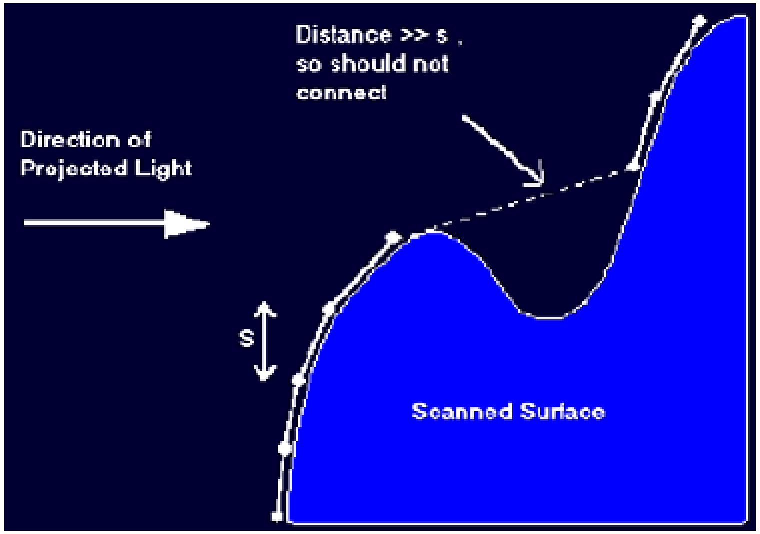
# Steps (1)

- Find quadruples of lattice points
- Form triangles
  - Find shortest diagonal
  - Form two triangles (test depth)



# Steps (2)

- Avoid connecting depth discontinuities:
  - Test 3D distance between points when generating triangles
  - Do not generate if depth >> S

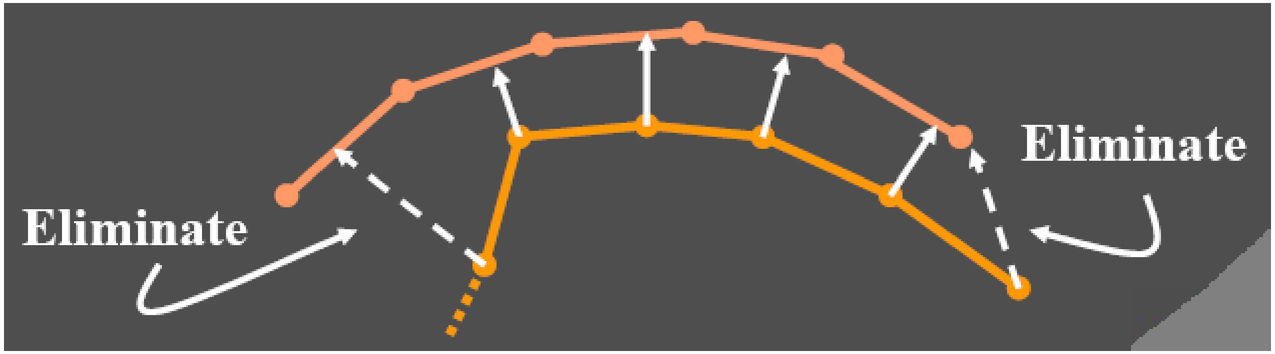


# Registration of Range Images

- Align corresponding portions of different range images
- Modified *iterated closest-point* (ICP) algorithm
- Initial alignment from camera positions (user)

# Alignment (ICP)

- Find nearest position on mesh A to each vertex of mesh B
- Discard pairs of points that are too far apart
- Find rigid transformation that minimizes weighted least-squared distance
- Iterate until convergence



# Point Matching

- Input:
  - 2 matching sets of 3D points (M, D)
- Find rigid transformation (rotation+translation) which minimizes the distance between M and D
- Use Least-Squares

$$\min_{R,T} \sum_i \left\| M_i - (RD_i + T) \right\|^2$$

# Weight Assignment

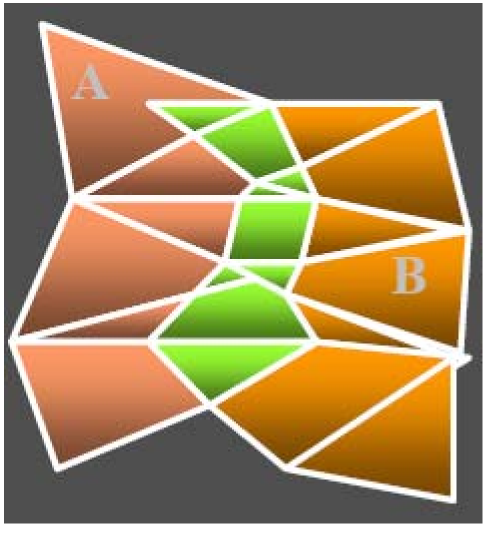
- Final surface will be weighted combination of range images
- Weights assigned at each vertex to:
  - Favor views with higher sampling rates
    - View direction is parallel to surface normal
  - Encourage smooth blends between range images

# Weights for Smooth Blending

- To assure smooth blends, weights are forced to taper in the vicinity of boundaries

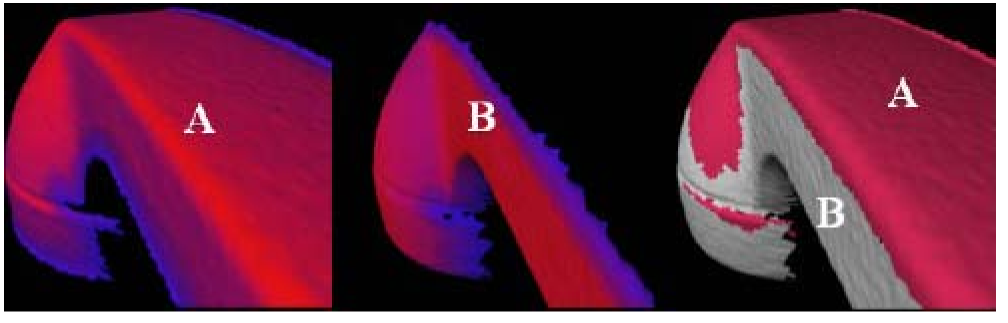
# Integration: Mesh Zippering

- After registration have two overlapping meshes
- Need to combine into single connectivity
- Zippering
  - Remove overlapping portion of the mesh
  - Clip one mesh against another
  - Remove small triangles introduced during clipping

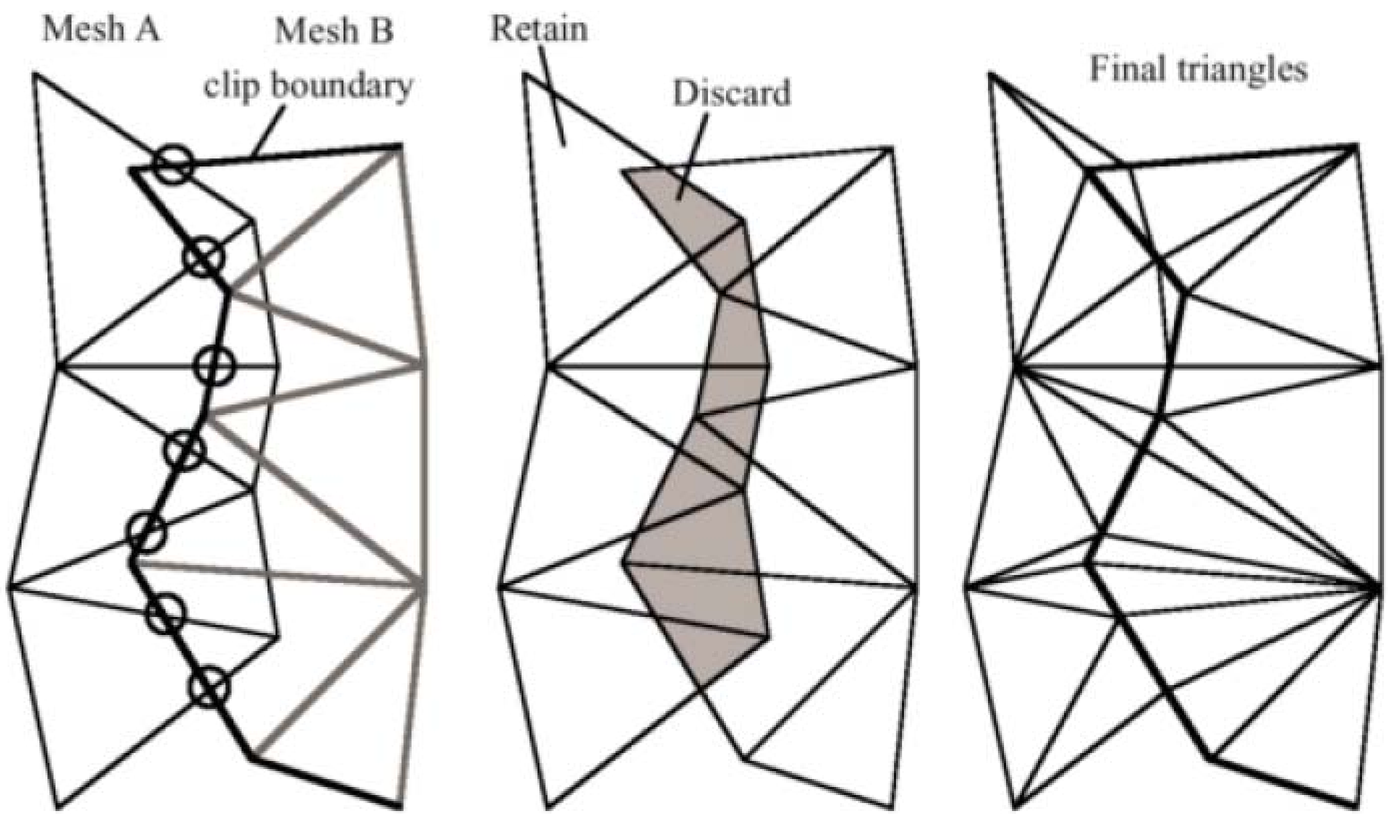


# Removing Redundant Surfaces

- Remove "redundant" triangles until the meshes just meet
  - Triangle T in mesh A is redundant if
    - Hausdorf distance from it to B is within tolerance
    - Nearest points on B are not on boundary

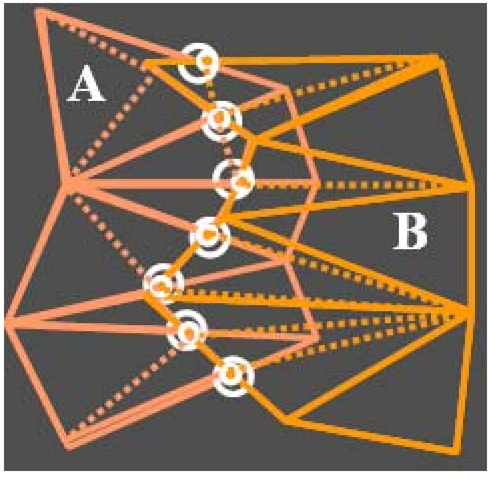


# Redundancy Removal

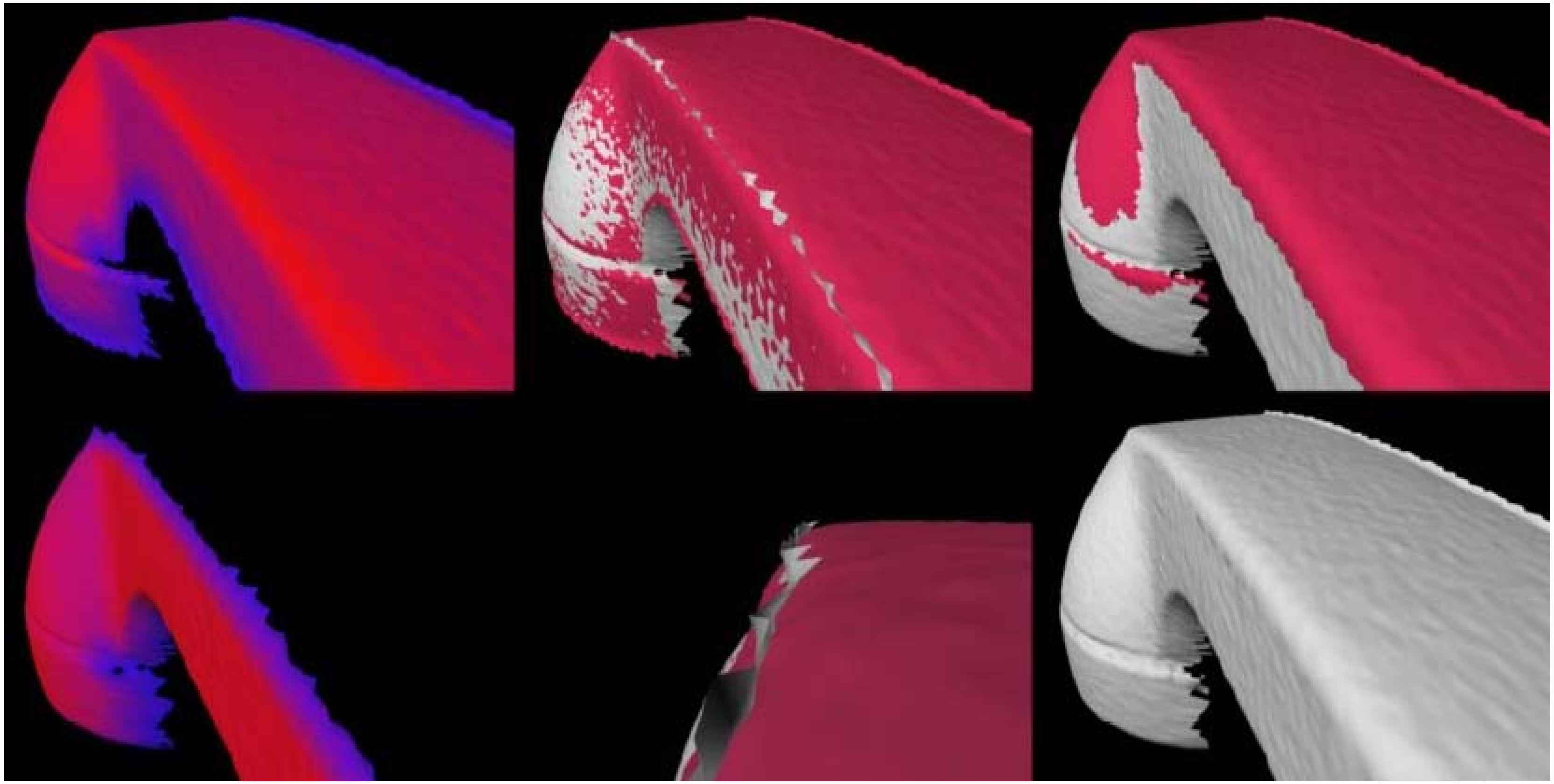


# Mesh Clipping

- Find intersection between boundary of B and mesh A
- Add intersection nodes to A and B
- Discard overlapping triangles



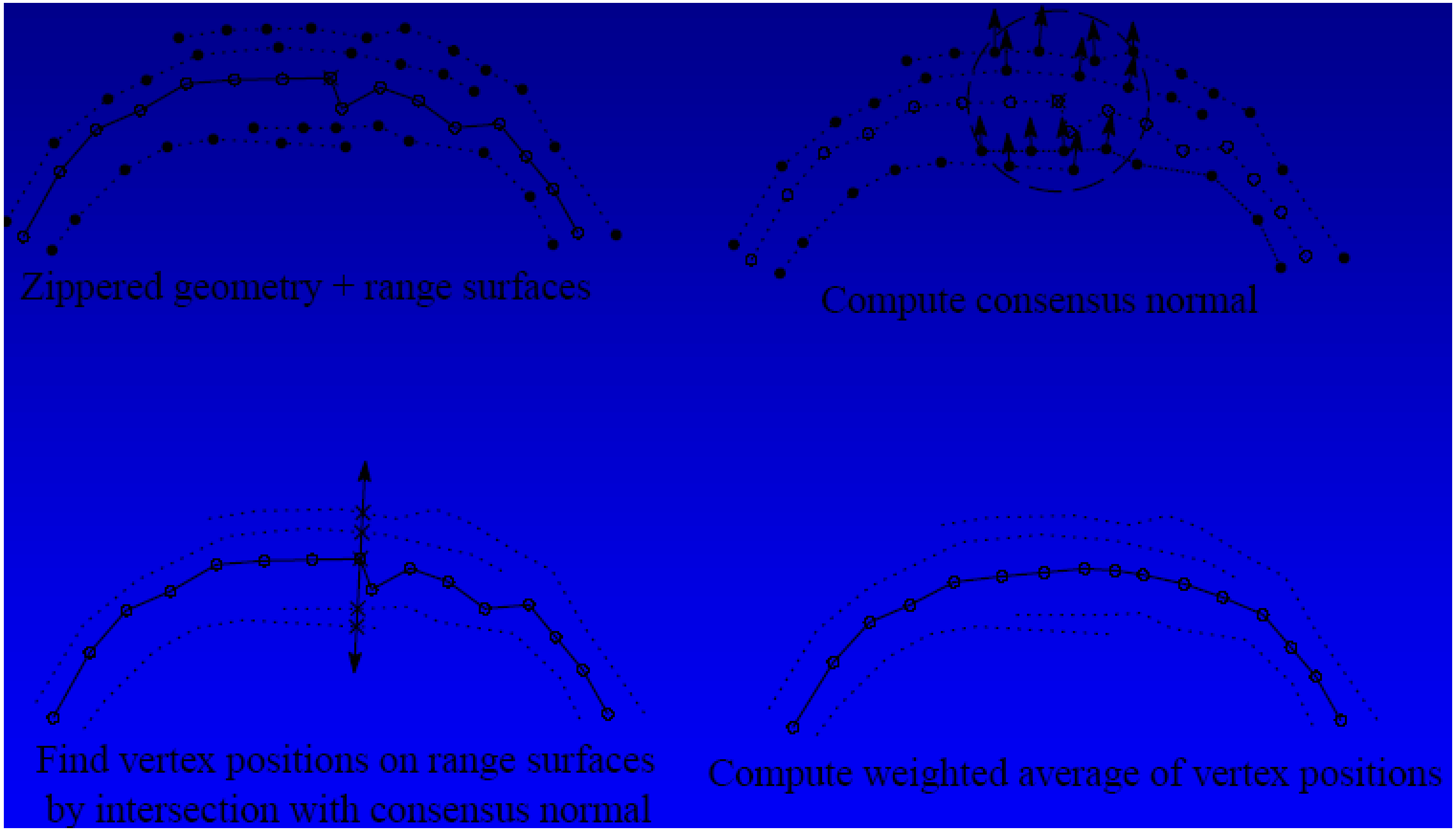
# Example



# Post-Processing

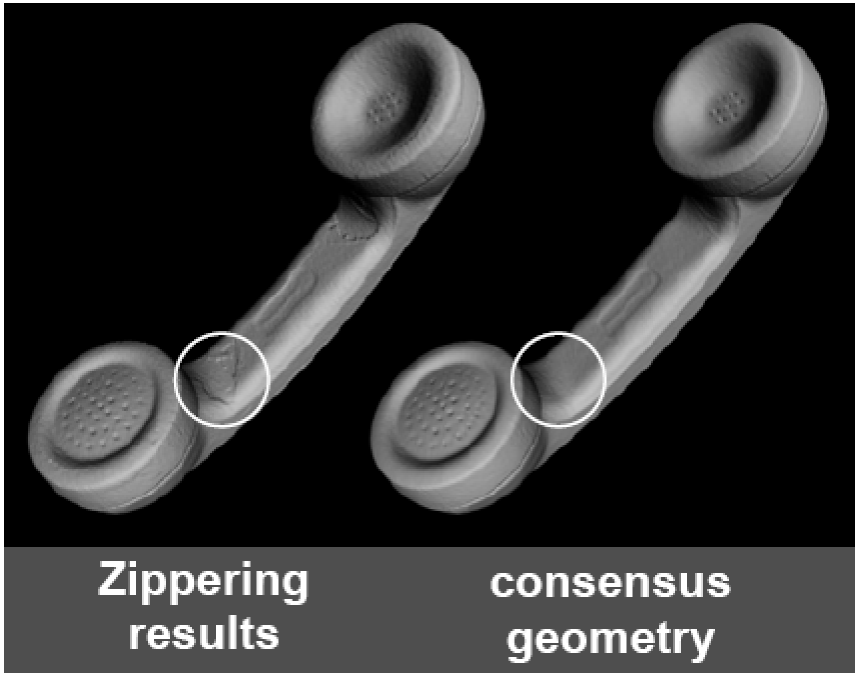
- Fill holes – local triangulation
- Remove small triangles – vertex removal
- Consensus geometry
  - Move each vertex to consensus position given by weighted average of positions from original range images

# Consensus Geometry



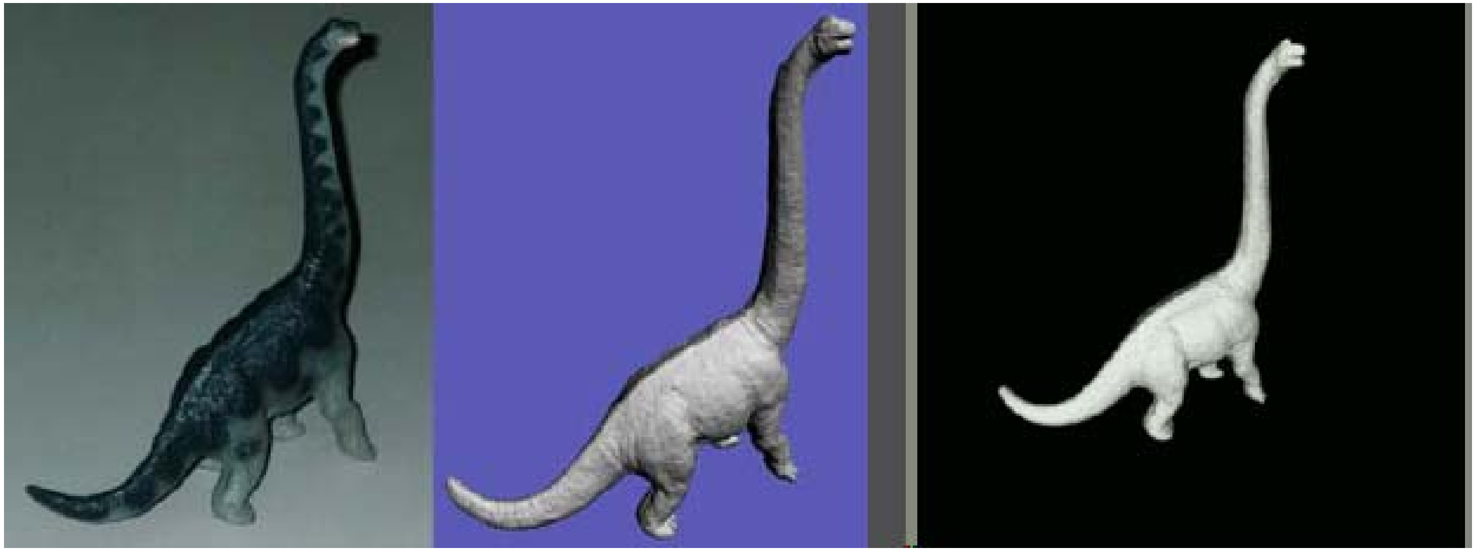
# Examples (1)

- 10 range images, more than 160,000 triangles



# Examples (2)

- 14 range images, more than 360,000 polygons



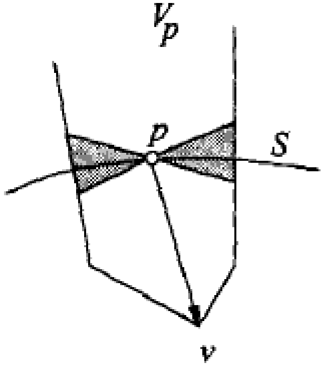
# Comparison

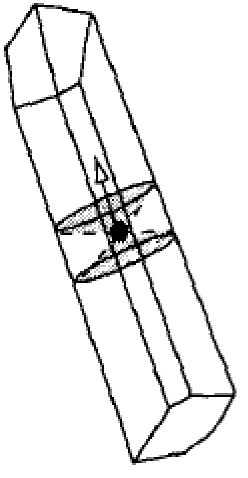
- Crust is more formal (no heuristics)
  - Proven to generate good results
  - Slow
  - Require “correct” data
- Zippering – heuristic
  - Fast
  - Lot of small “fixes” / “tricks”
  - In practice works well

# Discussions

# Revised (1)

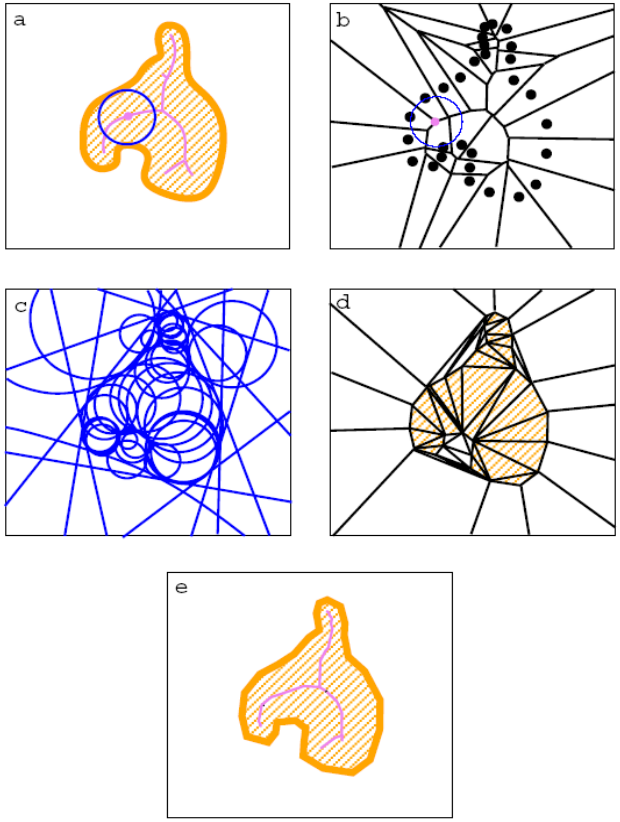
- Co-cones
  - Cone with apex at sample point and aligned with poles
  - Algorithm only requires one Voronoi diagram computation
  - Eliminates normal trimming step
  - Still does not support sharp edges



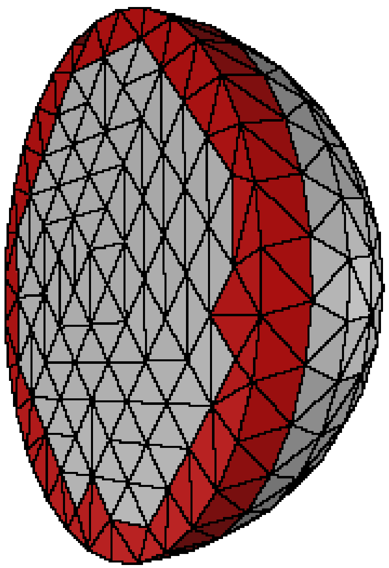
# Revised (2)

- The power crust
  - Use polar balls and power diagrams to separate the inside and outside of the surface
  - Approximates medial axis



# Revised (3)

- Detecting Undersampling
  - Fat Voronoi cells or dissimilarly oriented neighboring Voronoi cells imply undersampling.  Add sample points to accommodate
  - This accounts for sharp edges and boundaries
- Tight Co-cone
  - After detecting undersampling, stitch up holes



# Discussions

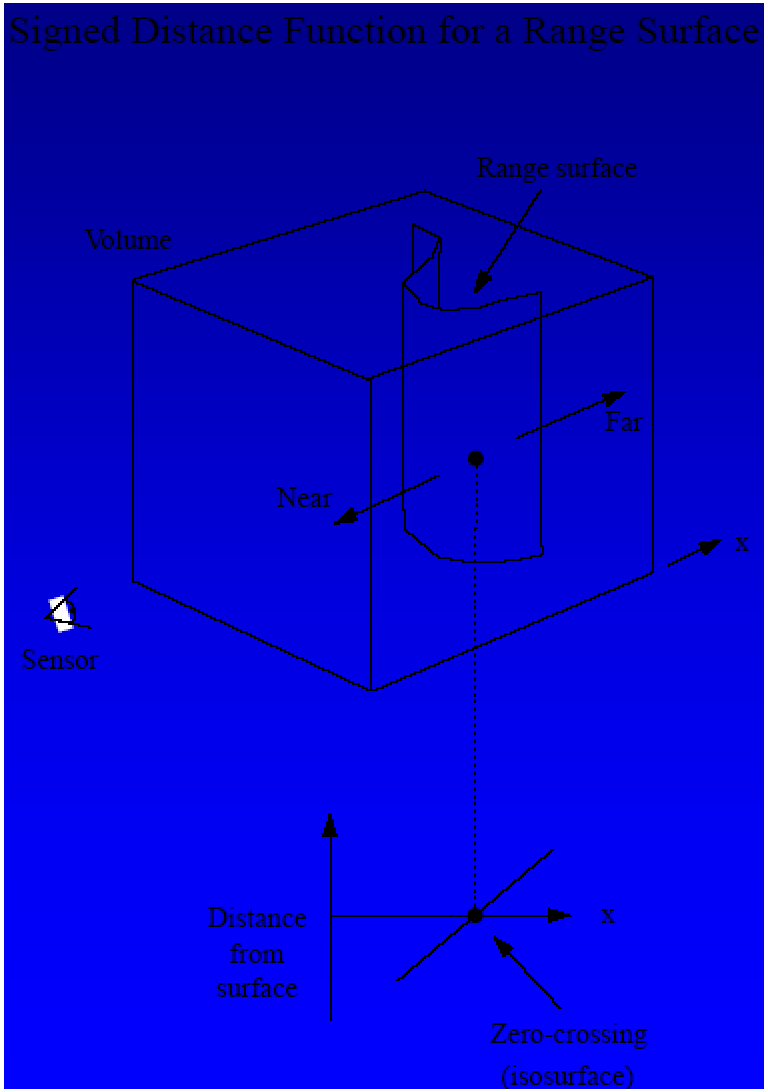
# Volumetrically Combining Range Images

Siggraph 1996

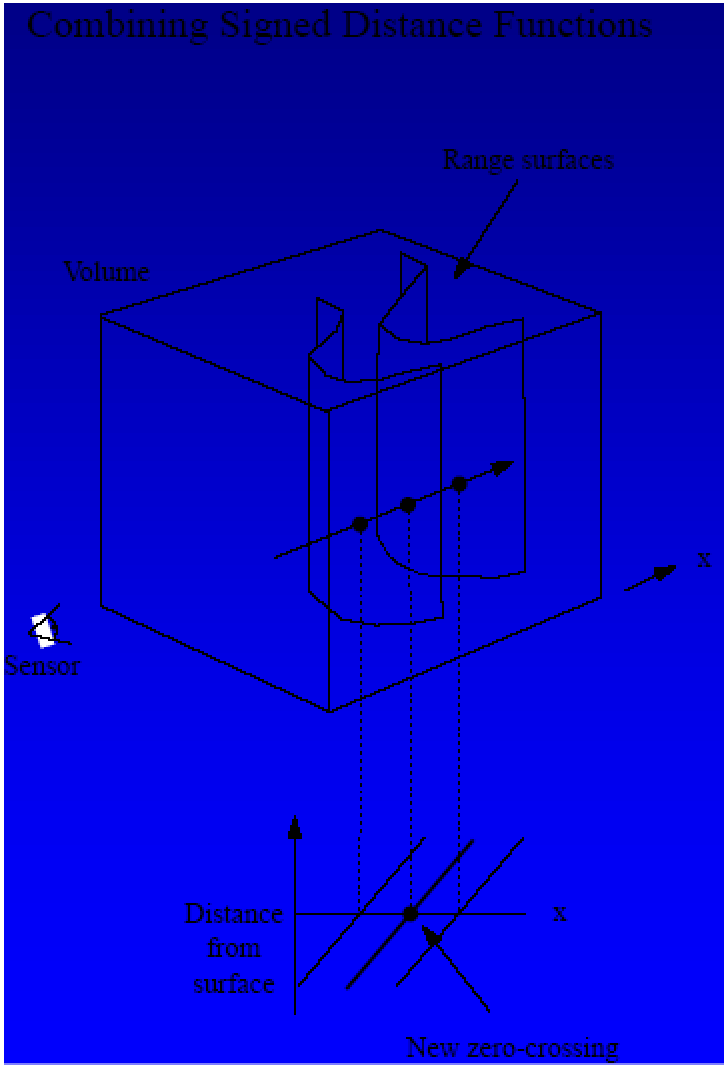
# Overview

- Convert range images to signed distance functions
- Combine signed distance functions
- Carve away empty space
- Extract hole-free isosurface

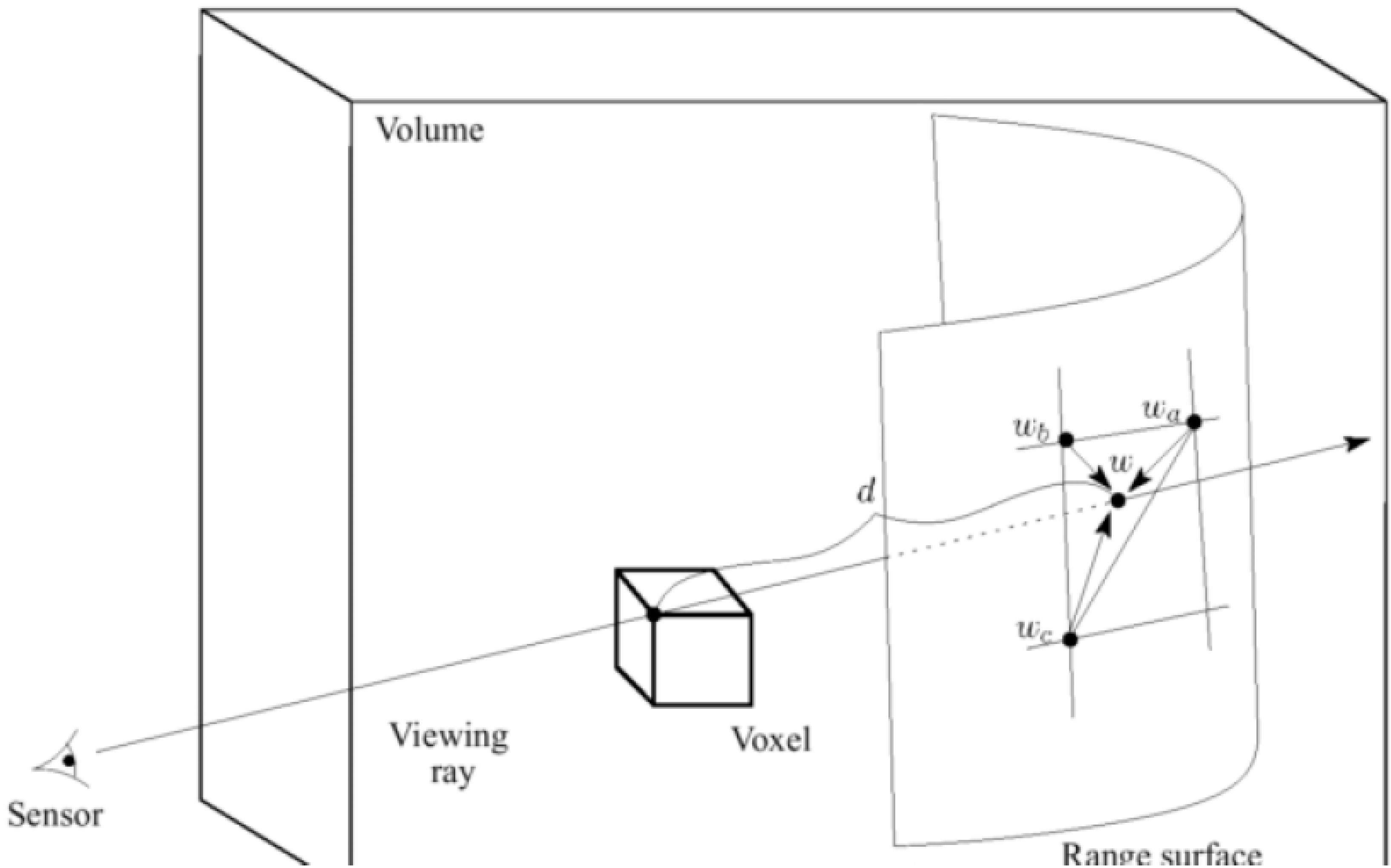
# Signed Distance Function



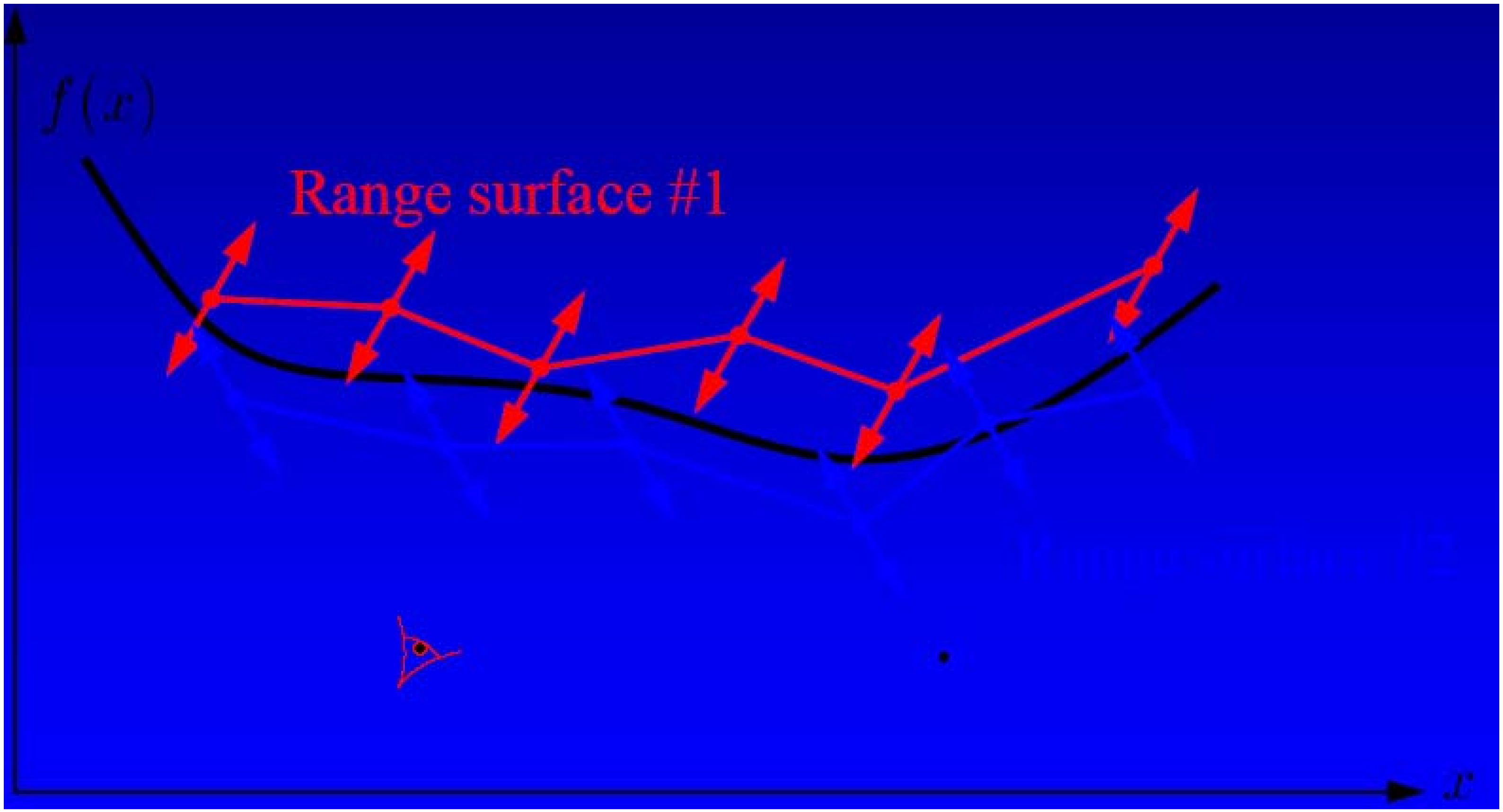
# Combining SDF



# Merging Surface



# Least Squares Solution



# Least Squares Solution

Error per point

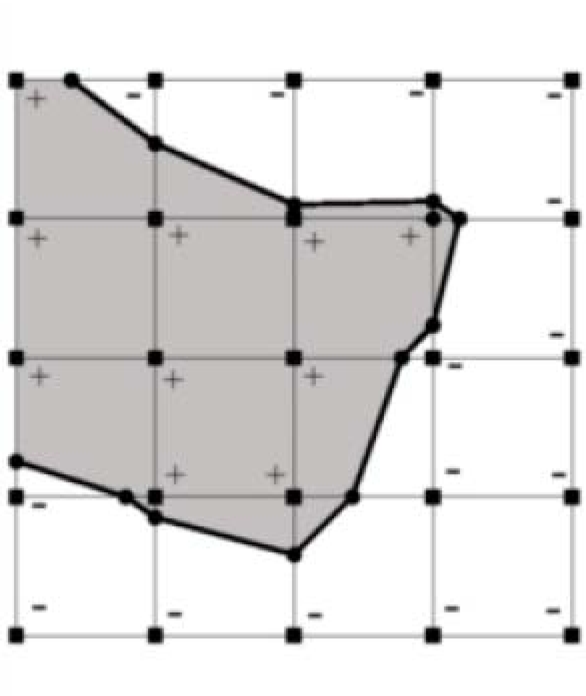
$$E(f) = \sum_{i=1}^{N} \int d_i^2(x, f)\,dx$$

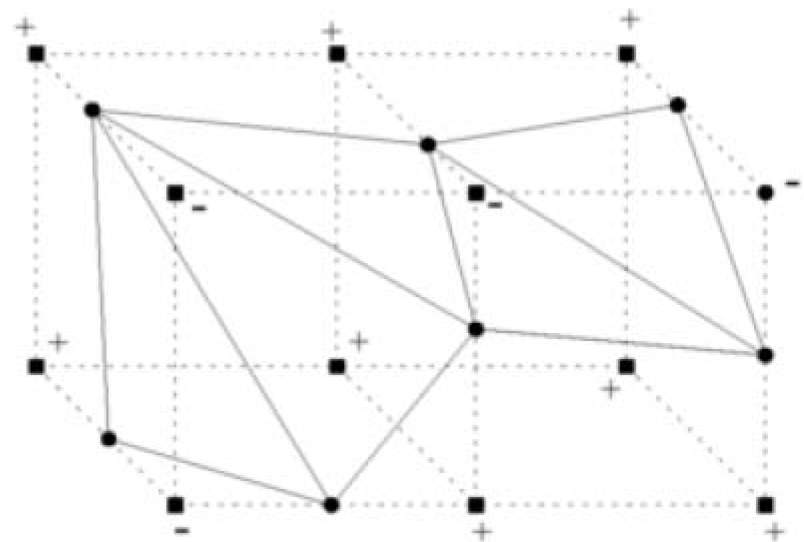
Error per range surface

**Finding the f(x) that minimizes E yields the optimal surface.**

**This f(x) is exactly the zero-crossing of the combined signed distance functions.**

# Isosurface Extraction



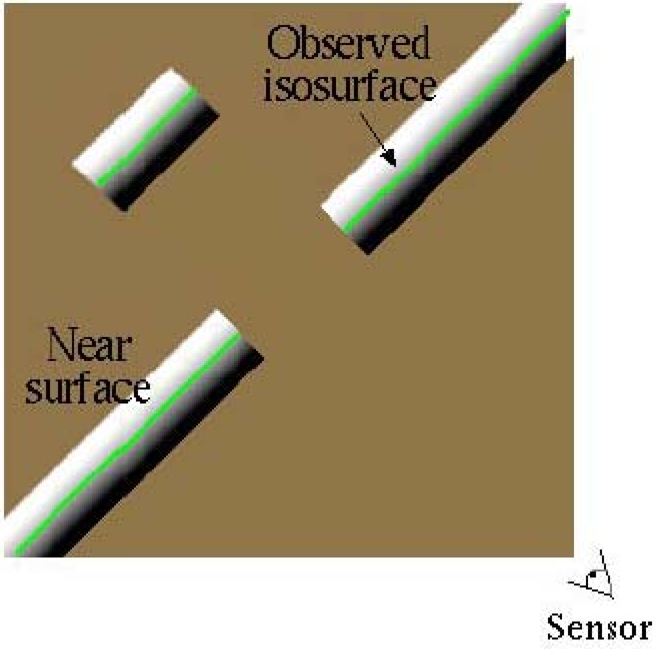


# Hole Filling

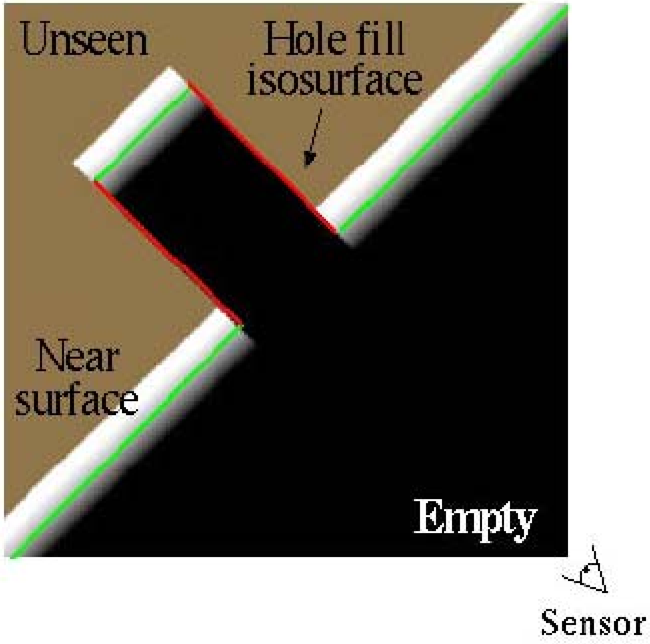
- The procedure so far will reconstruct a mesh from the observed surface. Unseen portions will appear as holes in the reconstruction

- We can fill holes in the polygonal model directly, but such methods:
  – are hard to make robust
  – do not use all available information

# Space Carving

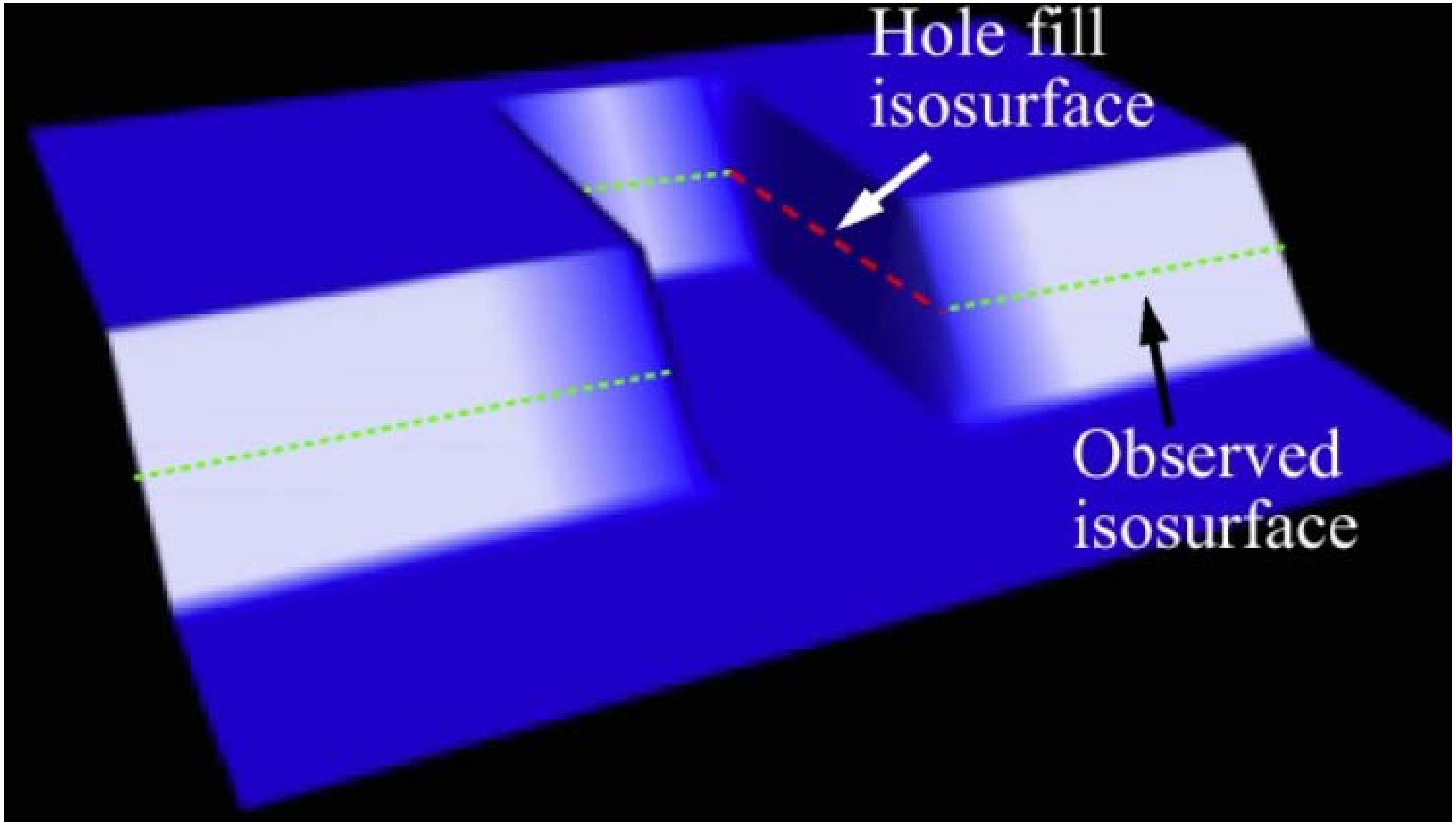



*Without* space carving

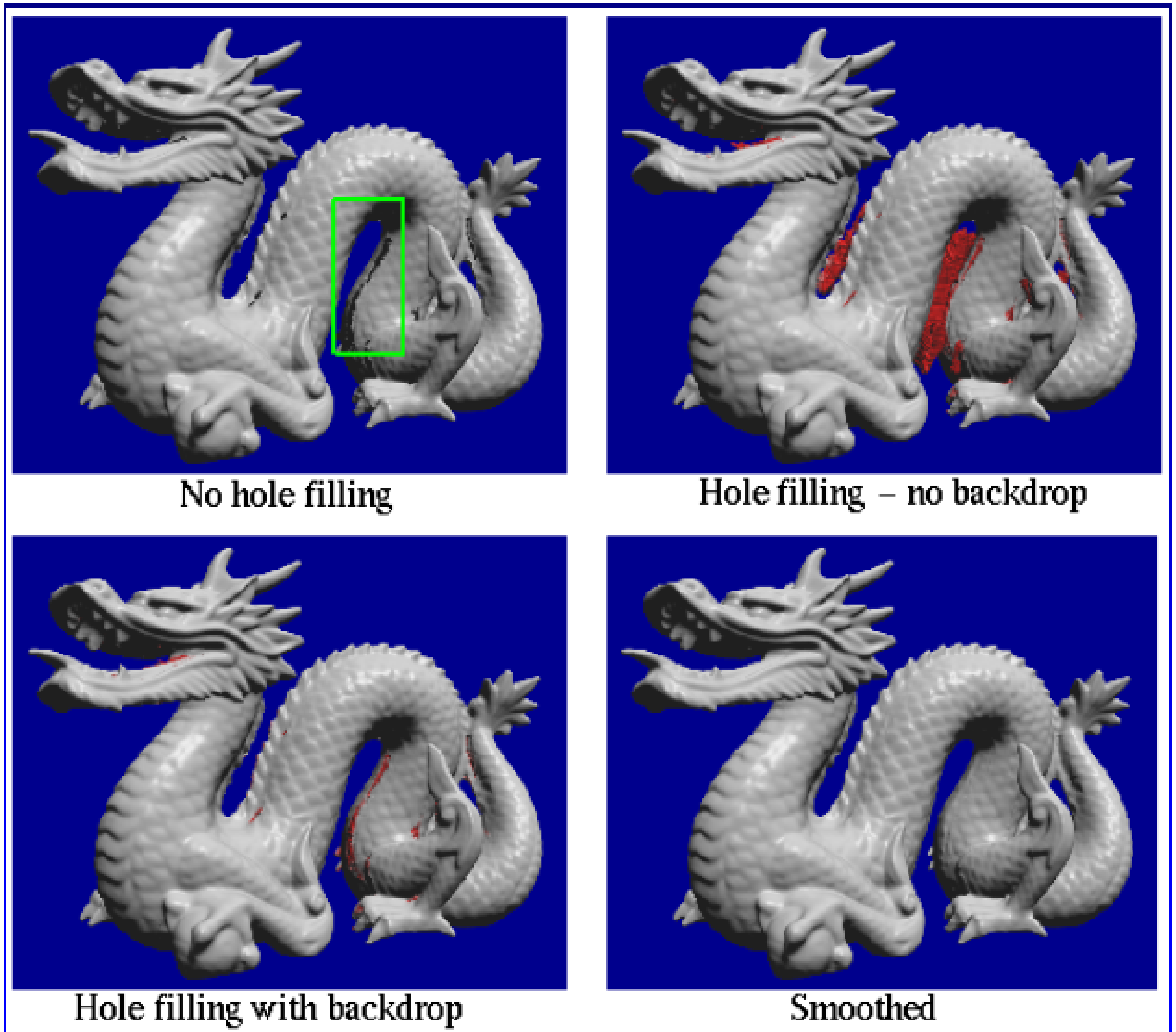



*With* space carving

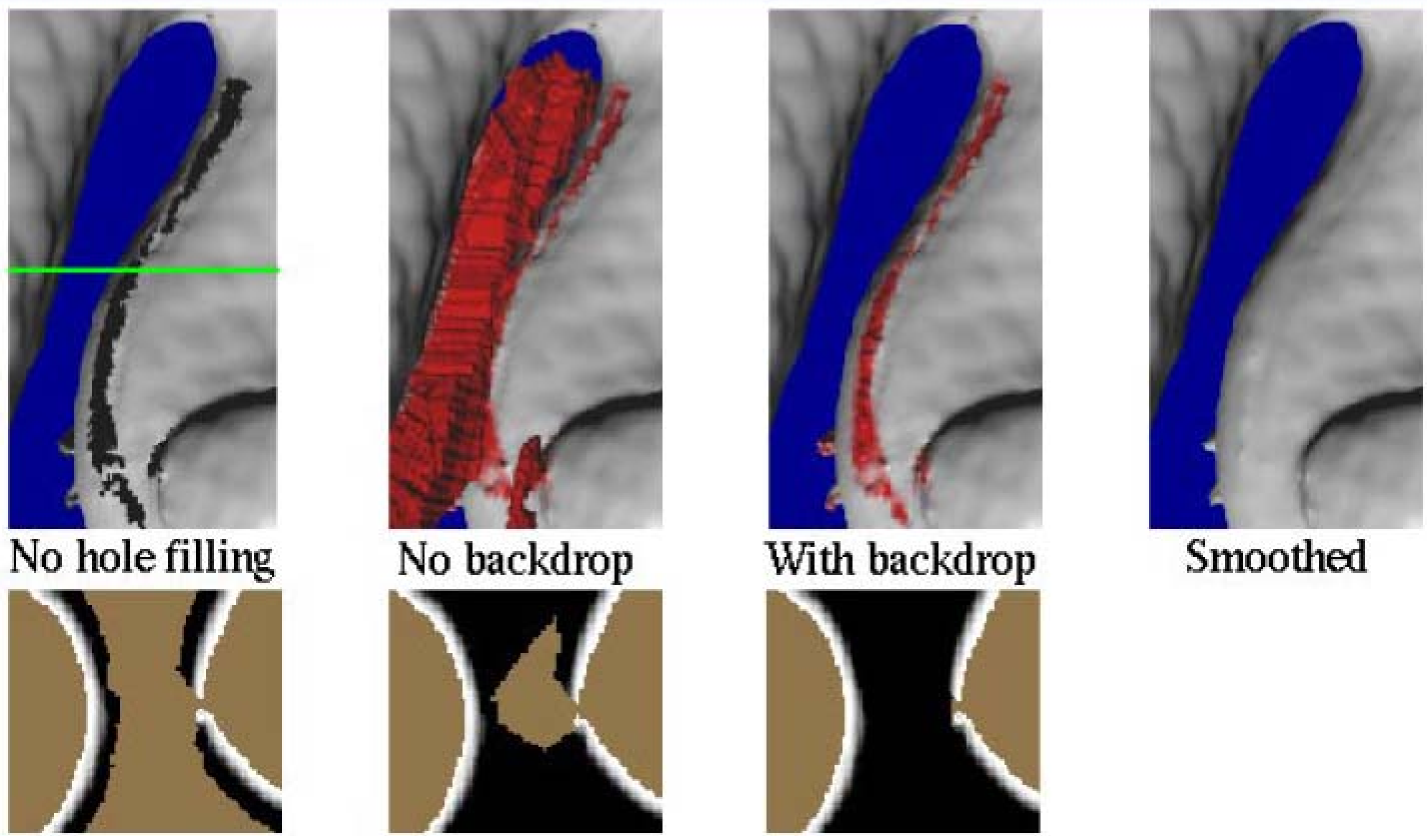
# Space Carving



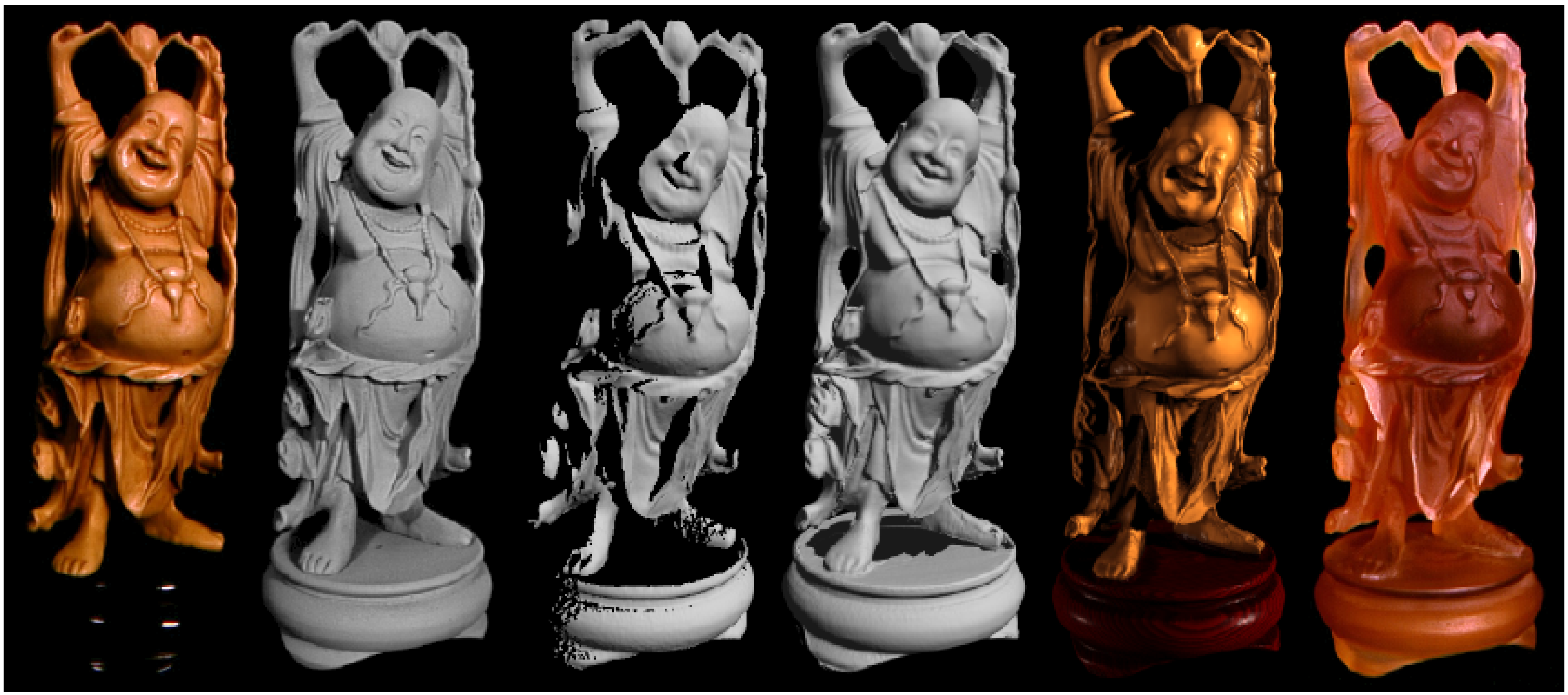
# Results (1)


No hole filling
Hole filling – no backdrop
Hole filling with backdrop
Smoothed

# Local Result

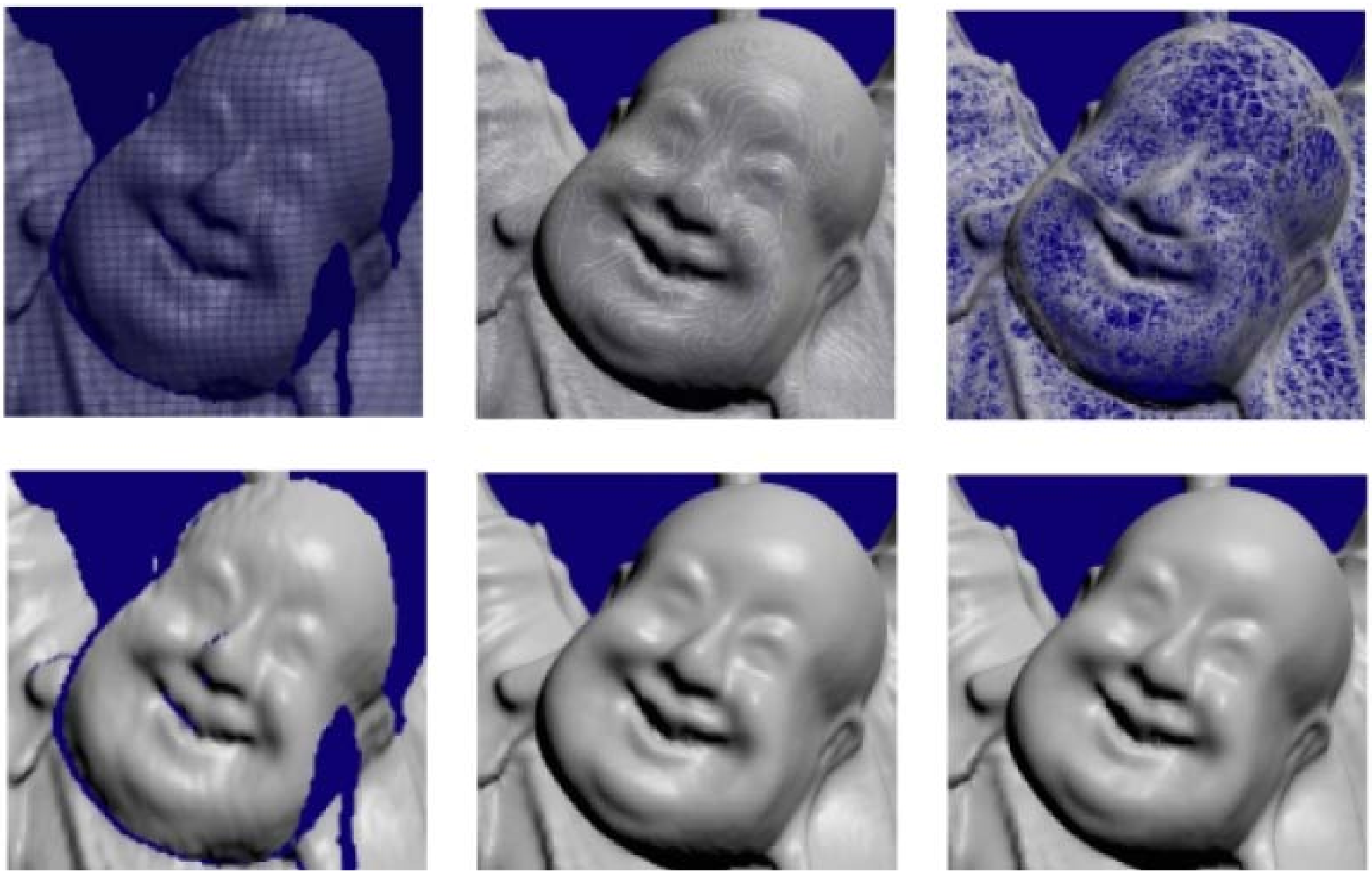


# Results (2)



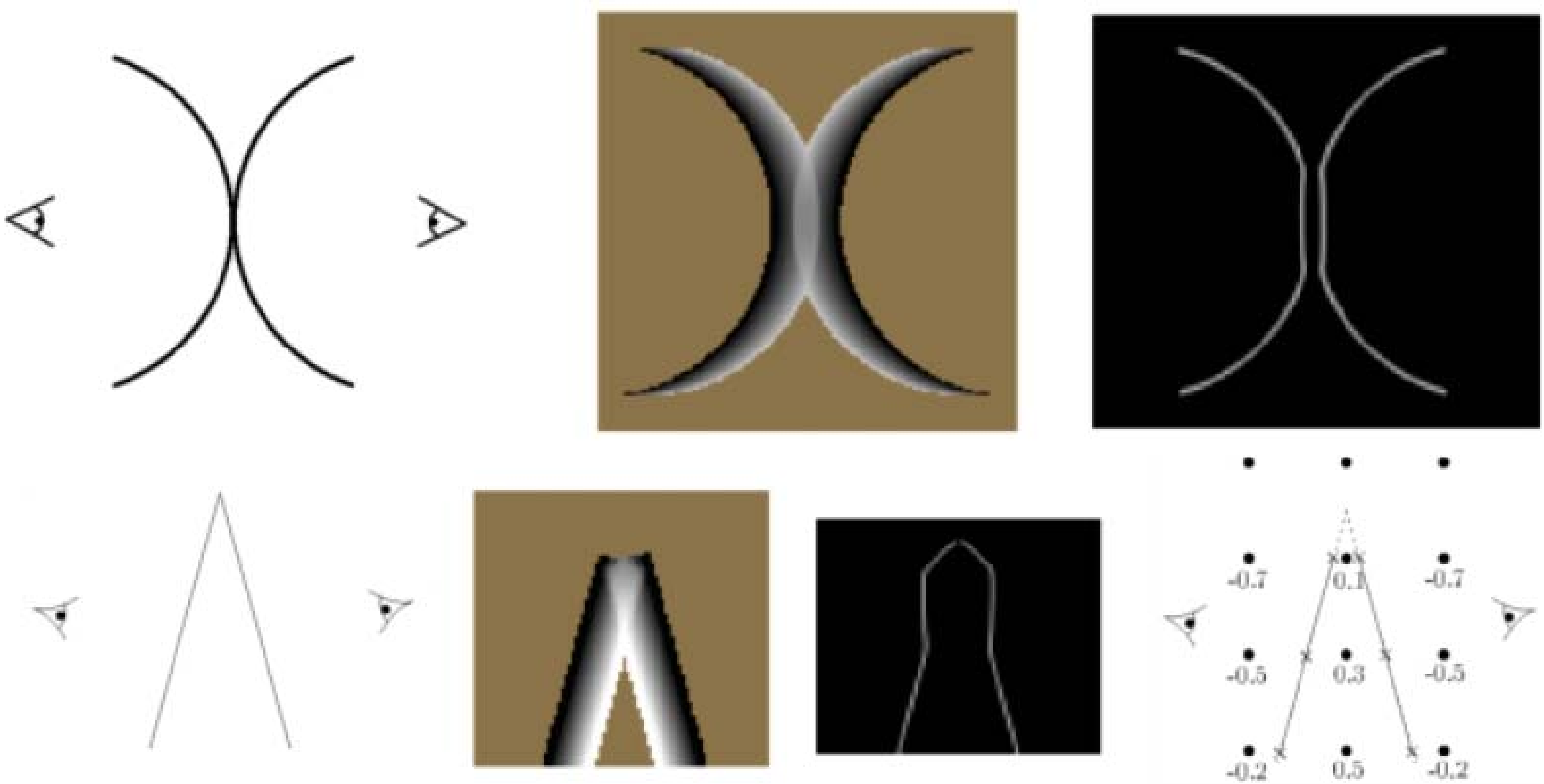
Photograph of original model | Photograph of painted original | Range surface from one scan | Reconstruction before hole-filling | Reconstruction after hole-filling | Hardcopy

# Local Result

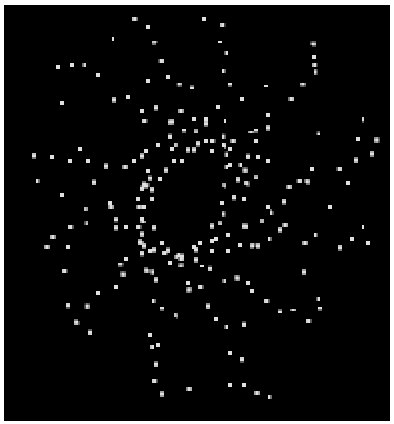


# Limitations

- Minimum thickness and edge sharpness have limits

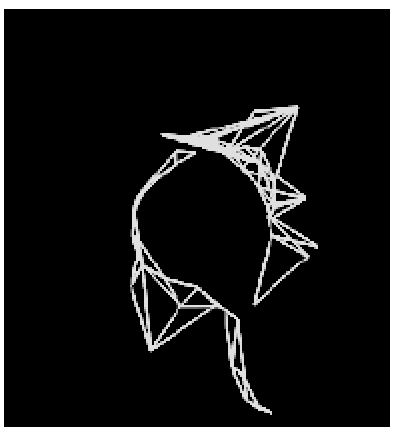


# Merging 12 views of a drill bit

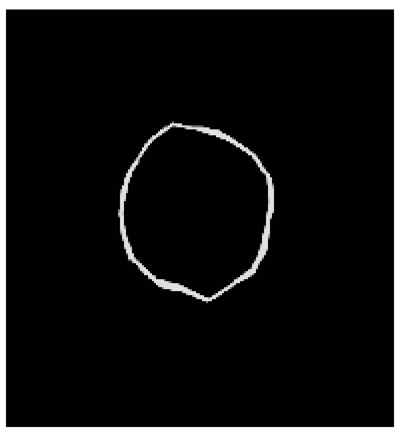


Scattered points

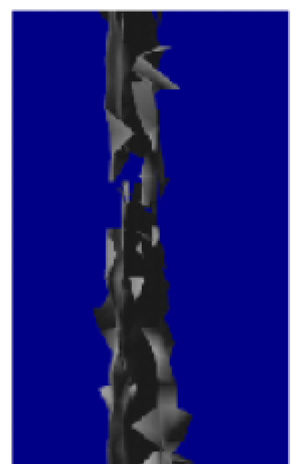
Range surfaces



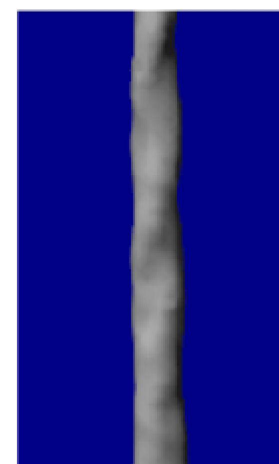
Zippered mesh



Volumetric mesh



Zippered mesh



Volumetric mesh

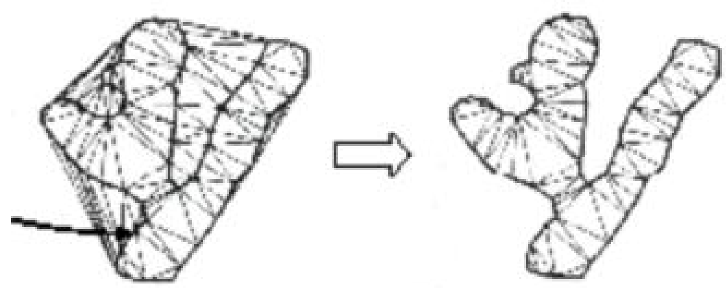
# Discussions

# More (1)

- Zero sets
  - Using input points, define implicit signed distance function
  - Distance function is interpolated and polygonized using marching cubes
  - Approximation rather than interpolation

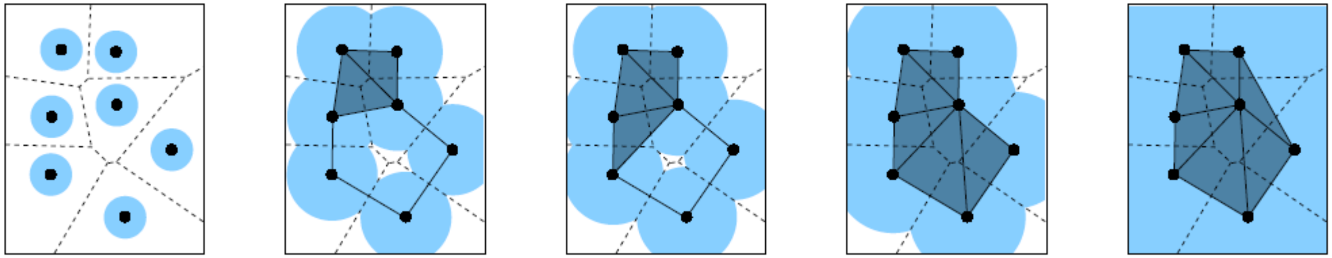
# More (2)

- Delaunay Sculpting
  - Remove tetrahedra from Delaunay triangulation
  - Associate values to tetrahedra and eliminate largest valued ones



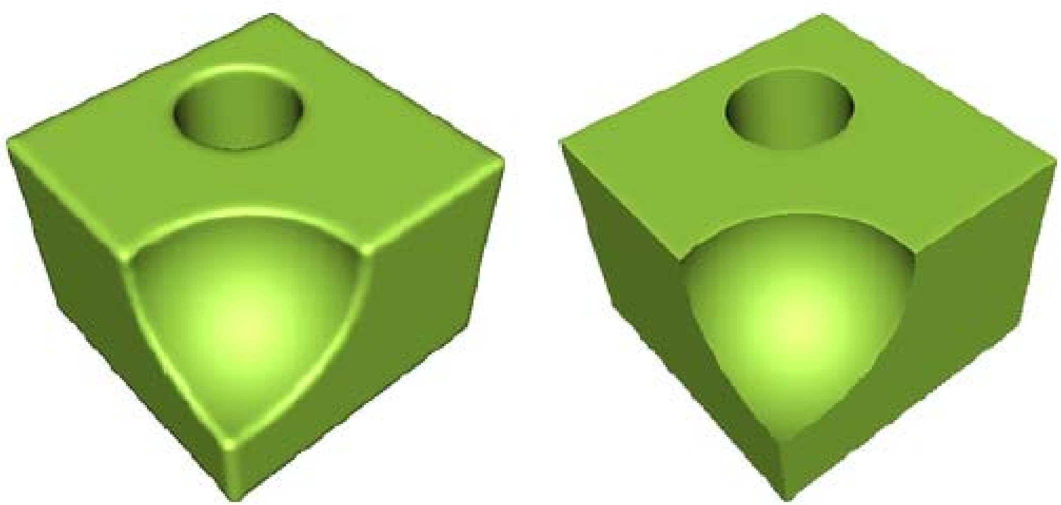
# More (3)

- Alpha Shapes
  - Given a parameter, α, connect vertices within α units
  - Subset of Delaunay triangulation
  - Generalized convex hull

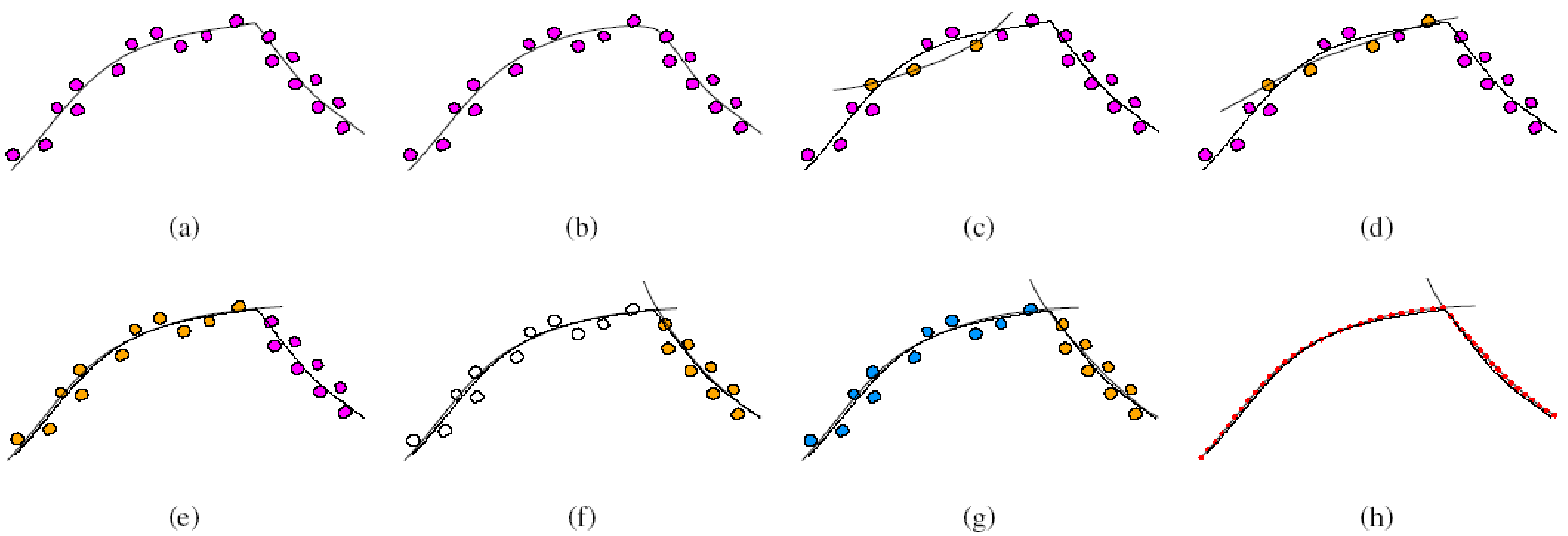


# More (4)

- Moving least square fitting – Siggraph05



(a) (b)

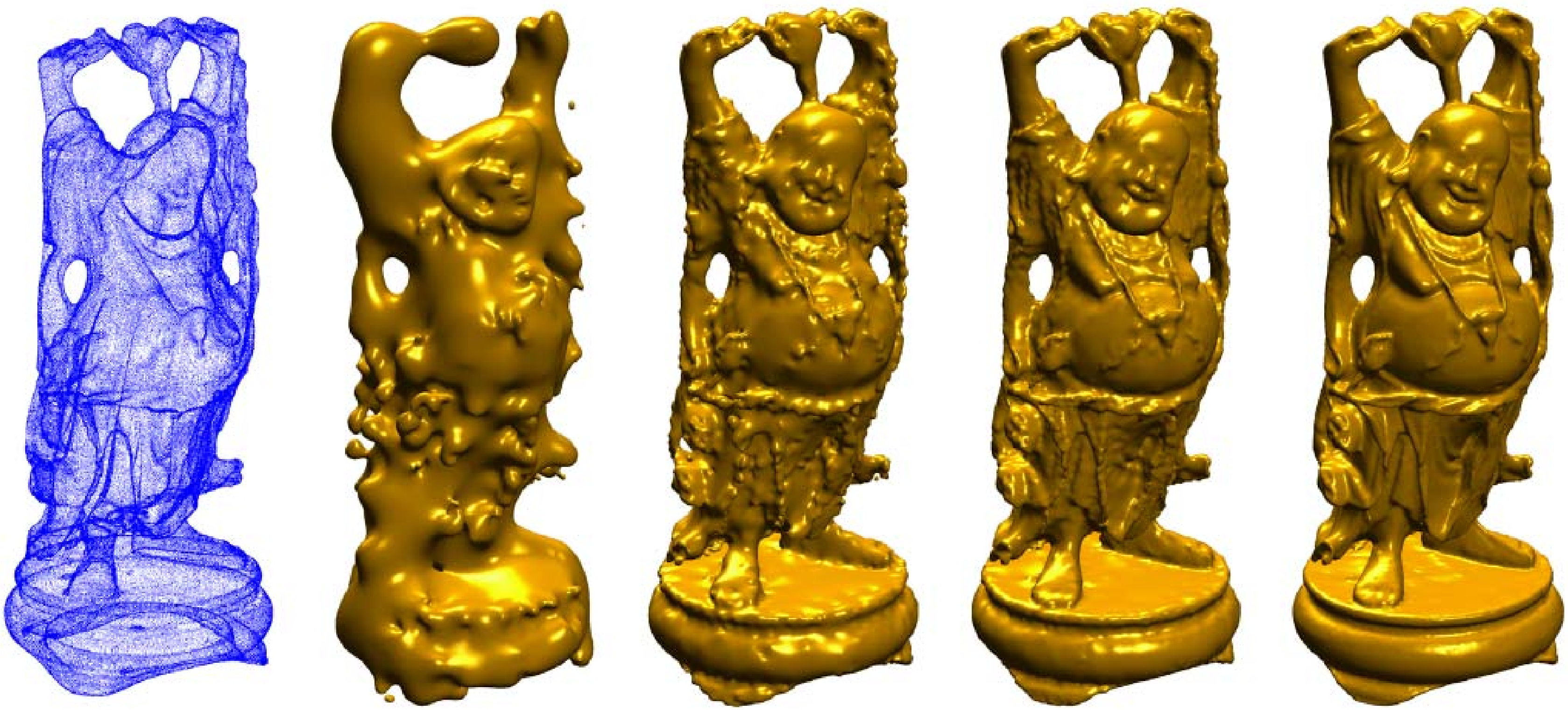


(a) (b) (c) (d)

(e) (f) (g) (h)

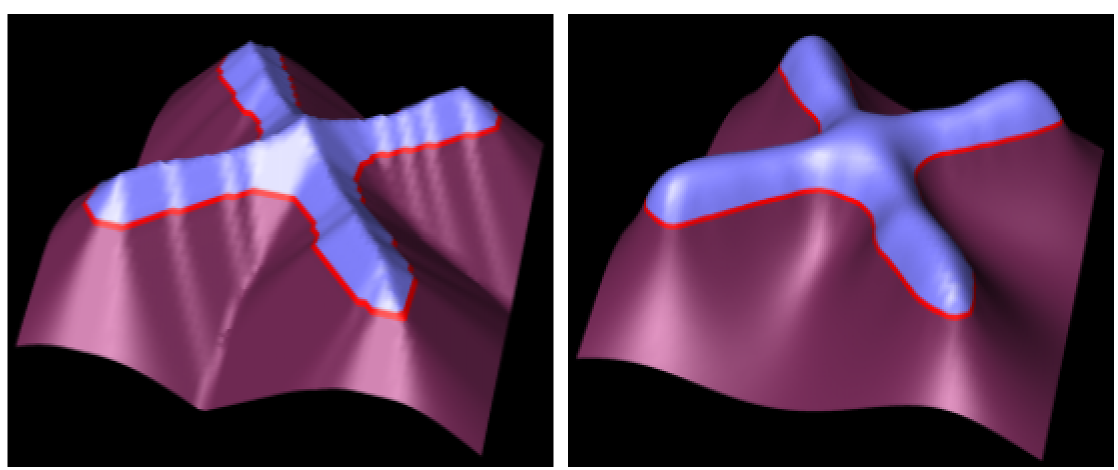
# More (5)

- RBF fitting – Siggraph01



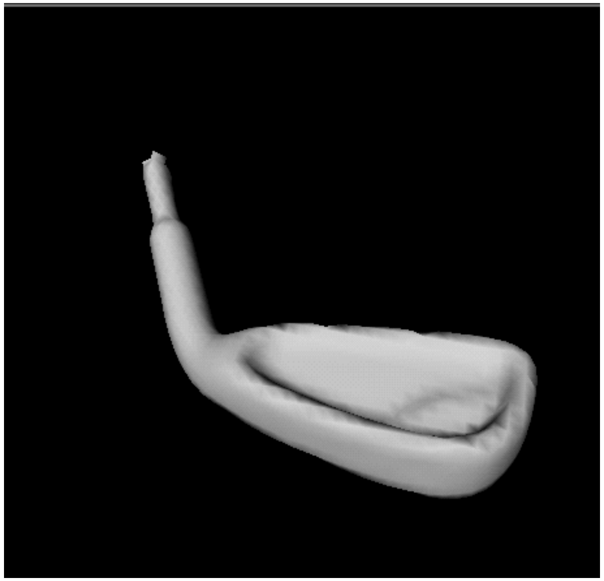
# More (6)

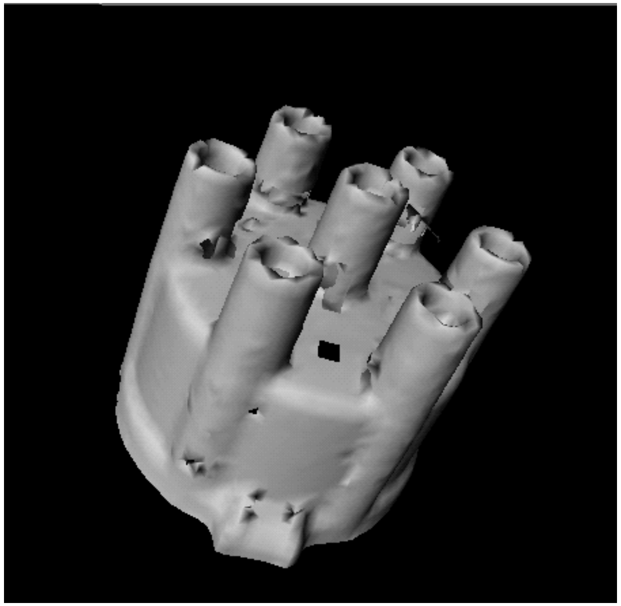
- Greg Turk – Siggraph99, TOG01

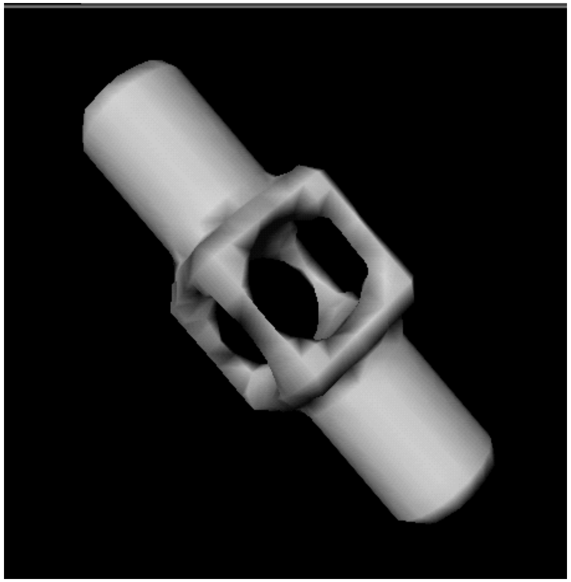


# More (7)

- Hughes Hoppe – Siggraph92







# More

- So many papers

# Summary

| Technique | Assumptions |
|---|---|
| Fit parametric surface | Data fits model |
| Piece together parallel contours | Data from known device |
| Fit Gaussian Kernels | Given normal at each point |
| $\alpha$-shape Triangulation | Noise-free |
| "Mesh" methods | Sample Dense Near Features |
| Crust Methods | Sample Dense Near Features |

# Marching Cubes

# Overview

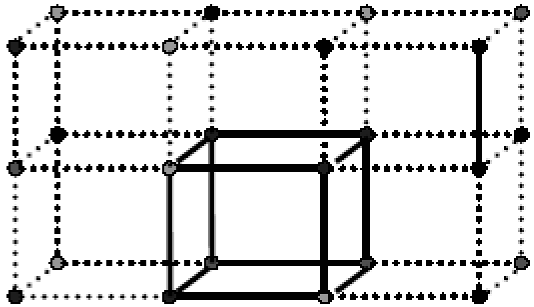
- Marching cubes: method for approximating surface defined by isovalue $\alpha$, given by grid data

- <u>Input:</u>
  - Grid data (set of 2D images)
  - Threshold value (isovalue) $\alpha$

- <u>Output:</u>
  - Triangulated surface that matches isovalue surface of $\alpha$

# Voxels

- Voxel – cube with values at eight corners
  - Each value is above or below isovalue $\alpha$
  - Method processes one voxel at a time
- $2^8$=256 possible configurations (per voxel)
  - reduced to 15 (symmetry and rotations)
- Each voxel is either:
  - Entirely inside isosurface
  - Entirely outside isosurface
  - Intersected by isosurface



# Algorithm

- First pass
  - Identify voxels which intersect isovalue
- Second pass
  - Examine those voxels
  - For each voxel produce set of triangles
    - approximate surface inside voxel

# Configurations

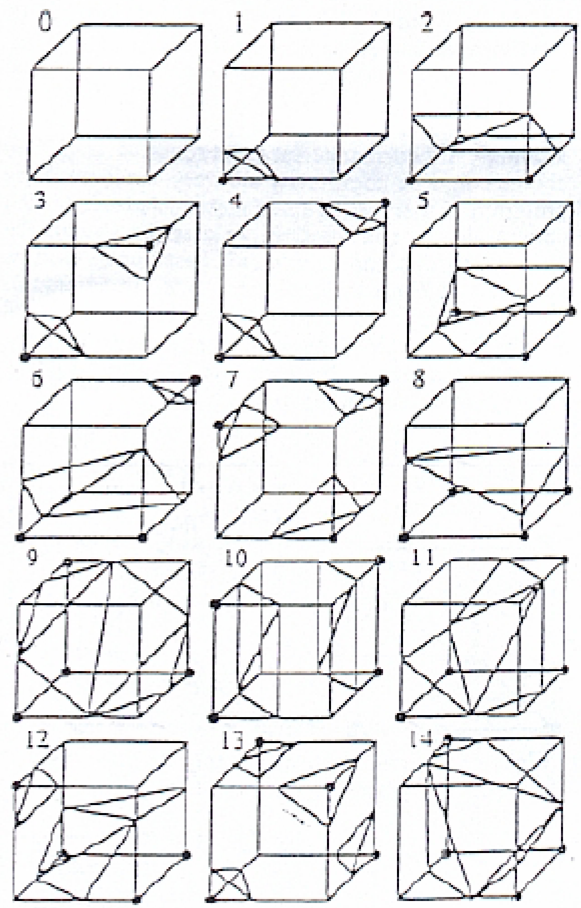
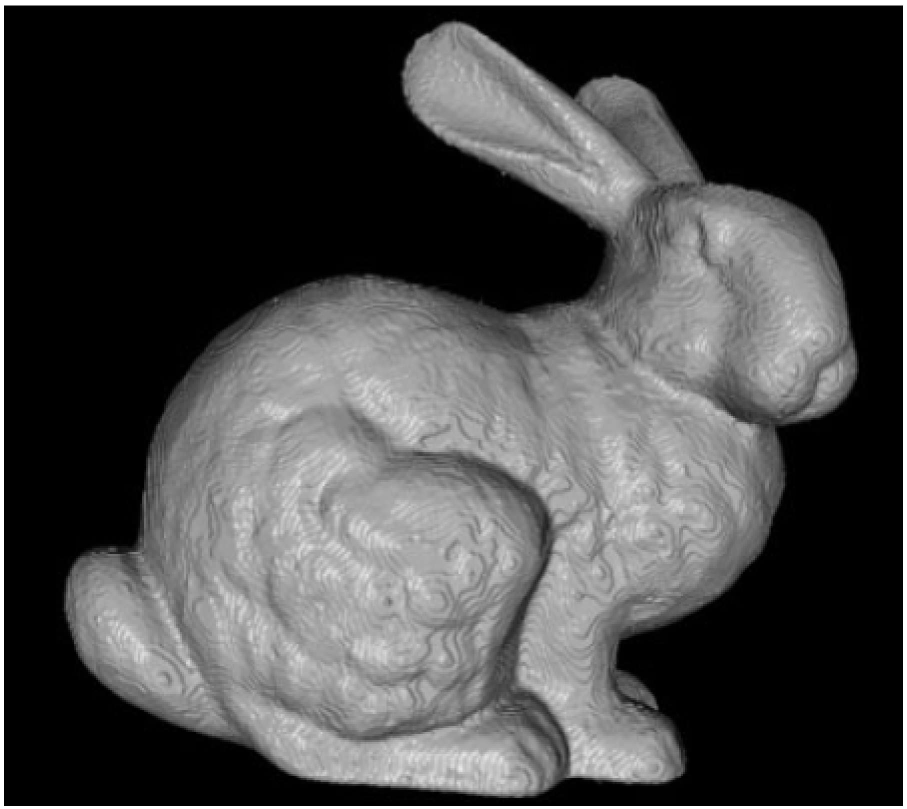


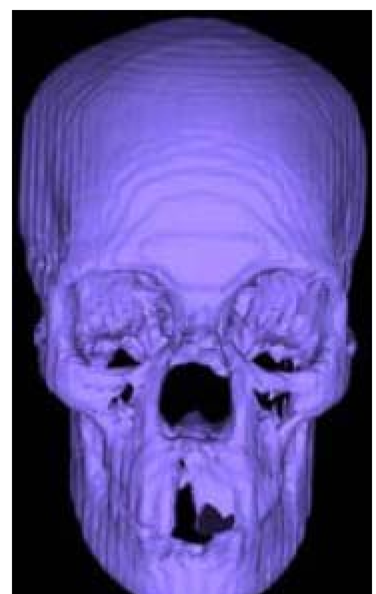

Figure 2. Configurations.

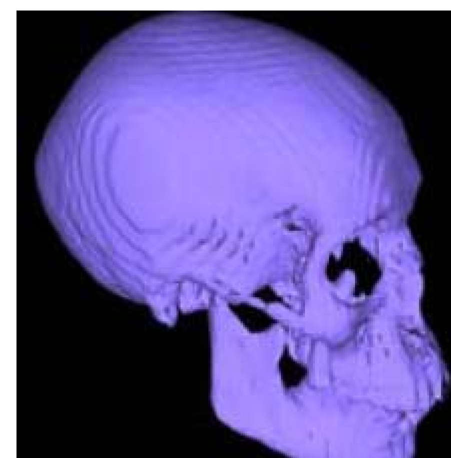
# Configurations

- For each configuration add 1-4 triangles to isosurface
- Isosurface vertices computed by:
  - Interpolation along edges (according to pixel values)
    - better shading, smoother surfaces
  - Default – mid-edges

# Example







# MC Problems

- Marching Cubes method can produce erroneous results
  - E.g. isovalue surfaces with "holes"
- Example:
  - voxel with configuration 6 that shares face with complement of configuration 3:

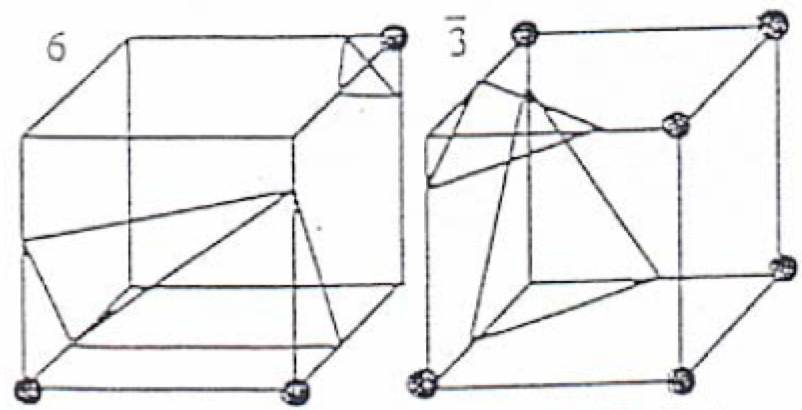
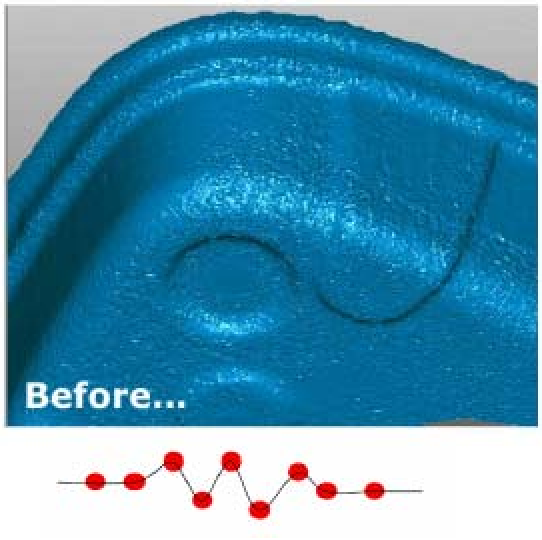


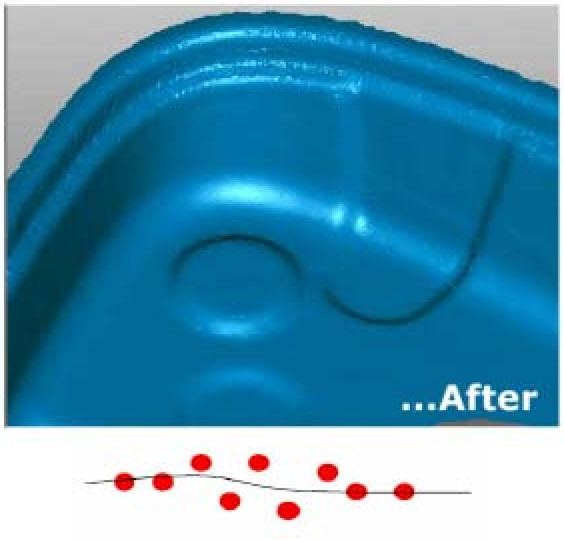

Figure 3. An example illustrating the flaw in the marching cubes method.

# Discussions

# Postprocessing

- Smoothing
  - Coming soon…



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# Q&A