



Warping and Morphing

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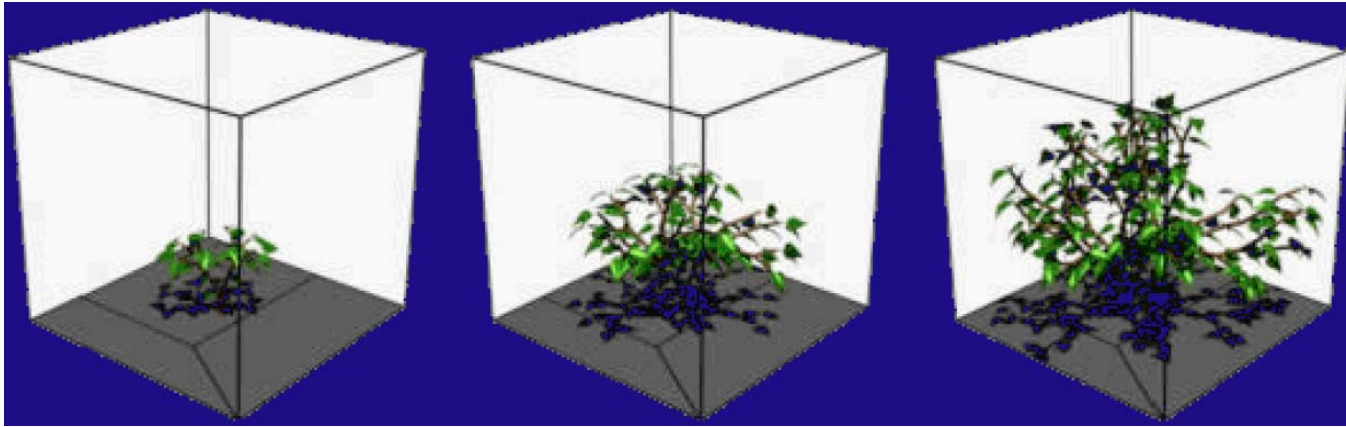
<http://staff.ustc.edu.cn/~lgliu>

Metamorphosis

*"transformation of a shape
and its visual attributes"*

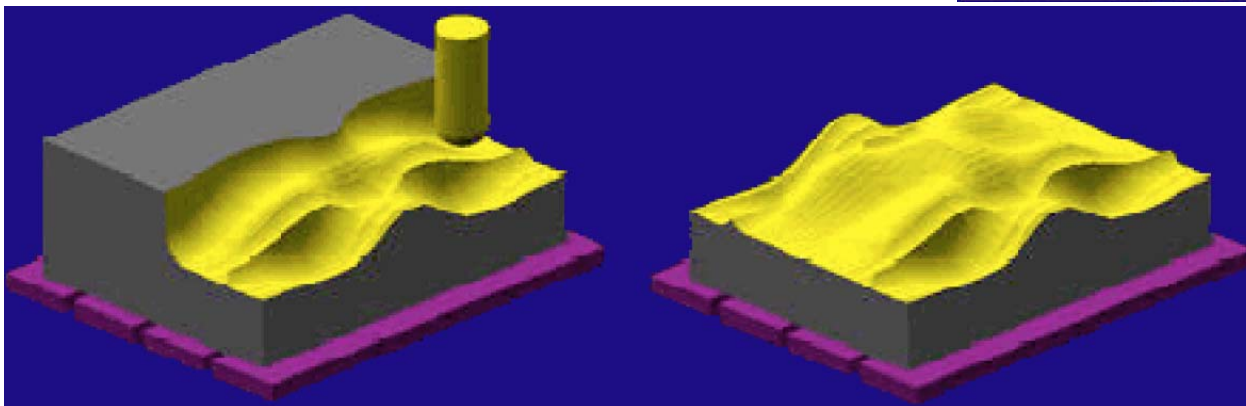
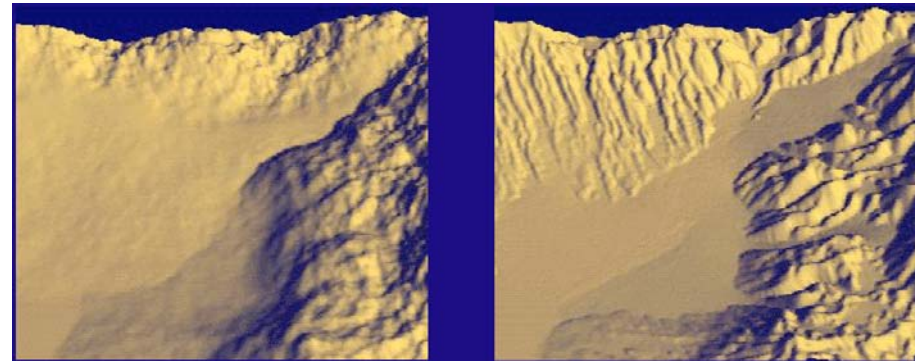
- Intrinsic in our environment
 - Deformations in nature
 - Man-made deformations
- Powerful concept
 - Comparison of shapes

Shape Change In Nature



Plant
Growth

Erosion



Industrial object

Graphical Objects

$$f: U \subset \mathbb{R}^n \rightarrow \mathbb{R}^k$$

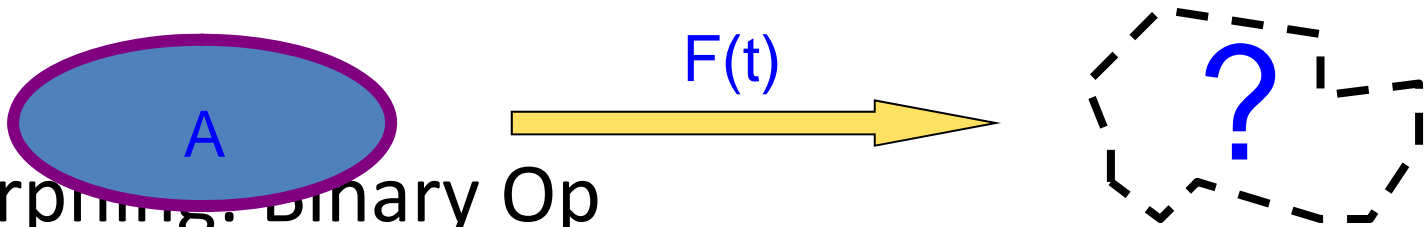
- Components
 - Shape
 - geometric support (vertex coordinates...)
 - Attributes
 - visual properties (color, texture...)
- Examples
 - Particle systems, drawings, images, surfaces, volumes, animations

Warping and Morphing

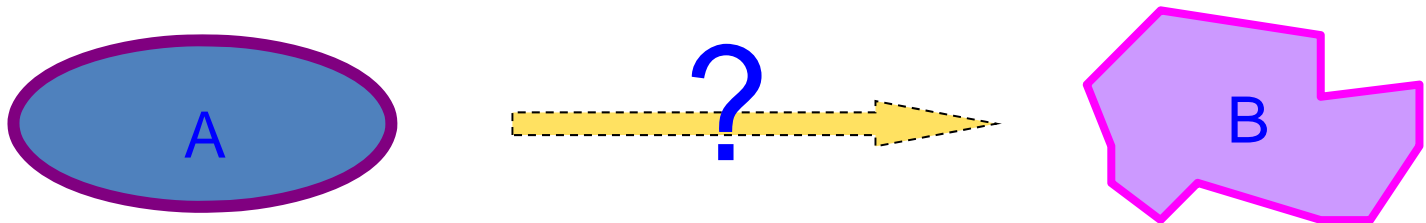
- Warping
 - continuous family of transformations of a graphical object
 - Source object, no target object
- Morphing
 - warping between two graphical objects
 - Source object, target object

Shape Transformations

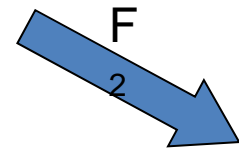
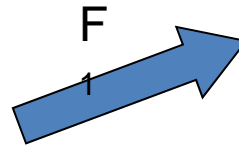
- Warping: Unary Op
 - Given Object A and $F(t)$, find Object B



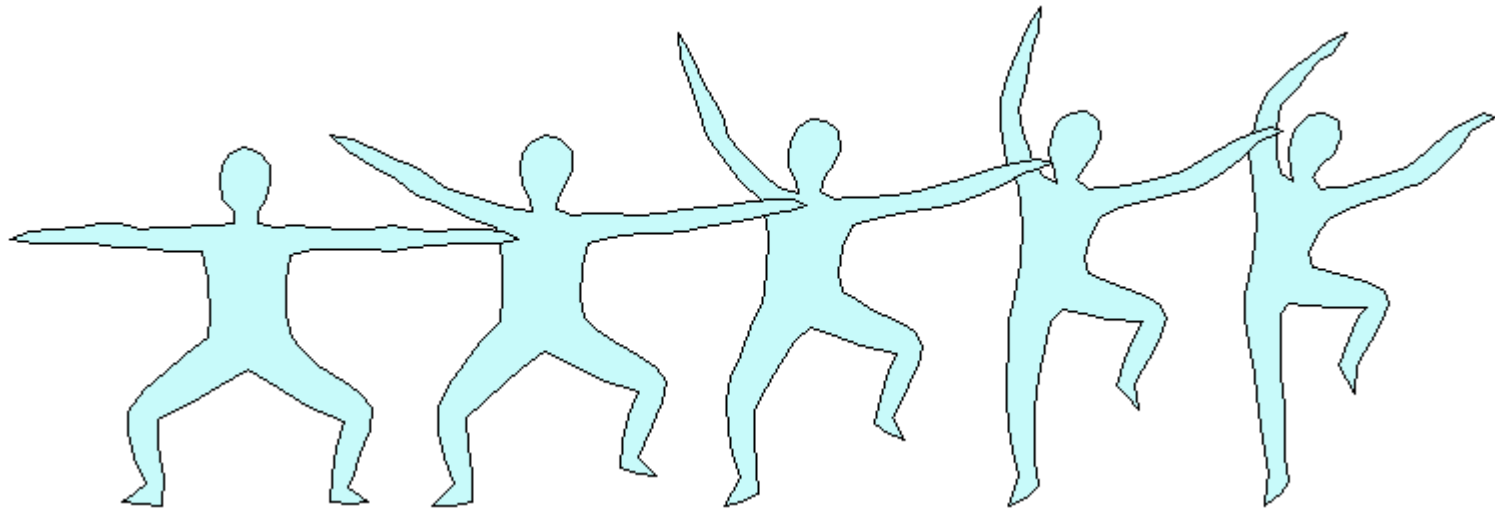
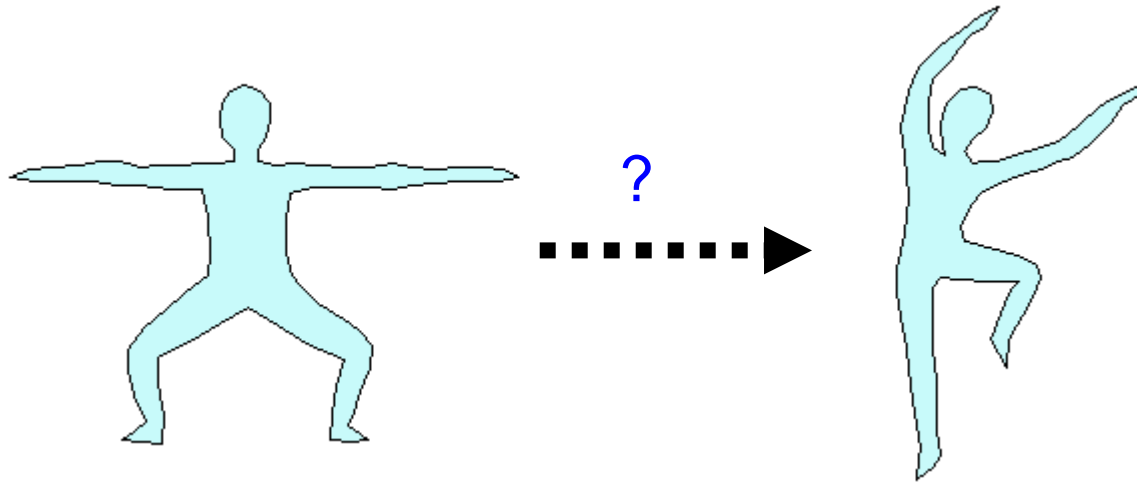
- Morphing: Binary Op
 - Given Object A and Object B, find $F(t)$



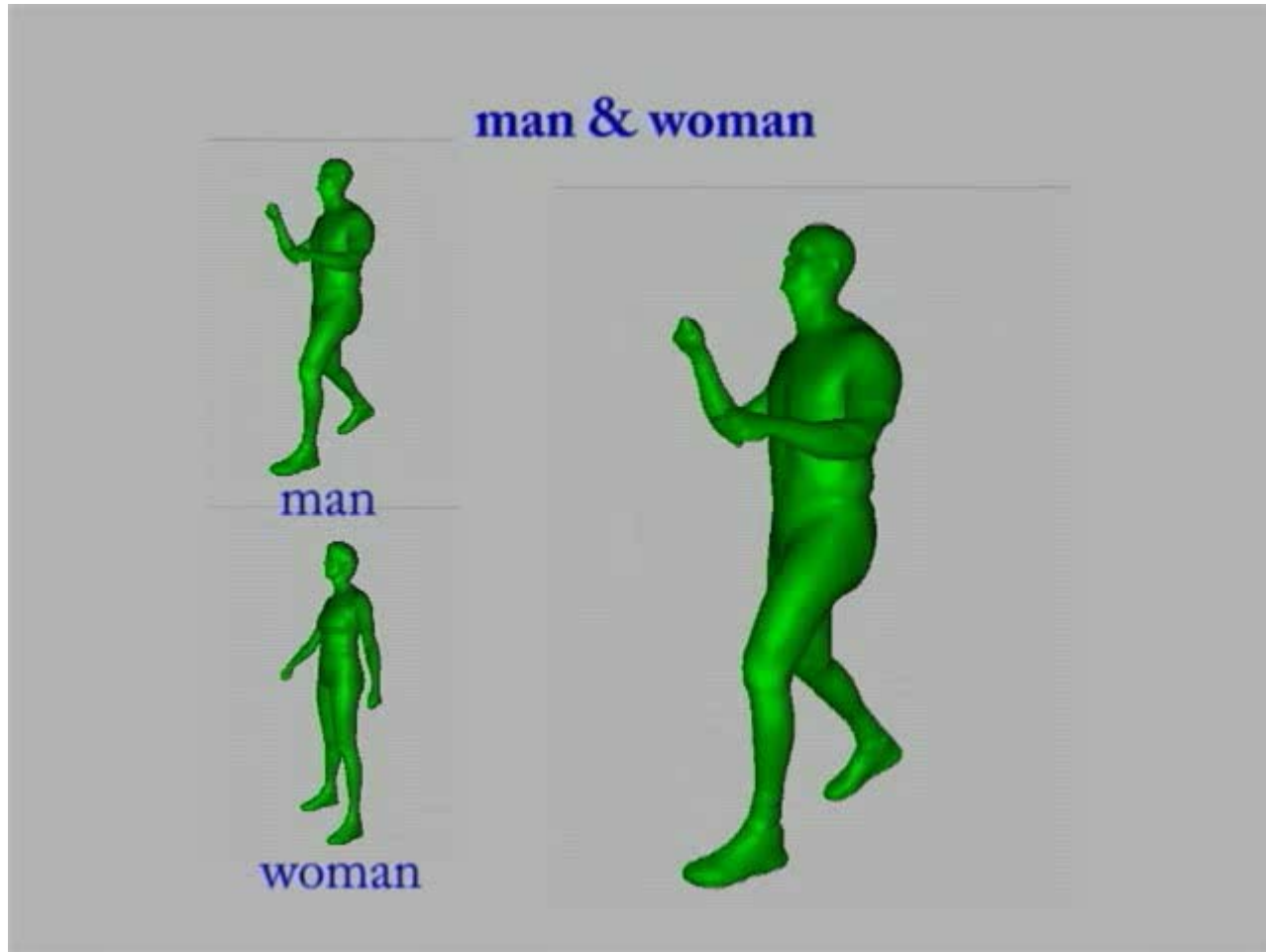
Warping Examples



Morphing Example



Morphing: Example - 1



Morphing: Example - 2

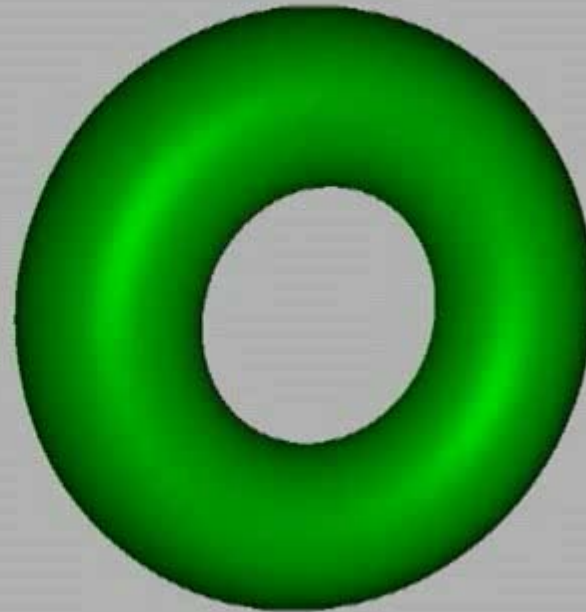
High genus morphing (torus & mug)



torus



mug



Applications

- Modeling
- Animation
- Special Effects
- Shape Analysis

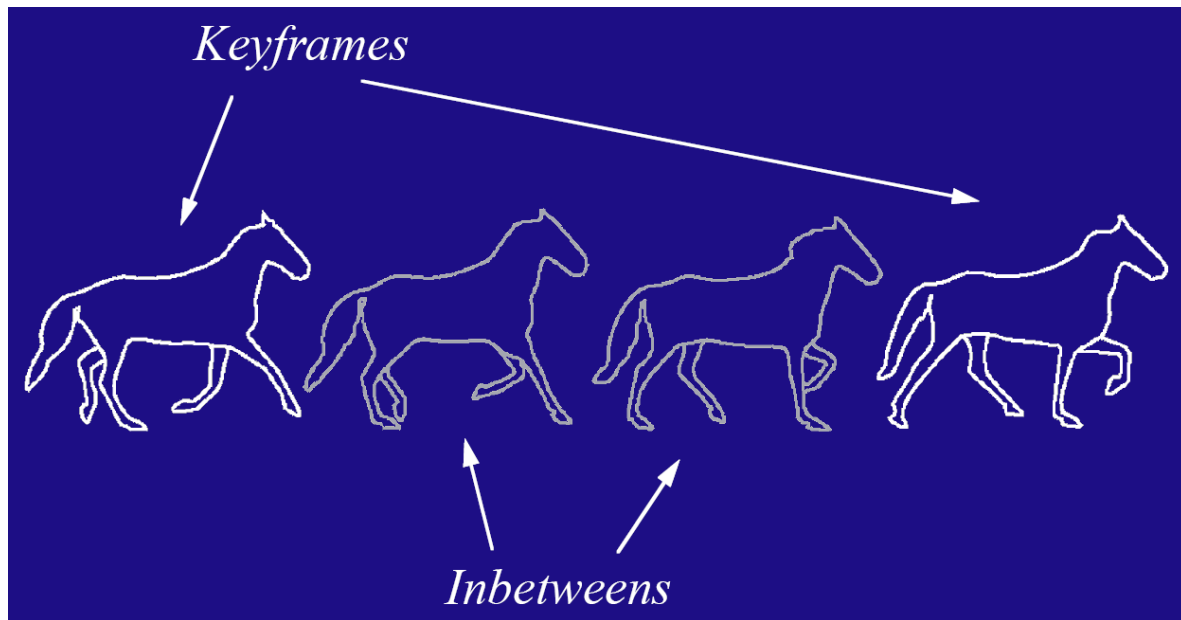
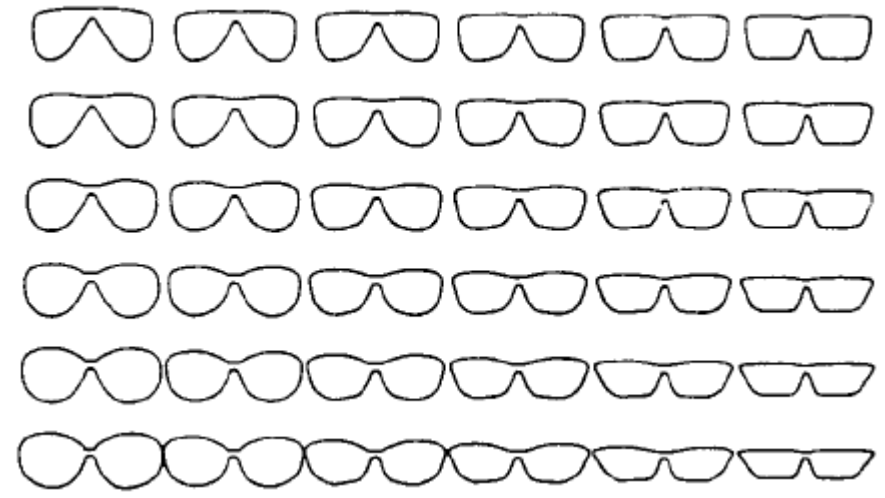
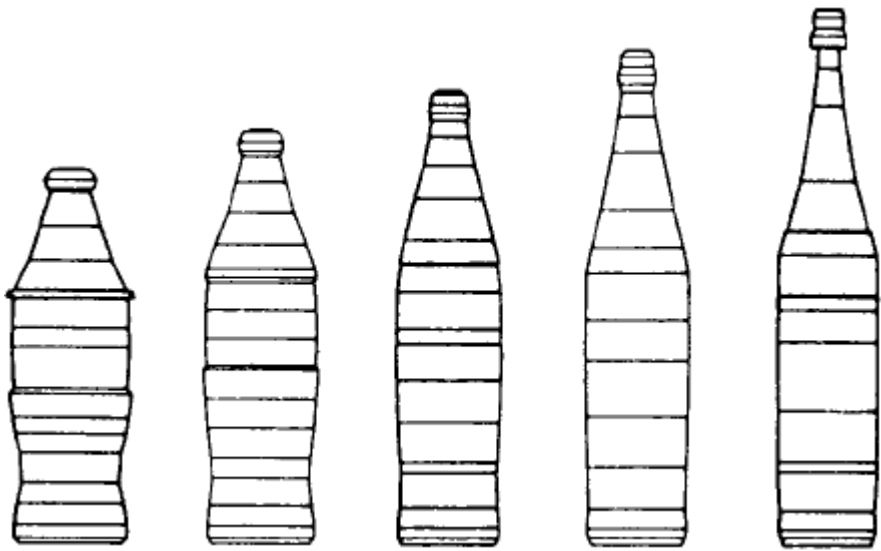
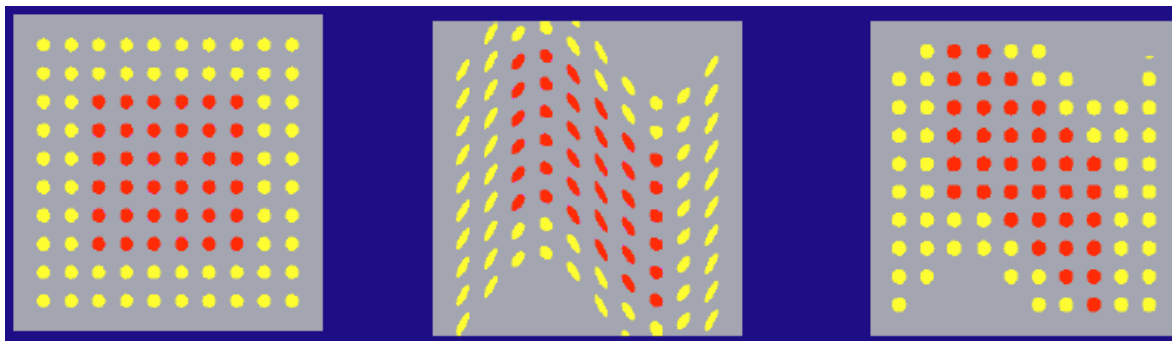


Image Warping and Morphing

Demo

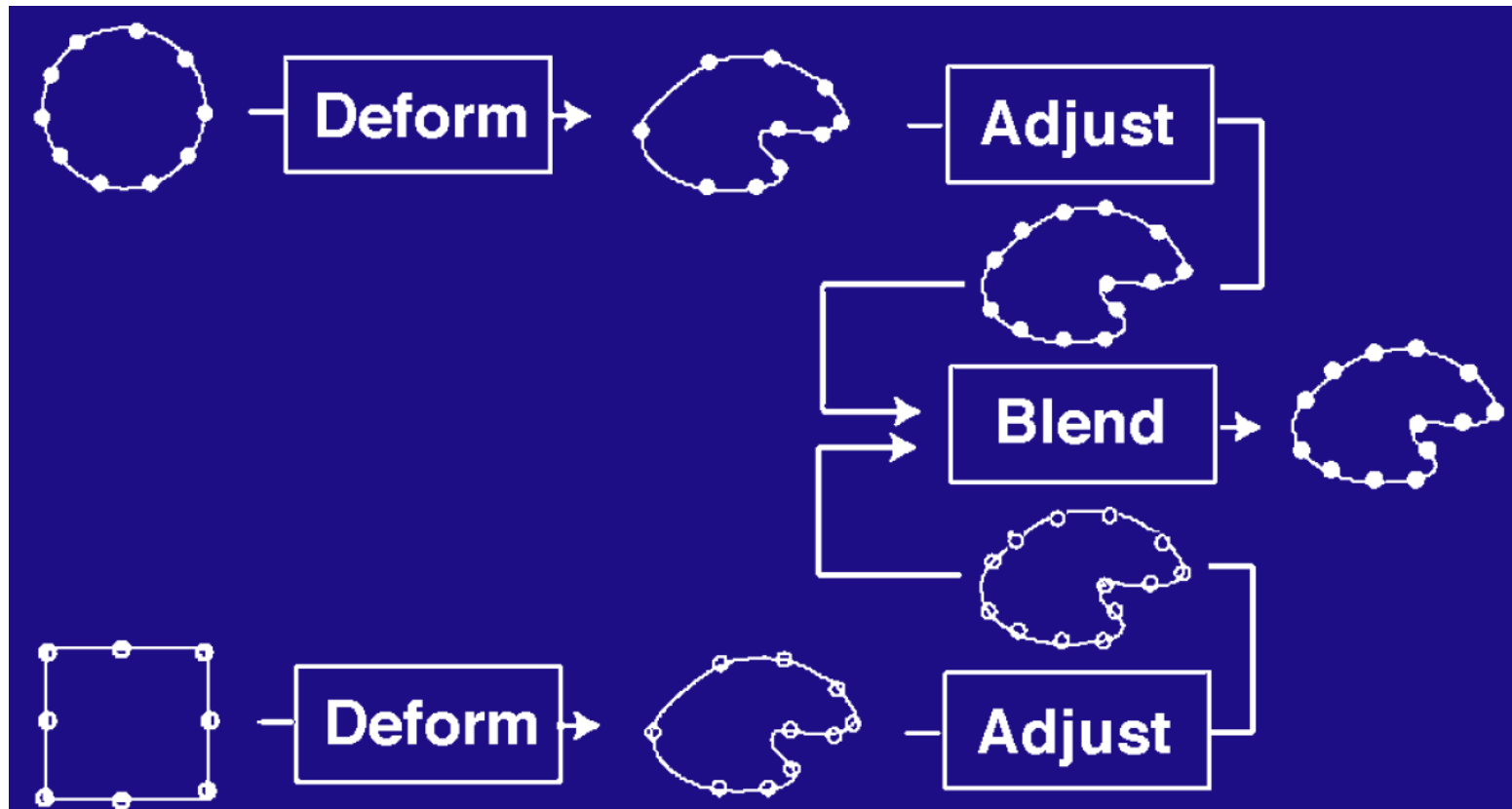
Warping Pipeline

- Geometry deformation
- Attribute generation

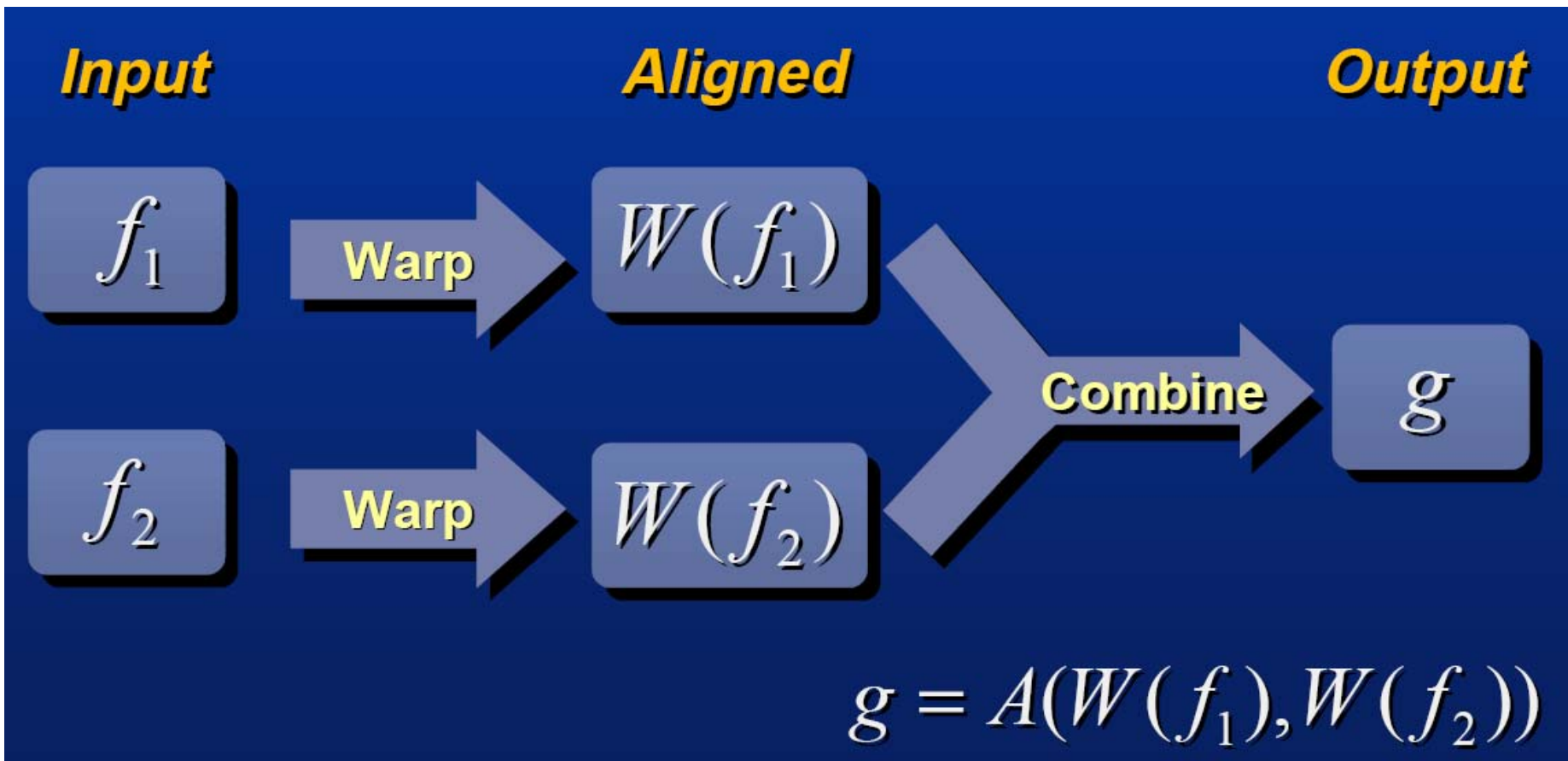


Morphing Pipeline

- Morphing = warping + blending



Morphing Pipeline



Cross Dissolve

- Linear color interpolation

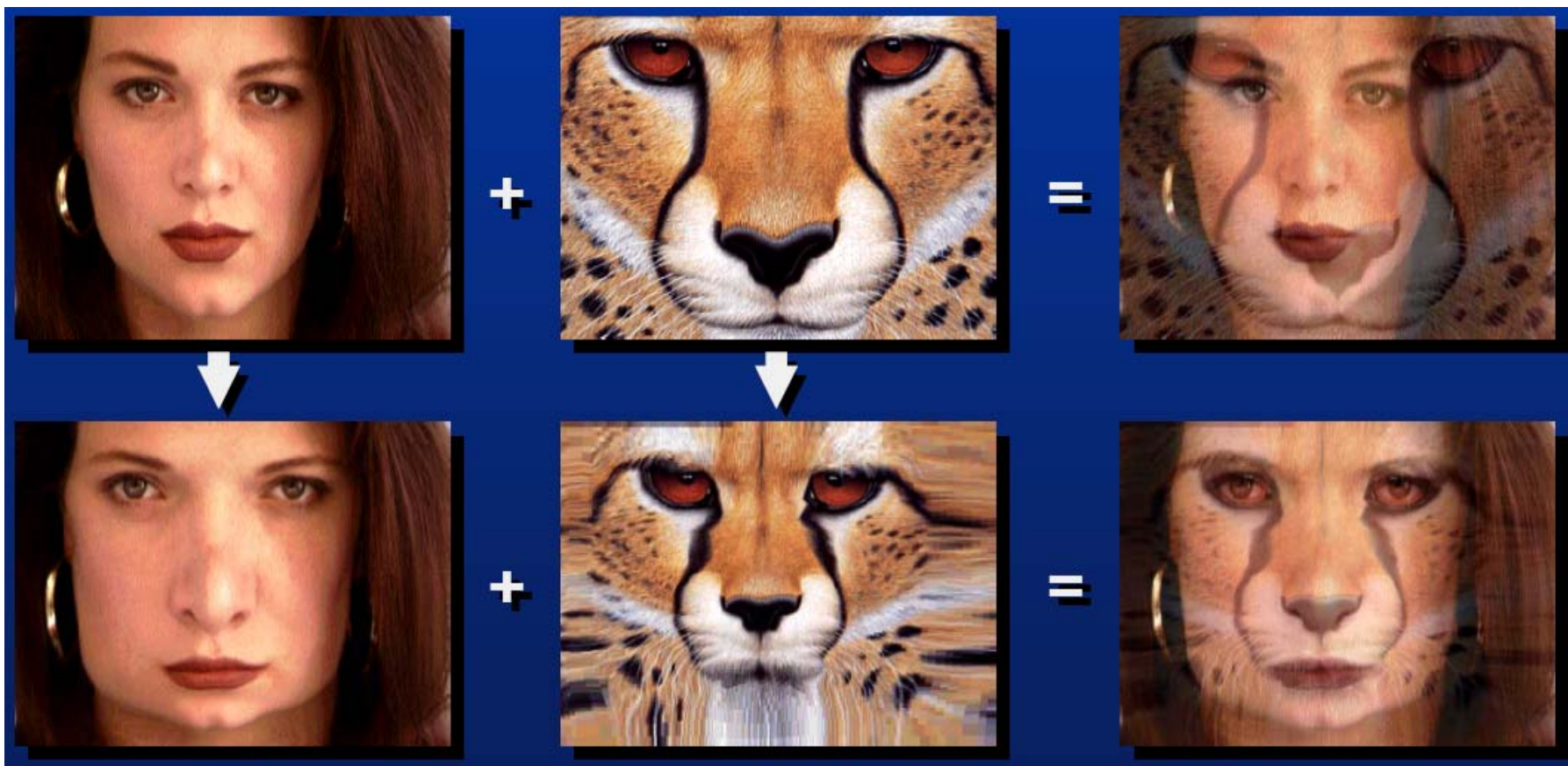


Feature Warping

- Shape warp + attribute blending

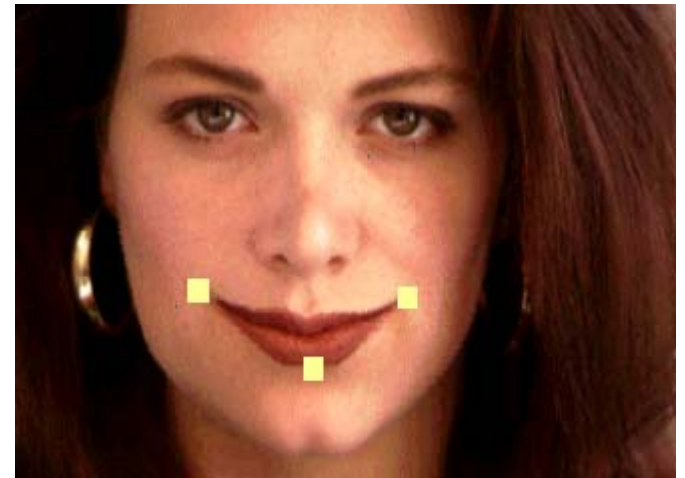
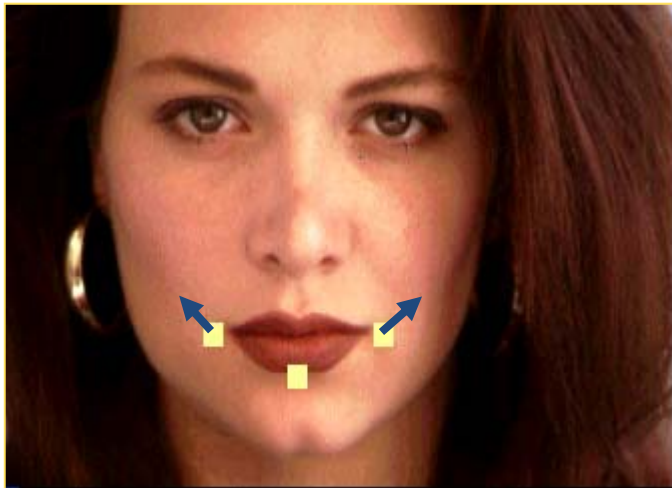


Aligning Features



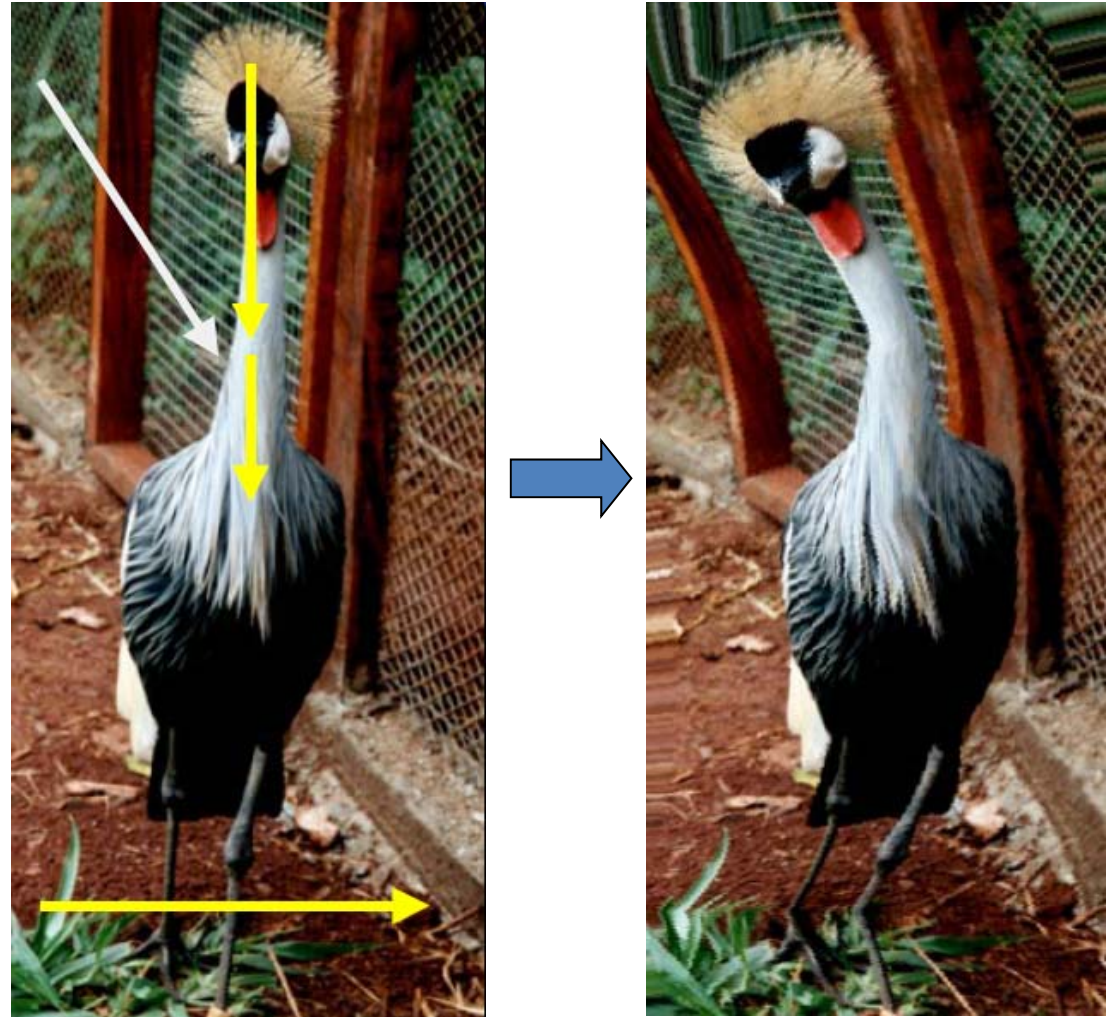
User Specifications: Warping

- Point features



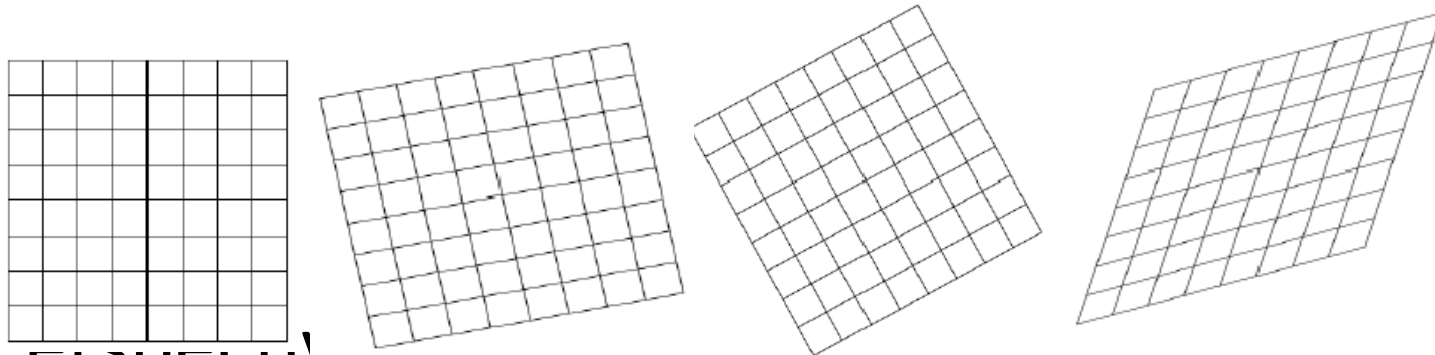
User Specifications: Warping

- Vector features

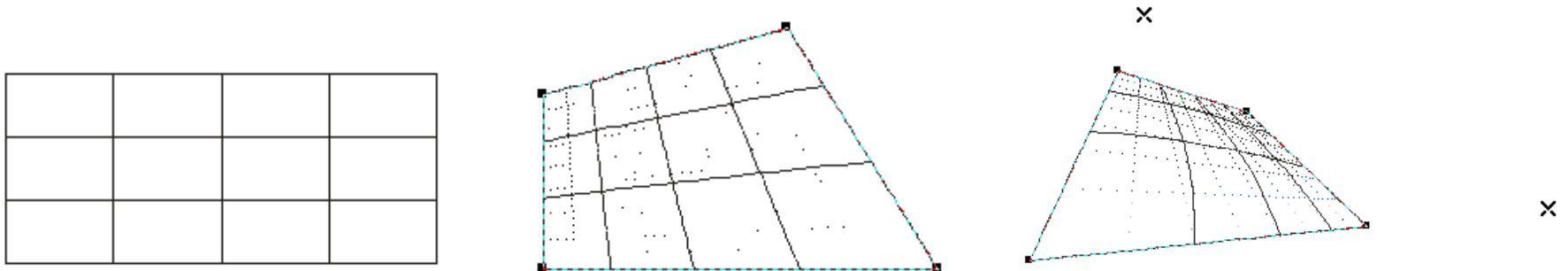


Simple Transformations

- Affine transformation

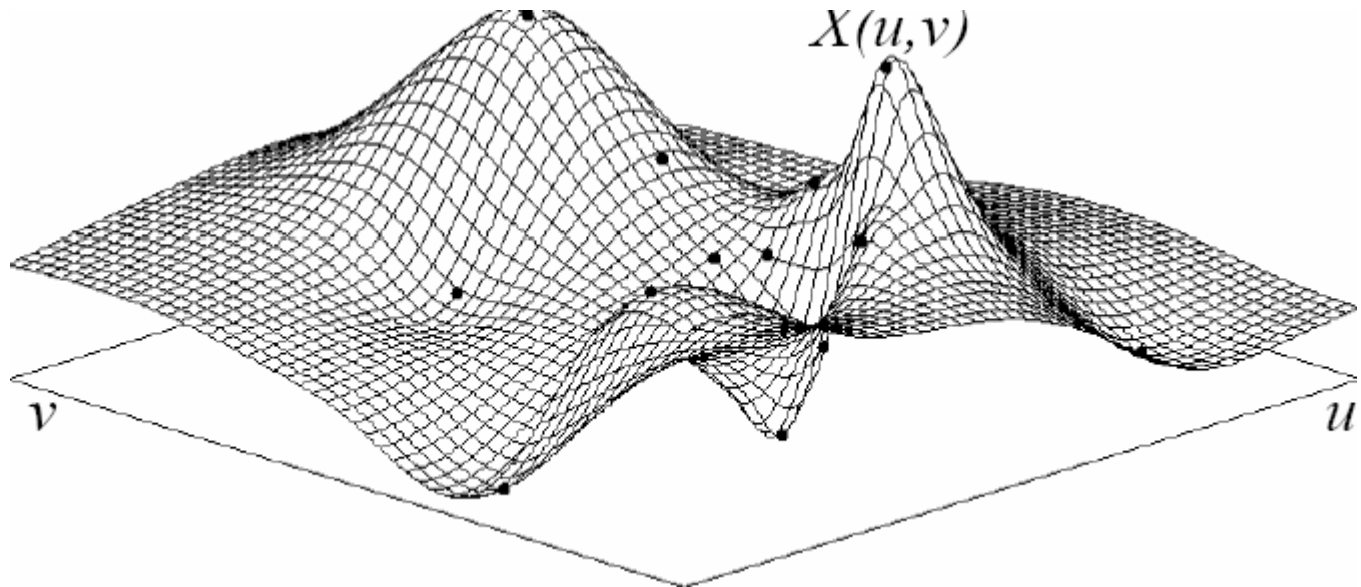


- Perspective transformation



Warping Problem

- Scattered point interpolation
 - Given the values on some points, compute the value on arbitrary other point



Interpolation Methods

- Inverted distance method
 - Shepard, 1965
- Spline surface fitting
- Thin-plate spline
- Finite element method
- ...

Radial Basis Functions

■ Affine + Radial:

$$W(p) = A(p) + R(p) \quad A(p) = Mp + b$$

$$R(p) = (R_x(p), R_y(p))$$

■ Radial basis function:

- ◆ Function of distance to anchors

$$g : R \rightarrow R$$

Radial Basis Functions

- Radial component:

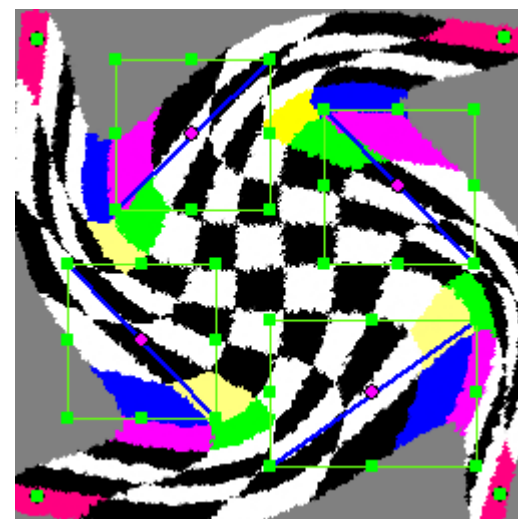
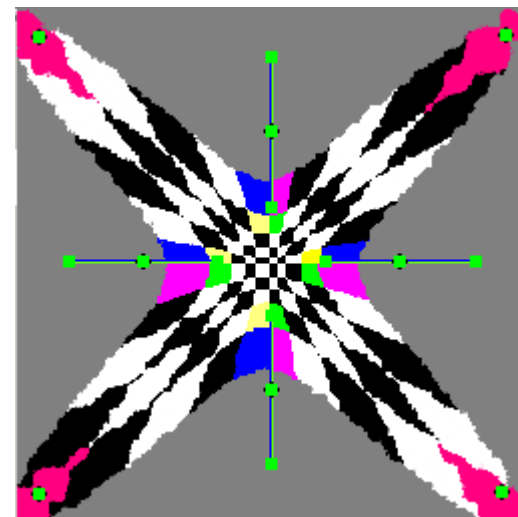
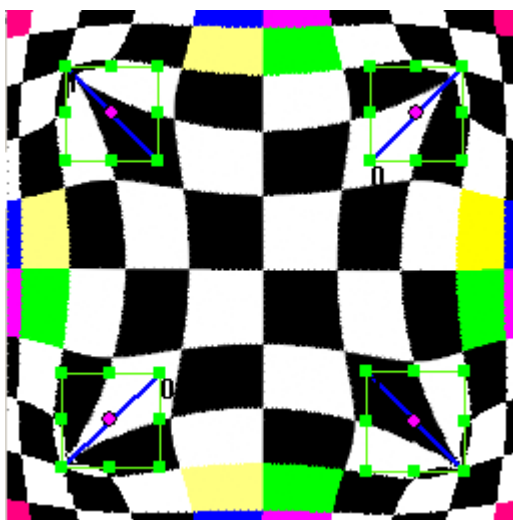
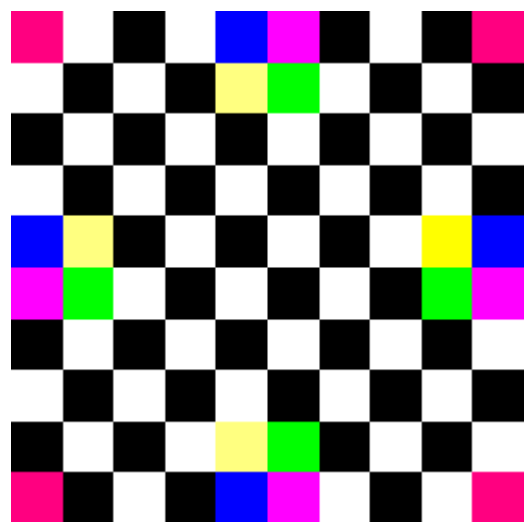
$$R_i(p) = \sum_{i=1}^N a_i g(\|p - p_i\|)$$

- Given specification with N anchors p_i, p_i'

- ◆ $W(p_i) = p_i'$

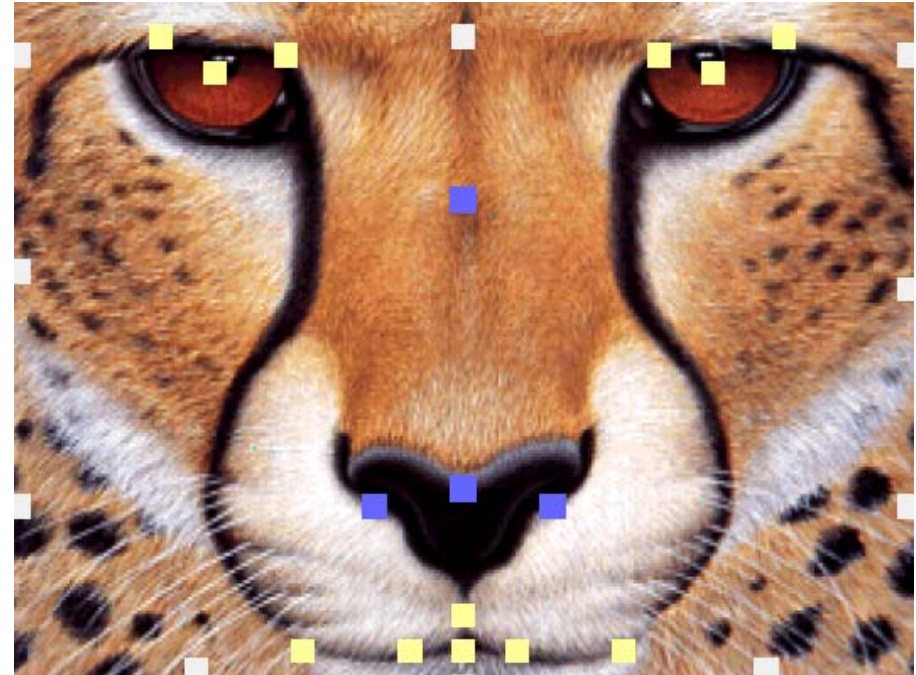
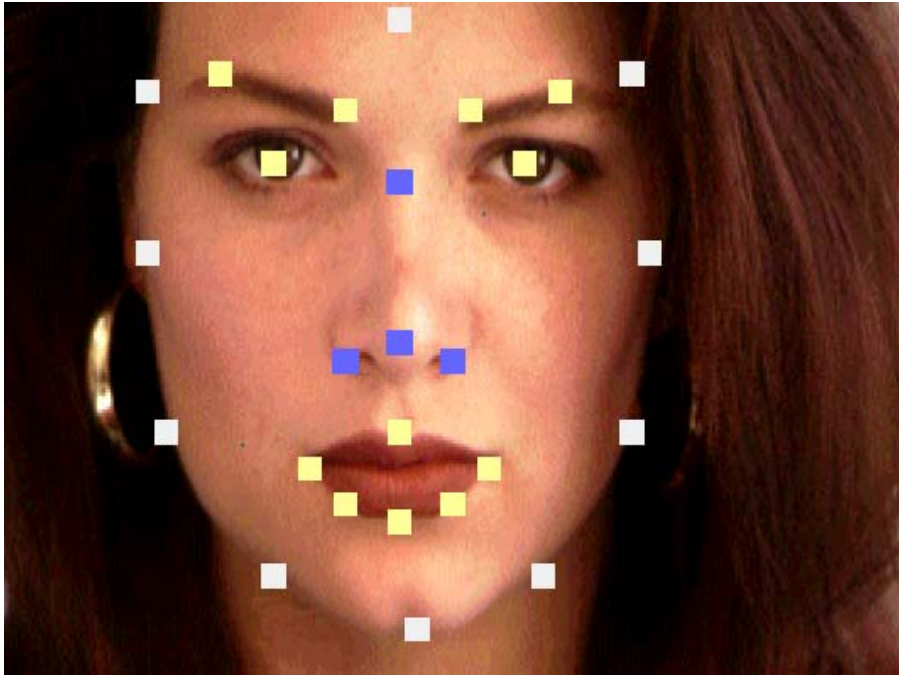
- ◆ Solve linear system to obtain coefficients a_i

RBF Examples



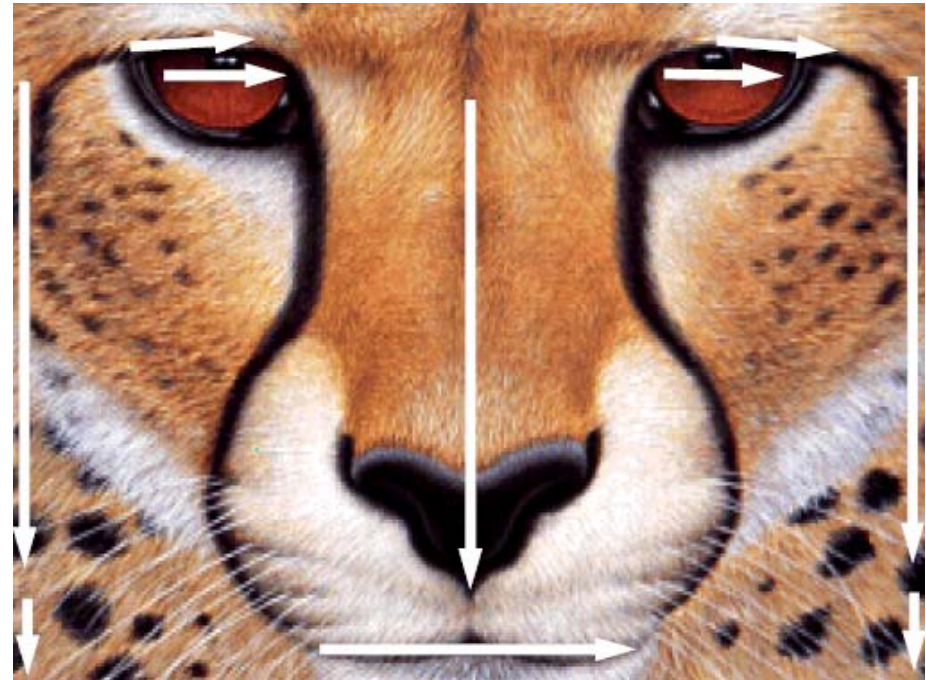
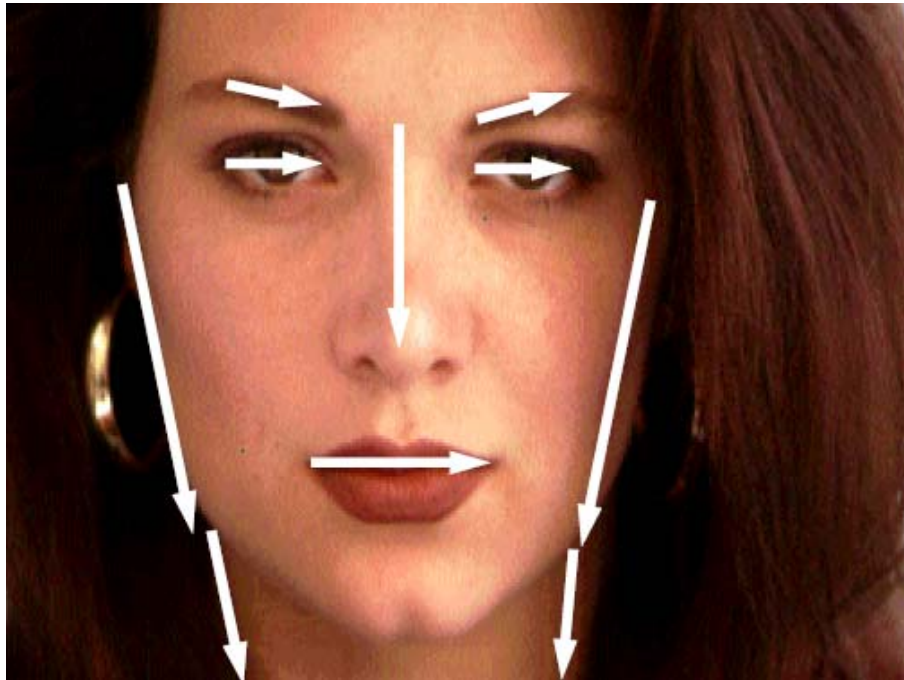
User Specifications: Morphing

- Point features



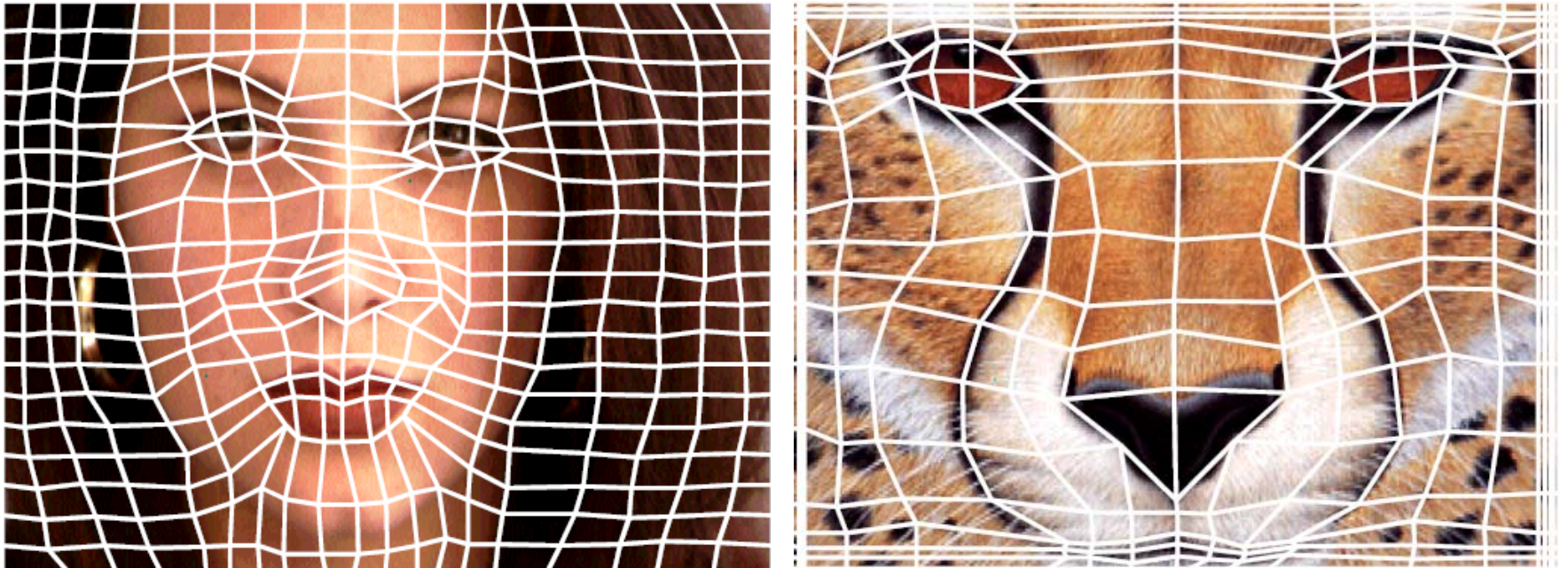
User Specifications: Morphing

- Vector features



User Specifications: Morphing

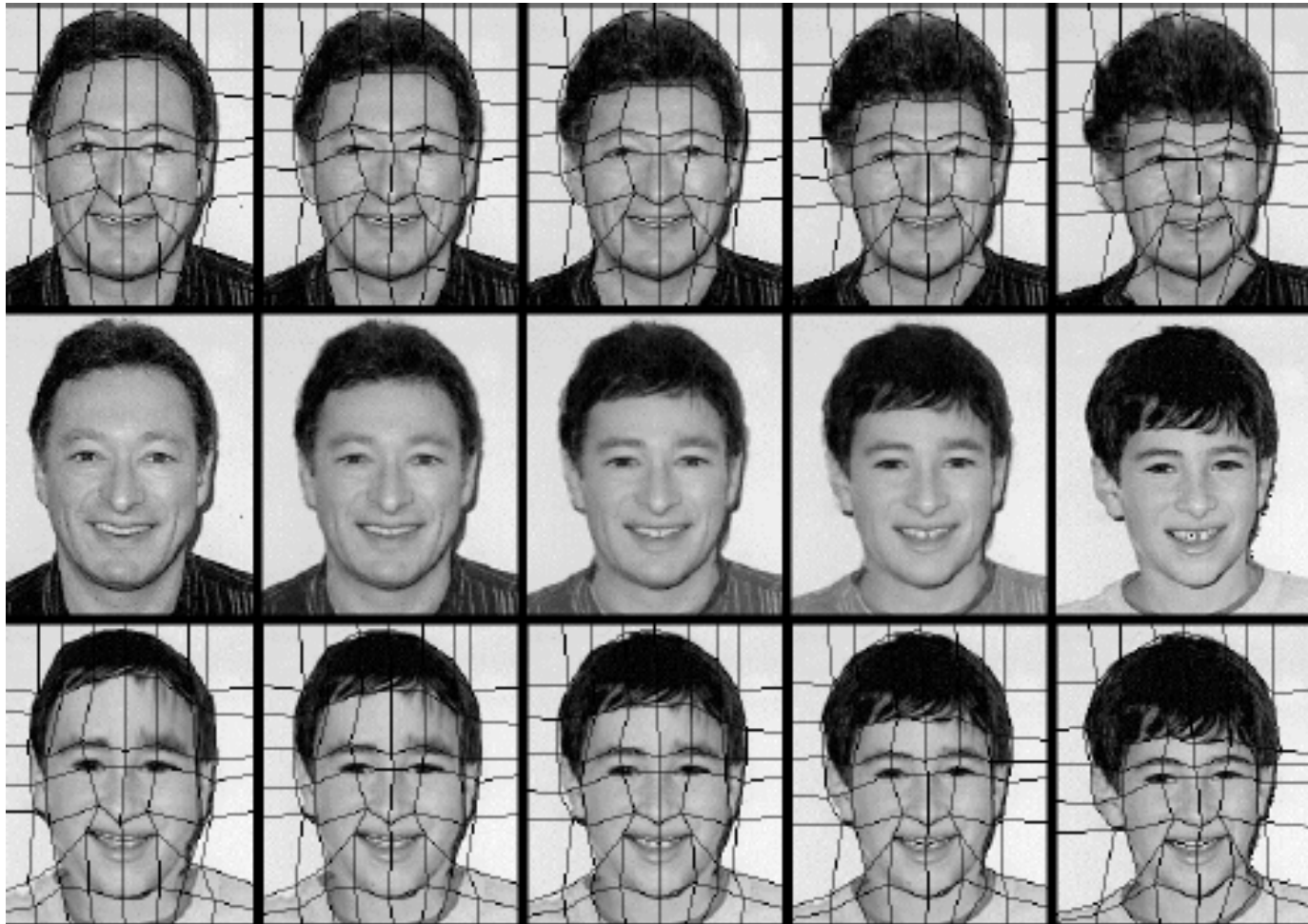
- Partition



Morphing Example



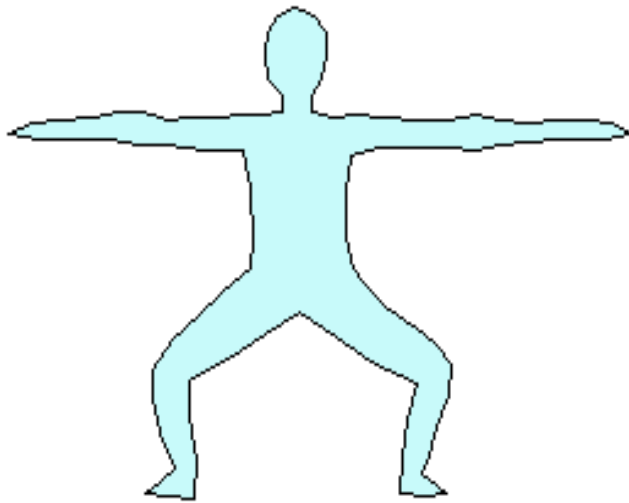
Morphing Example



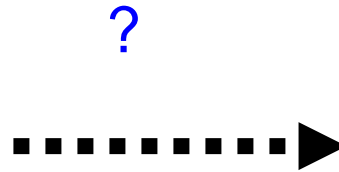
Discussions

Polygon Morphing

Problem



Source polygon
shape



Target polygon
shape

Two Subproblems

- Correspondence problem
 - find a correspondence between vertices of the two shapes
- Path problem
 - find paths that the corresponding vertices traverse during the morphing process
- Dependent on each other

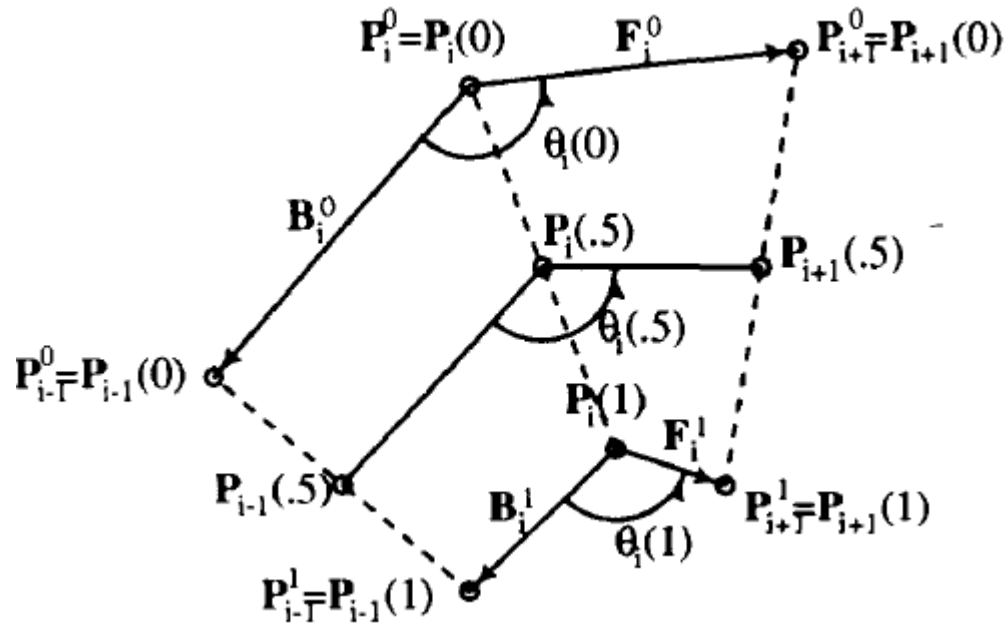
1. Correspondence Problem

Matching Problem

Physically Based Method

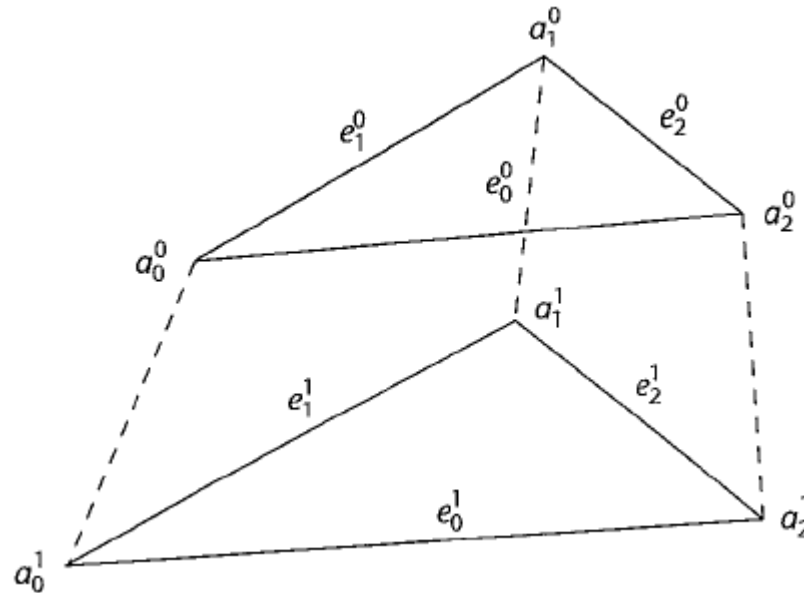
- Stretching work
- Bending work

Sederberg et al. 1992



Fuzzy Approach

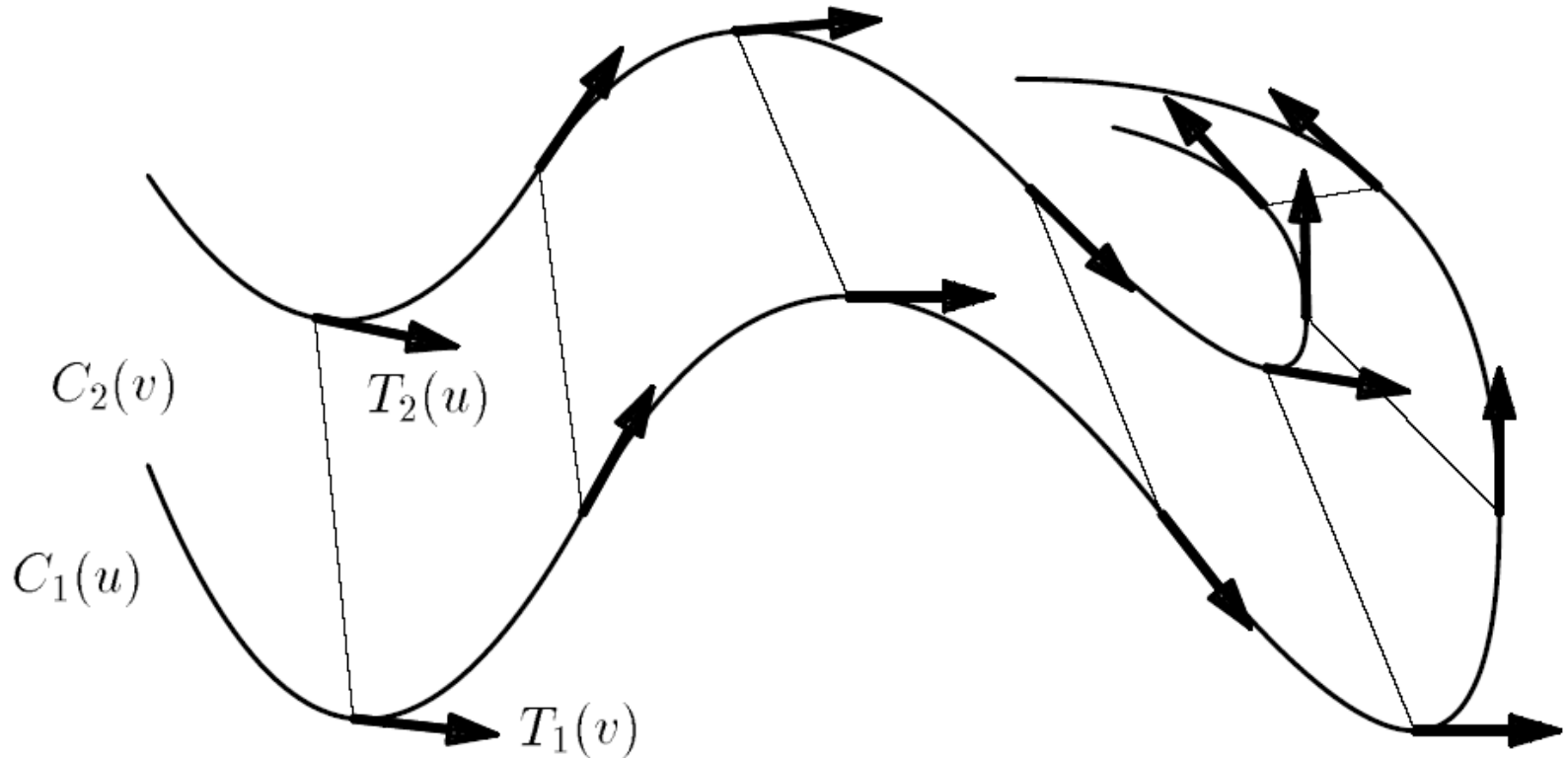
Zhang 1996



$$\begin{aligned} \text{sim}_t = & w_1 \times \left(1 - \frac{|e_1^0 \times e_2^1 - e_1^1 \times e_2^0|}{e_1^0 \times e_2^1 + e_1^1 \times e_2^0} \right) \\ & + w_2 \times \left(1 - \frac{|a_1^0 - a_1^1|}{360 \text{ deg.}} \right) \end{aligned}$$

Curve Matching

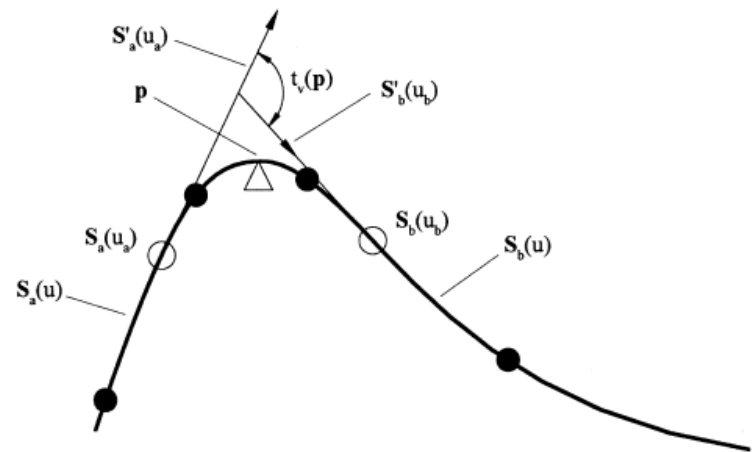
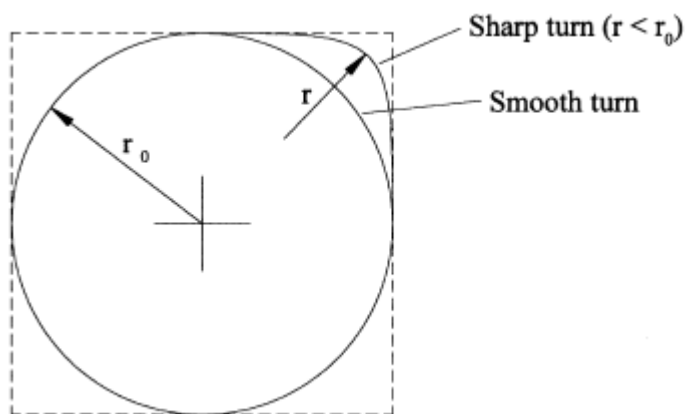
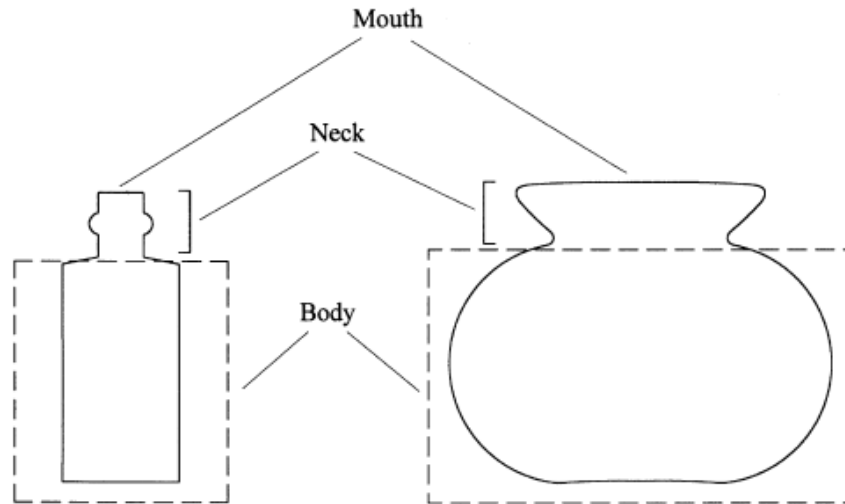
Cohen et al. 1997



$$\max_{v(u)} \int_{u_0}^{u_1} \frac{\langle T_1(u) \times (C_1(u) - C_2(v(u))), T_2(v(u)) \times (C_1(u) - C_2(v(u))) \rangle}{\| (C_1(u) - C_2(v(u))) \|^2} du$$

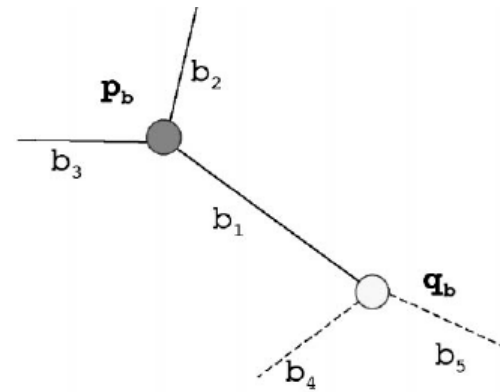
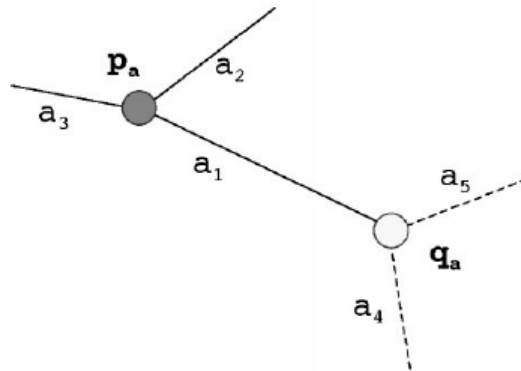
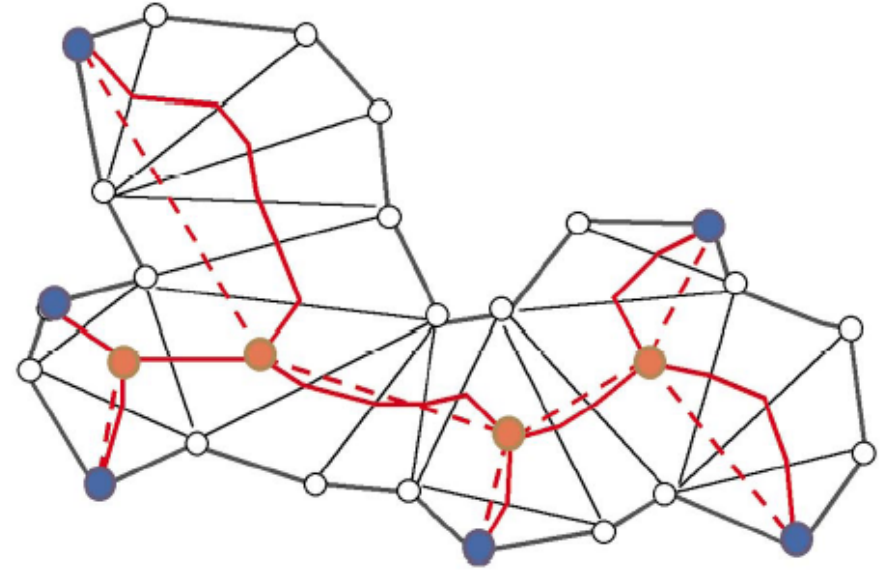
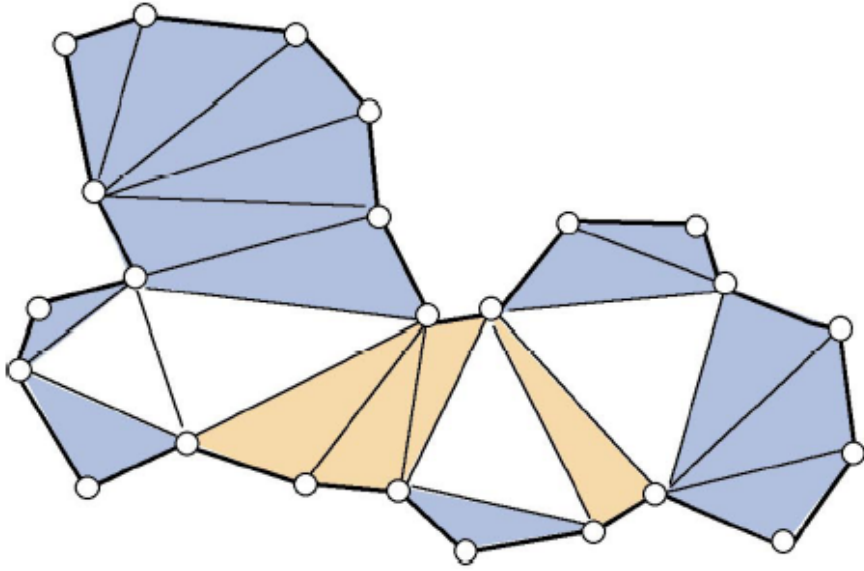
Feature Based Method

Hui and Li 1998



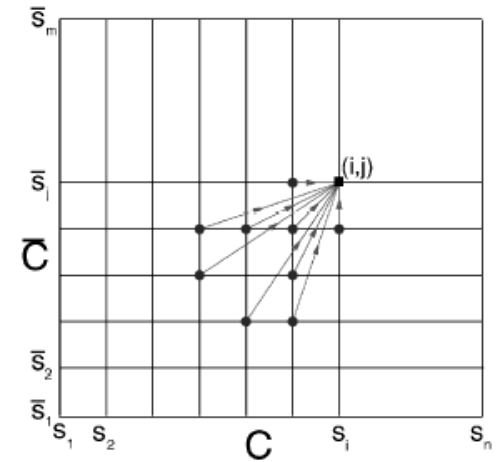
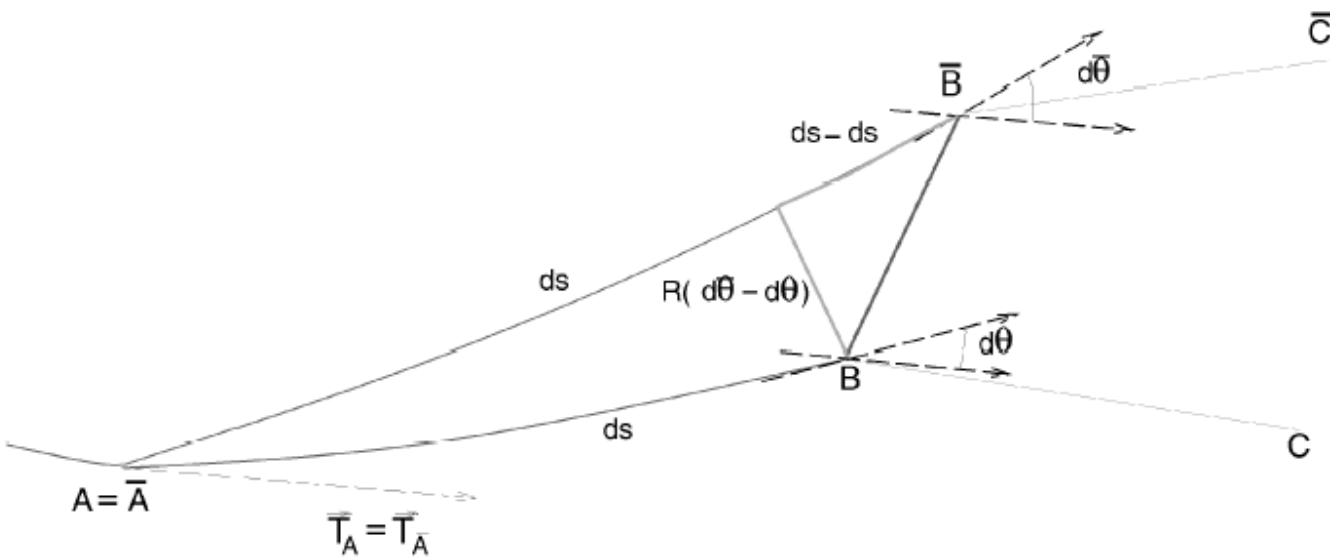
Approximated Skeleton

Mortara et al. 2001



Curve Aligning

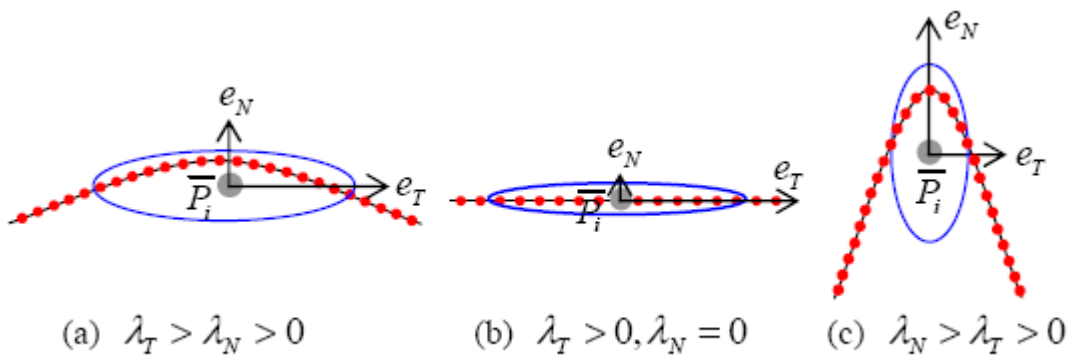
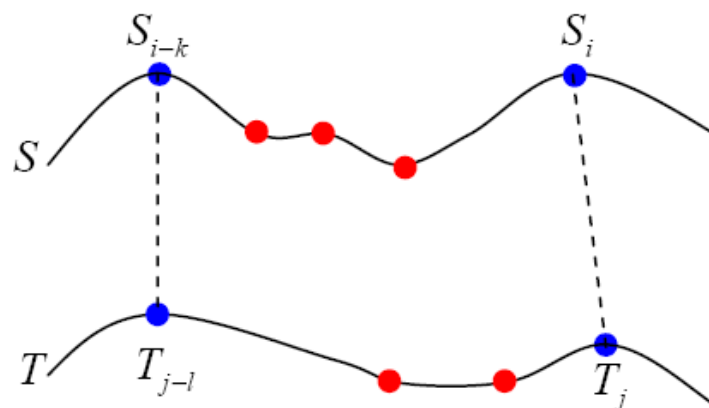
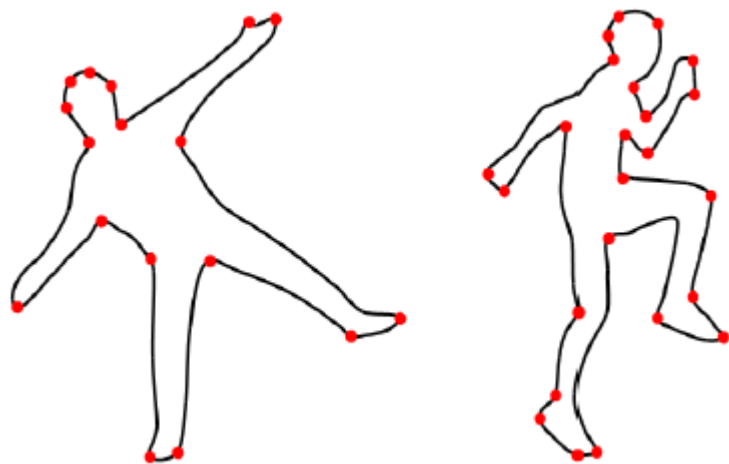
Sebastian et al. 2003



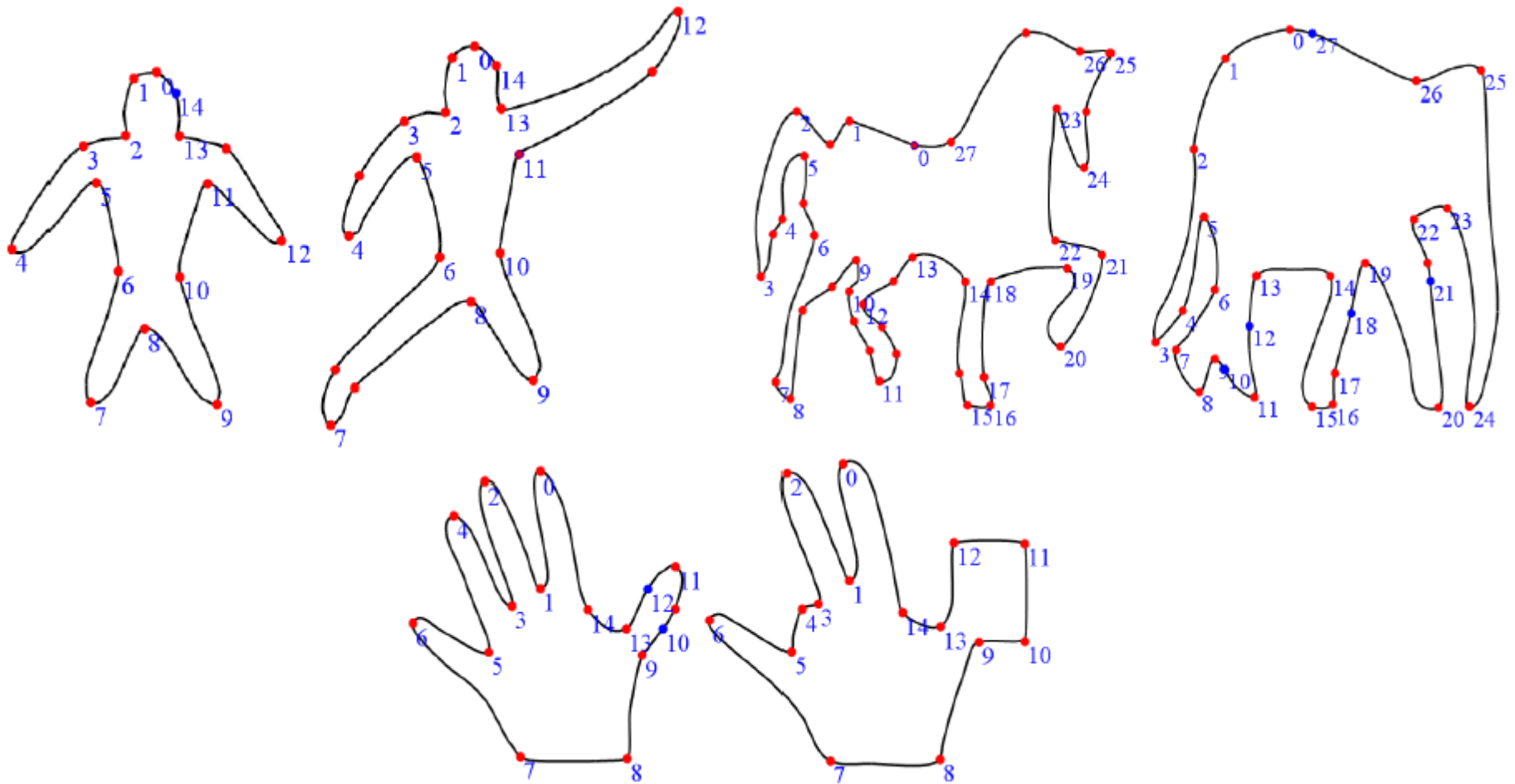
$$\mu[g] = \int_C \left| \frac{\partial}{\partial s} (\bar{C}(\bar{s}) - C(s)) \right|^2 ds + R \int_C (\kappa(s) - \bar{\kappa}(\bar{s}))^2 ds,$$

Perceptually Based Method

Liu et al. 2004



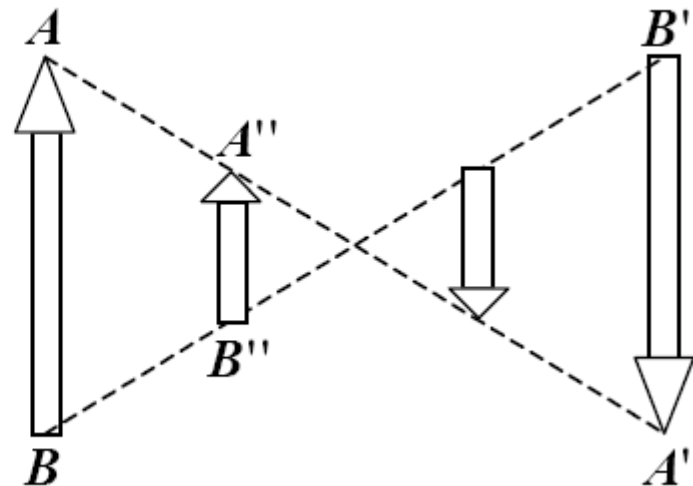
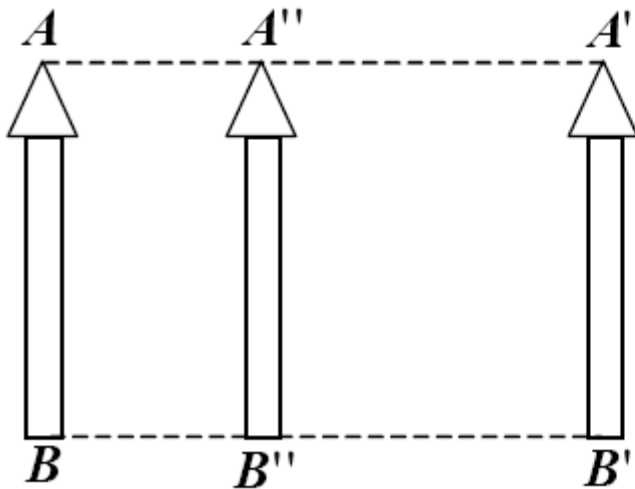
Matching Results



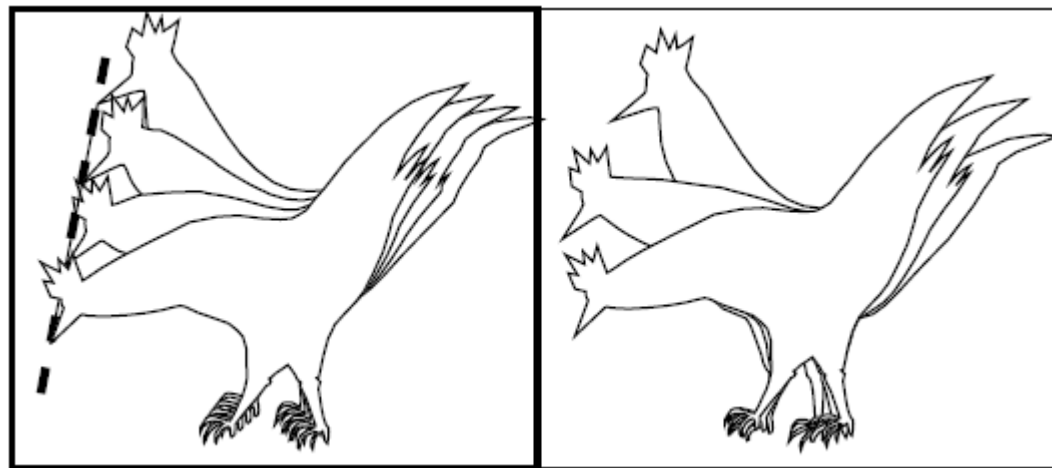
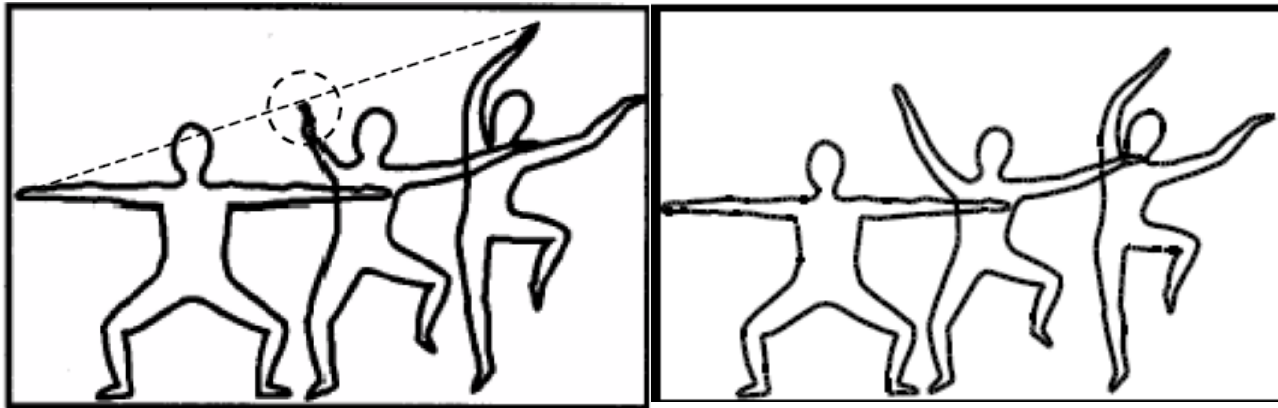
2. Path Problem

Linear Interpolation

- Simple and easy
- Drawbacks
 - Shrinkage



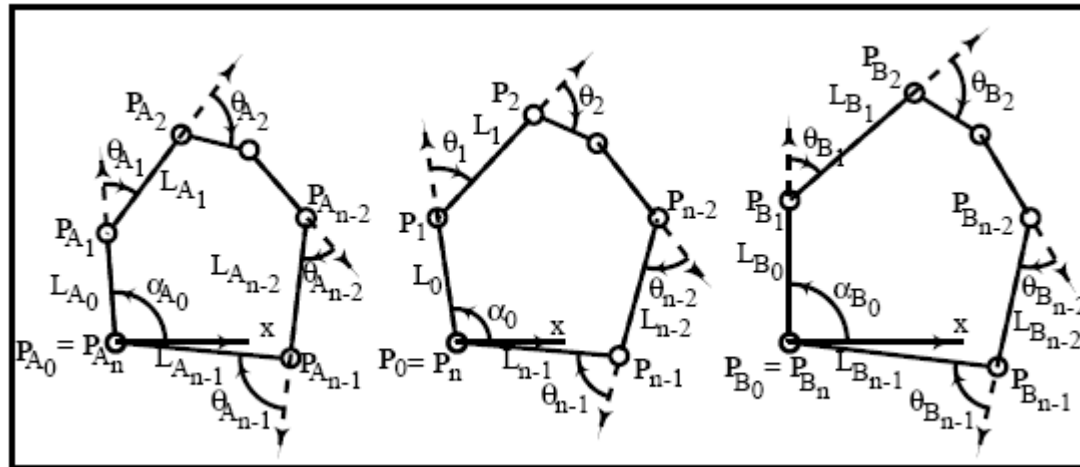
Unnatural Results in Linear Interpolation



Intrinsic Approach

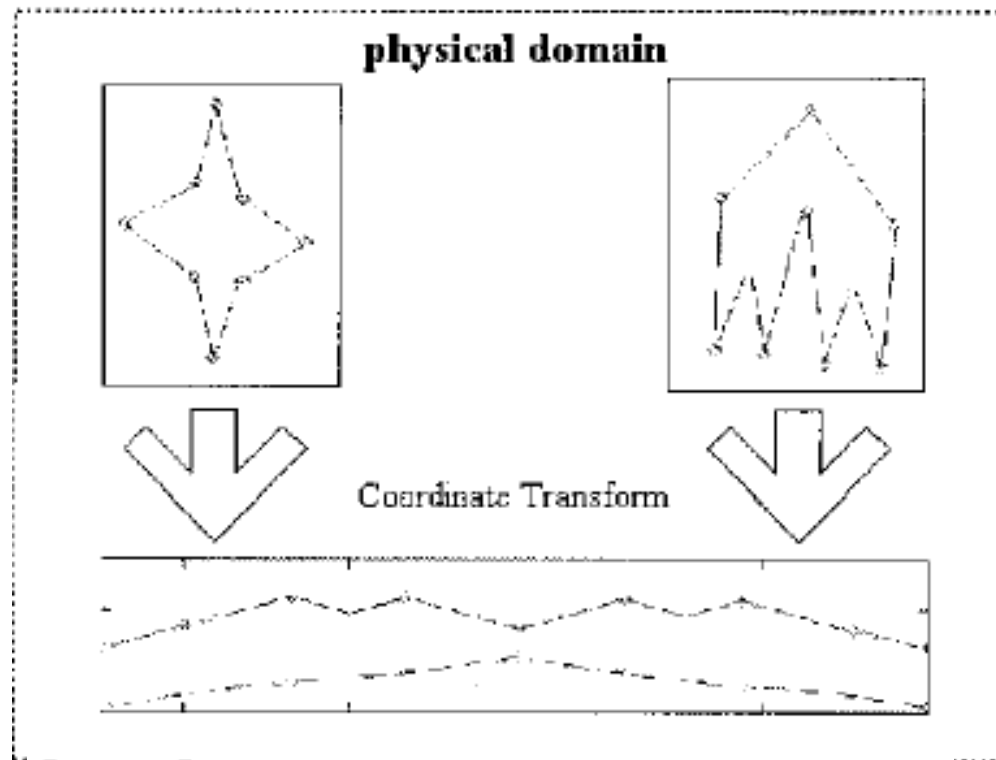
- Intrinsic variables
 - edge lengths
 - turning angles

Sederberg et al. 1993



Fourier Approach

Chen et al. 2001

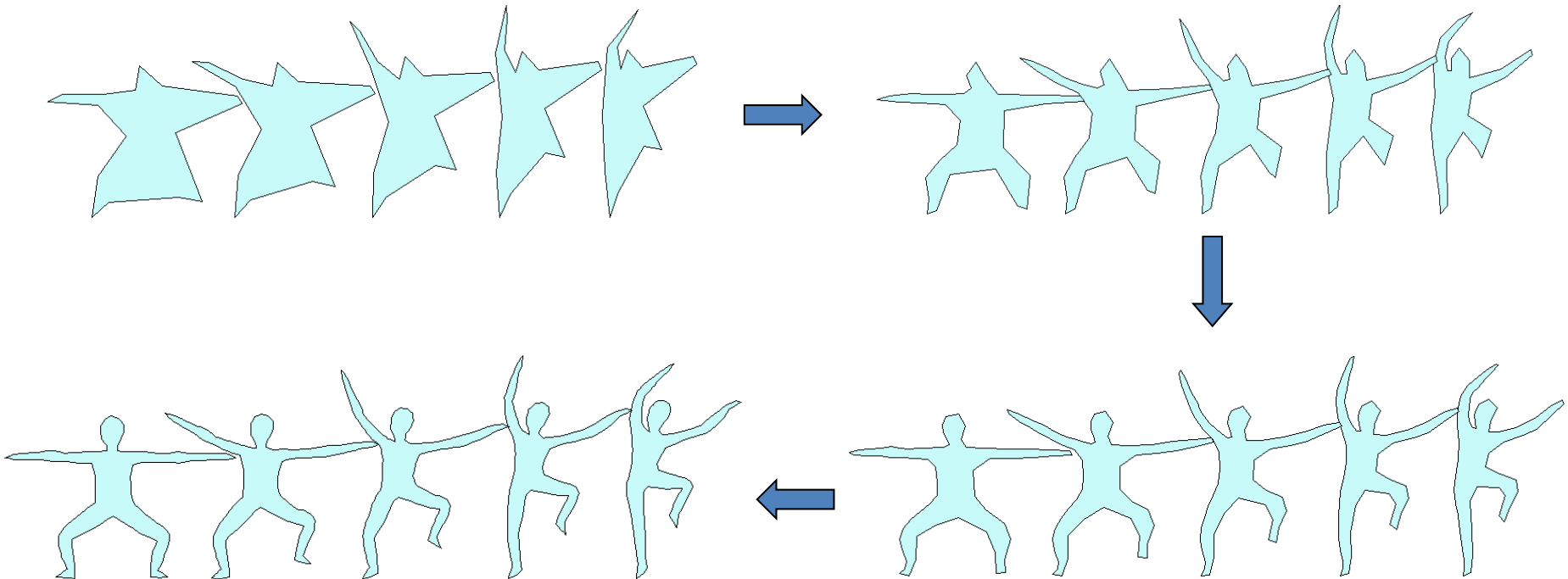


$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{k=1}^{\infty} \begin{bmatrix} a_k & b_k \\ c_k & d_k \end{bmatrix} \begin{bmatrix} \cos(2\pi kt) \\ \sin(2\pi kt) \end{bmatrix}$$

Wavelet Approach

- Wavelet decomposition

Zhang et al. 2000



Warp Guided Method

- Warp + linear interpolation

Camel et al. 1997



a



c



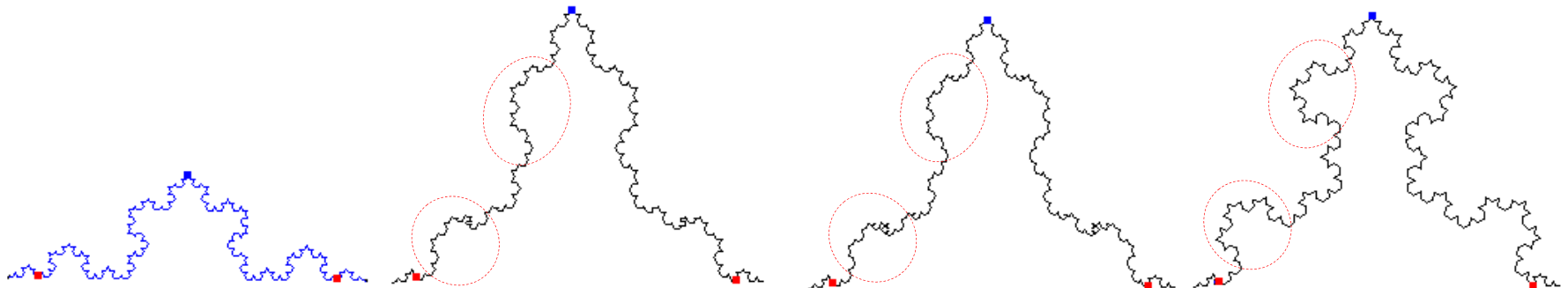
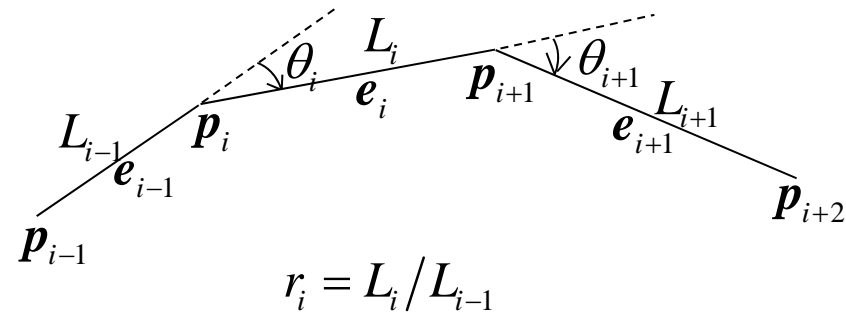
b



Scale-Invariant Intrinsic Approach

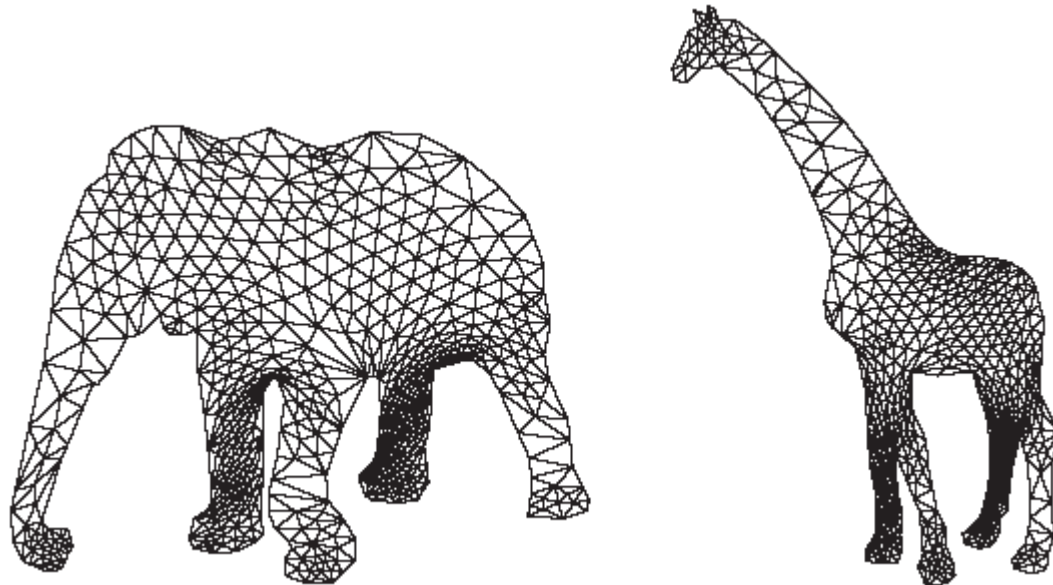
Ren et al. 2006

- Scaling-invariant intrinsic variables



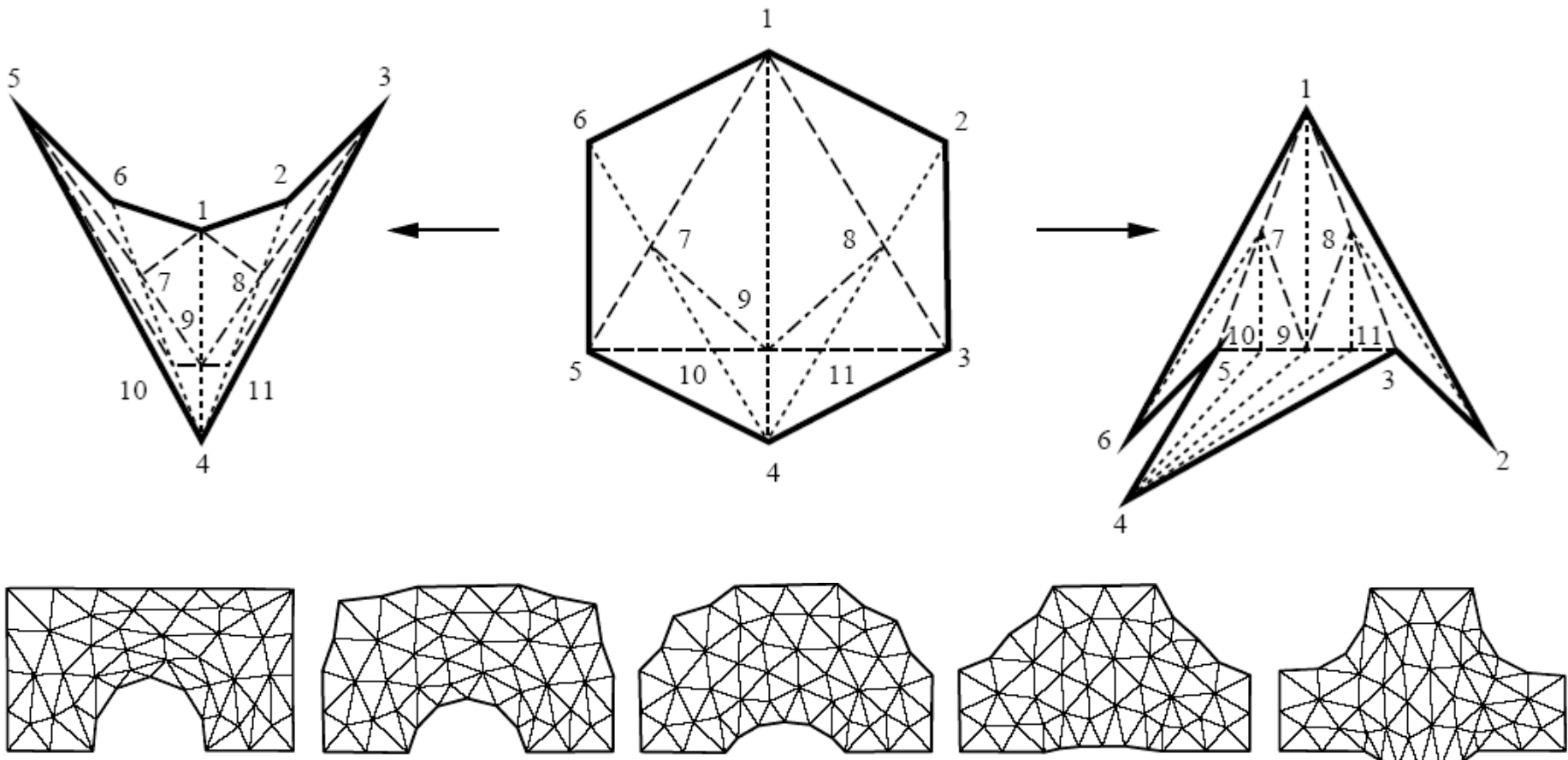
Interior Based Approach

- Based on compatible triangulation
 - [Gotsman and Surazhsky, 1999-2001]
 - As-rigid-as-possible [Alexa et al. 2000]

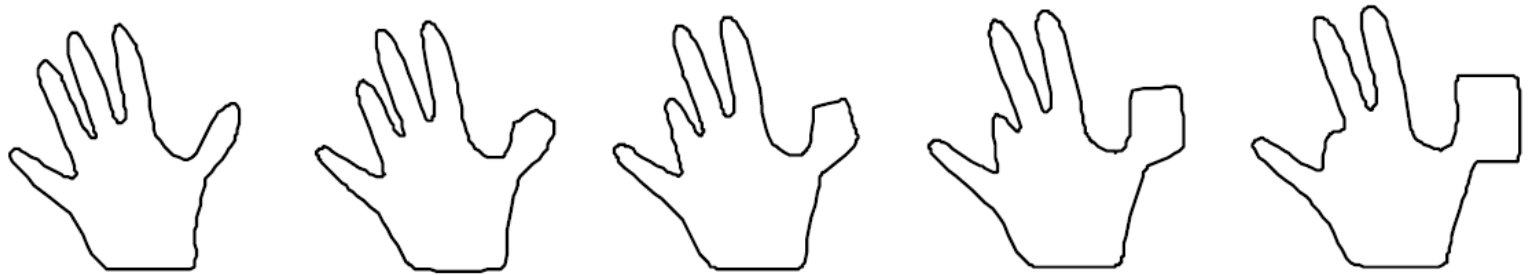


Compatible Triangulation

[Aronov et al. 1994]

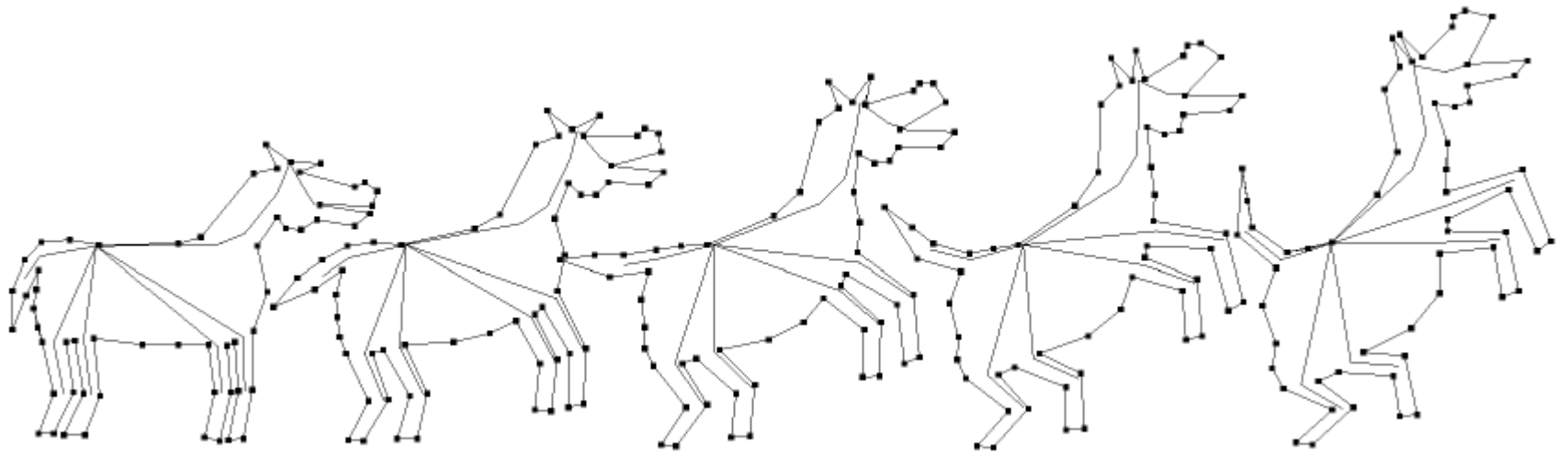


Examples



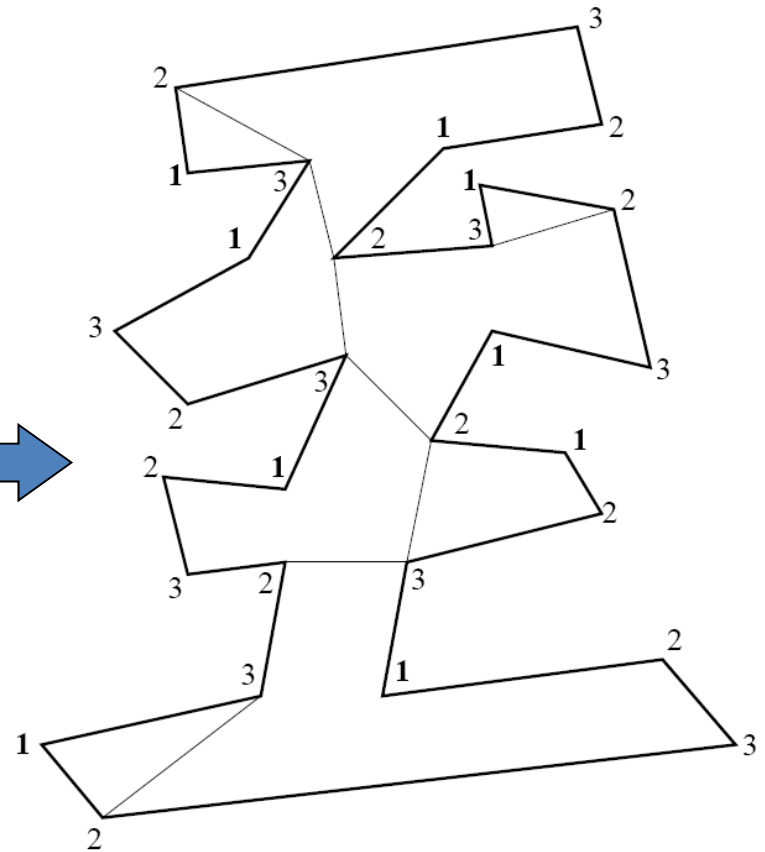
Star Skeleton Representation

[Shapira et al. 1995]

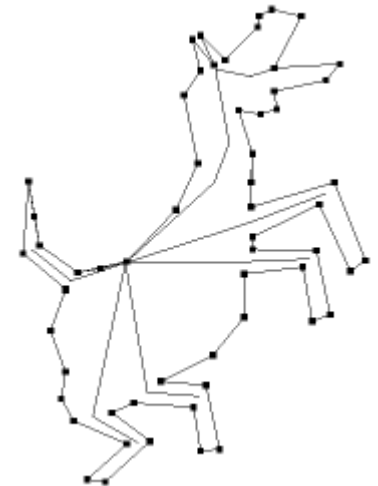
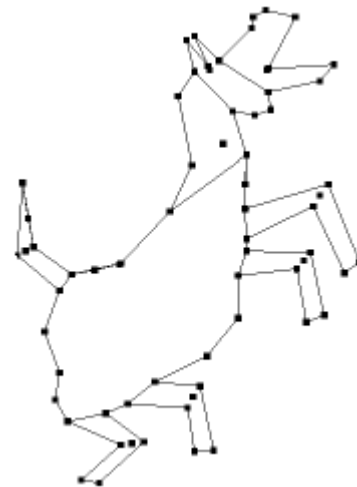
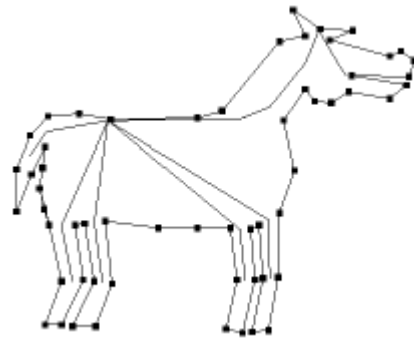
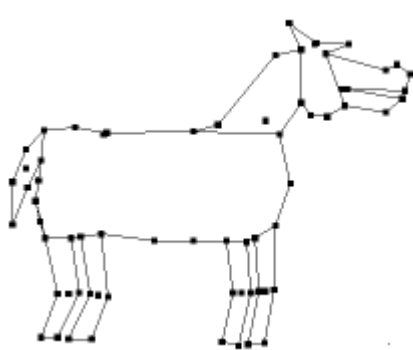
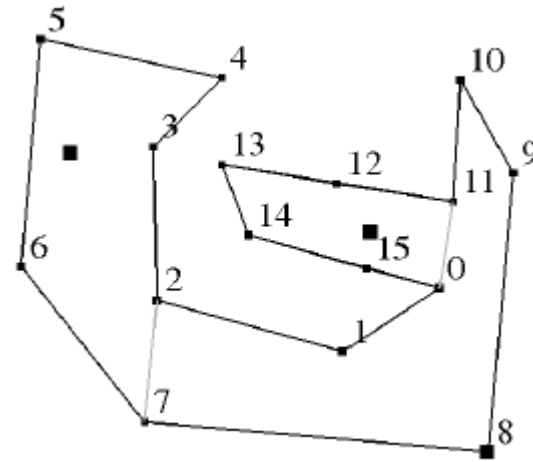
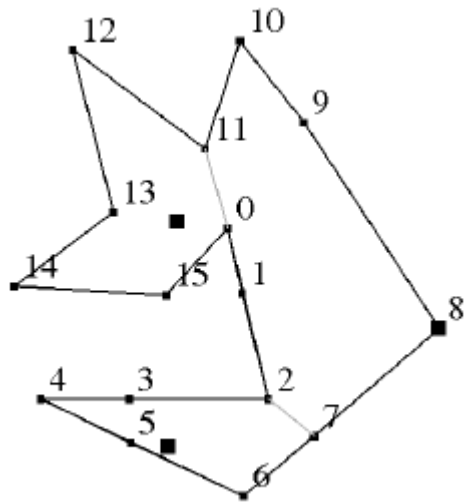


Star-Shaped Polygon Decomposition

- Star shaped, star points

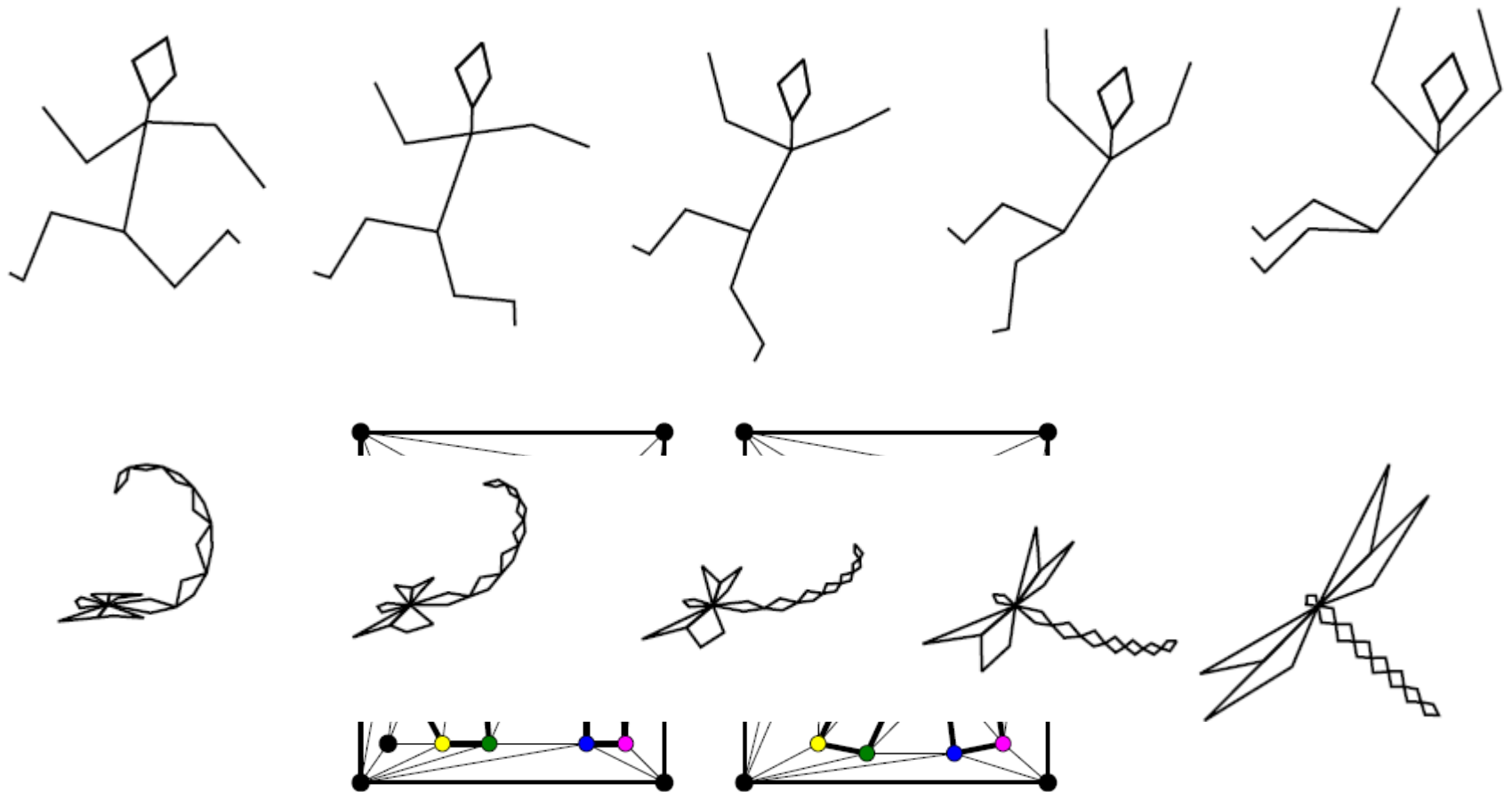


Compatible Star Decompositions



Morphing Stick Figures

[Surazhsky et al. 2001]



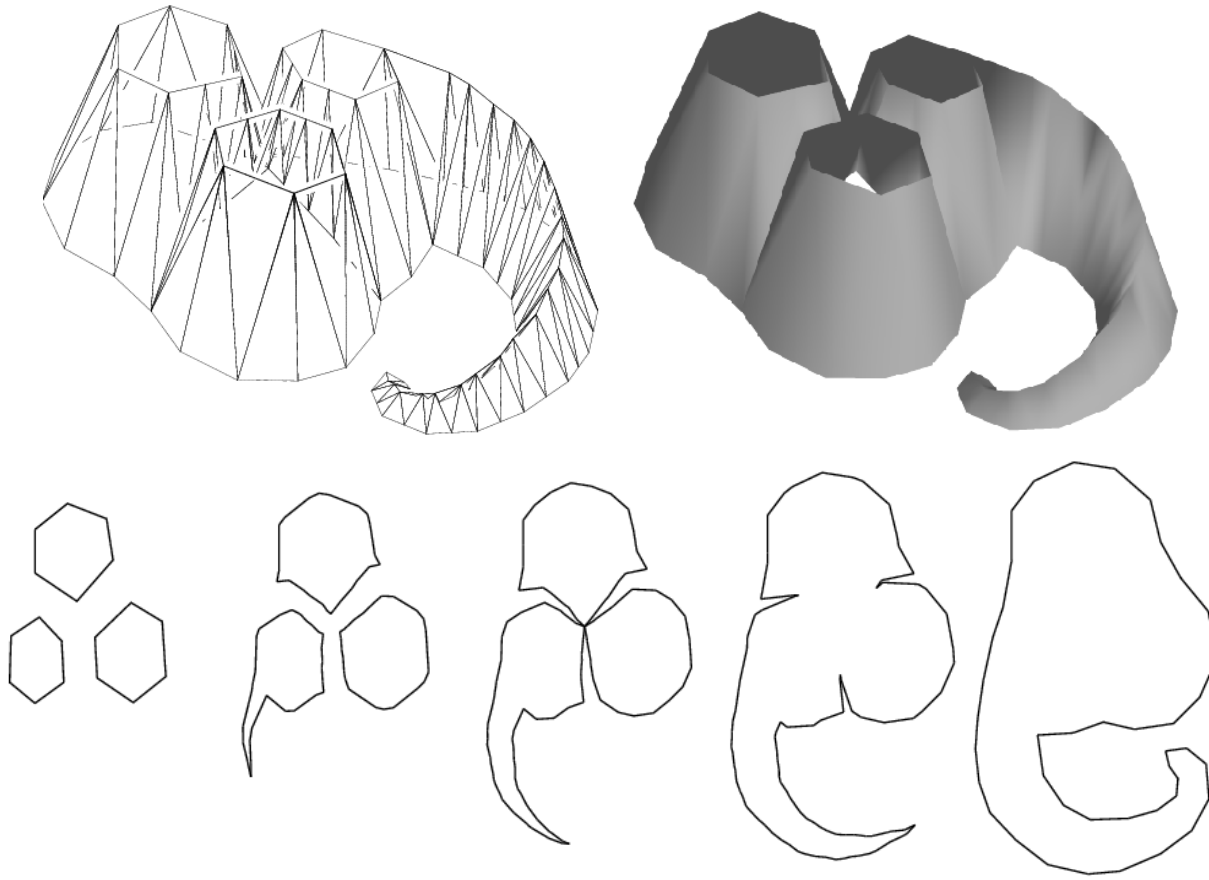
Morphing between Different Topologies

Liu et al. 2005

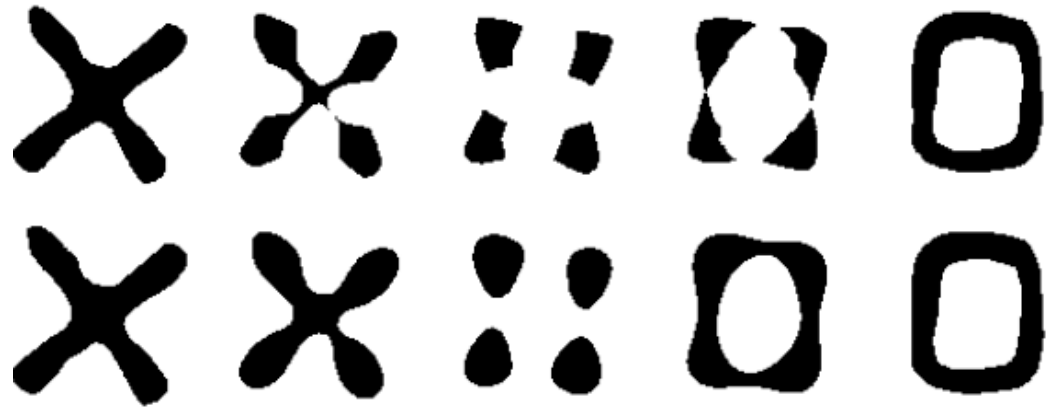
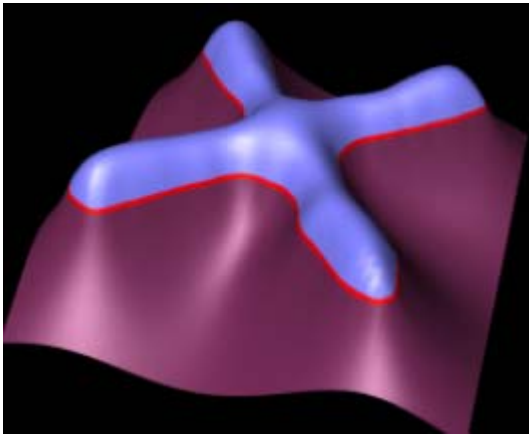


More...

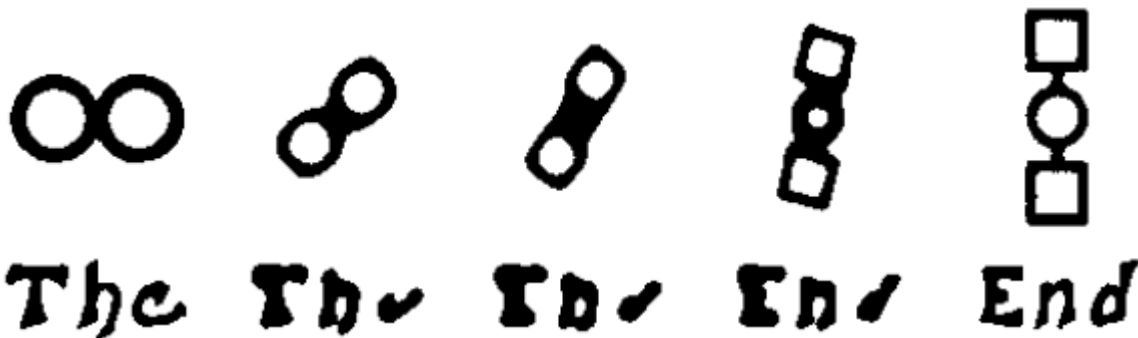
- Surface reconstruction from slices



Implicit Approaches



Implicit function from an X shape



Discussions