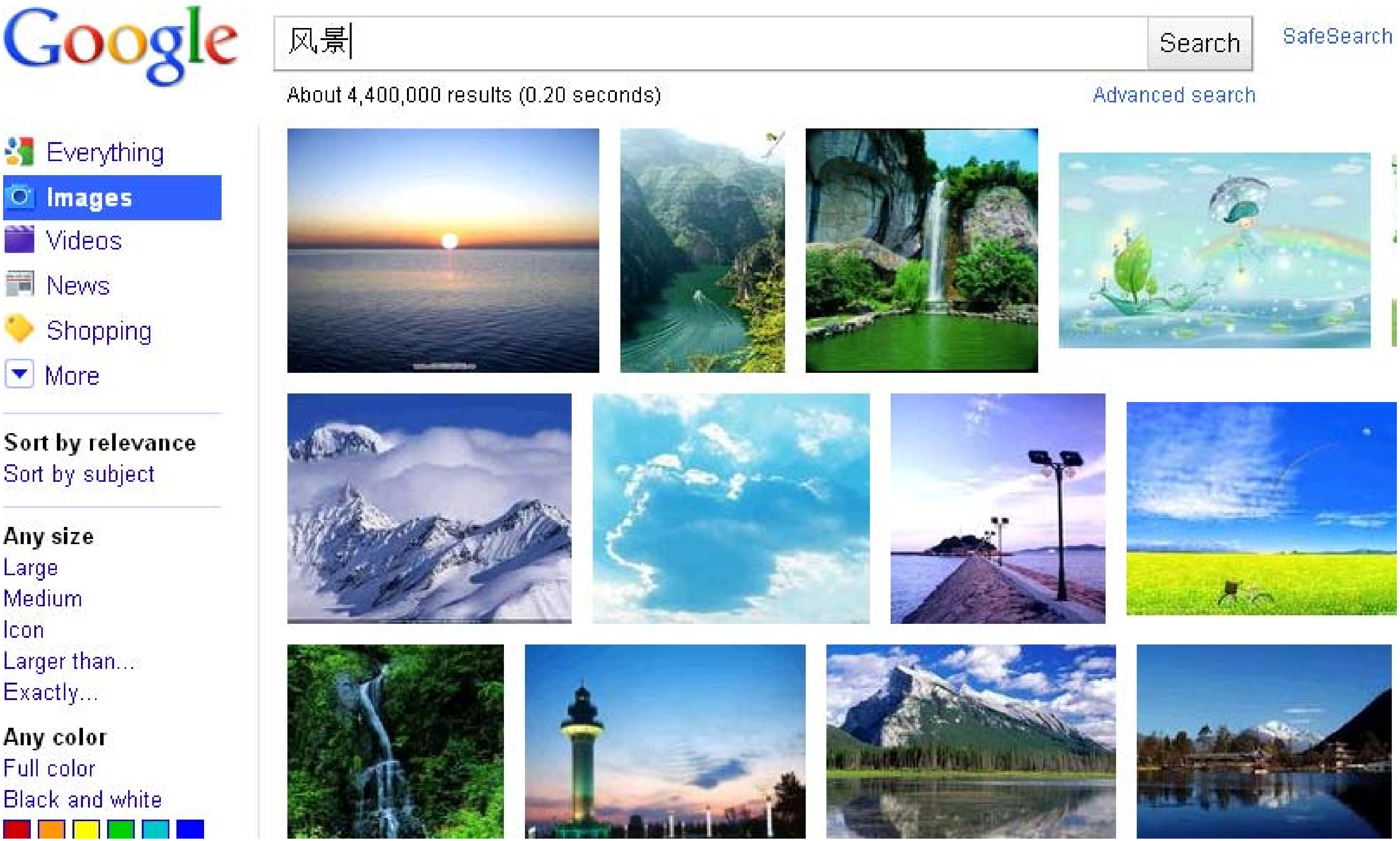
Graphics&Geometric Computing Lab @USTC

Poisson Image Editing

Ligang Liu
Graphics&Geometric Computing Lab
USTC
http://staff.ustc.edu.cn/~lgliu


Google
风景
Search
SafeSearch
About 4,400,000 results (0.20 seconds)
Advanced search
Everything
Images
Videos
News
Shopping
More
Sort by relevance
Sort by subject
Any size
Large
Medium
Icon
Larger than...
Exactly...
Any color
Full color
Black and white

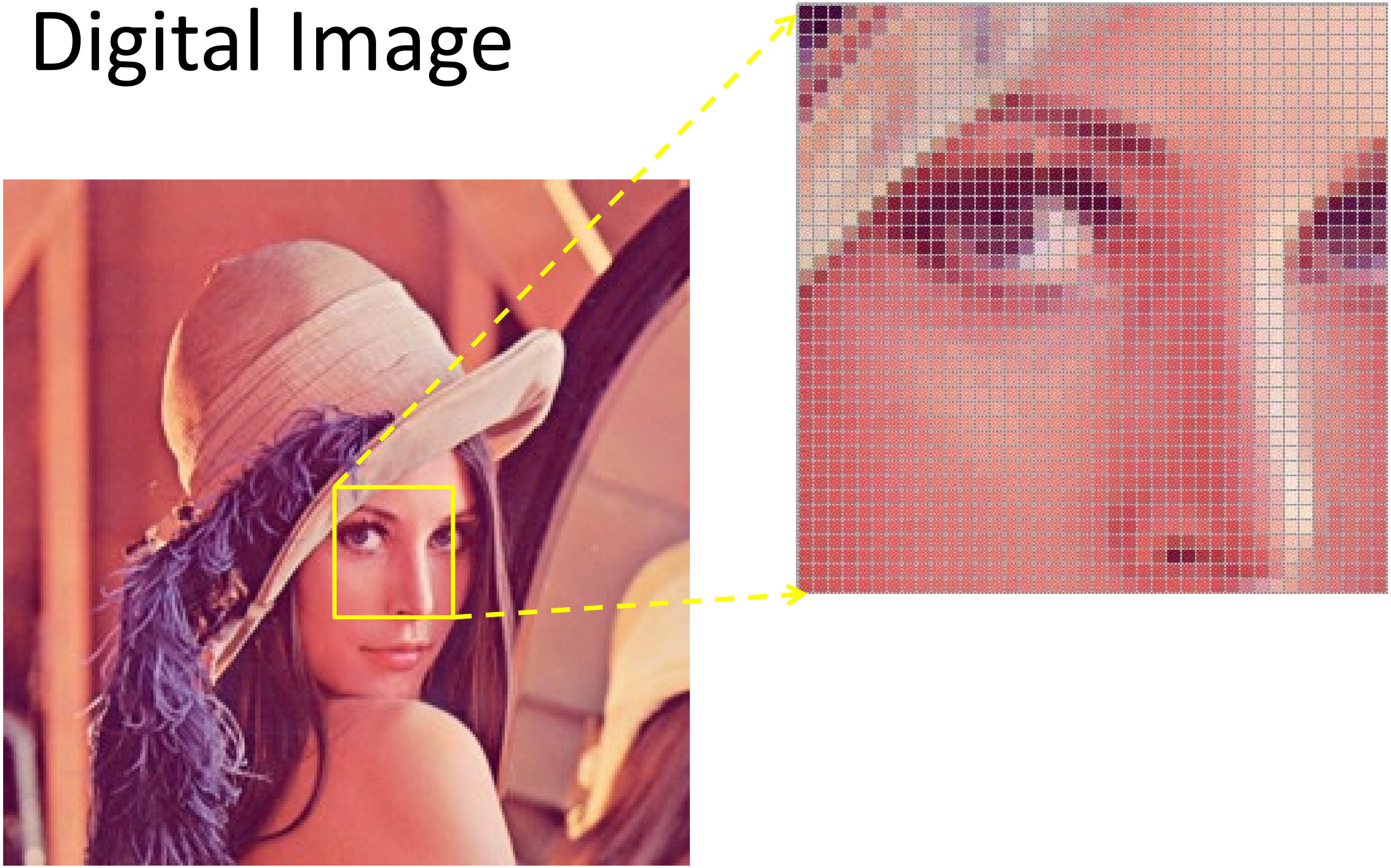
Images in the Internet

Images everywhere!

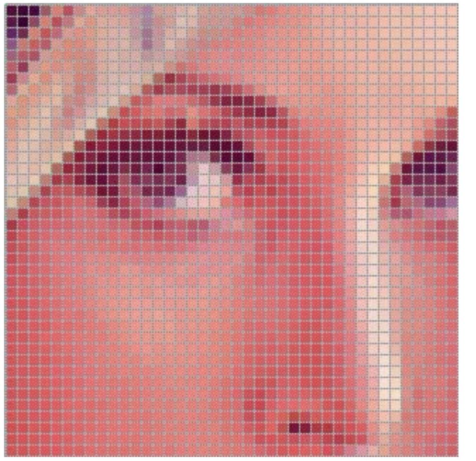
Images Everywhere

- Digital camera
- Personal digital album
- Photoshop

Digital Image



Math Model of Image



- Continuous
  - f(x,y)
- Discrete
  - I (i,j)
  - Pixel

    (R, G, B)

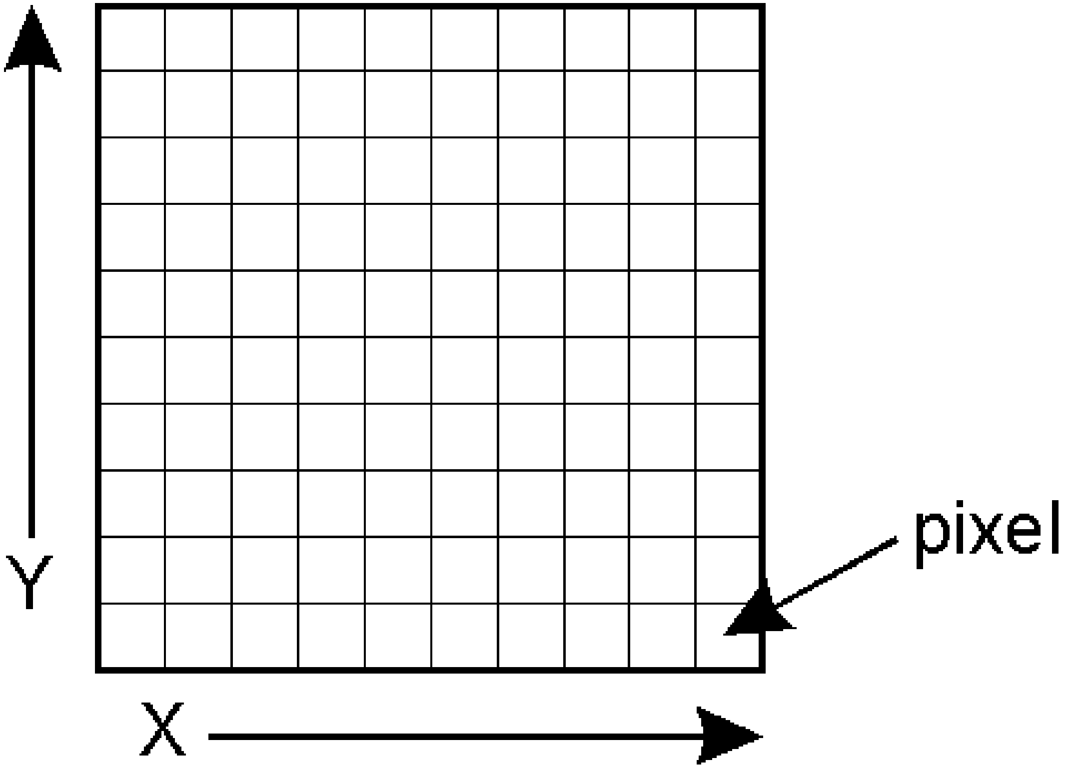


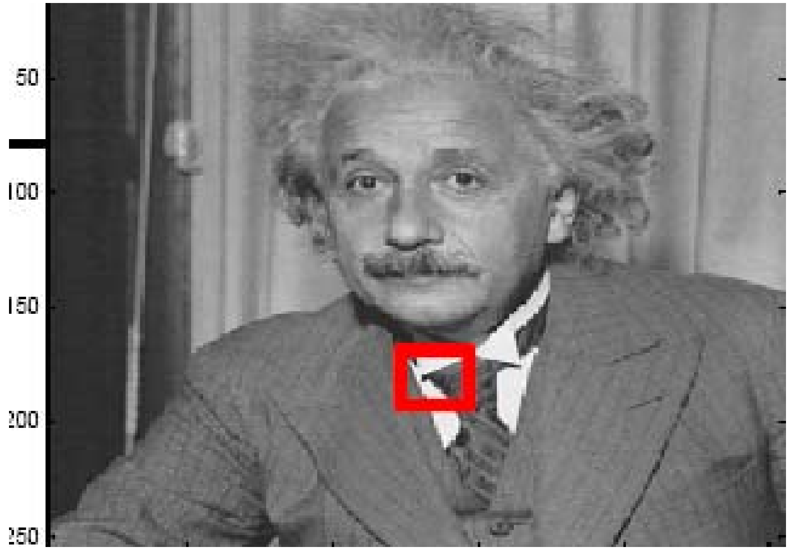
Image Processing

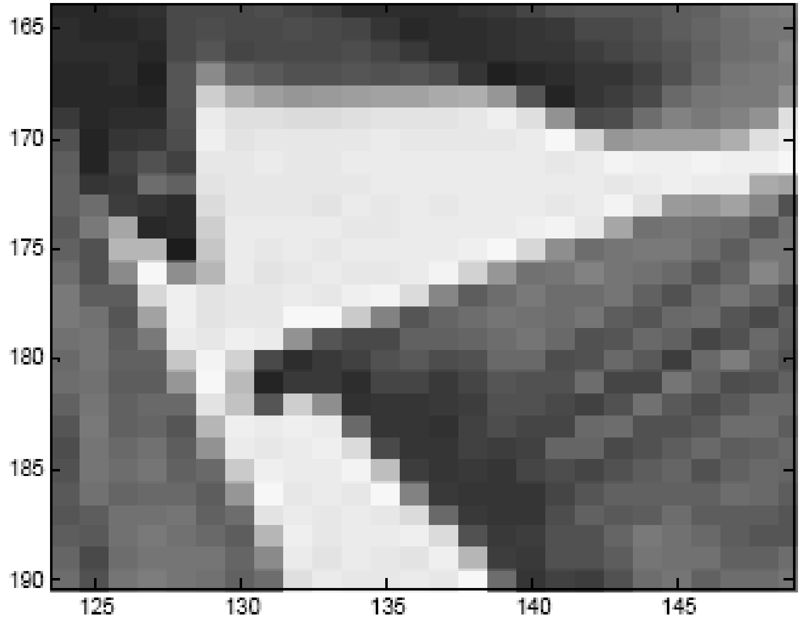
- Image filtering
- Image coding
- Image transformation
- Image enhancement
- Image segmentation
- Image understanding
- Image recognition
- …

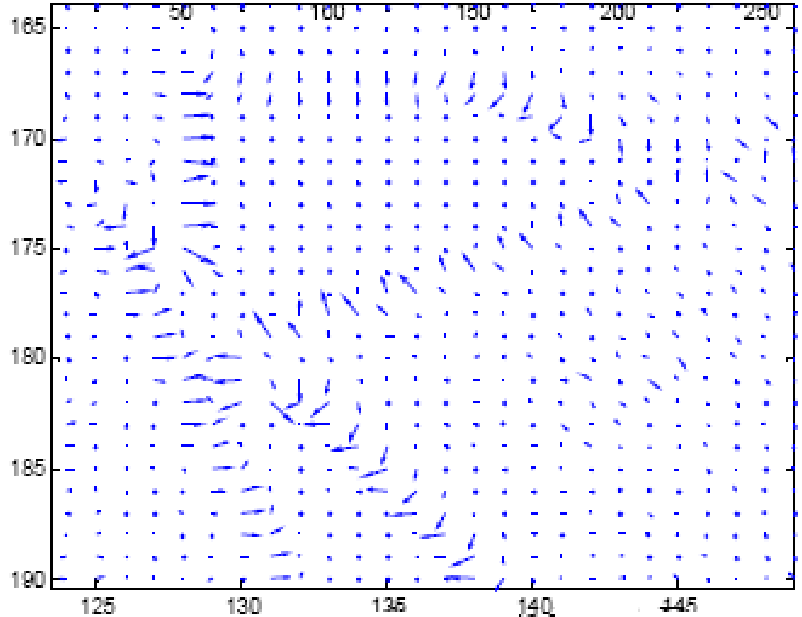
Gradient Domain Image Editing

- Motivation:
  - Human visual system is very sensitive to gradient
  - Gradient encode edges and local contrast quite well
- Approach:
  - Edit in the gradient domain
  - Reconstruct image from gradient

Gradient Domain: View of Image



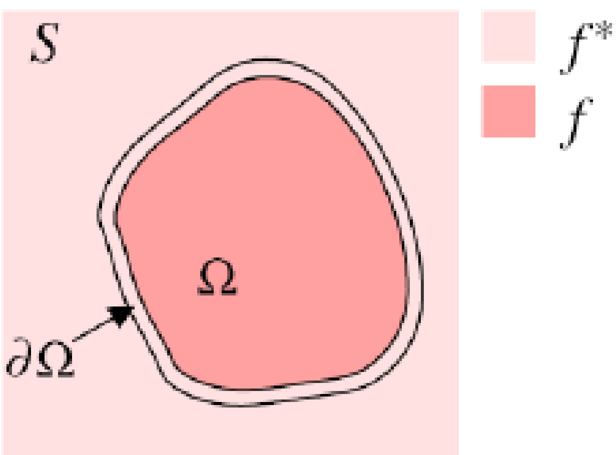


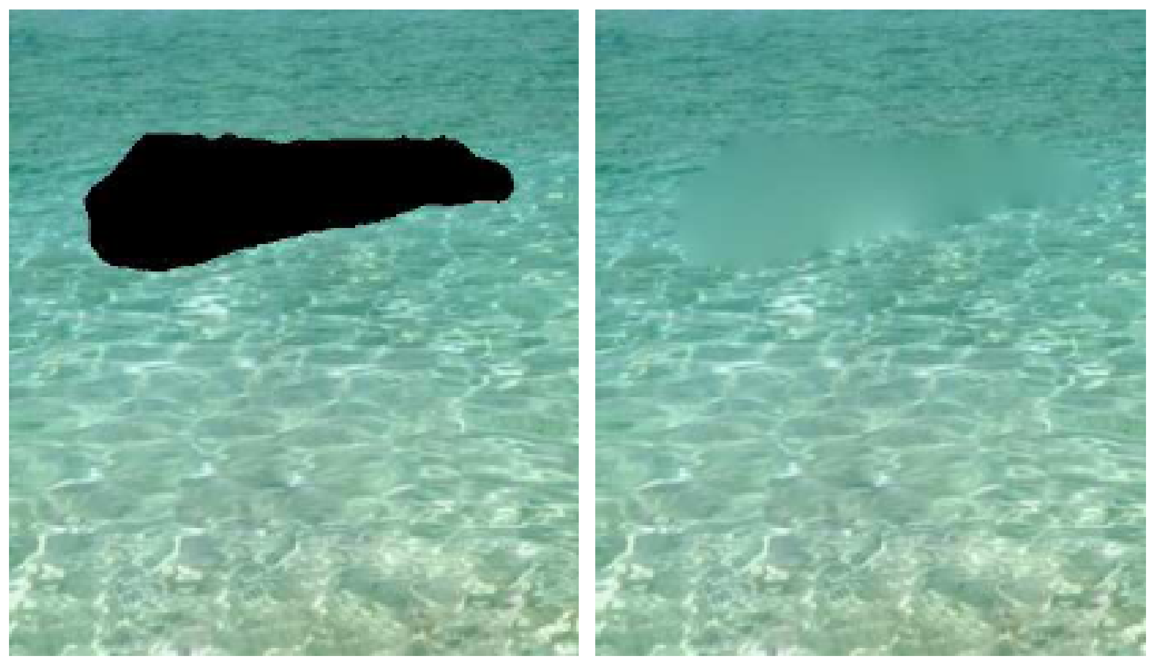


Membrane Interpolation

Laplace equation (a.k.a. membrane equation)

$$\min_f \iint_\Omega |\nabla f|^2 \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$





Membrane Interpolation

Laplace equation (a.k.a. membrane equation)

$$\min_f \iint_\Omega |\nabla f|^2 \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

Mathematicians will tell you there is an Associated Euler-Lagrange equation:

$$\Delta f = 0 \text{ over } \Omega \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$

- Where the Laplacian Δ is similar to -1 2 -1in 1D

Kind of the idea that we want a minimum, so we kind of derive and get a simpler equation

Poisson Equations

$$\Delta\Phi = -4\pi G\rho(\mathbf{x})$$

Siméon Denis Poisson

- His teachers: *Laplace*, *Lagrange*, …
- Poisson's terms:
  - Poisson's equation
  - Poisson's integral
  - Poisson distribution
  - Poisson brackets
  - Poisson's ratio
  - Poisson's constant

1781-1840, France

"Life is good for only two things: to study mathematics and to teach it."

Background:

- Partial Differential Equations (PDE)

$$E(f, f_x, f_y, f_{xx}, f_{xy}, f_{yy}) = 0$$

- The PDE's which occur in physics are mostly second order and linear:

$$A \cdot f_{xx} + 2B \cdot f_{xy} + C \cdot f_{yy} + D \cdot f_x + E \cdot f_y + F \cdot f + G = 0$$

$$A\cdot f_{xx} + 2B\cdot f_{xy} + C\cdot f_{yy} + D\cdot f_x + E\cdot f_y + F\cdot f + G = 0$$

$A\cdot C < B^2 :$ – Hyberbolic

- wave equation: $$\Delta f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

$A\cdot C = B^2 :$ – Parobolic

- heat equation: $$\frac{\partial f}{\partial t} = k\cdot\Delta f$$

$A\cdot C > B^2 :$ – Elliptic

- Laplace equation: $$\Delta f = 0$$
- Poisson equation: $$\Delta f = -\rho$$

Poisson Equation

$$\Delta f = -\rho$$

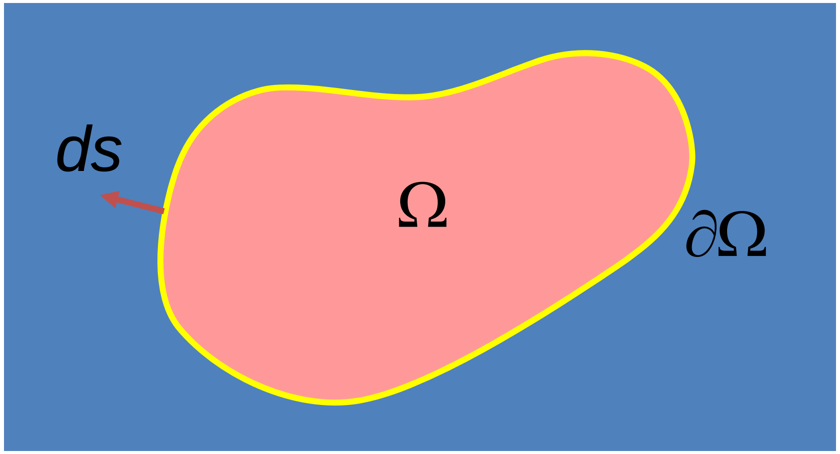
$$\Delta \equiv \frac{\partial^2}{\partial^2 x} + \frac{\partial^2}{\partial^2 y}$$

$$\rho = \rho(x, y)$$

Boundary conditions

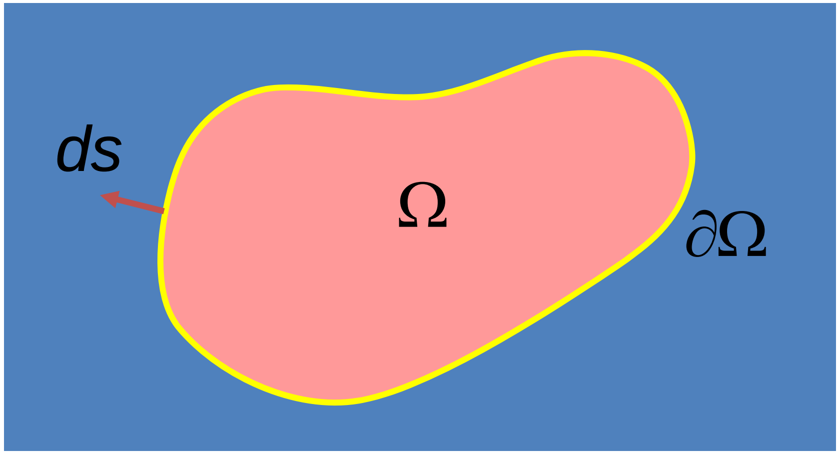
- *Dirichlet* boundary conditions: $f\ |_{\partial\Omega}$

- *Neumann* boundary conditions: $\frac{\partial f}{\partial \mathbf{s}}\ |_{\partial\Omega}$

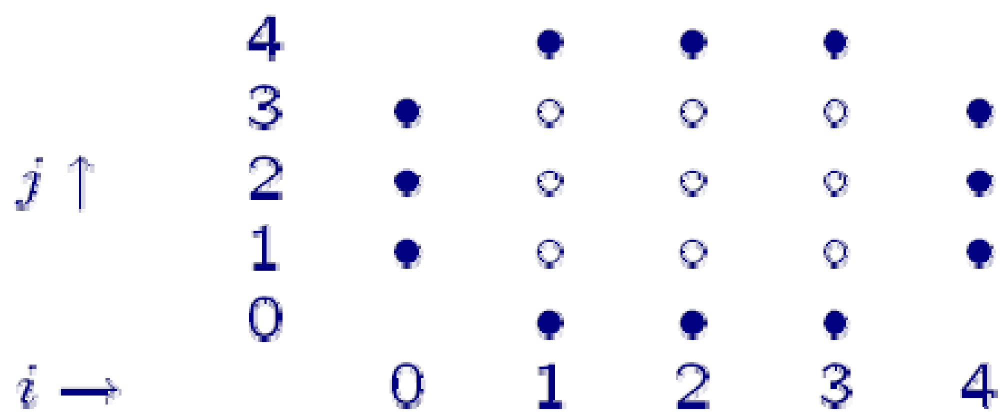


Existence of solution

The solution of an Poisson Equation is uniquely determined in Ω , if *Dirichlet* boundary conditions or *Neumann* boundary conditions are specified on $\partial\Omega$



Discrete Poisson Equation



$$\Delta f = -\rho$$

$$\frac{f_{i+1,j} + f_{i-1,j} - 2f_{i,j}}{h^2} + \frac{f_{i,j+1} + f_{i,j-1} - 2f_{i,j}}{h^2} = \rho_{i,j}$$

Matrix Nature of Poisson Equation

$$\begin{bmatrix} -4 & 1 & & & & & & & \\ 1 & -4 & 1 & & & & & & \\ & 1 & -4 & 1 & & & & & \\ & & 1 & -4 & 1 & & & & \\ & & & 1 & -4 & 1 & & & \\ & & & & 1 & -4 & 1 & & \\ & & & & & 1 & -4 & 1 & \\ & & & & & & 1 & -4 & 1 \\ & & & & & & & 1 & -4 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{21} \\ f_{31} \\ f_{12} \\ f_{22} \\ f_{32} \\ f_{13} \\ f_{23} \\ f_{33} \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \\ b_{12} \\ b_{22} \\ b_{32} \\ b_{13} \\ b_{23} \\ b_{33} \end{bmatrix}$$

Large sparse linear system

Poisson Equation Solver

- Direct method
- Iterative methods
  - Jacobi, Gauss-Seidel, SOR
- Multigrid method

Variational interpretation

$$f^* = \arg\min_f \iint_\Omega \underbrace{\left\|\nabla f - \mathbf{v}\right\|^2}_{F} \quad \text{s.t } f^*|_{\partial\Omega} = f|_{\partial\Omega}$$

Euler Equation: $F_f - \frac{\partial}{\partial x} F_{f_x} - \frac{\partial}{\partial y} F_{f_y} = 0$

$$\Delta f = div(\mathbf{v}) \quad \text{s.t } f^*|_{\partial\Omega} = f|_{\partial\Omega}$$

$\mathbf{v}$ is a guidance field, needs not to be a gradient field.

Physical Origins of Poisson Equation

- Electrostatic potential

$$\Delta\Phi = -\frac{\rho(\mathbf{x})}{\varepsilon_0}$$

- Gravitational potential

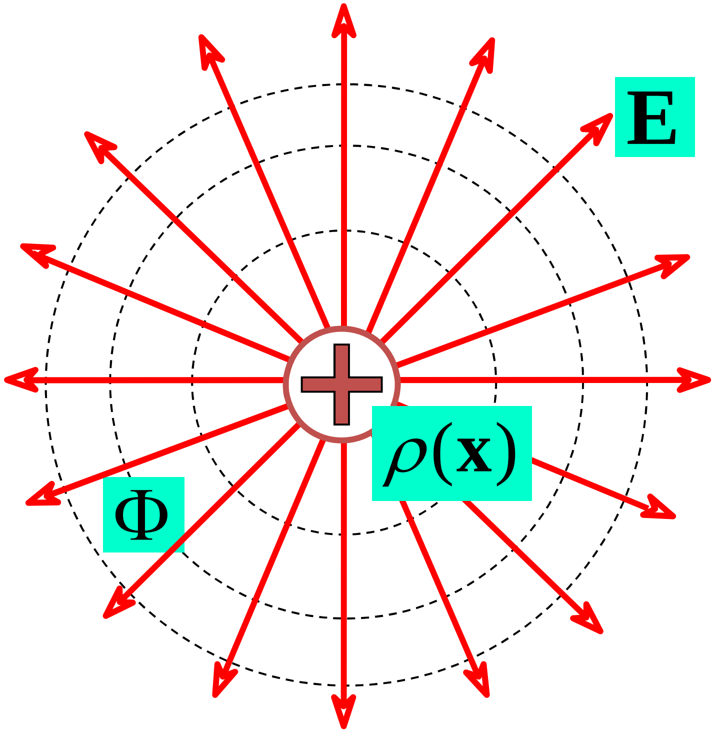
$$\Delta\Phi = -4\pi G\rho(\mathbf{x})$$

Electrostatic potential $\mathbf{F} = \frac{q_1 q_2 \mathbf{r}}{4\pi\varepsilon_0 r^3}$

$\rho(\mathbf{x})$ Charge Density

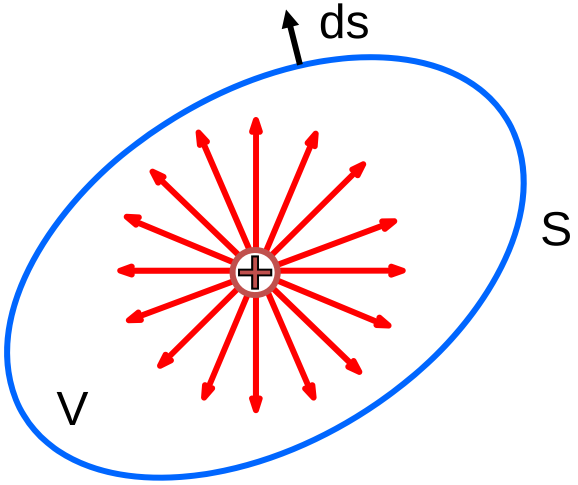
Φ Electric Potential

$\mathbf{E}$ Electric Field



$$\mathbf{E} = -\nabla\Phi$$

Derivations



Gauss's Law:

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \int_V \frac{\rho(\mathbf{x})}{\varepsilon_0} dv$$

Gauss's theorem:

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \int_V \nabla \cdot \mathbf{E} dv$$

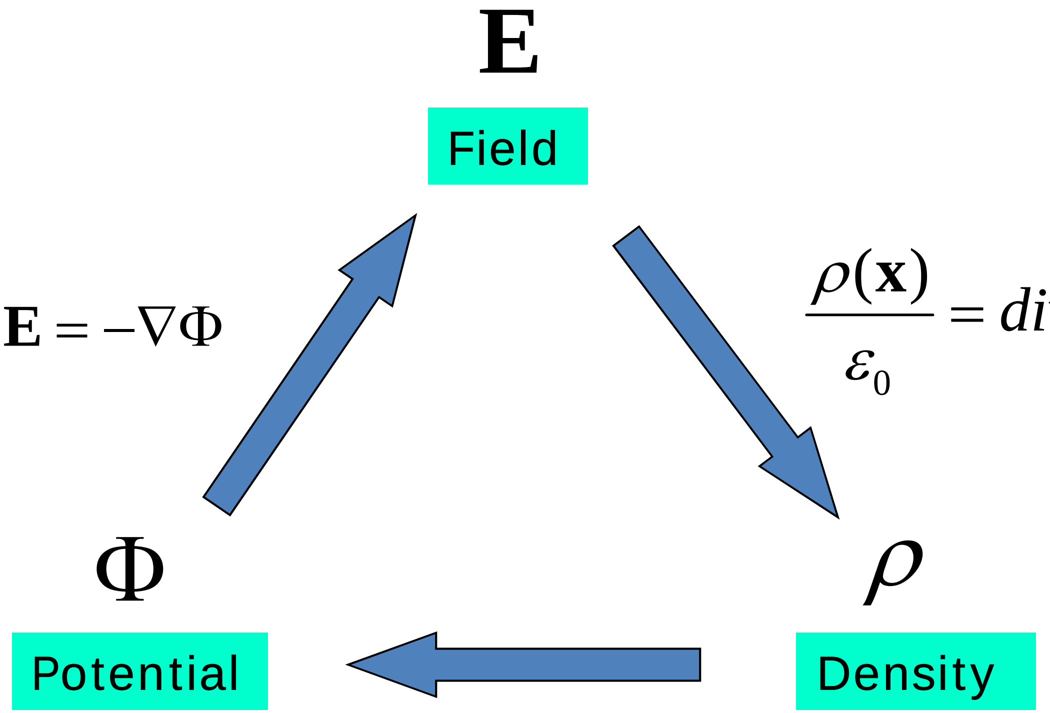
$$\nabla \cdot \mathbf{E} = \frac{\rho(\mathbf{x})}{\varepsilon_0}$$

$$\mathbf{E} = -\nabla \Phi$$

$$\Delta \Phi = -\frac{\rho(\mathbf{x})}{\varepsilon_0}$$

Poisson Equation

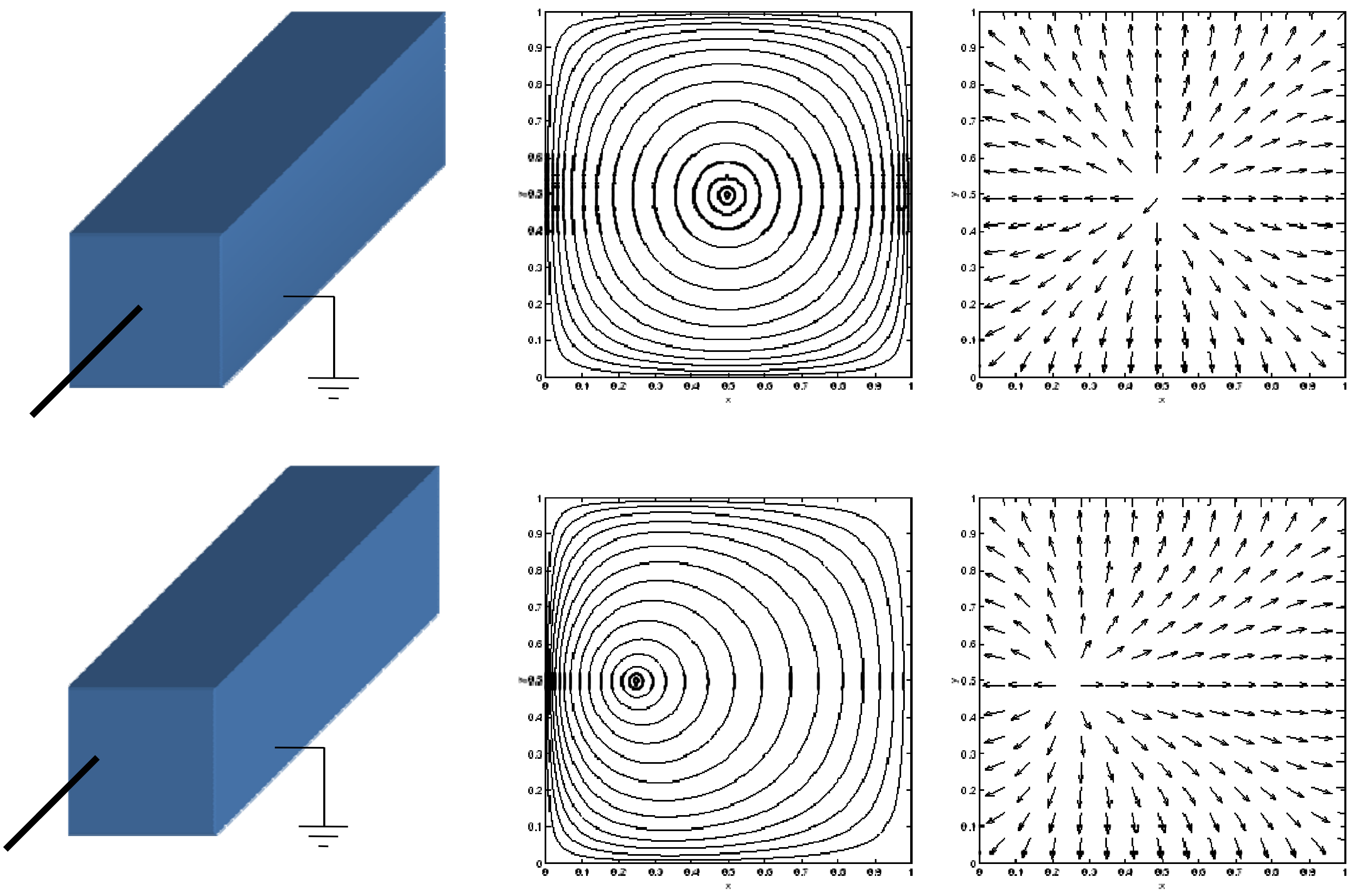
Relationships



$$\mathbf{E} = -\nabla\Phi$$

$$\frac{\rho(\mathbf{x})}{\varepsilon_0} = div(\mathbf{E}) \equiv \lim_{V\to 0} \frac{\oint_S \mathbf{E}\cdot d\mathbf{s}}{V}$$

$$\Delta\Phi = -\frac{\rho(\mathbf{x})}{\varepsilon_0}$$



$$\rho(\mathbf{x}) = \delta(x_0, y_0)$$

Φ $\mathbf{E}$

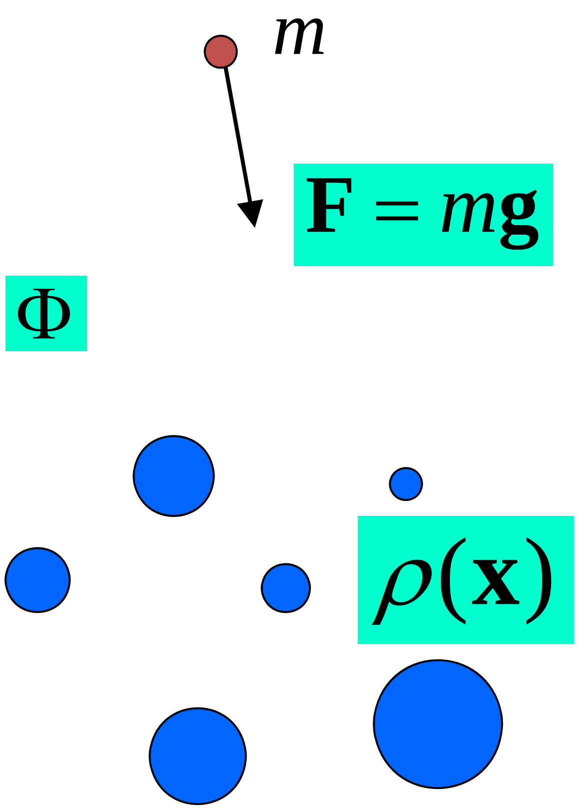
Gravitational potential $\mathbf{F} = \frac{mMG\mathbf{r}}{r^3}$

$\rho(\mathbf{x})$ Mass Density

Φ Gravitational Potential

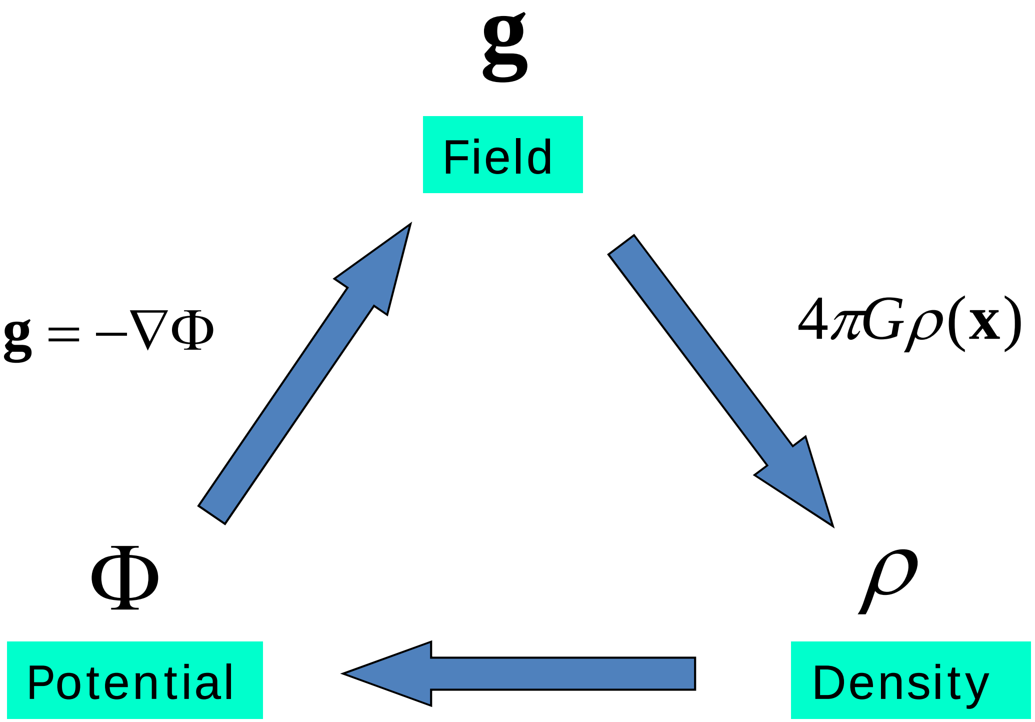
$\mathbf{g}$ Force Field

(acceleration)



$$\mathbf{g} = -\nabla\Phi$$

Relationships



$$\mathbf{g} = -\nabla\Phi$$

$$4\pi G\rho(\mathbf{x}) = div(\mathbf{g}) \equiv \lim_{V \to 0} \frac{\oint_S \mathbf{g} \cdot d\mathbf{s}}{V}$$

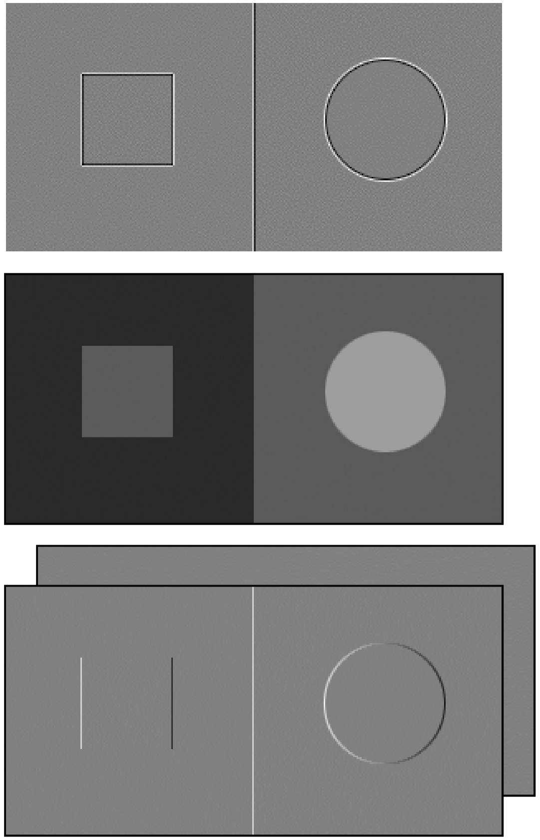
$$\Delta\Phi = -4\pi G\rho(\mathbf{x})$$

Analogy for Image

$\rho(\mathbf{x})$ Image Density

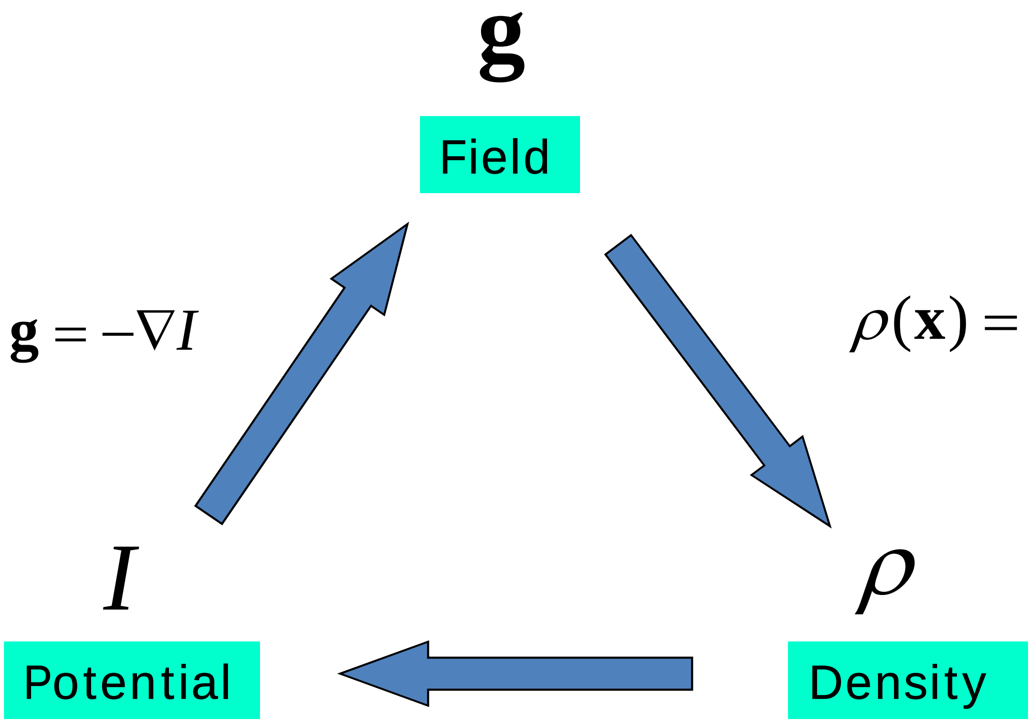
I Image (Potential)

$\mathbf{g}$ Image Gradient



$$\mathbf{g} = -\nabla I$$

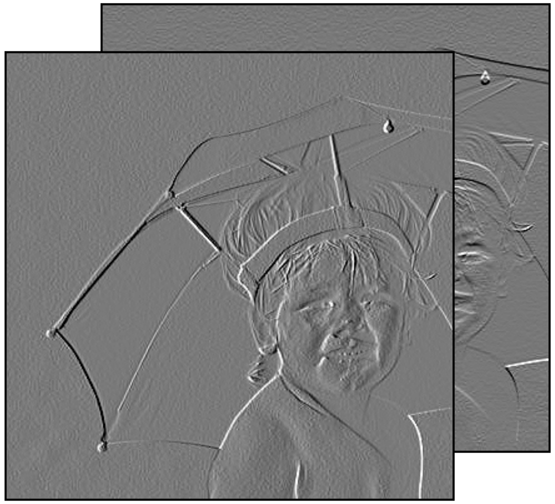
Relationships



$$\mathbf{g} = -\nabla I$$

$$\rho(\mathbf{x}) = div(\mathbf{g}) \equiv \lim_{V \to 0} \frac{\oint_S \mathbf{g} \cdot d\mathbf{s}}{V}$$

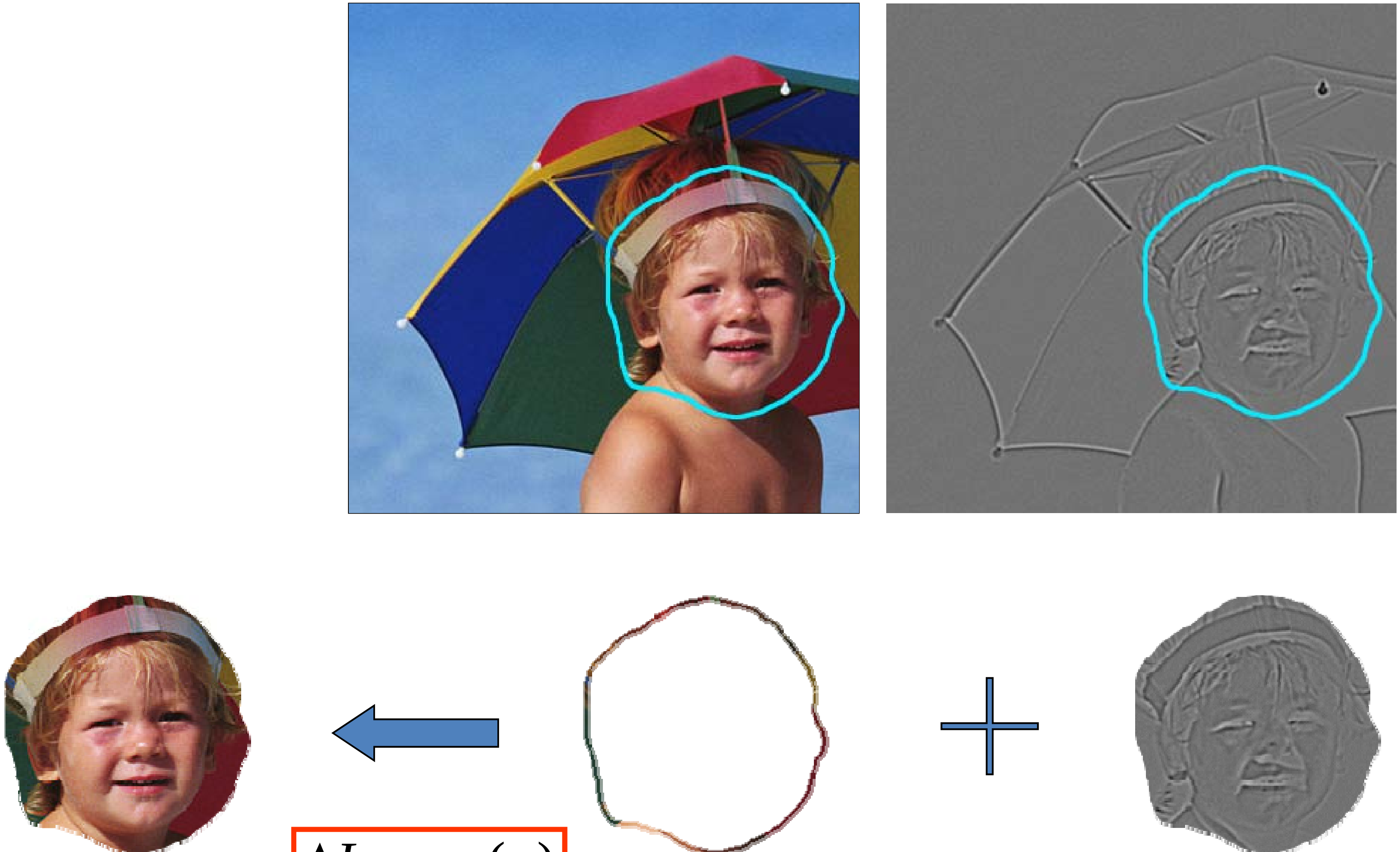
$$\Delta I = -\rho(\mathbf{x})$$



$$\mathbf{g} = -\nabla I$$

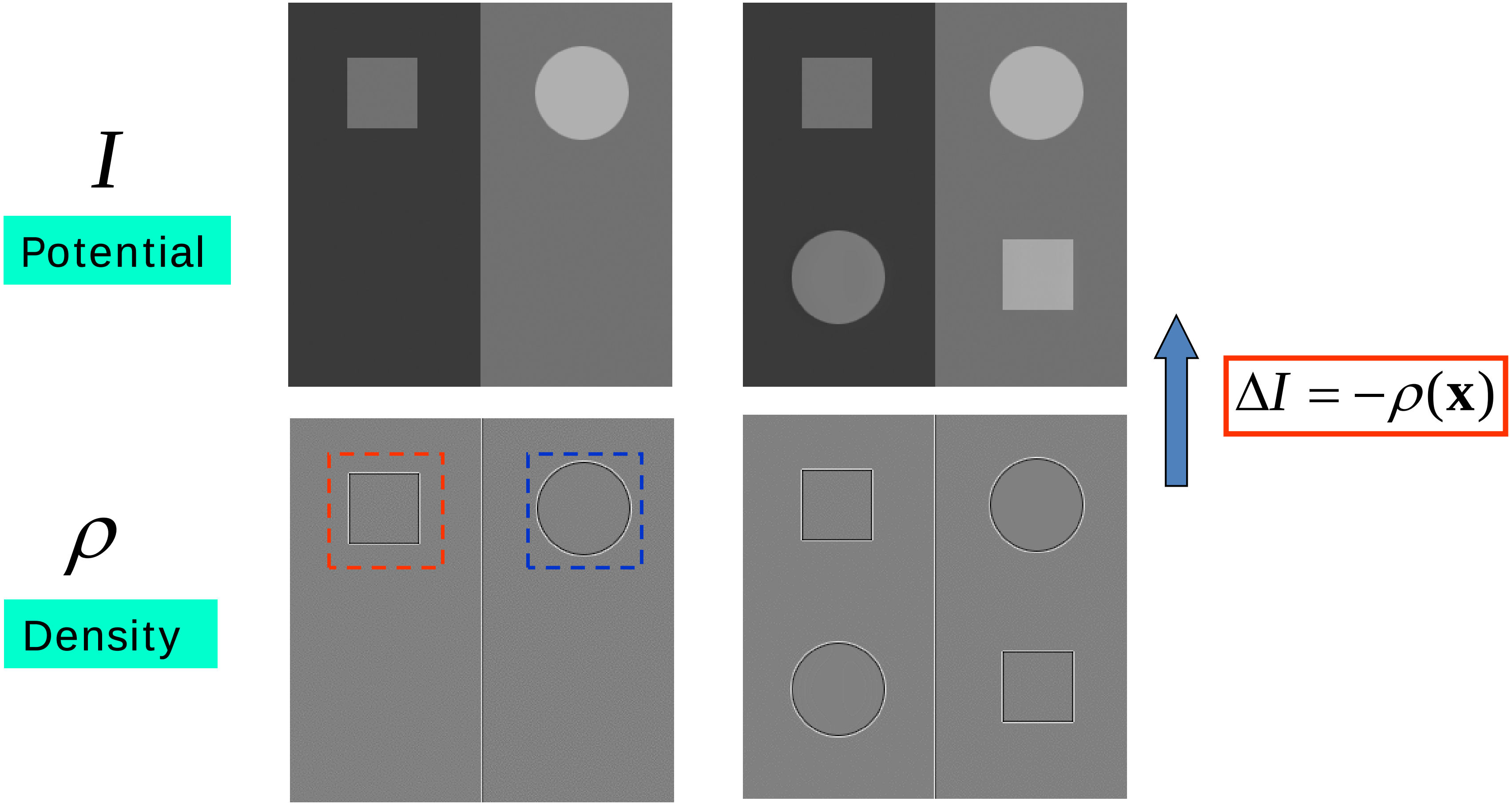
$$\rho(\mathbf{x}) = div(\mathbf{g}) \equiv \lim_{V \to 0} \frac{\oint_S \mathbf{g} \cdot d\mathbf{s}}{V}$$

$$\Delta I = -\rho(\mathbf{x})$$



$$\Delta I = -\rho(\mathbf{x})$$

Motivation for Image Editing



Poisson Image Editing

P. Pérez, M. Gangnet, and A. Blake

SIGGRAPH 2003

Seamless Cloning

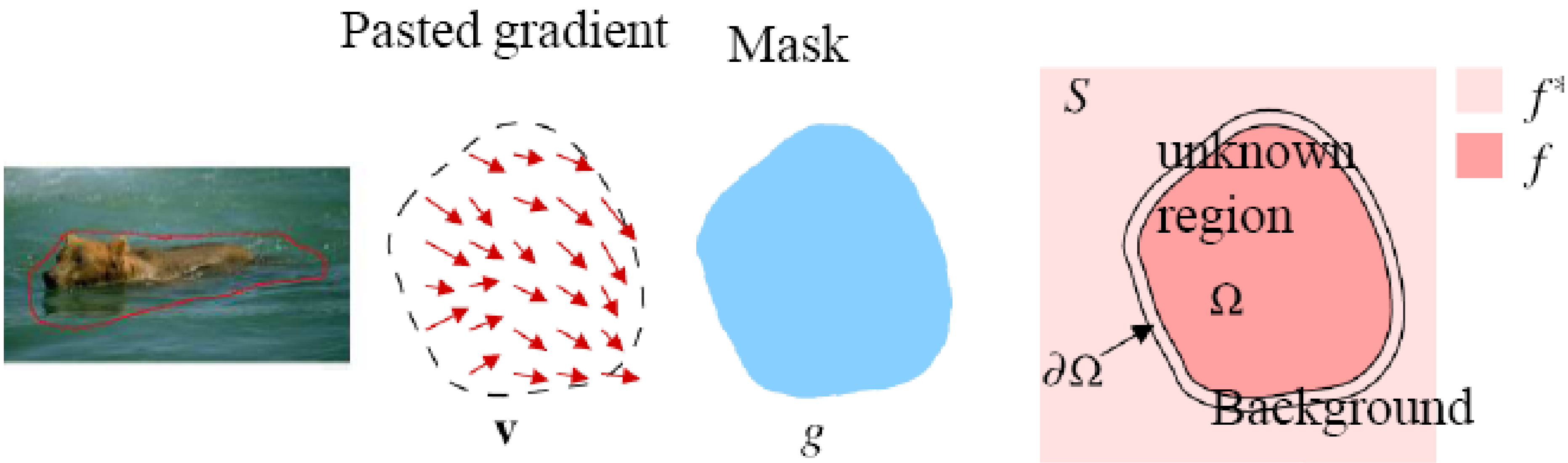
- Precise selection: tedious and unsatisfactory
- Alpha-Matting: powerful but involved
- Seamless cloning: loose selection but no seams?

Seamless Poisson Cloning

Given vector field *v* (pasted gradient), find the value of *f* in unknown region that optimize:

Previously, v was null

$$\min_{f} \iint_{\Omega} |\nabla f - \mathbf{v}|^2 \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$$



Cloning by solving Poisson Equation

$$\Delta I = div(\nabla I_A) \qquad \text{s.t.} \quad I\,|_{\partial\Omega} = I_B\,|_{\partial\Omega}$$

I_A $\qquad$ I_B

Why we do analogy for image?

- Easier in the image gradient domain
  - *Local* editing ➡ *global* effects
  - *Seamless* - cloning, editing, tilling

Variational Interpretation

$$I^* = \arg\min_f \iint_\Omega \underbrace{\|\nabla I - \nabla I_A\|^2}_{F} \quad \text{s.t } I^*|_{\partial\Omega} = I_B|_{\partial\Omega}$$

Euler Equation: $F_f - \frac{\partial}{\partial x} F_{f_x} - \frac{\partial}{\partial y} F_{f_y} = 0$

$$\Delta I = div(\nabla I_A) \quad \text{s.t.} \quad I|_{\partial\Omega} = I_B|_{\partial\Omega}$$

∇I_A is a gradient field to be cloned.

Discrete Poisson solver

Minimize variational problem $\min_f \iint_\Omega |\nabla f - \mathbf{v}|^2 \text{ with } f|_{\partial\Omega} = f^*|_{\partial\Omega}$,

Discretized gradient

$$\min_{f|_\Omega} \sum_{\langle p,q\rangle \cap \Omega \neq \emptyset} (f_p - f_q - v_{pq})^2, \text{ with } f_p = f_p^*, \text{ for all } p \in \partial\Omega$$

(all pairs that are in Ω)

Discretized v: g(p)-g(q)

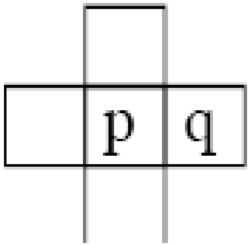
Boundary condition

Rearrange and call N_p the neighbors of p

$$\text{for all } p \in \Omega, \quad |N_p| f_p - \sum_{q \in N_p \cap \Omega} f_q = \sum_{q \in N_p \cap \partial\Omega} f_q^* + \sum_{q \in N_p} v_{pq}$$

Only for boundary pixels

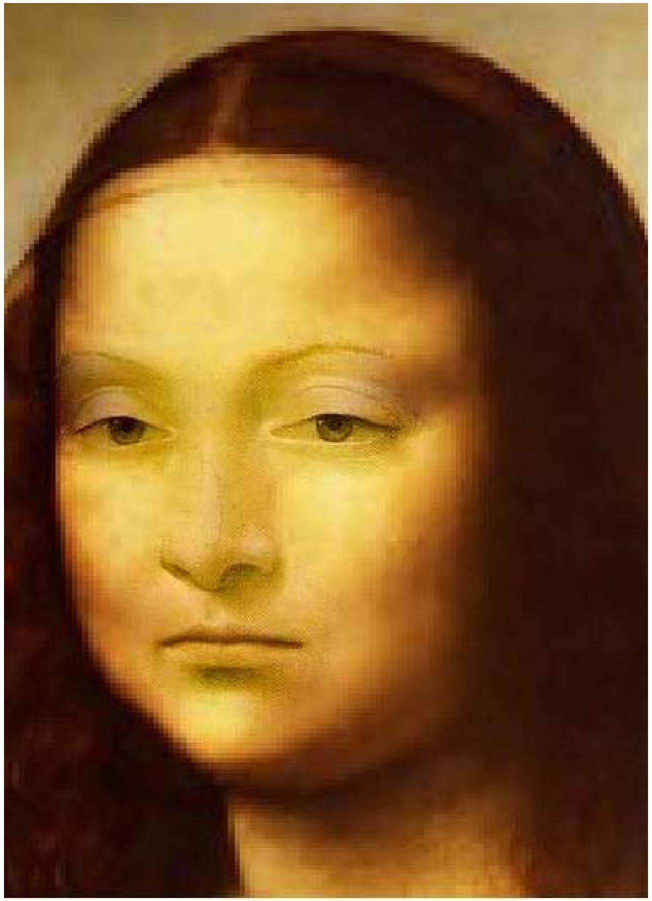
Big yet sparse linear system



Demo

Compose

Change Features



Change Texture

Conceal

Mix Lights

Change Colors

Change Colors

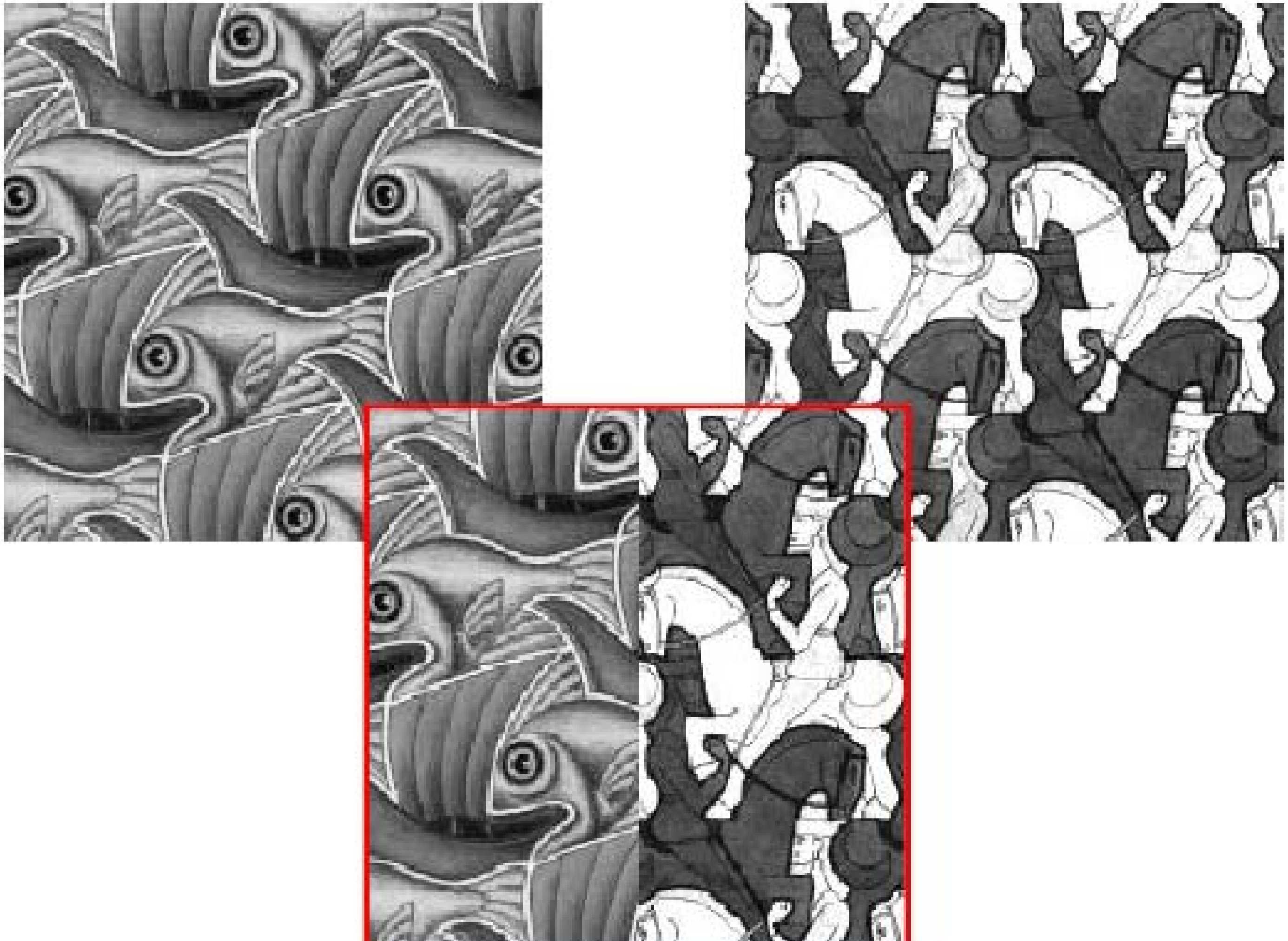
Seamless Tiling

Single image

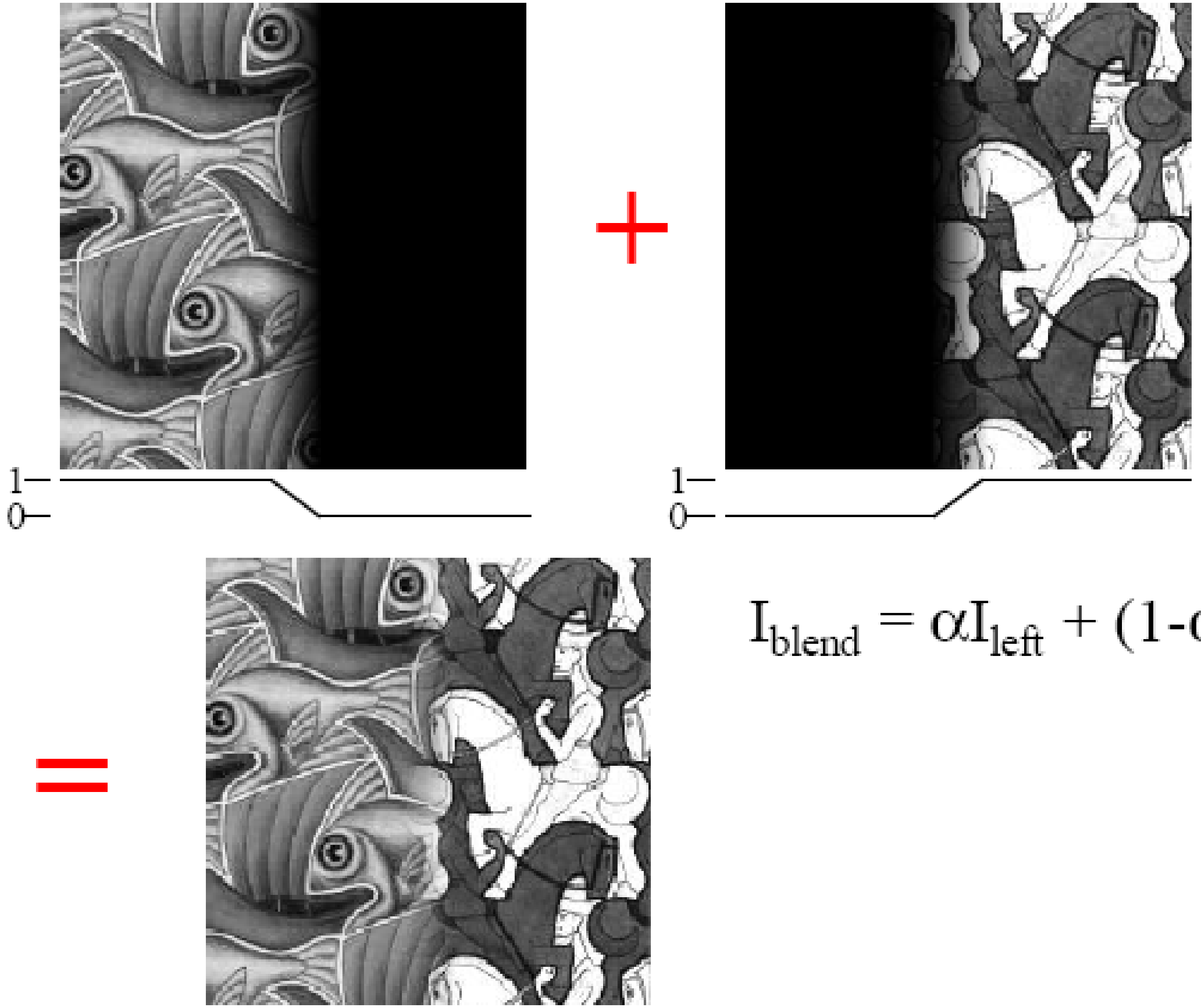
Seamless Tiling

Multiple images tiled at random

Image Stitching

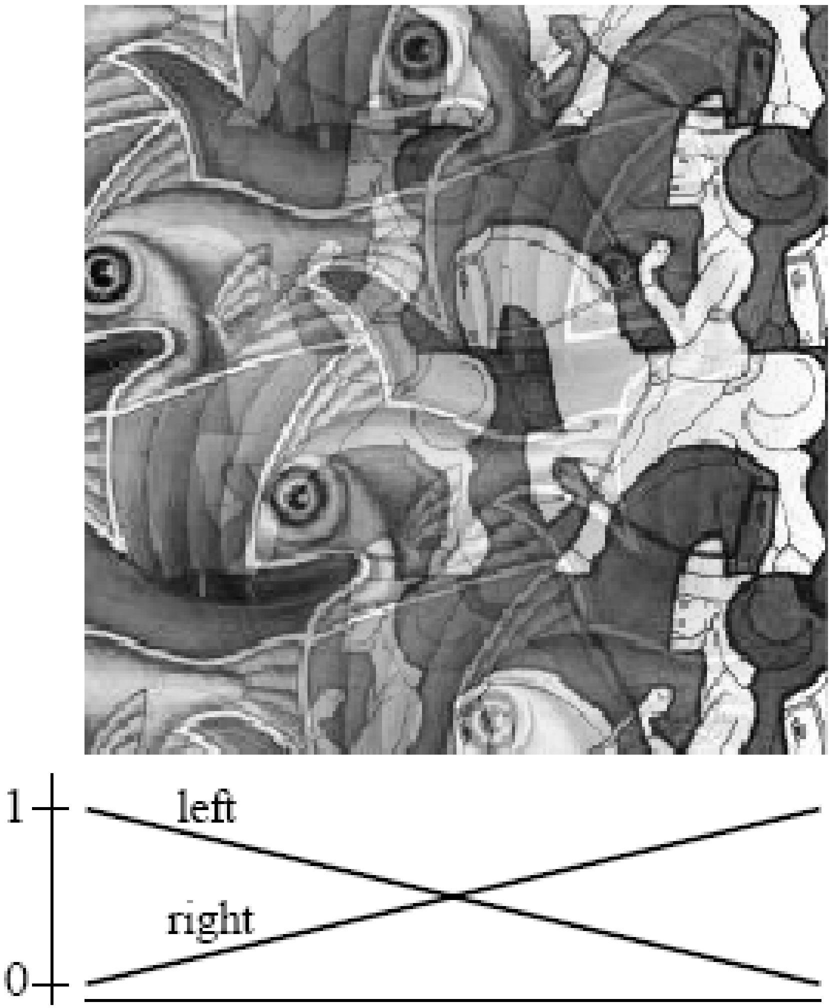


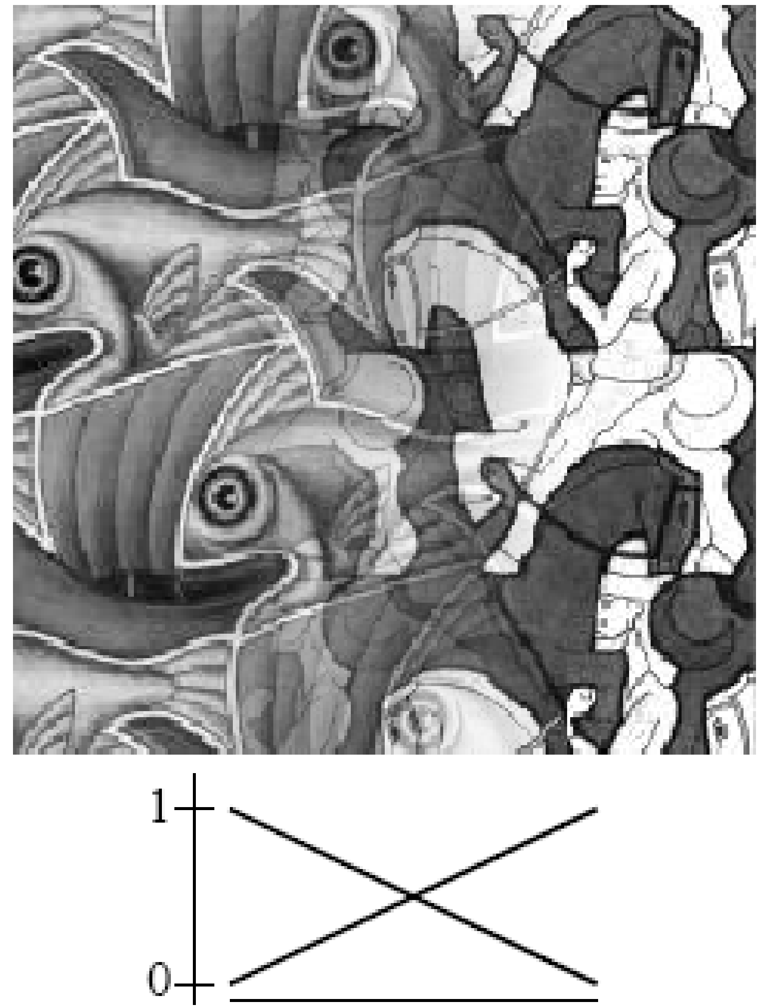
Alpha Blending / Feathering



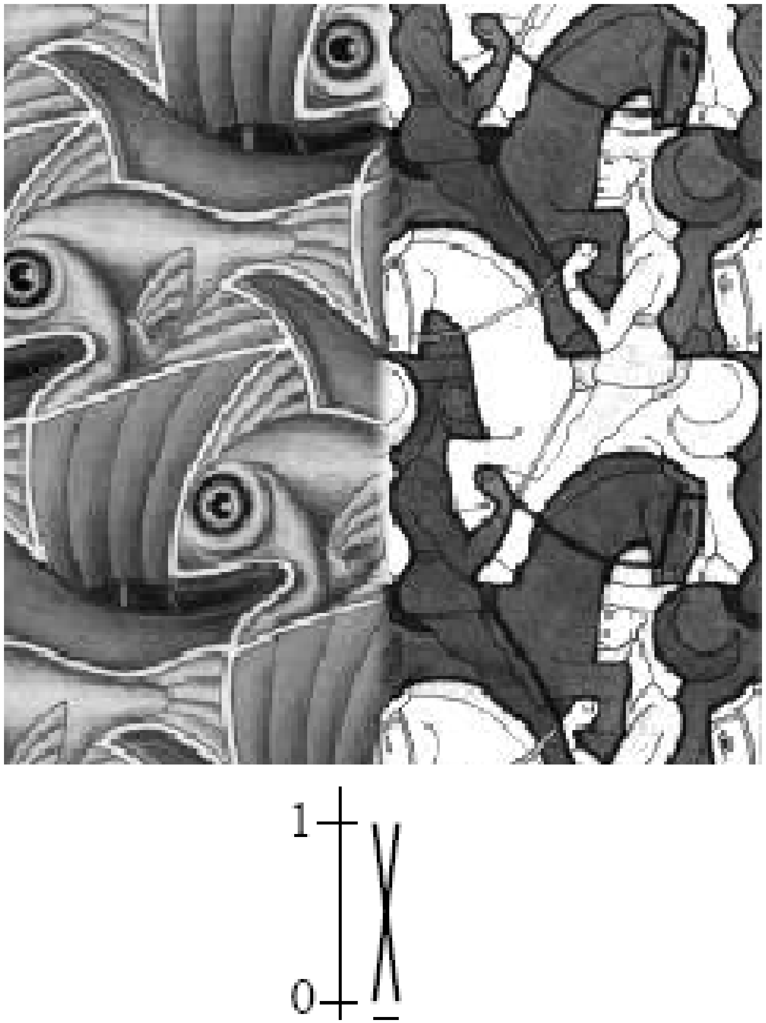
$$I_{blend} = \alpha I_{left} + (1-\alpha) I_{right}$$

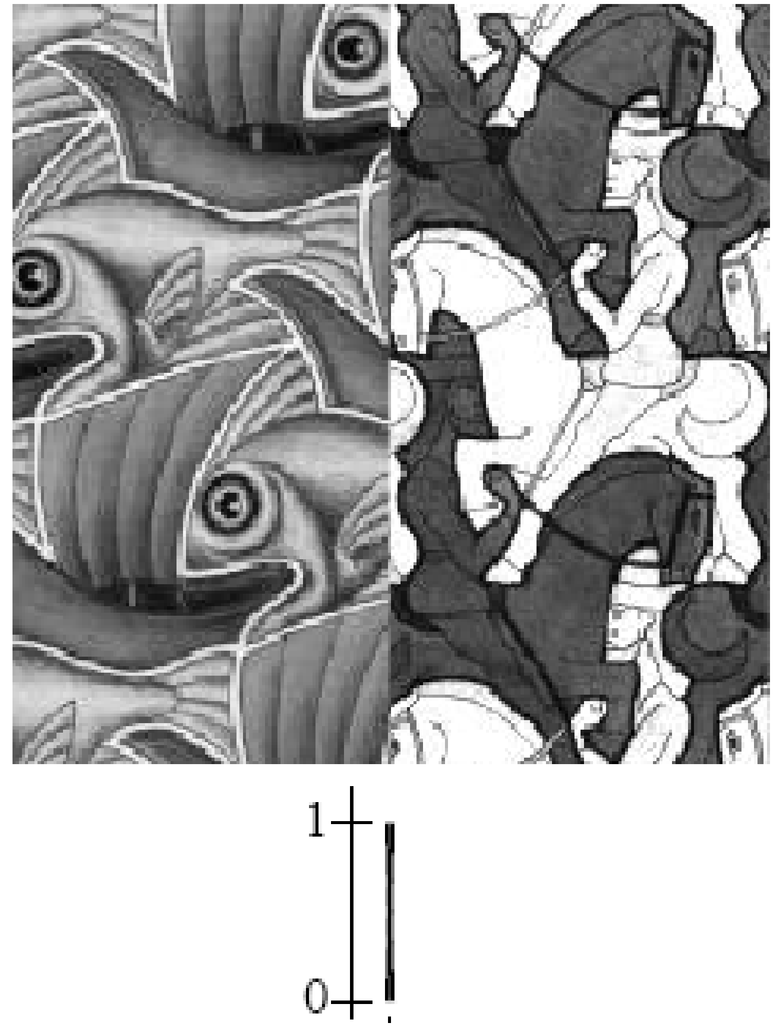
Affect of Window Size



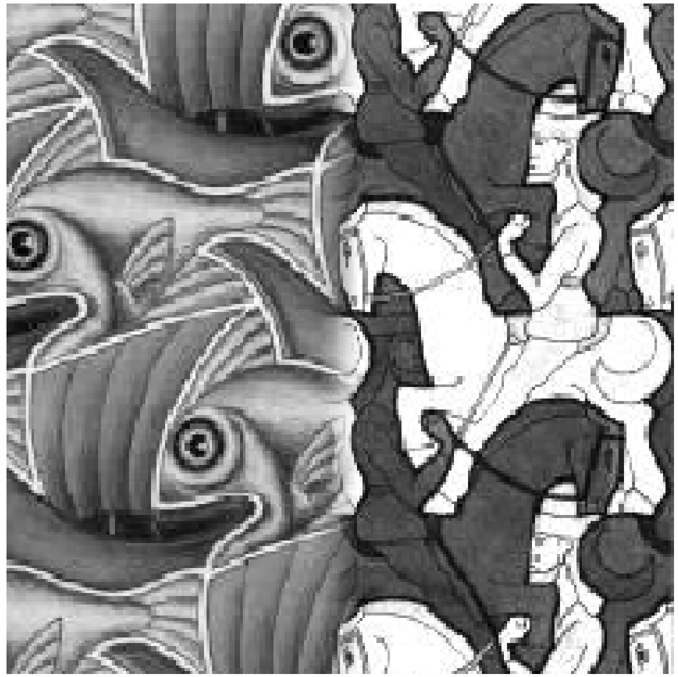


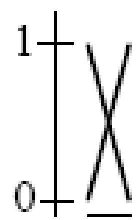
Affect of Window Size





Good Window Size

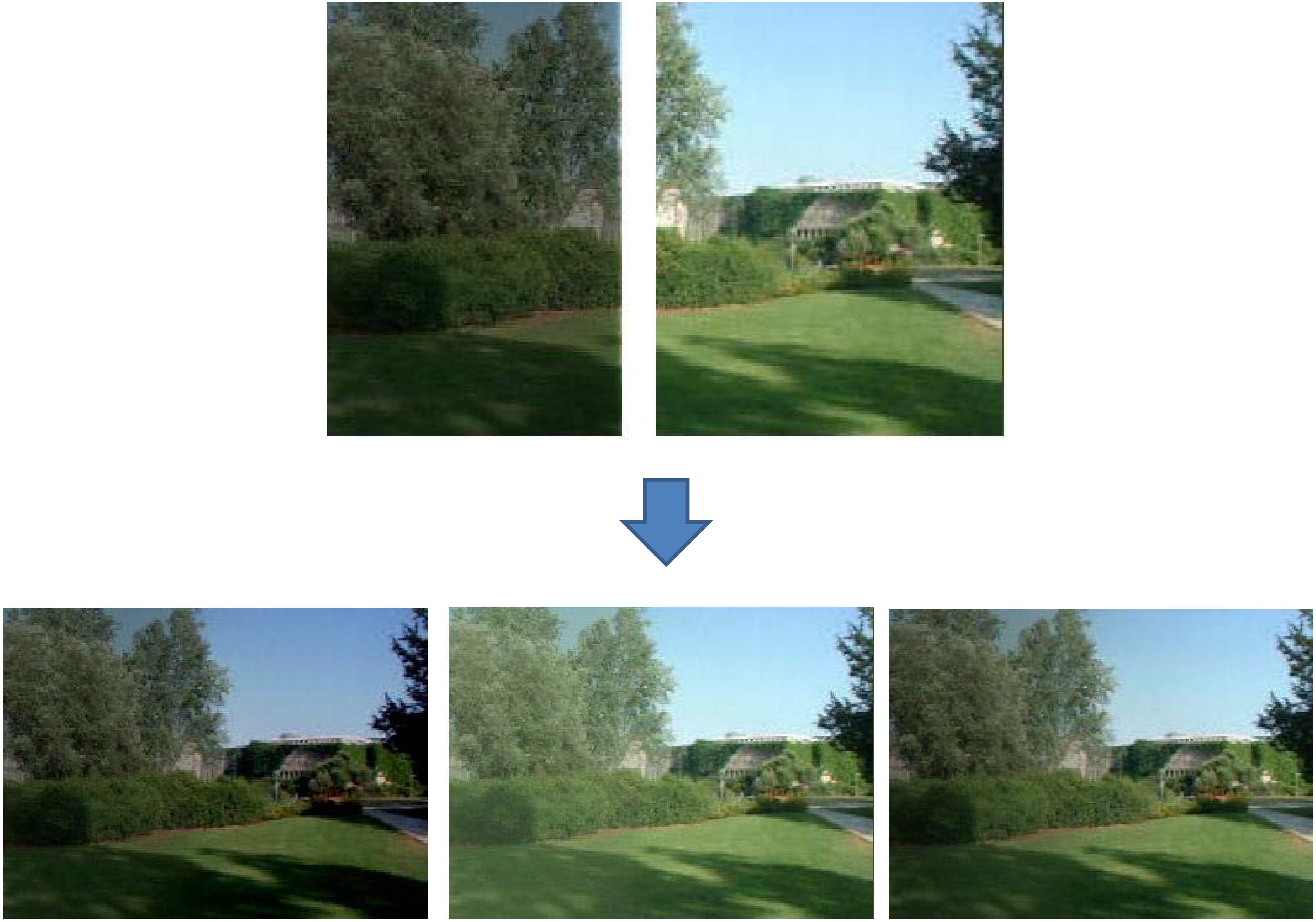




“Optimal” Window: smooth but not ghosted

Gradient based Stitching

- Input images



Recap: Gradient Domain Image Editing

- Motivation:
  - Human visual system is very sensitive to gradient
  - Gradient encode edges and local contrast quite well
- Approach:
  - Edit in the gradient domain
  - Reconstruct image from gradient

Q&A