



# Surface Completion

Ligang Liu

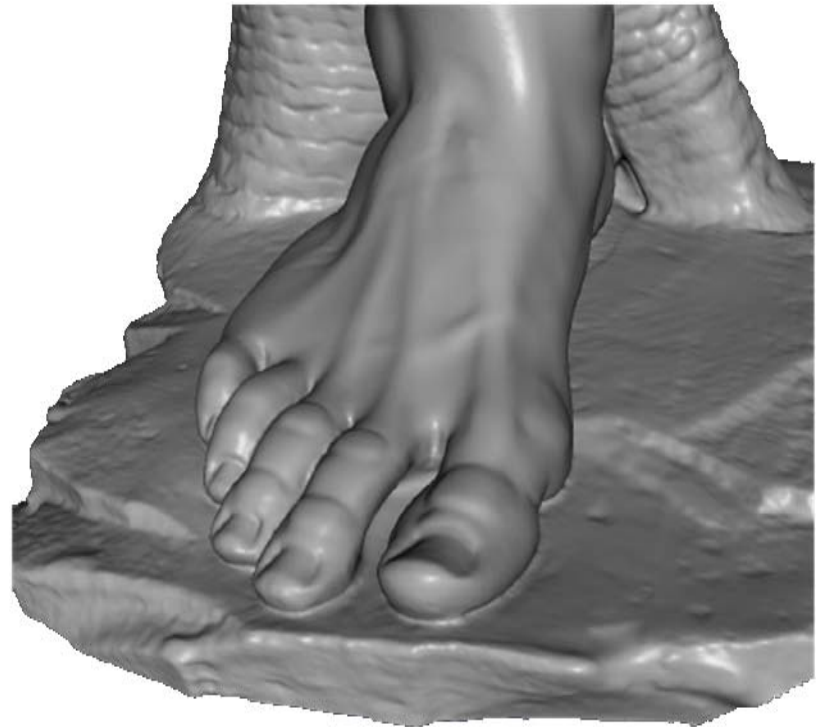
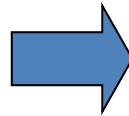
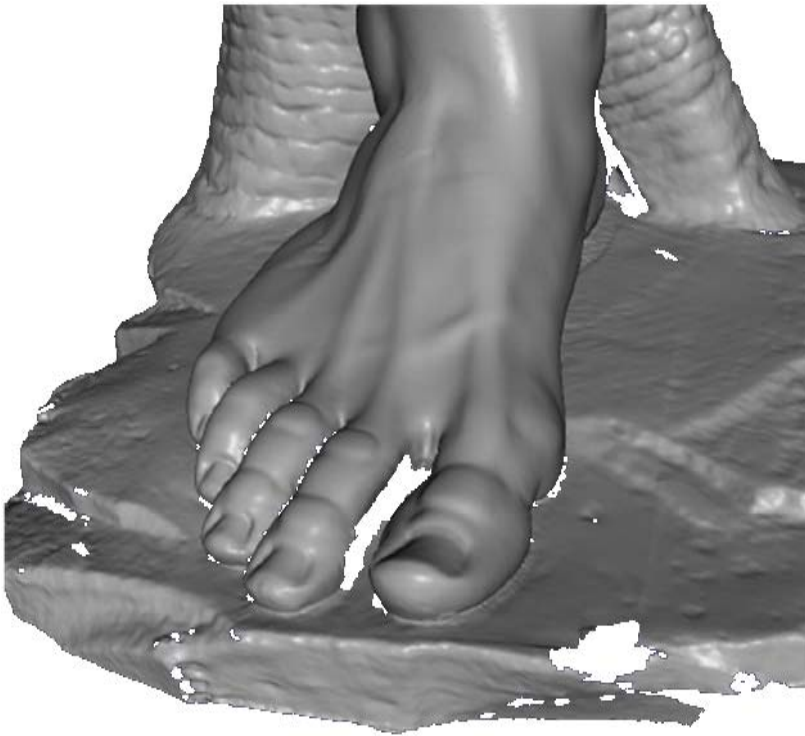
Graphics&Geometric Computing Lab

USTC

<http://staff.ustc.edu.cn/~lgliu>

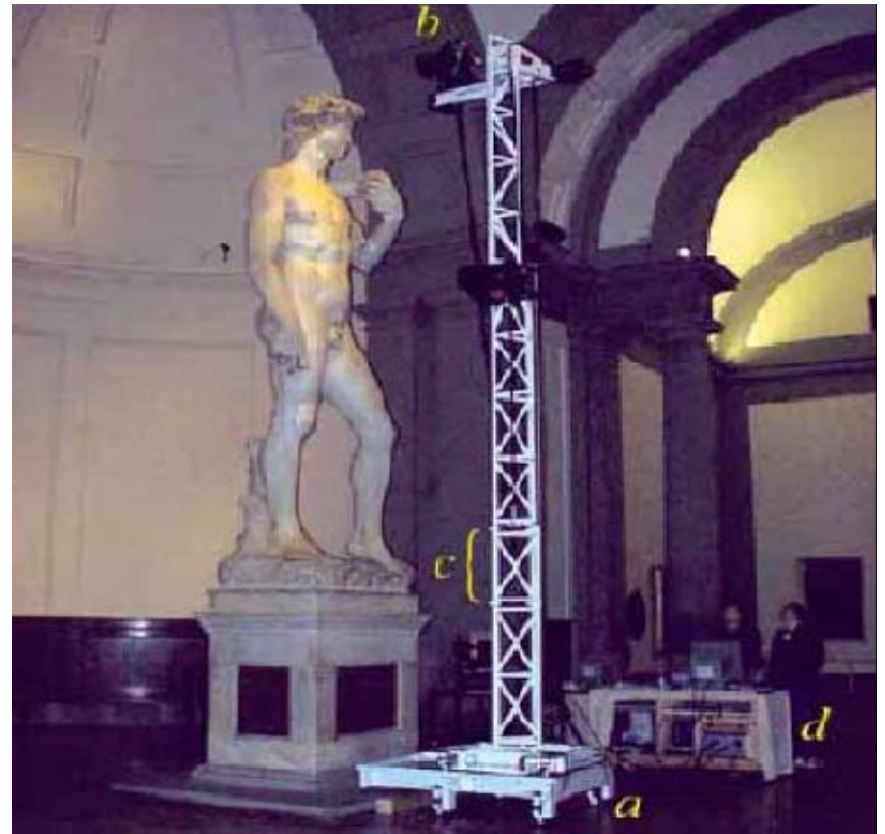
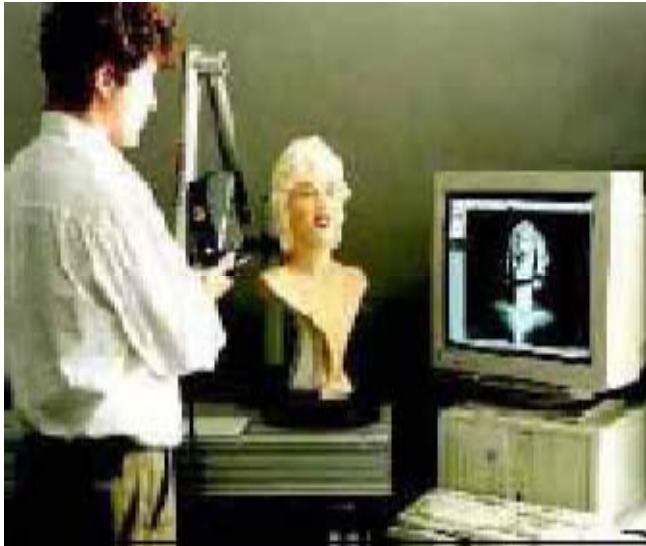
# Problem

- Filling holes in surface



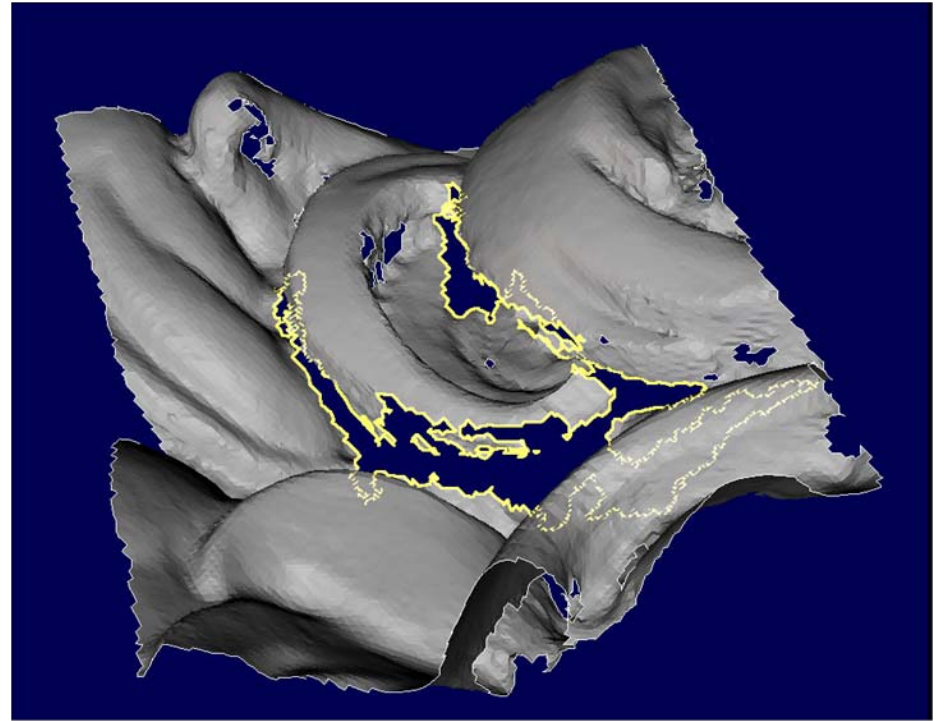
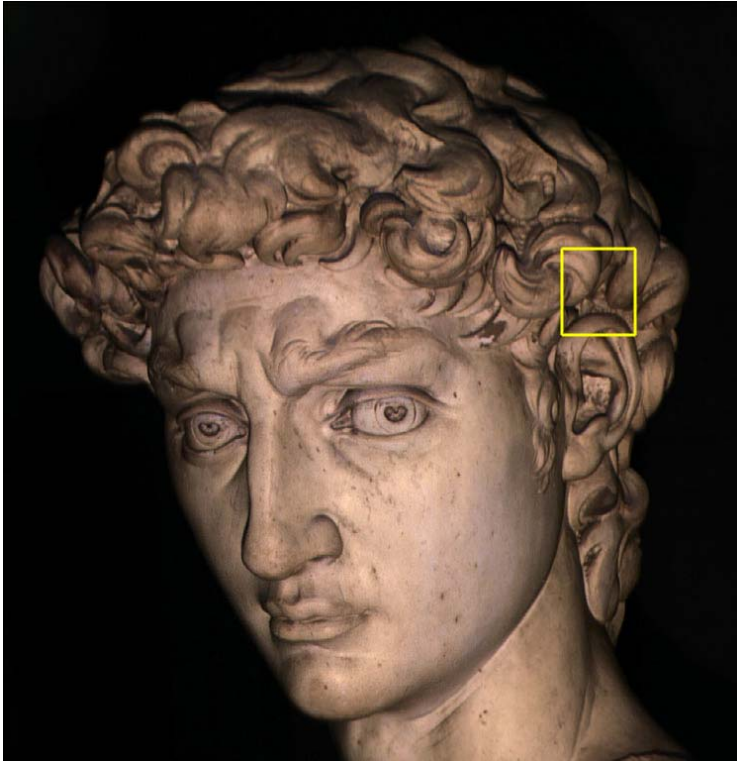
# Background

- Data acquisition



# Background

- Surface reconstruction



# Surface Completion

- Integrated into surface reconstruction algorithm
- Considered as a post-processing

# Other Terminologies

- Surface completion
- Surface inpainting
- Surface repairing
- Surface hole filling
- Surface restoration

# Outline

- Image inpainting
- Surface completion
  - Geometric method
  - Volumetric method
  - Texture synthesis based method

# 1. Image Inpainting

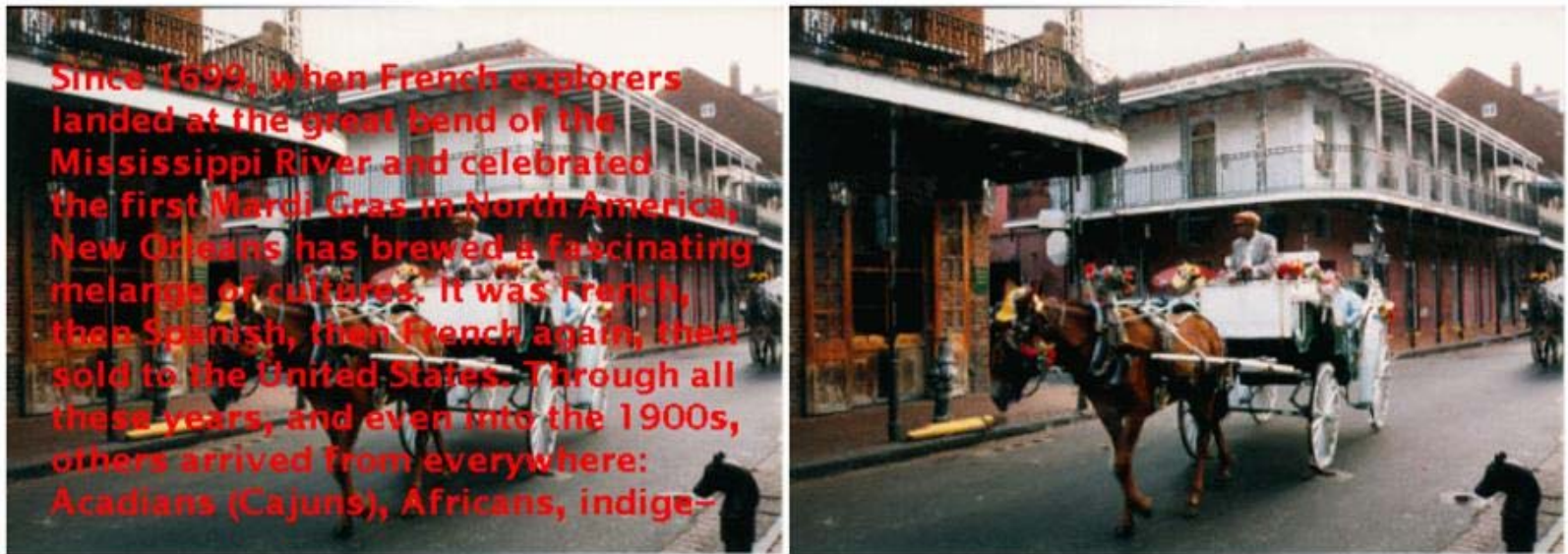


# Image Inpainting

- Objectives
  - Modify the image in a way that is non-detectable by an observer (fill holes in image)
  - Remove objects from the image
- Originated from museum restoration artists
  - Also known as “retouching”
- Less information present within the region to be inpainted

# Revised Definition

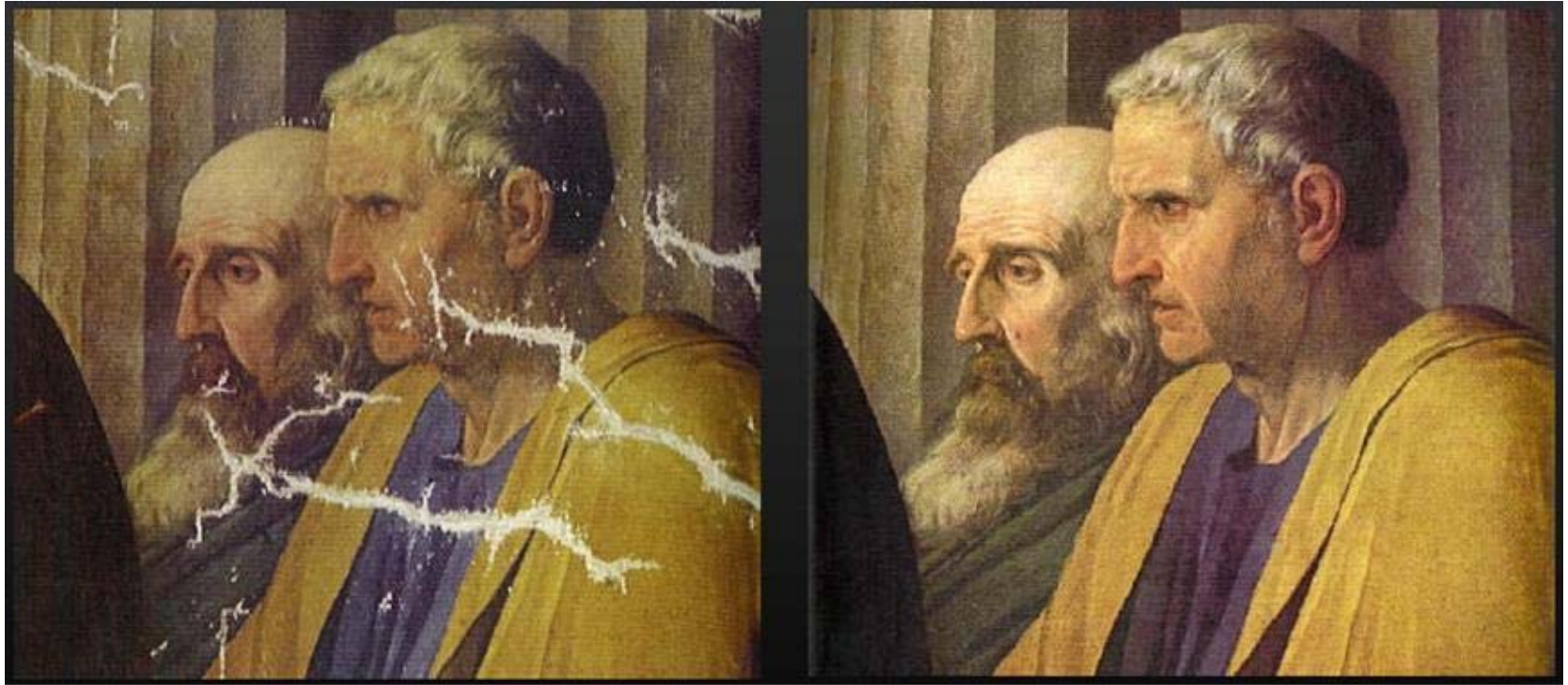
“Digital Image Inpainting is an iterative method for repairing damaged pictures or removing unnecessary elements from pictures”



***“Fast Digital Image Inpainting”***,

Manuel M. Oliveira, Brian Bowen, Richard McKenna and Yu-Sung Chang

# Photo Restoration



***“Image Inpainting : An Overview”***,  
Guillermo Sapiro

# Object Removal



# Related Work: Films

- e.g. Kokaram et al.



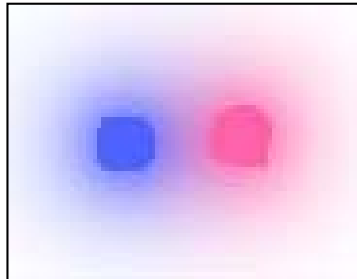
- Doesn't work for stills or static objects

# In the Literature

- Image Inpainting
  - Bertalmio and Sapiro , Siggraph 2000
- Level Lines Based Disocclusion
  - Masnou and Morel , ICIP 1998
- Fast Digital Image Inpainting
  - Oliveira et al., ICVIP 2001
- Image Inpainting: An Overview,
  - Sapiro 2002
- Fragment-based Image Completion
  - Drori et al., Siggraph 2003
- Image Completion with Structure Propagation
  - Sun et al., Siggraph 2005

# Photoshop

- Photoshop has some utilities for image inpainting
- Diffusion based inpainting
  - ❑ Simplest of all
  - ❑ Colors diffuse into the missing areas of the image
  - ❑ Repeated blurring
  - ❑ Colors of each pixel are averaged with a small portion of the color from the neighboring pixels.

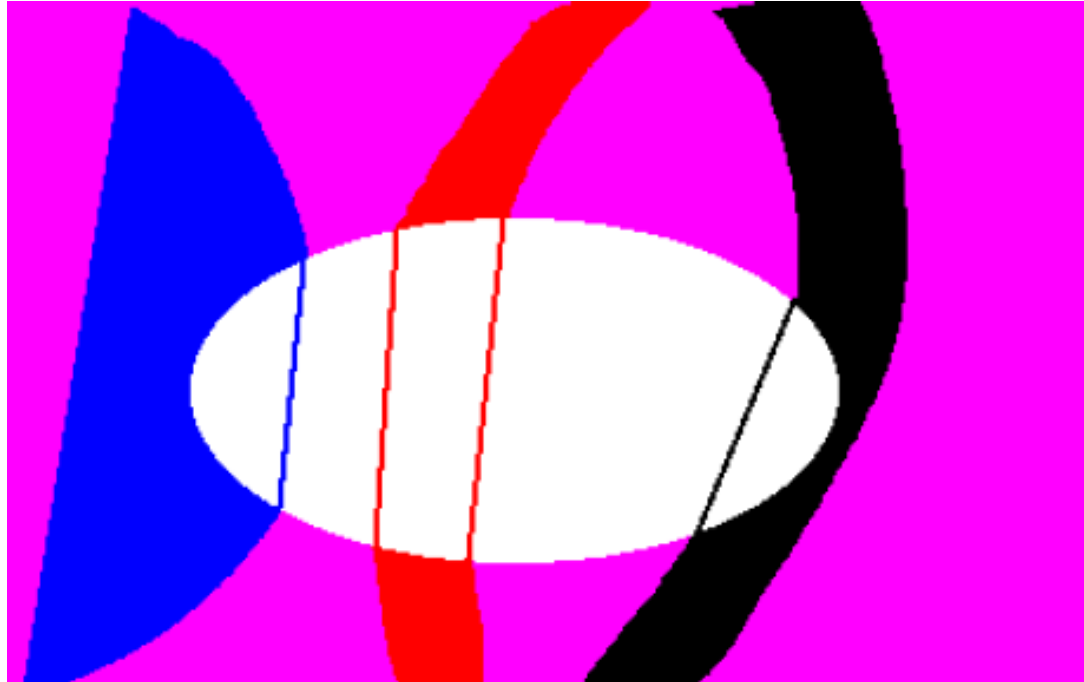


# 1.1 Level Line Based Method

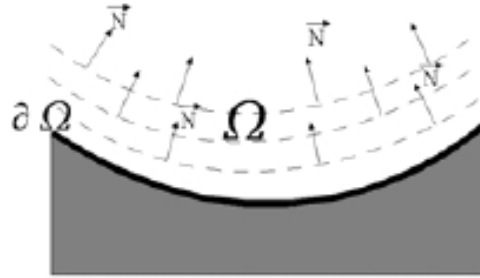
- Joining with geodesic curves the points of the isophotes (lines of equal gray values) arriving at the boundary of the region to be inpainted
- Drawbacks
  - Inpainted region should have simple topology
  - Angle of level lines is not preserved



# Example



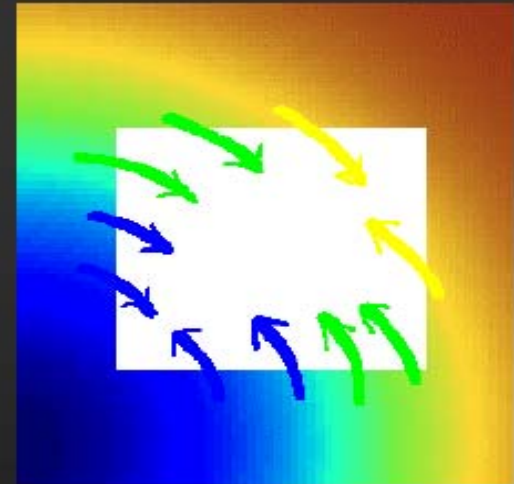
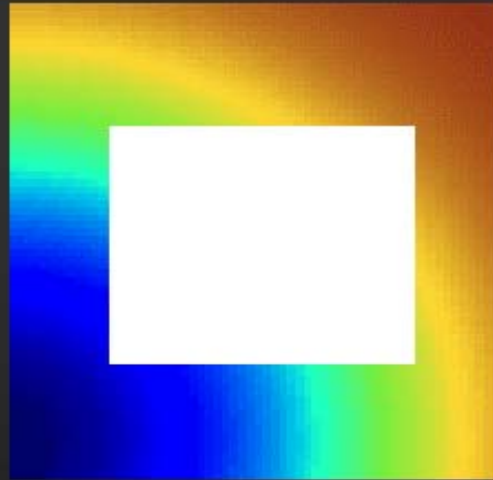
# Extension



[Siggraph 2000]

- Apply diffusion to the original image to avoid noise
- Updates to the values of pixels inside the region are made, information propagated in the direction of the isophotes
- After every few iterations, diffusion process is applied
- Propagation of gray values and the isophotes direction is critical
- Color images are considered as a group of 3 images and this technique is applied independently to them

# Idea



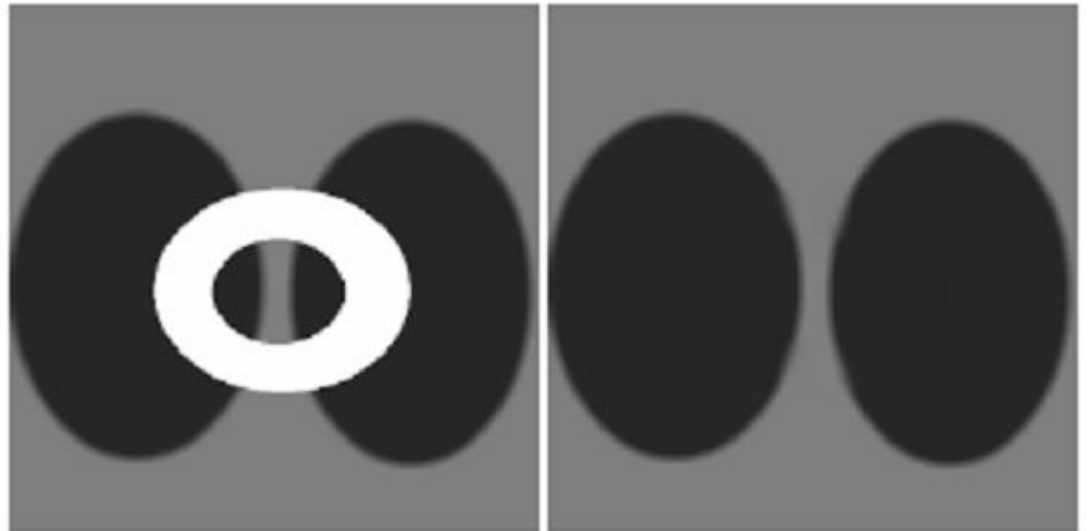
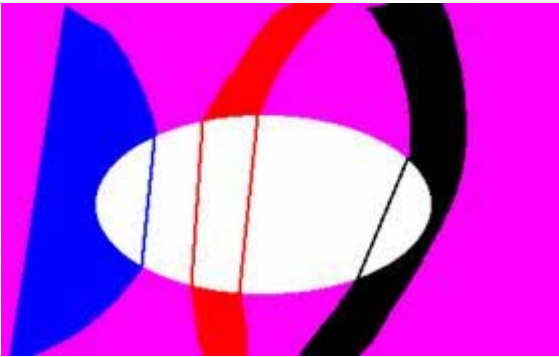
- **Propagate information**

$$\nabla \mathbf{L} \cdot \vec{\mathbf{N}} = 0$$

- **Evolutionary form**

$$\frac{\partial \mathbf{I}}{\partial \mathbf{t}} = \nabla \mathbf{L} \cdot \vec{\mathbf{N}}$$

# Comparison



# Examples



# Examples



# 1.2 Texture Synthesis Based Method

- Use frequency and spatial domain information to fill a given region with a selected texture
- Requires the user to specify which texture to put where
- Will not be preferred when the region to be replaced covers several different structures

# Texture Synthesis



- **Hirani, Efros, Heeger, DeBonet, Simoncelli, Zhu, etc.**
- **Not practical for rich regions**
- **Not designed for structured regions**
- **“Copy” information instead of “see and interpolate”**



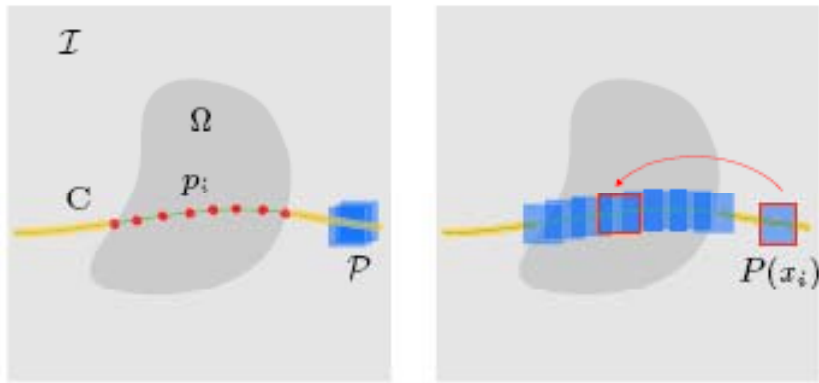
# 1.3 Structure Propagation

[Siggraph 2005]

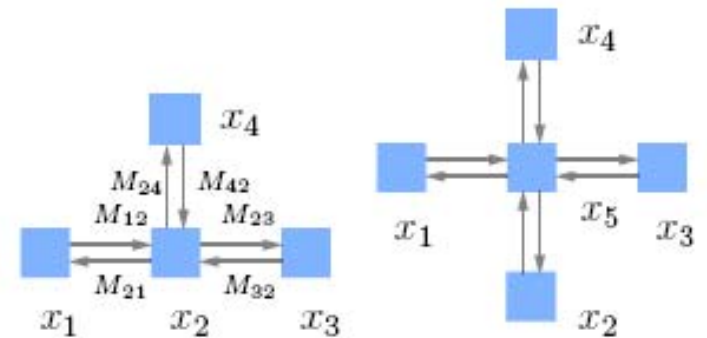
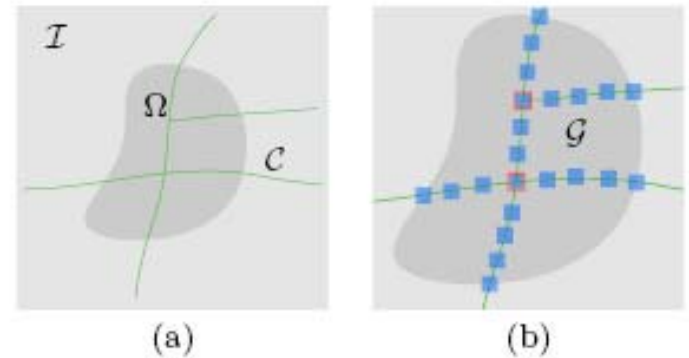
- Structure propagation
  - User specified structure curves
- Patch based texture synthesis



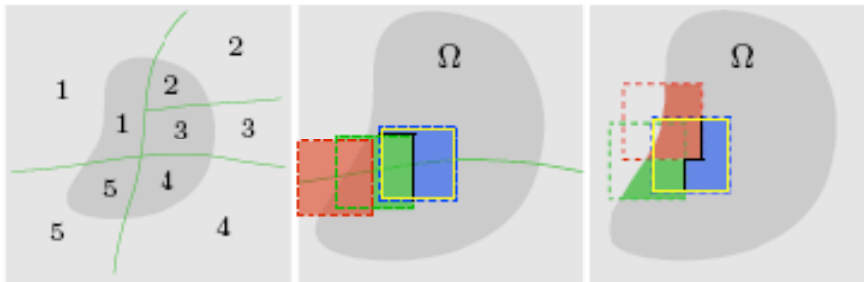
# Structure Propagation



1D chain propagation



2D graph propagation

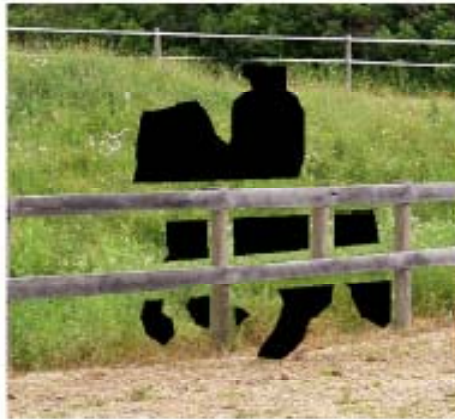
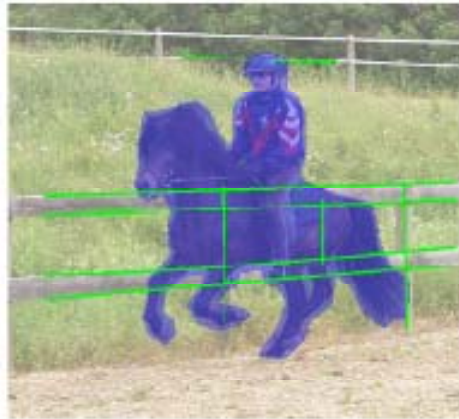


texture propagation

# Examples



# Examples



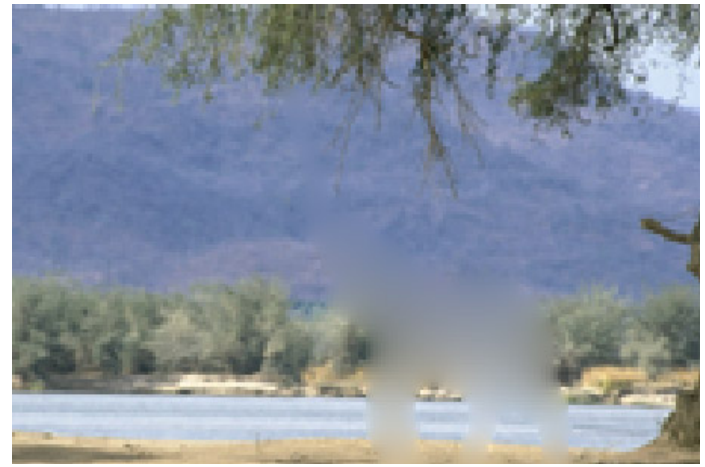
# 1.4 Fragment-based Method

[Siggraph 2003]

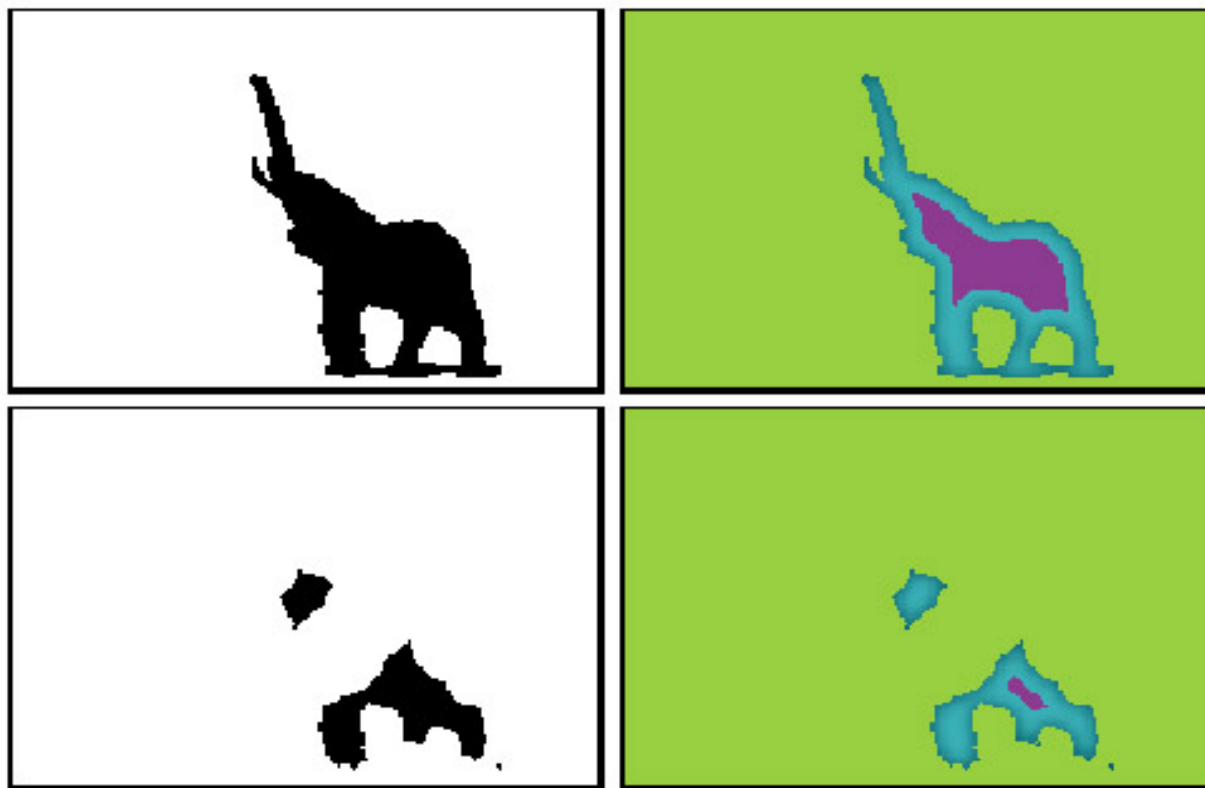
Multiply Image by inverse matte  
and add matte



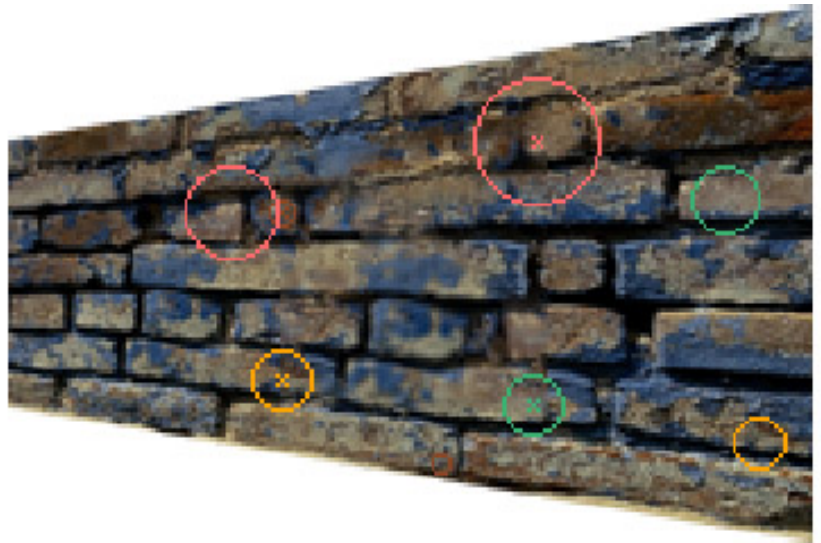
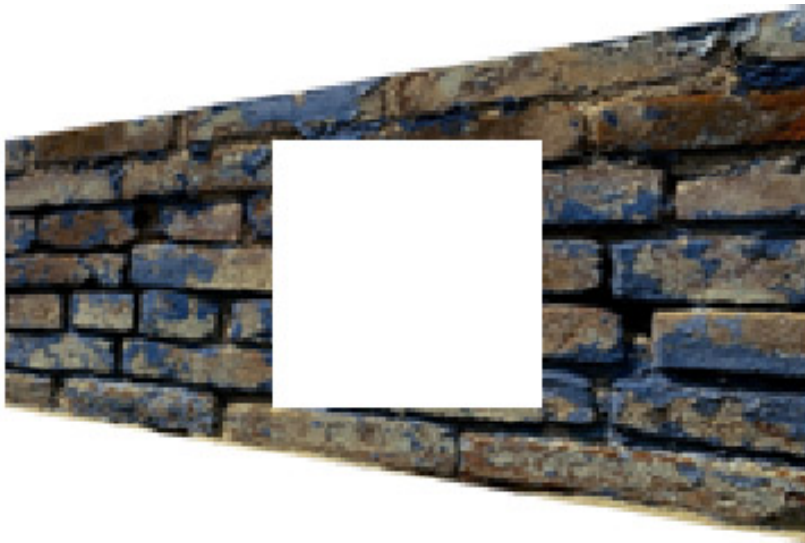
Downsample and upsample with  
kernel



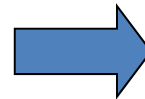
# Confidence Map



# Search

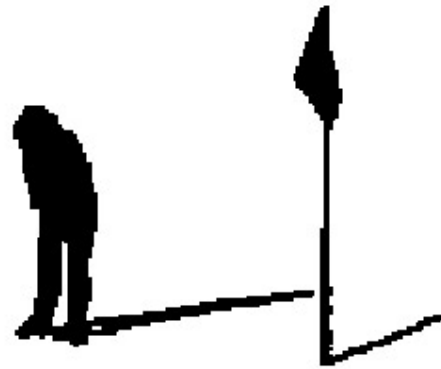


# Results





# Results



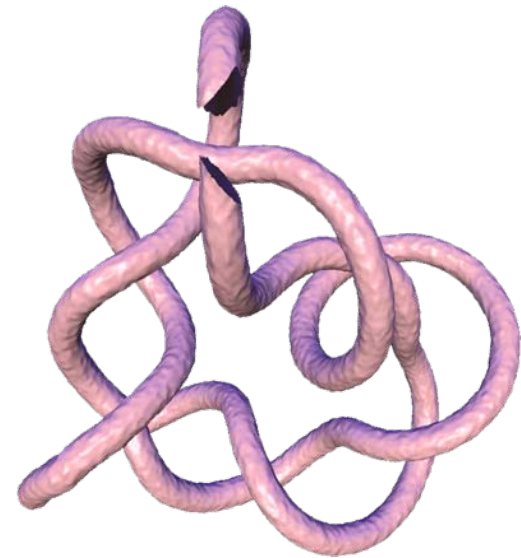
## 2. Surface Completion

# Requirements

- Robustness
  - Producing a watertight surface for any input model
- Efficiency
  - Processing huge models within reasonable time and space
- Accuracy
  - Preserving geometry
  - Boundary condition
  - Context condition

# Extend 2D Methods to 3D

- Images
  - A regular spatial structure domain
- Problems in 3D
  - Topology and geometry of missing region
  - Fitting a patch to the boundary of the missing region
- Definition of similarity of shapes
- Definition of a surface patch



# Classifications

- Geometric method
  - Operate directly on the polygons in the model
- Volumetric method
  - Convert a polygonal model into a volume representation
- Image-based
  - Convert a polygonal model into a geometry image

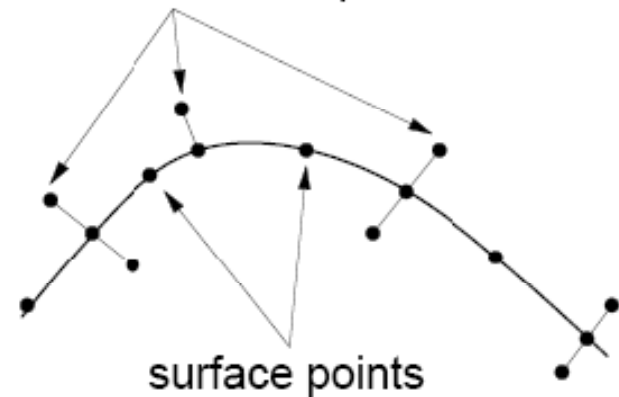
# 2.1 Radial Basis Function

[Carr et al., Siggraph 2001]

# Signed-distance Function

$$\begin{aligned} f(x_i, y_i, z_i) &= 0, & i &= 1, \dots, n & \text{(on-surface points),} \\ f(x_i, y_i, z_i) &= d_i \neq 0, & i &= n+1, \dots, N & \text{(off-surface points).} \end{aligned}$$

off-surface 'normal' points



# Interpolation Problem

Given:  $X = \{x_i\}_{i=1}^N \subset \mathbb{R}^3$

and  $\{f_i\}_{i=1}^N \subset \mathbb{R}$

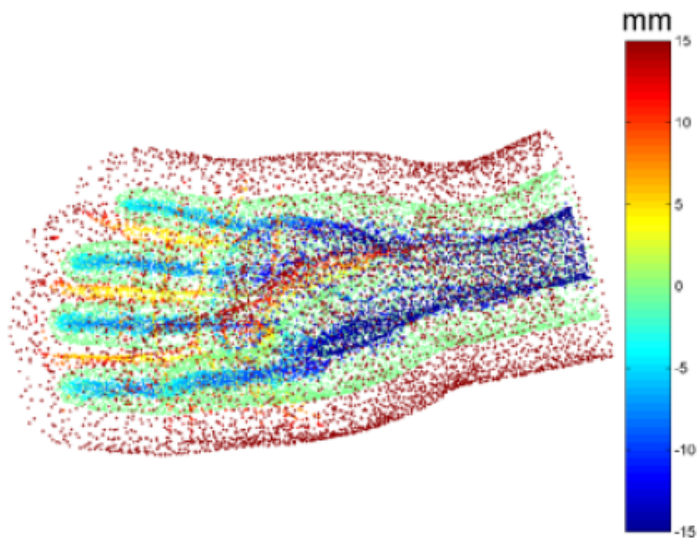
$s: \mathbb{R}^3 \rightarrow \mathbb{R}$



$s(x_i) = f_i, i = 1, \dots, N.$

Output:

$\{x \in \mathbb{R}^3 \mid s(x) = 0\}$





# Radial Basis Function

- General form

$$s(x) = p(x) + \sum_{i=1}^N \lambda_i \phi(|x - x_i|), \quad s \in \text{BL}^{(2)}(\mathbb{R}^3)$$

$p(x)$  is a polynomial of low degree  
the basic function  $\phi$  is a real valued function

- e.g.

$$p(\vec{x}) = c_1 + c_2x + c_3y + c_4z,$$
$$\phi(|x_i - x_j|) = |x_i - x_j|,$$

# Evaluation

$$s(x) = c_1 + c_2x + c_3y + c_4z + \sum_{i=1}^N \lambda_i |x - x_i|,$$

$$s \in \text{BL}^{(2)}(\mathbb{R}^3)$$

$$\sum_{i=1}^N \lambda_i = \sum_{i=1}^N \lambda_i x_i = \sum_{i=1}^N \lambda_i y_i = \sum_{i=1}^N \lambda_i z_i = 0.$$

# Evaluation

$$\begin{pmatrix} A & P \\ P^\top & 0 \end{pmatrix} \begin{pmatrix} \lambda \\ c \end{pmatrix} = \begin{pmatrix} f \\ 0 \end{pmatrix},$$

$$A_{i,j} = |x_i - x_j|, \quad i, j = 1, \dots, N,$$

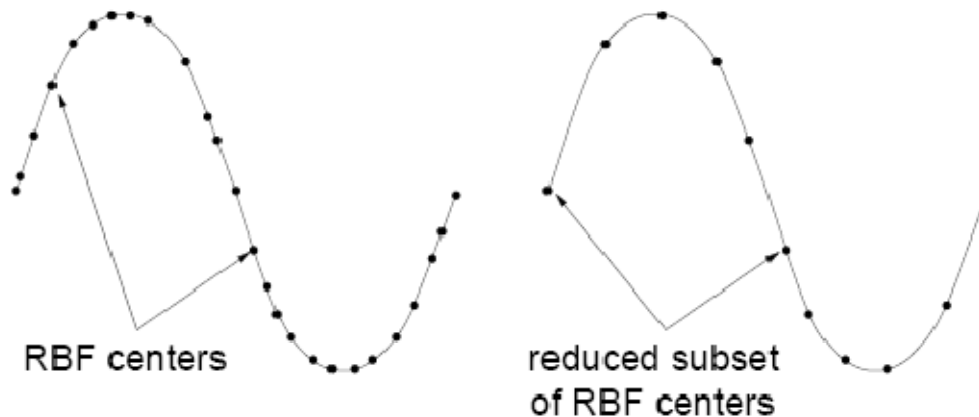
$$P = (1, x_i, y_i, z_i), \quad \lambda = (\lambda_1, \dots, \lambda_N)^\top$$

$$c = (c_1, c_2, c_3, c_4)^\top.$$

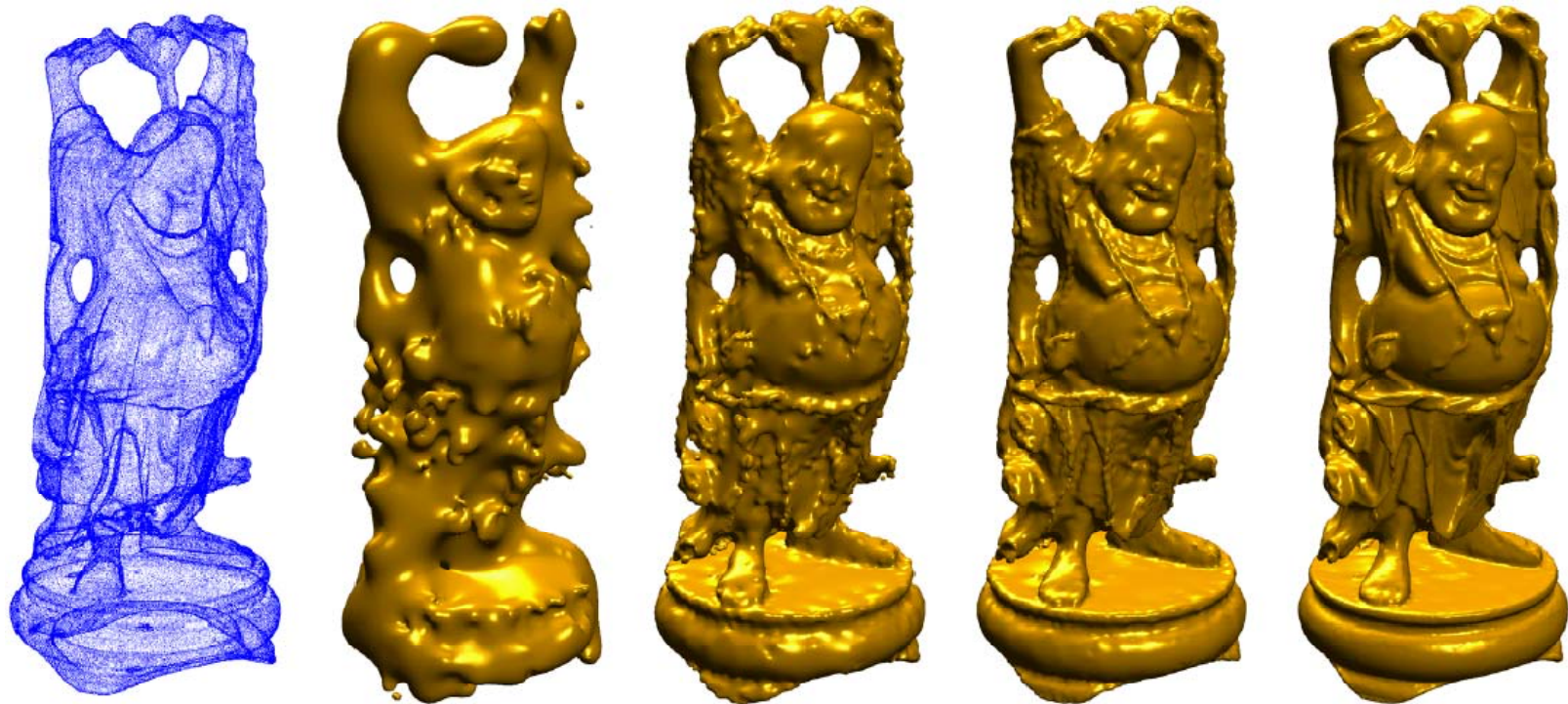
# Greedy algorithm

- Procedure

1. Choose a subset from the interpolation nodes  $X$  and fit an RBF only to these.
2. Evaluate the residual,  $\varepsilon_i = |f_i - s(x_i)|$  nodes.
3. If  $\max\{|\varepsilon_i|\} < \text{fitting accuracy } \rho$ .
4. Else append new centers where  $|\varepsilon_i|$  is large.
5. Re-fit RBF and goto 2.



# Results



# Summary

- Repair holes with arbitrary topology



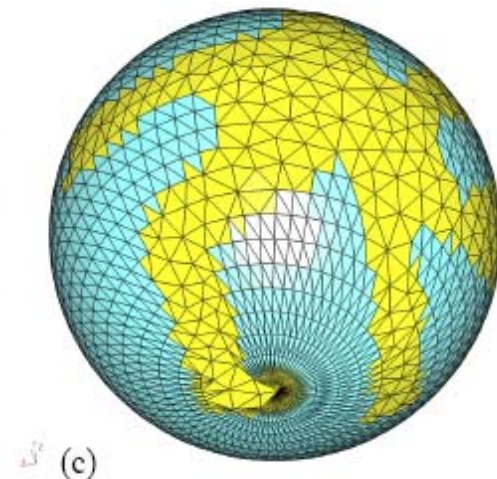
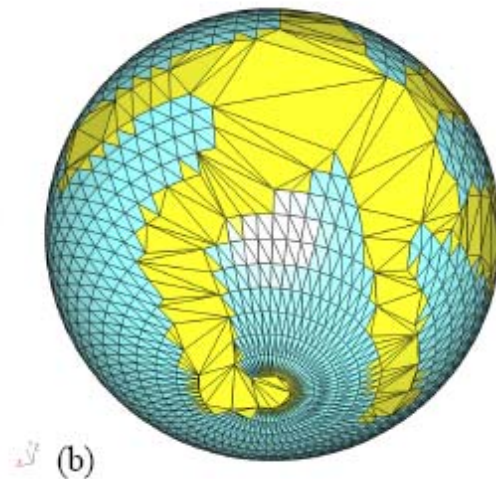
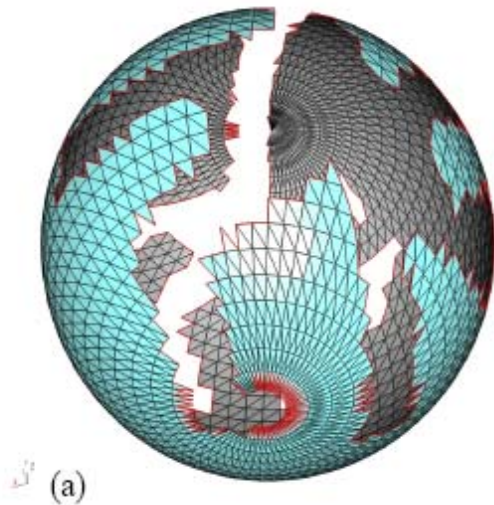
- Holes are smoothly filled
- Approximate original surface

## 2.2 Filling Holes in Meshes

[Liepa, SGP 2003]

# Pipeline

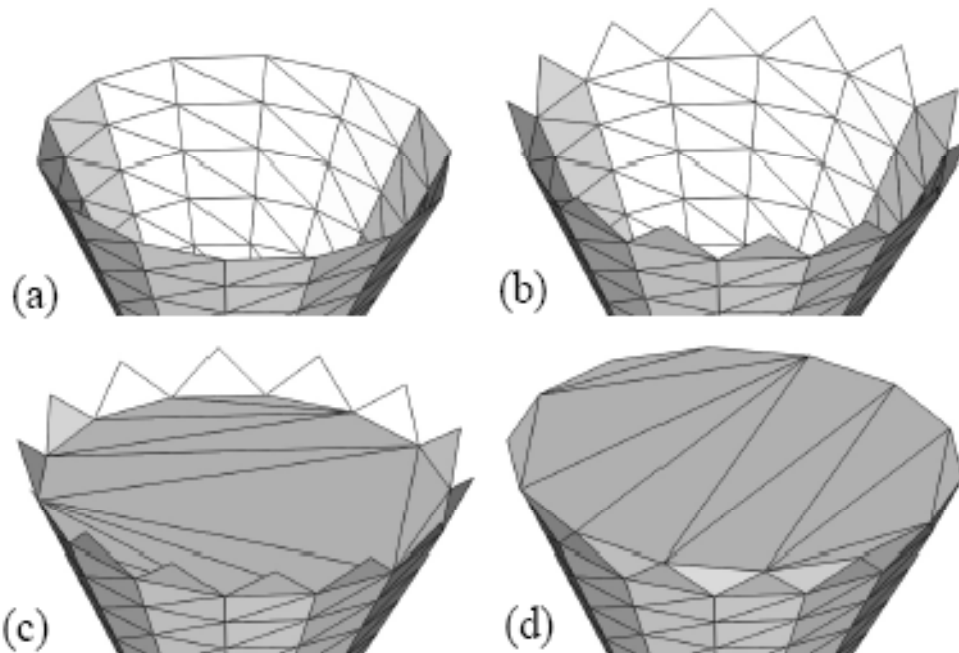
1. Hole identification
2. Hole triangulation
3. Mesh refinement
4. Mesh fairing



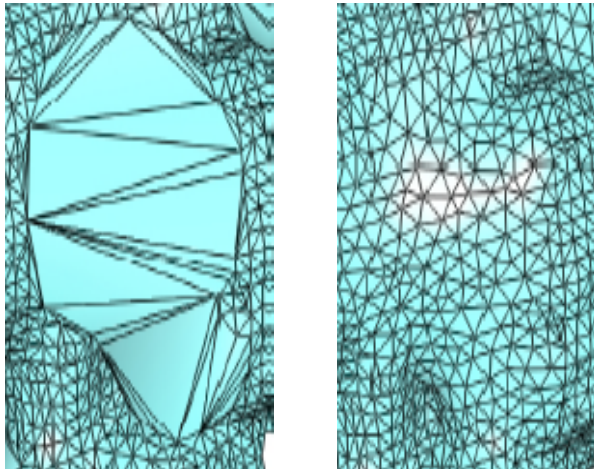


# Triangulation of 3D Polygons

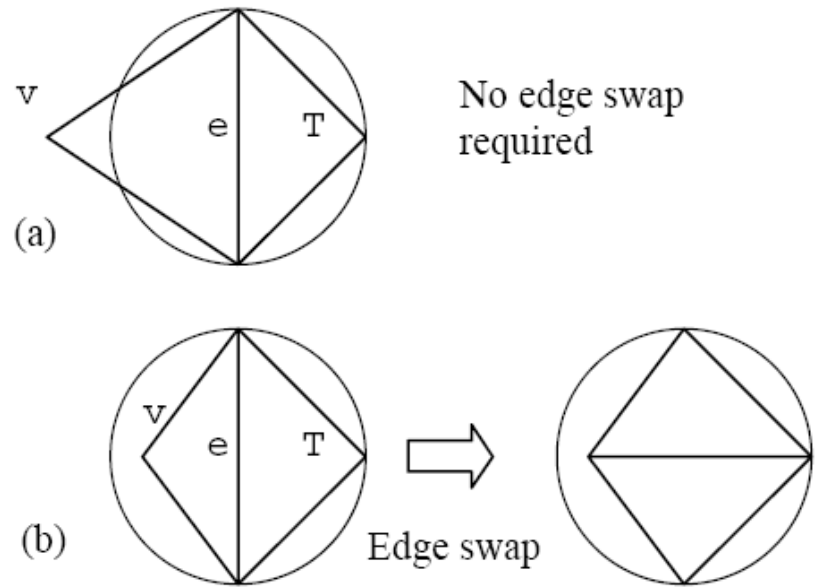
- Minimum area triangulation
- Min-max dihedral angle triangulation



# Mesh Refinement



1. Subdivision



2. Edge Relaxation

# Fairing

- Weighted umbrella-operator

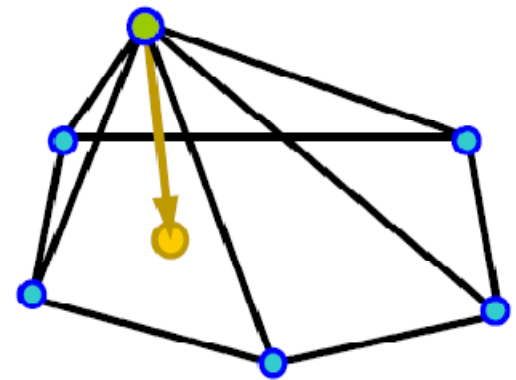
$$\mathbf{U}_\omega(\mathbf{v}) = -\mathbf{v} + \frac{1}{\omega(\mathbf{v})} \sum_i \omega(\mathbf{v}, \mathbf{v}_i) \mathbf{v}_i,$$

$$\mathbf{v} = \mathbf{v} + \mathbf{U}_\omega(\mathbf{v})$$

- Uniform :  $\omega(\mathbf{v}_i, \mathbf{v}_j) = 1$

- Scale-dependent :

$$\omega(\mathbf{v}_i, \mathbf{v}_j) = 1 / \|\mathbf{v}_i - \mathbf{v}_j\|$$



# Summary

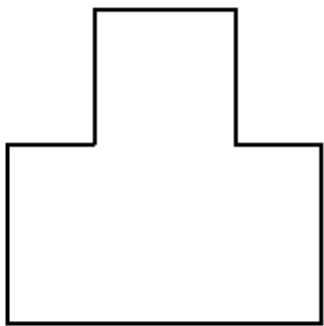
- Easy to implement
- Focus algorithm on holes
- Triangulation may self-intersect
- Can't fill holes with islands
- Fairing weaken original surface feature

## 2.3 Robust Repair of Polygonal Models

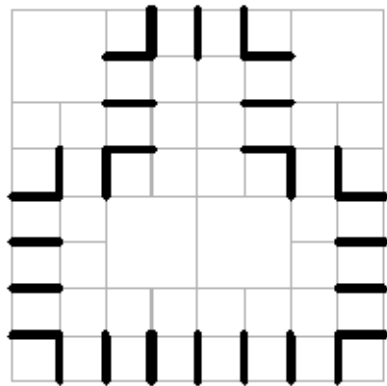
[Ju, Siggraph 2004]

# Pipeline

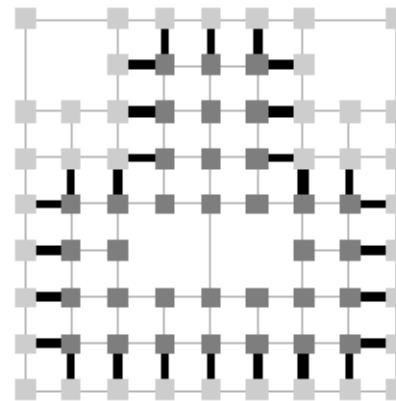
- I. Scan-conversion
- II. Sign generation
- III. Surface reconstruction



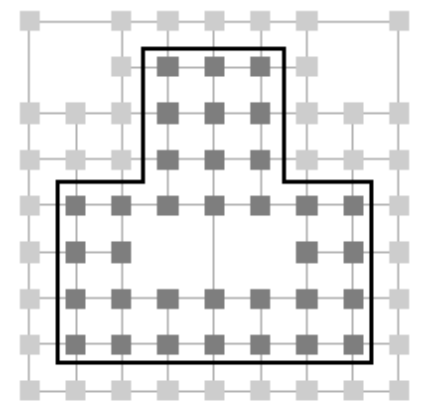
a



b



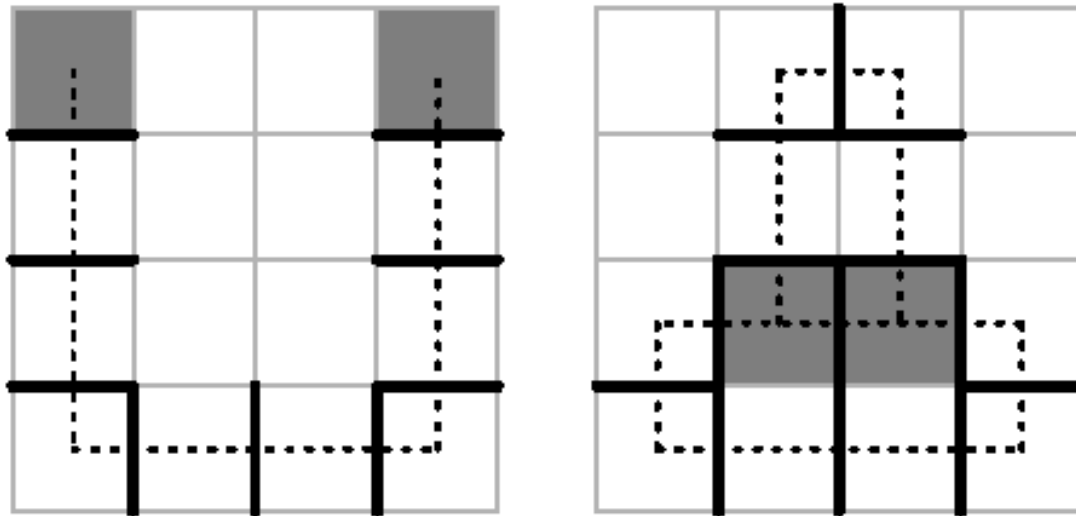
c



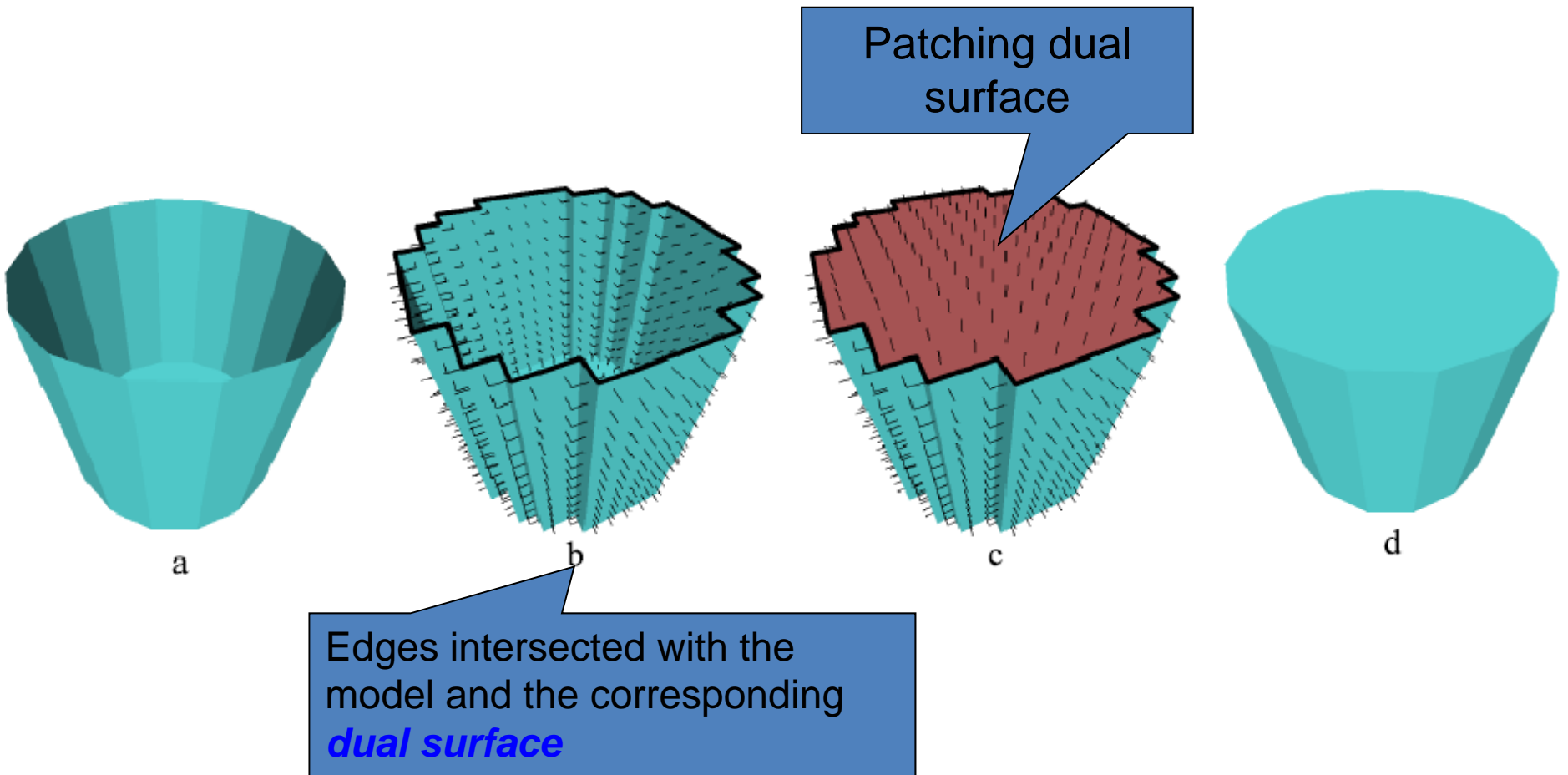
d

# Sign Generation

- Cell faces containing an odd number of intersection edges



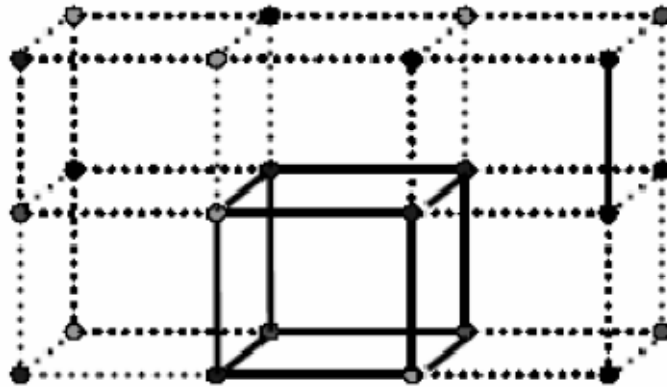
# Patch Boundary Circles



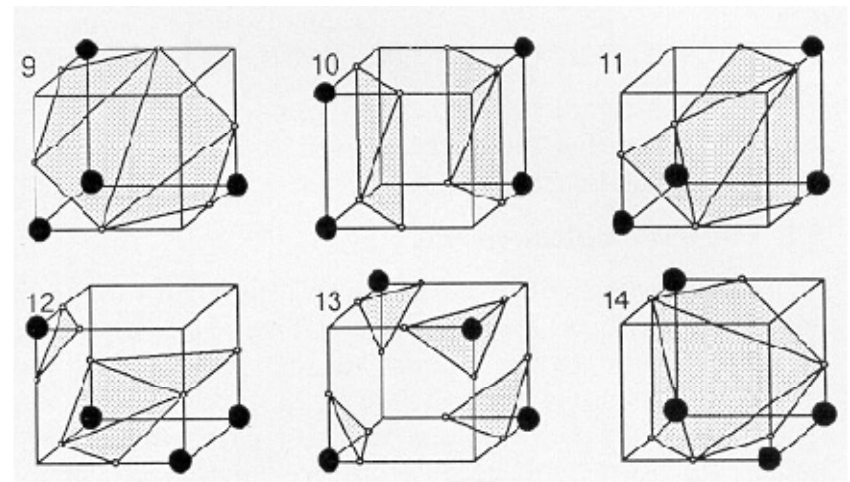
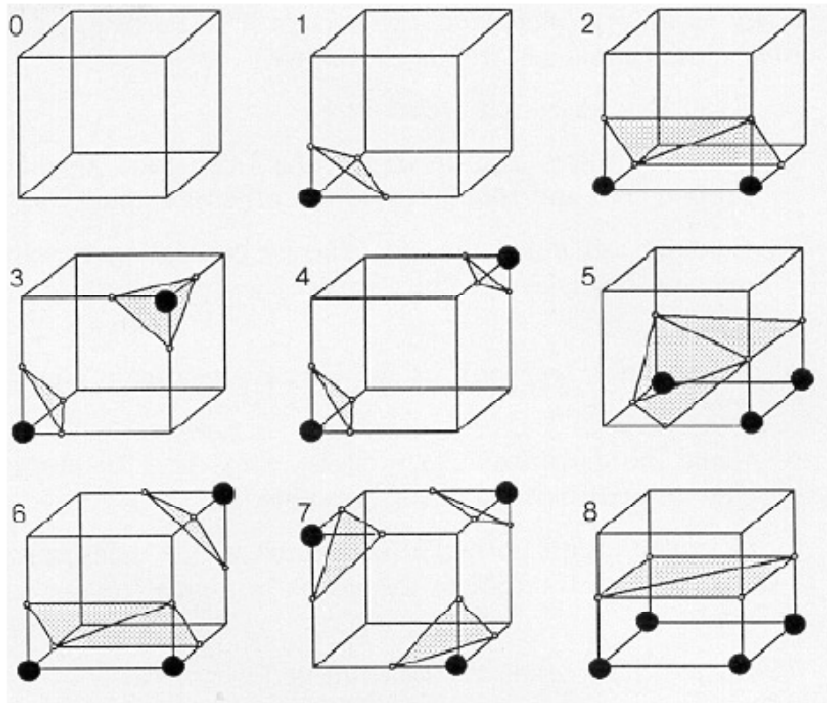


# Marching Cubes

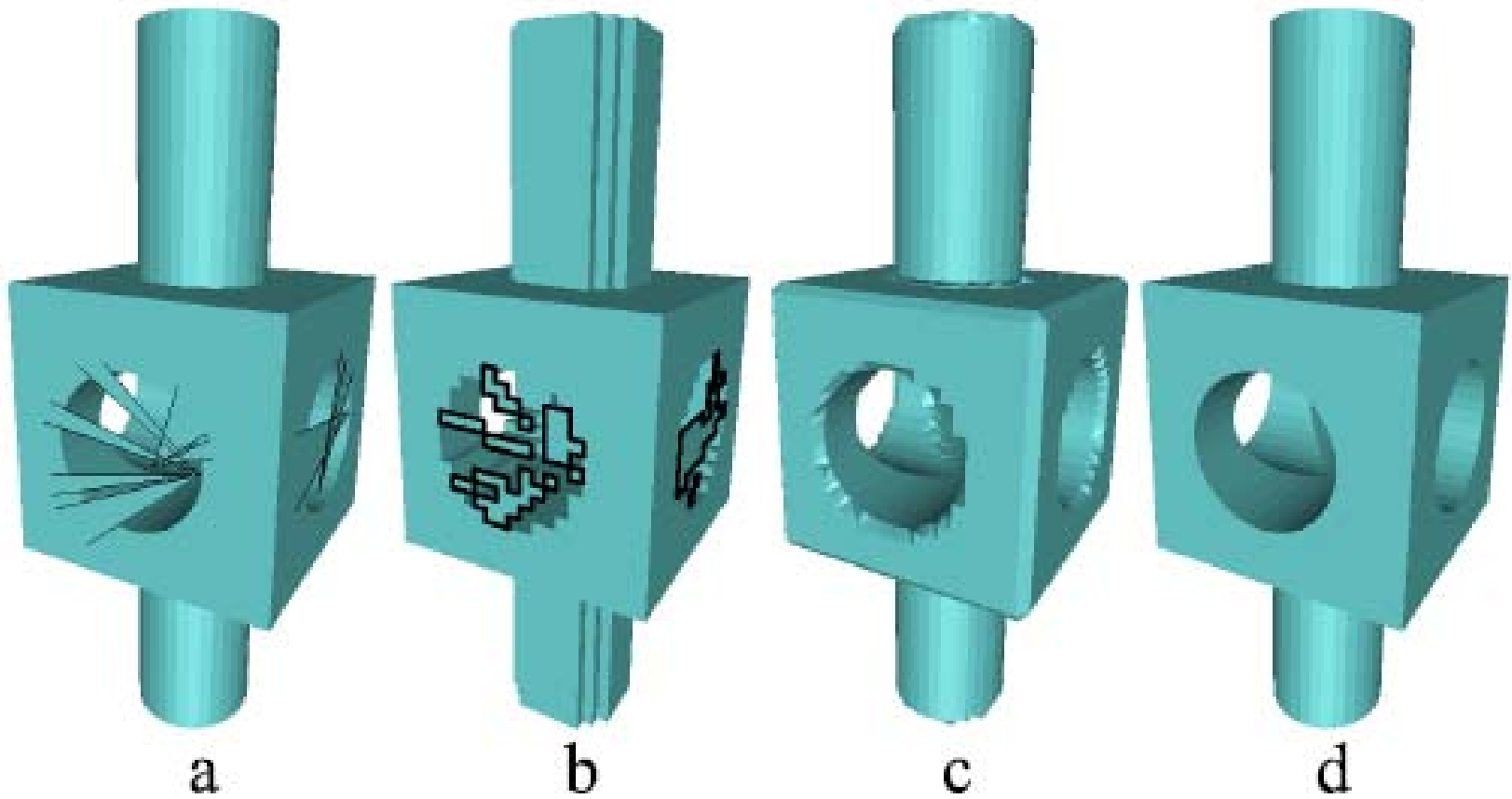
- Cube with signs at eight corners



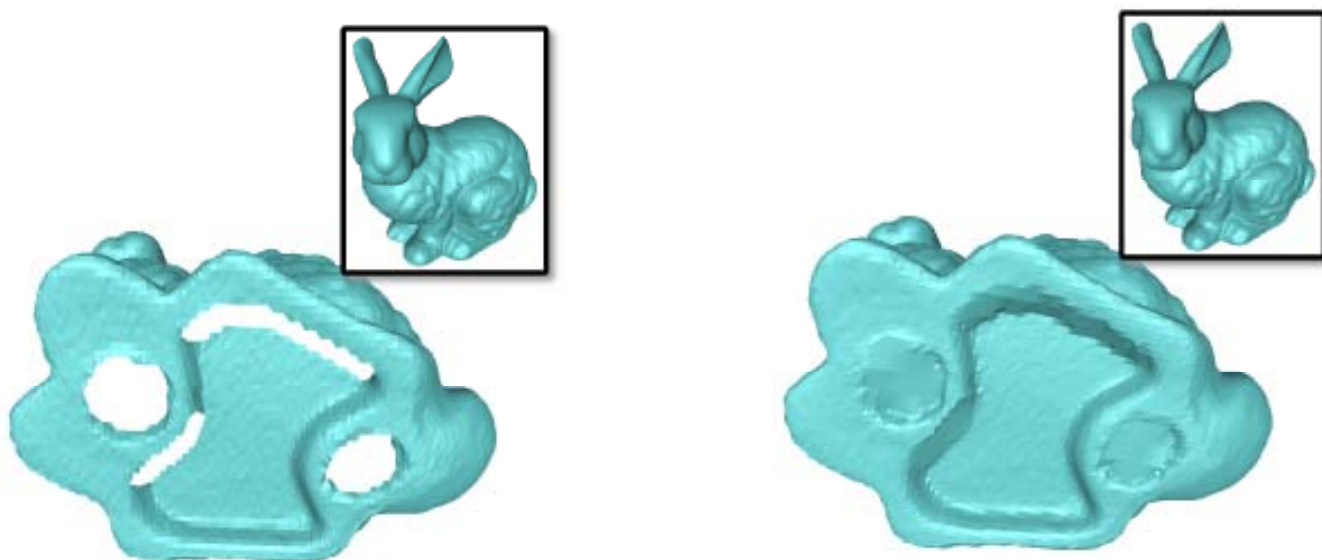
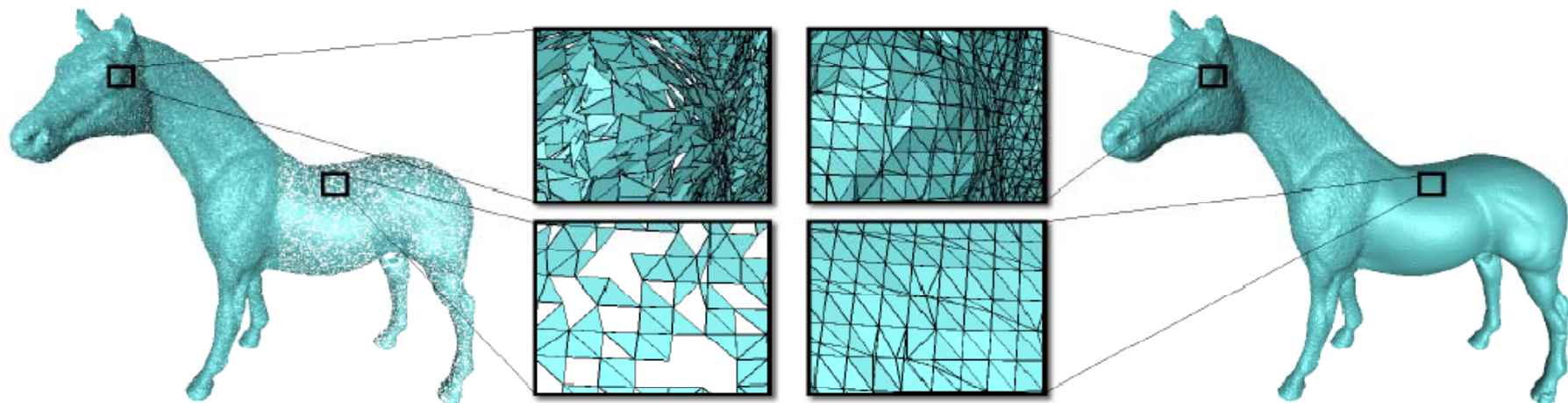
# Marching Cubes



# Results



# Results



# Summary

- Employ a space-efficient octree grid
- Produce closed, manifold surface for any input model

## 2.4 Context-based Surface Completion

[Sharf et al., Siggraph 2004]

# Motivation

Complete the missing region with patches that conform with its context



Smooth

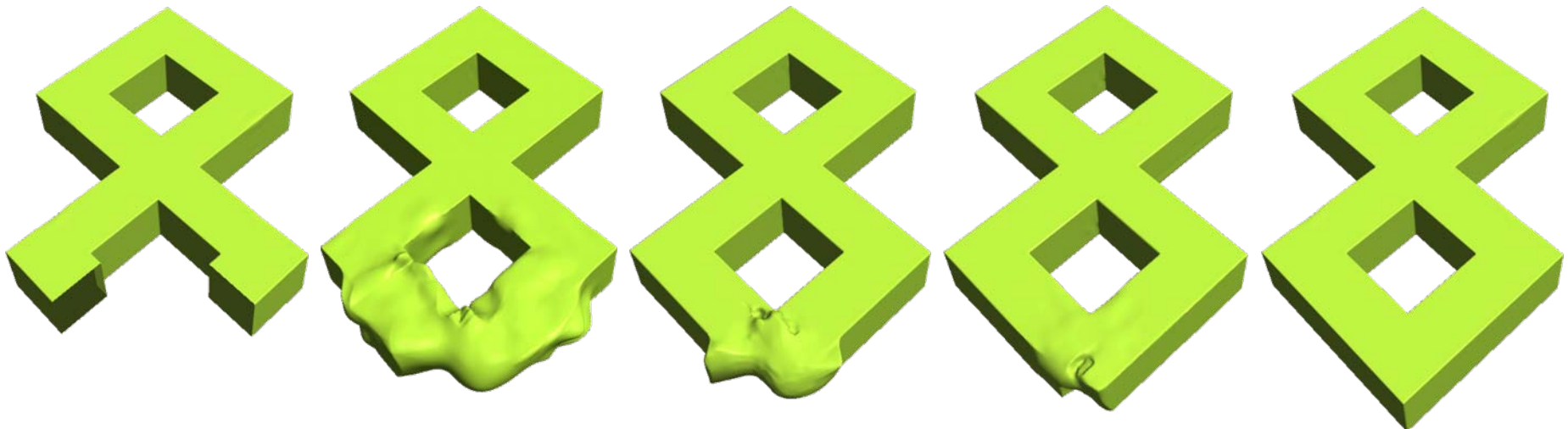


Context-based

# Method

Import patches with matching context from the surface itself :

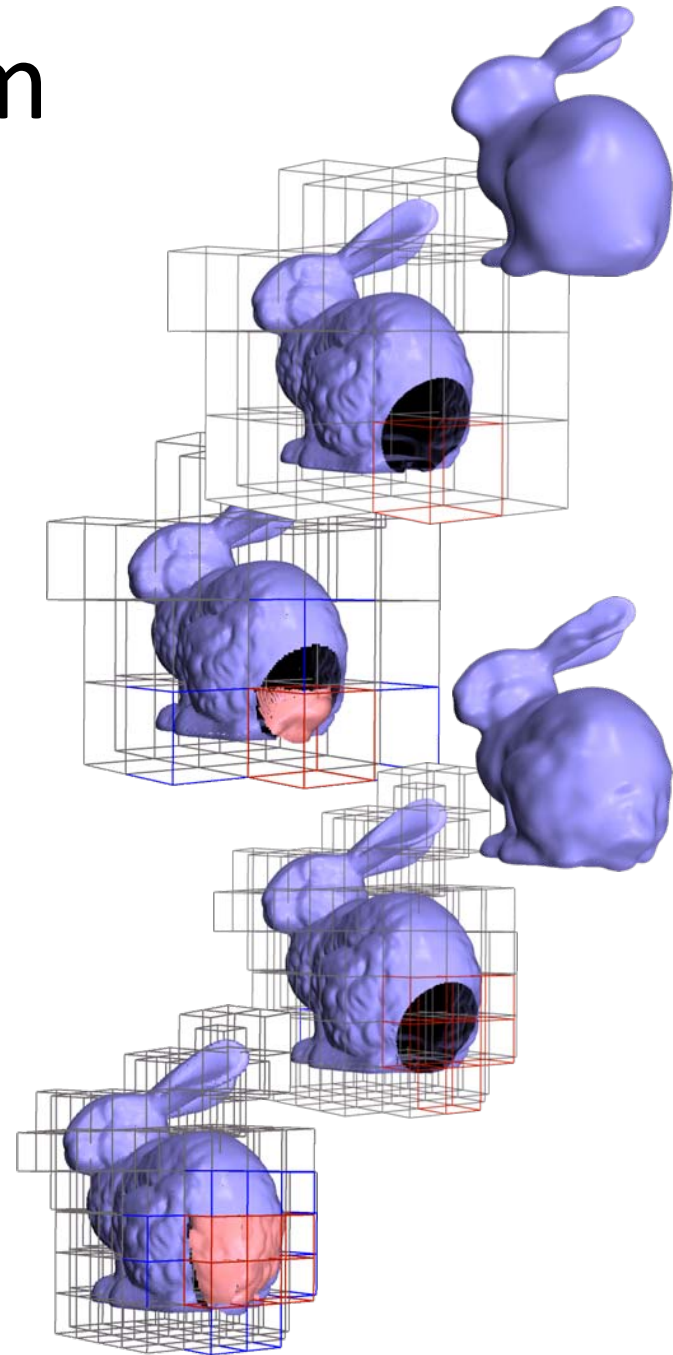
- Analyze surface characteristics.
- Find best matching patch.
- Fit imported patch to boundary.



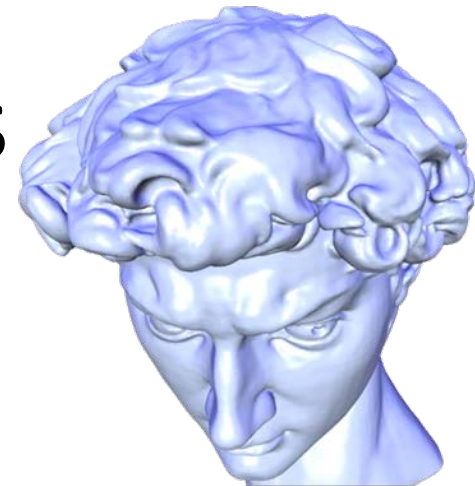


# Algorithm

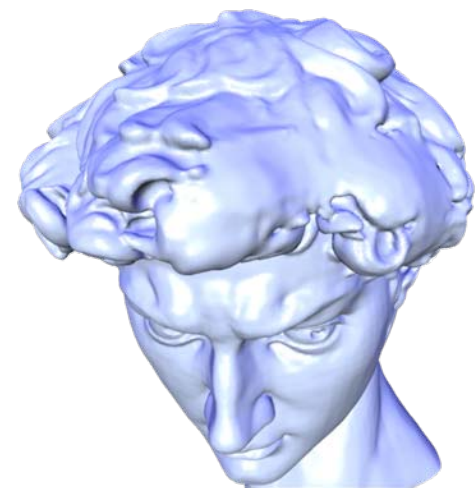
- Create initial spatial subdivision
- For each cell
  - Compute a local shape representation.
  - Compute a shape signature.
- For each empty cell:
  - Find matching nonempty cell  $\omega'$ .
  - Copy patch of  $\omega'$  into  $\omega$ .
- Subdivide cells and repeat
- Until completed region matches its neighborhood



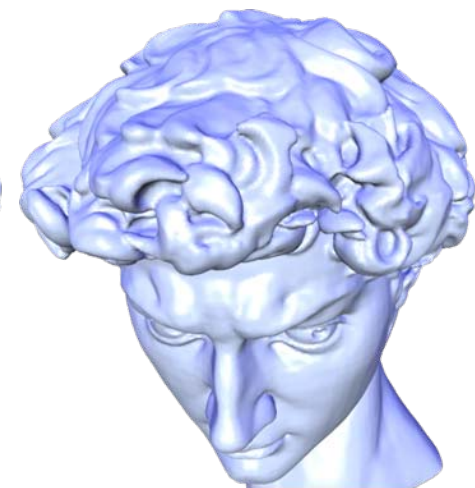
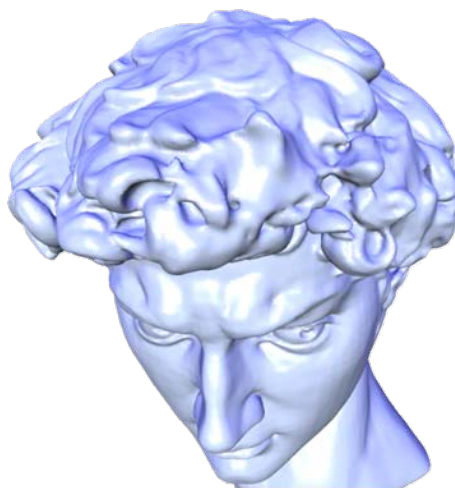
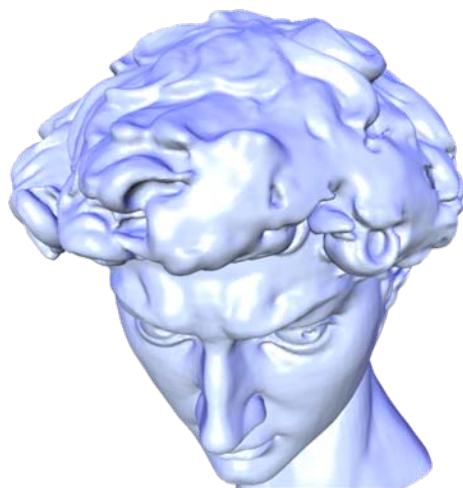
# Completion Process



Original

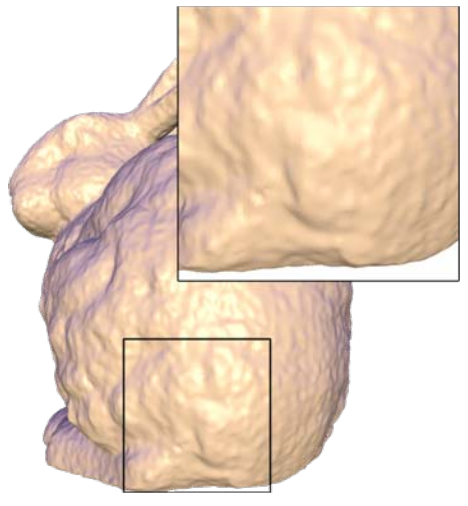
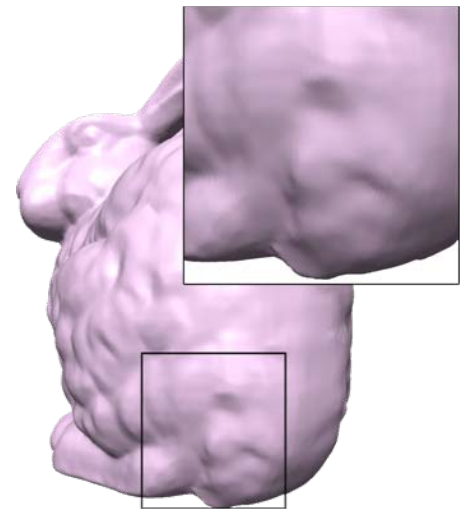
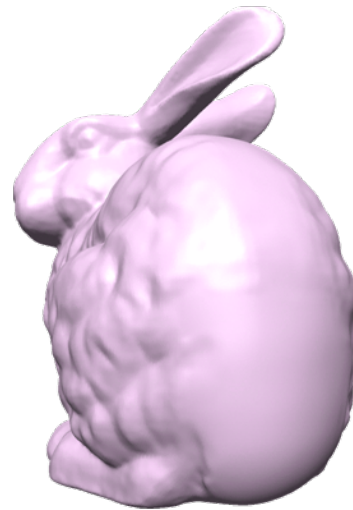


Initial  
approximation



Final result

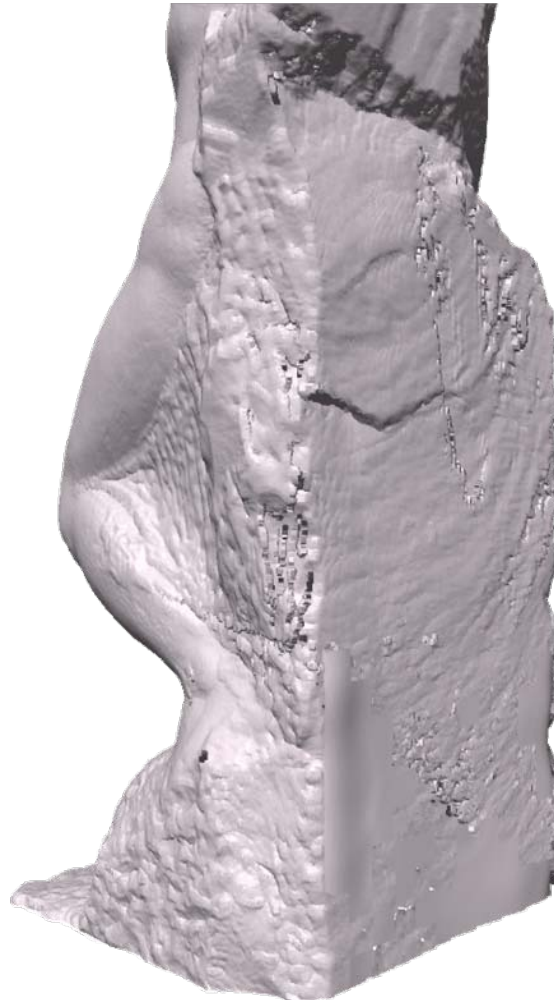
# Manual Editing of Bunny Model



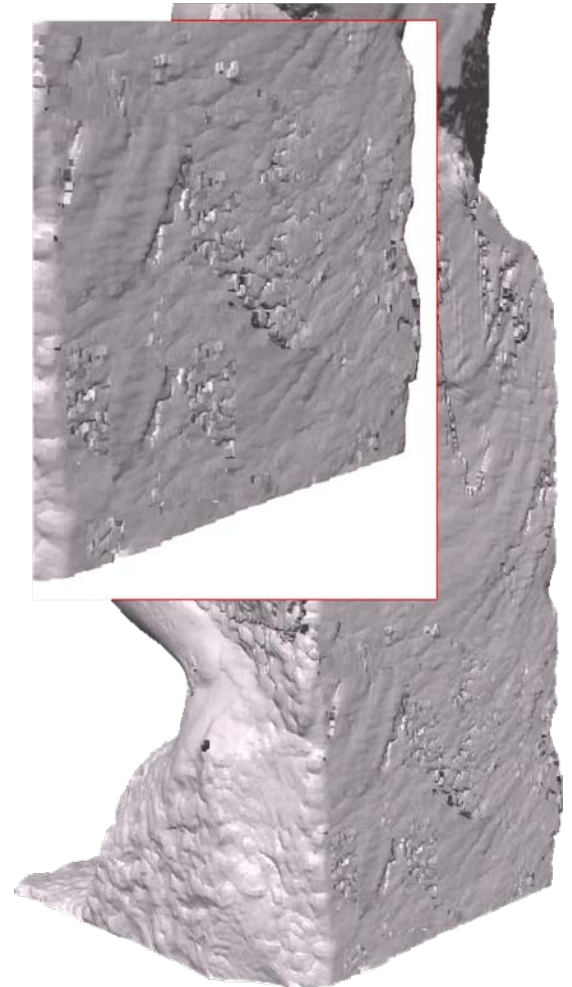
# Scan of “Youth” Statue



Original



Smooth



Result

# Scan of Human Bone



Original

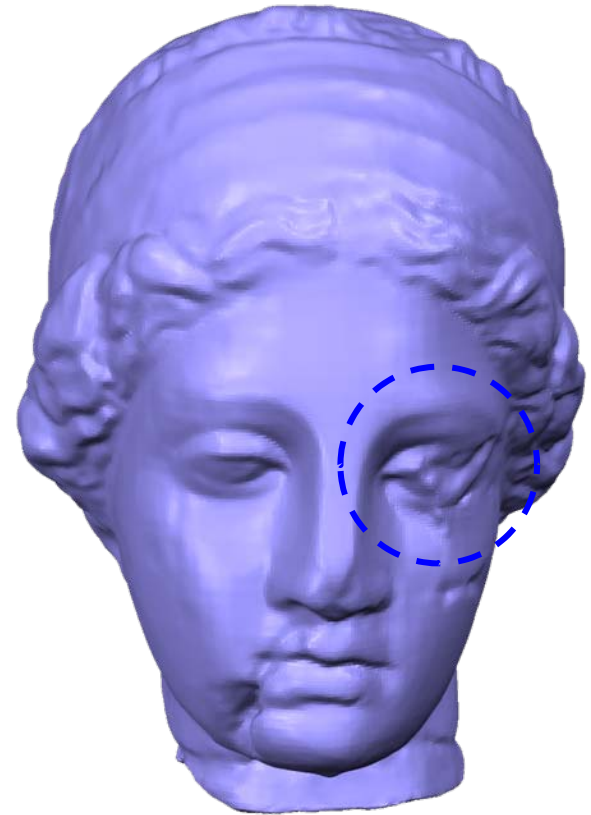
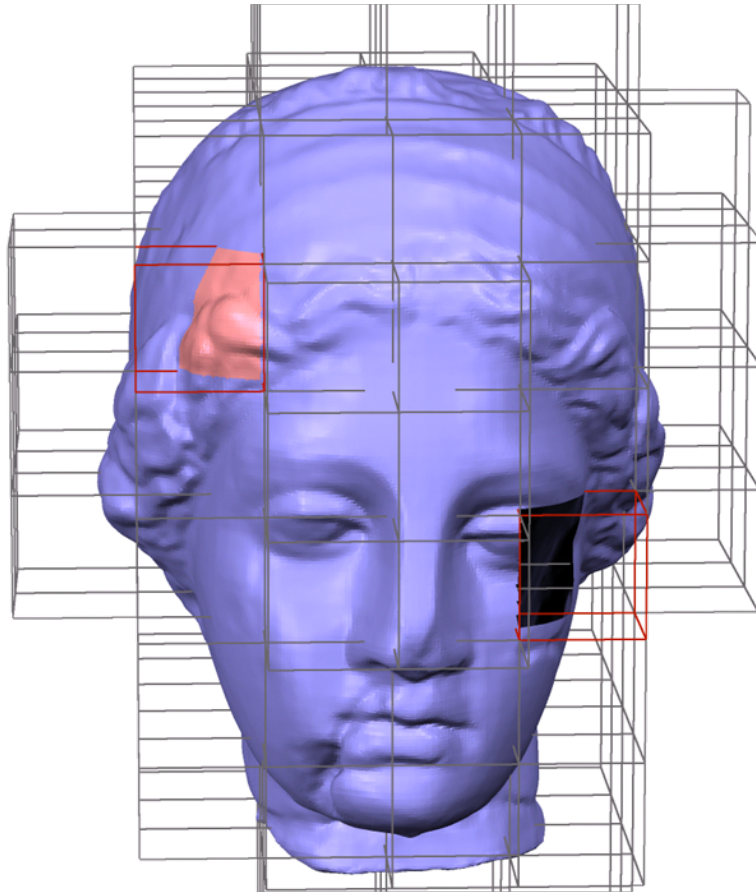


Smooth



Result

# Limitations: Semantics



# Summary

- A fully automatic method to complete a missing region in a surface from its context.
  - Completed patches geometrically conform with neighborhood.
  - Incremental scale-space framework for finer approximation of the unknown region.

## 2.5 Example-based Surface Completion

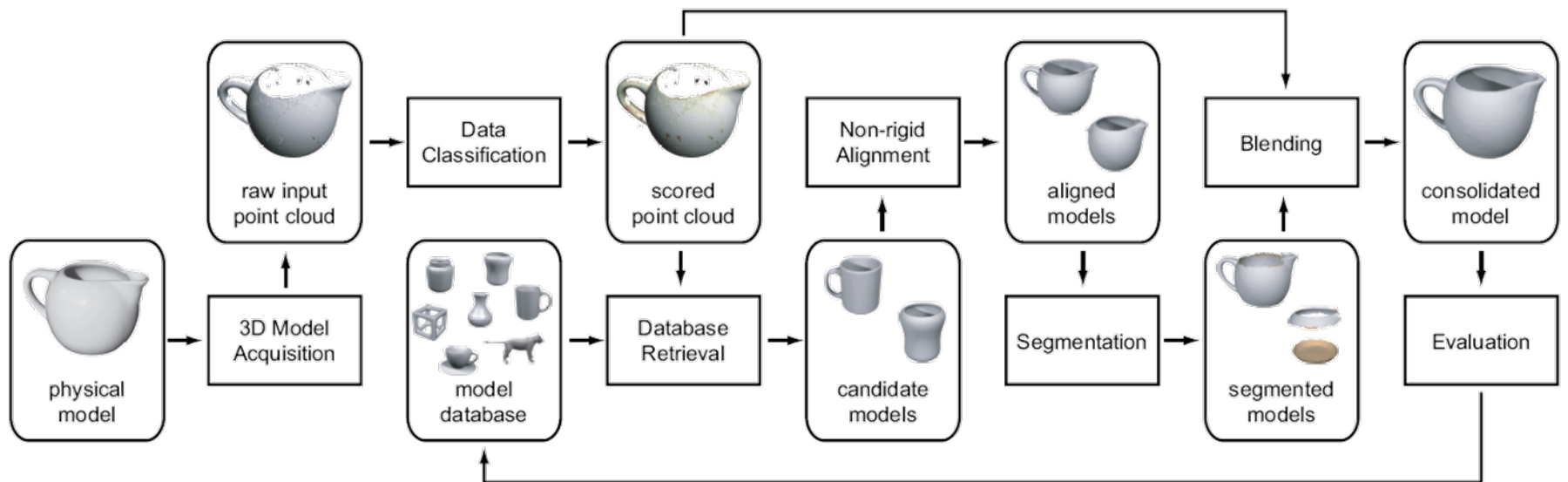
[Pauly et al., SGP 2005]



# Solution

- Use ***3D model database*** to provide geometric priors for shape completion
- Apply ***non-rigid transforms*** on the models
  - More deformation  $\Rightarrow$  less likely completion
- ***Consistently*** combine geometric information from multiple context models
- Final result comes with ***confidence values***

# Shape Completion Pipeline



# Data Classification



## Local analysis

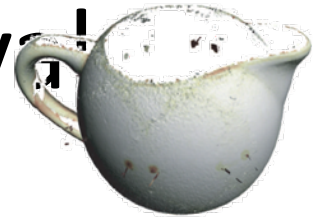
- quality of fit
- uniformity of sample distribution



## Scored Point Cloud

- confidence value assigned to each point

# Database Retrieval



1.93



1.71



1.46



1.27



1.0



# Non-rigid Alignment



Similar to the approaches proposed by:

- Allen, Curless and Popovic, 2003.
- Sumner and Popovic, 2004.

# Non-rigid Alignment



## Deformation Model

- Piecewise linear.  
Each vertex of the mesh assigned an independent displacement vector.

## Optimize for smallest ***Shape Matching Penalty***

- Distortion Measure
  - Geometric Error
- Derived in the continuous setting to allow *consistent comparison between different context models.*

## Feature Correspondence

# Warped Models



Context Model

Warped Model

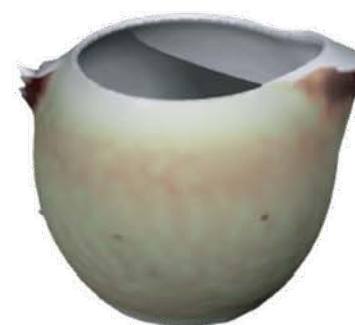


Matching Penalty

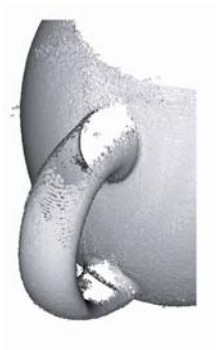
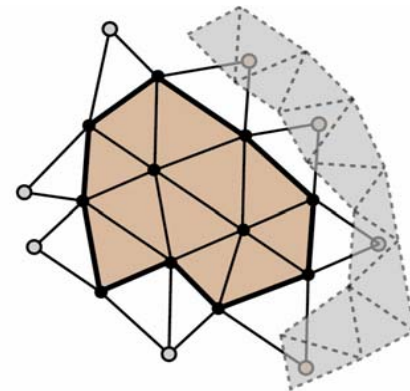
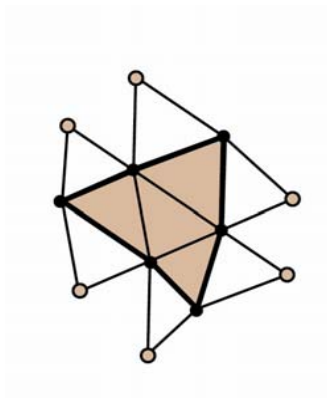
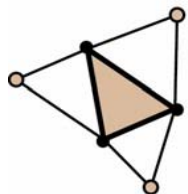
Low



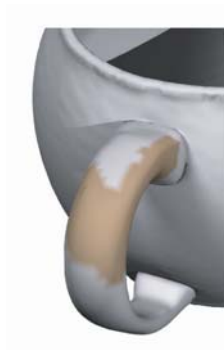
High



# Initial Segmentation



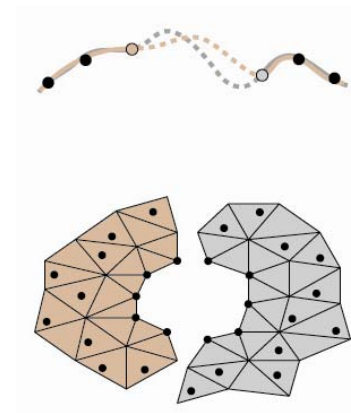
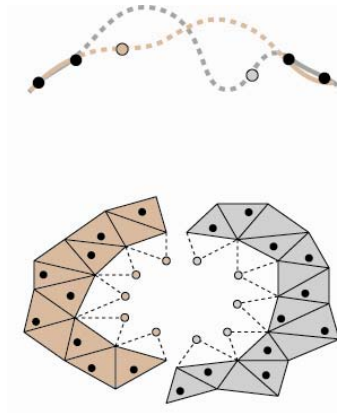
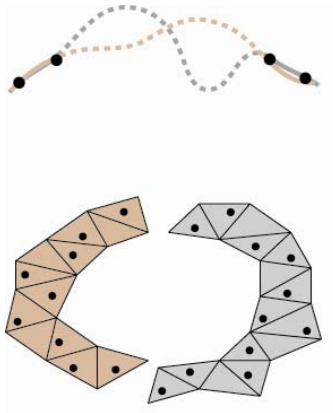
Input Data



Warped Context Model



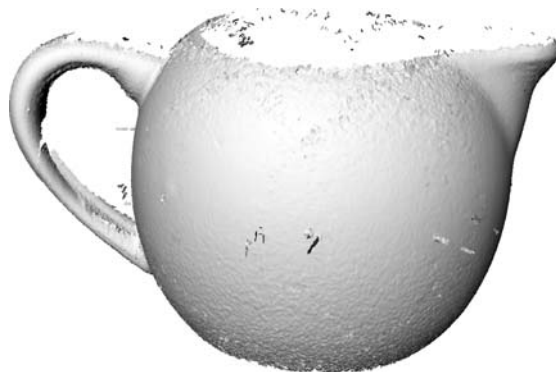
# Patch Growing



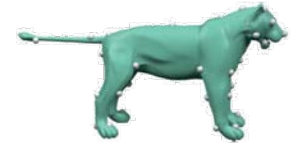
Initial Segmentation

Final Segmentation

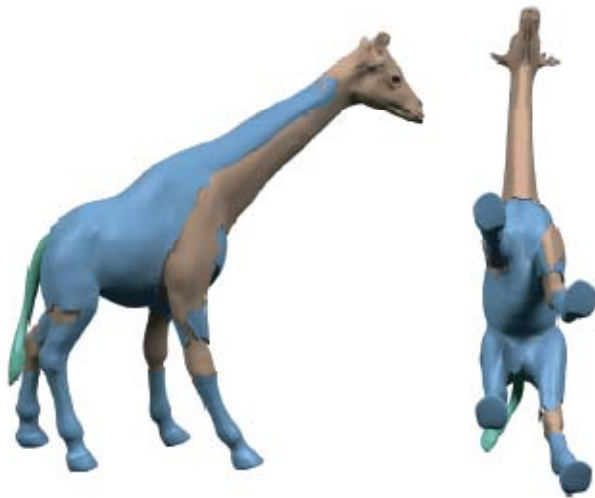
# Result



# Giraffe Example



Context Models

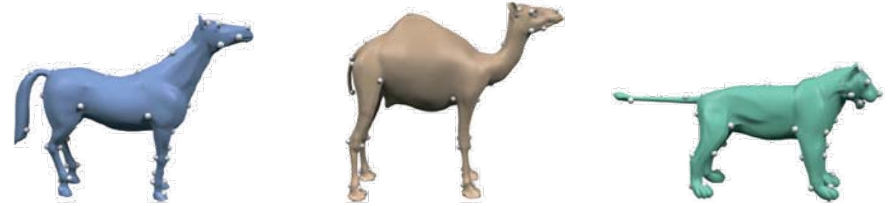


Segmentation



Deformed Models

# Giraffe Example



Context Models



Final Model



Deformed Models

# Evaluation



Input Data



Context Model



Final Model



Evaluation

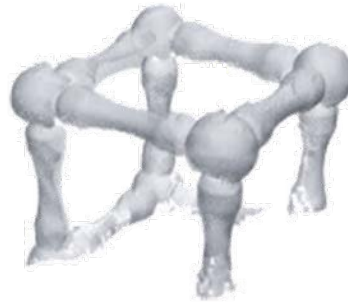
# Enriching the Database



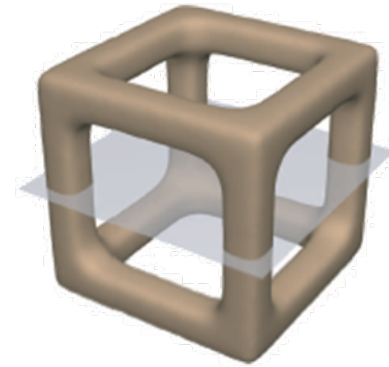
# Additional Constraints



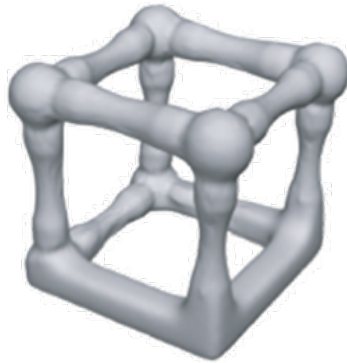
Physical Model



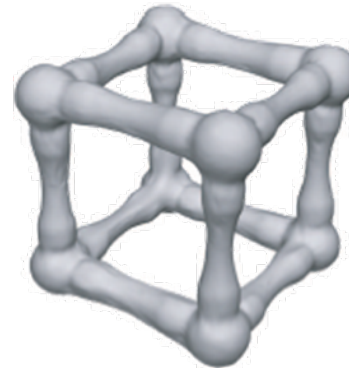
Acquired Data



Context Model



No Constraints



Symmetry Constraints

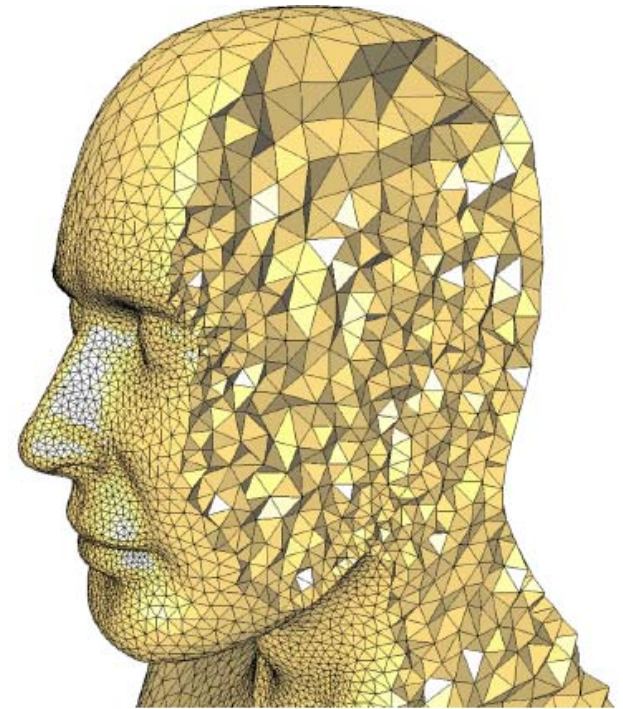
## 2.6 Atomic Volumes for Mesh Completion

[ Podolak and Rusinkiewicz, SGP 2005]

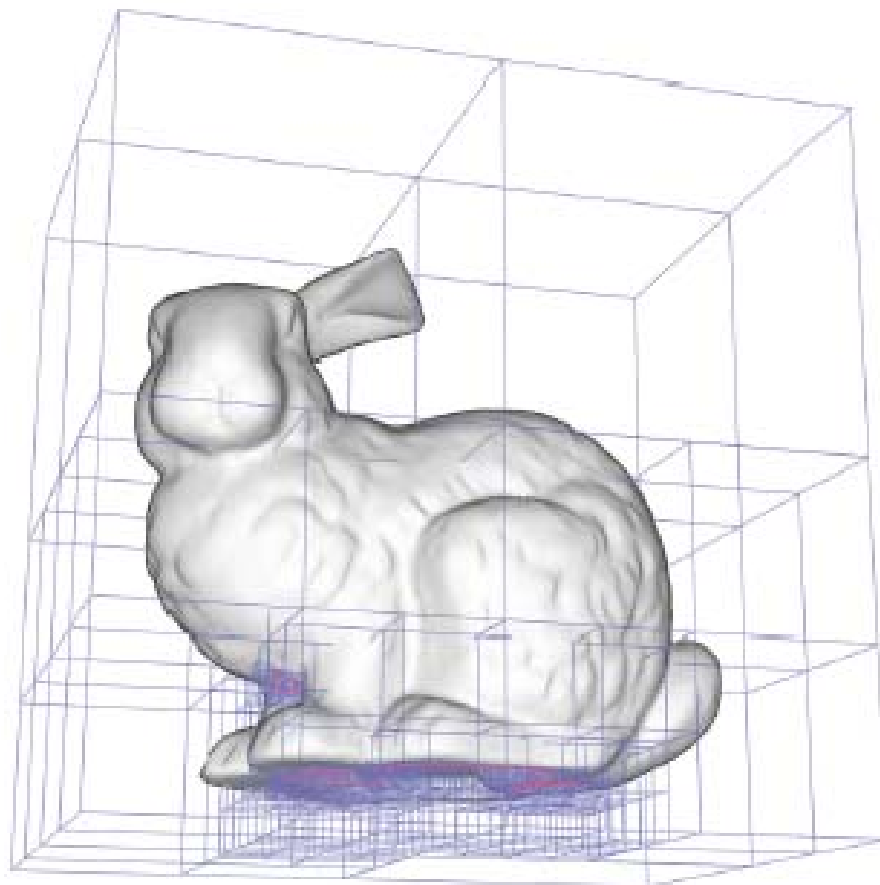


# Atomic Volumes

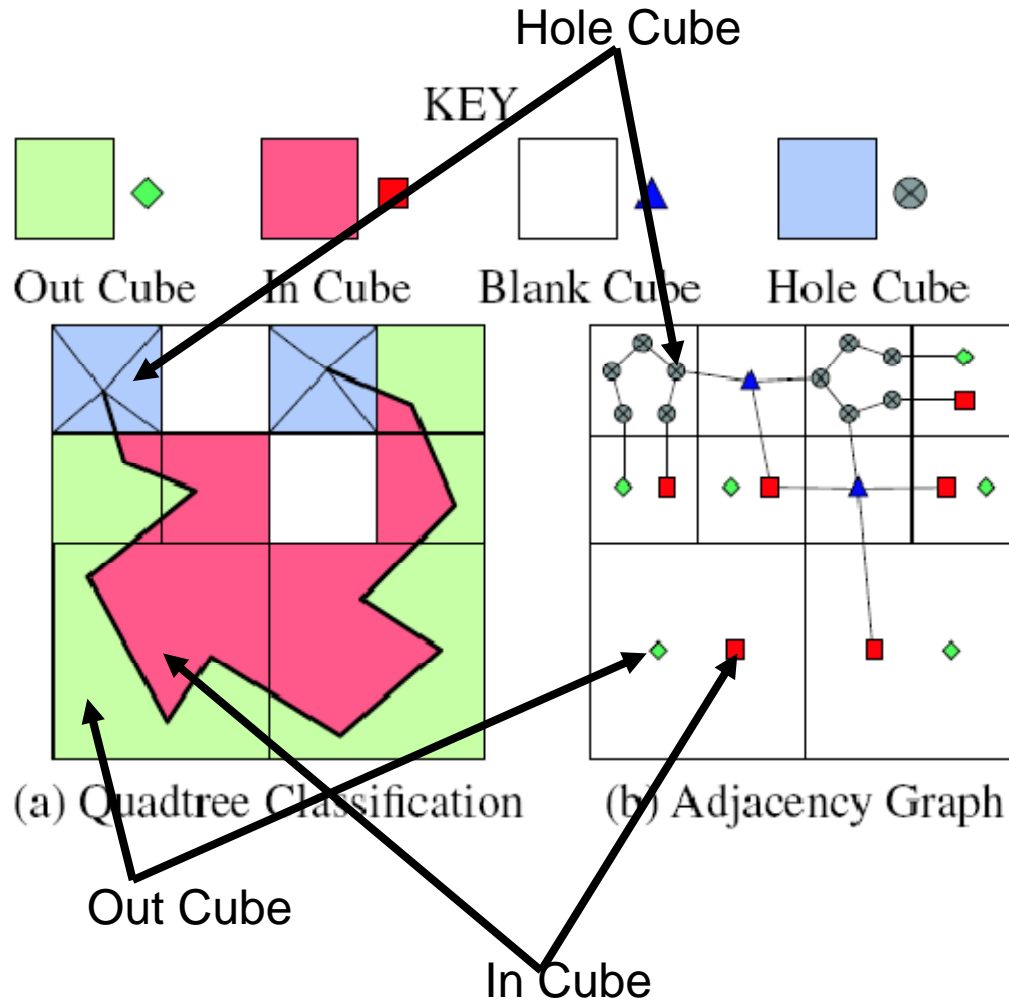
- A volume is *atomic* if it doesn't intersect the polygons of the mesh.



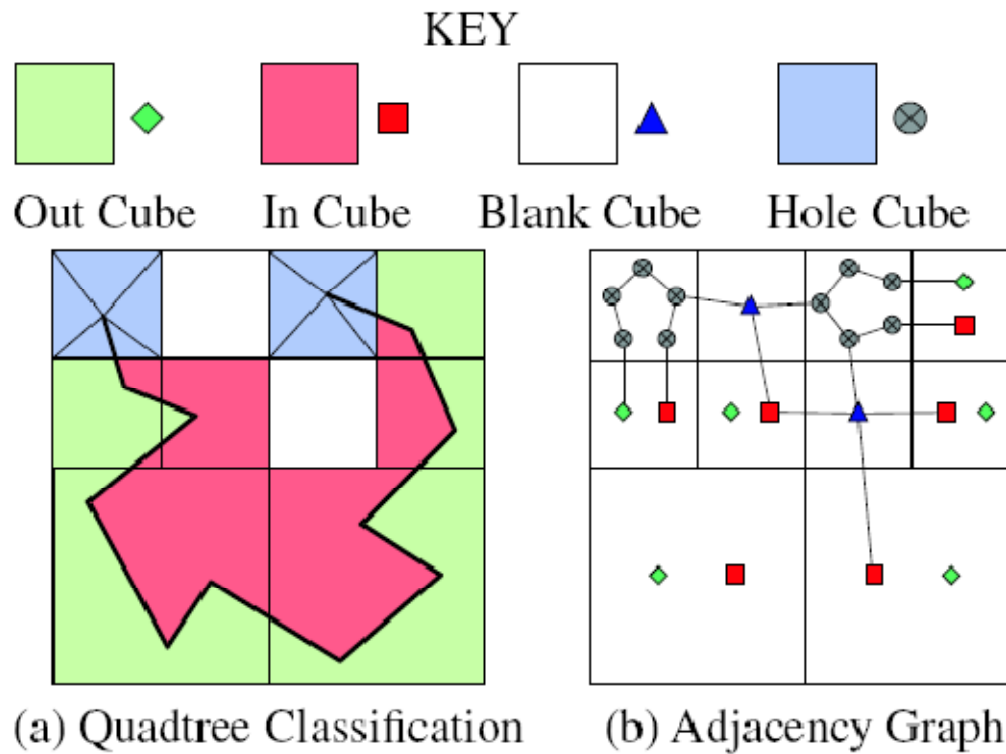
# Spatial Partitioning



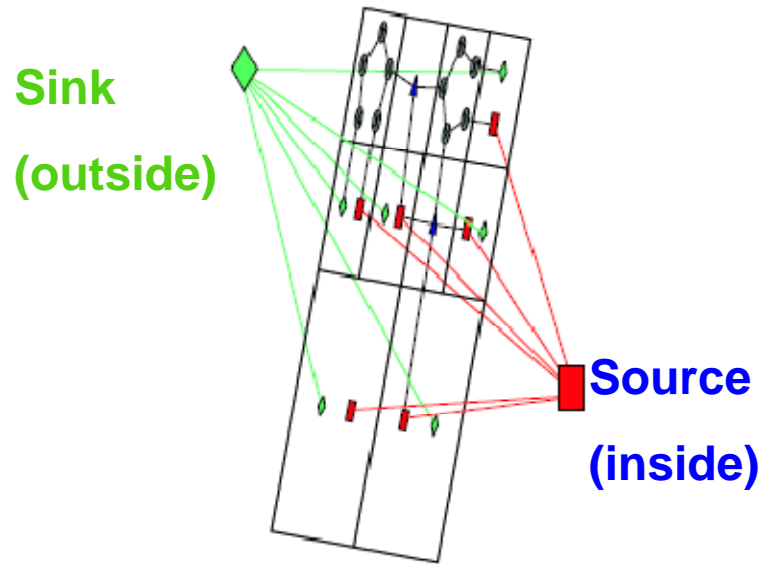
# Pipeline



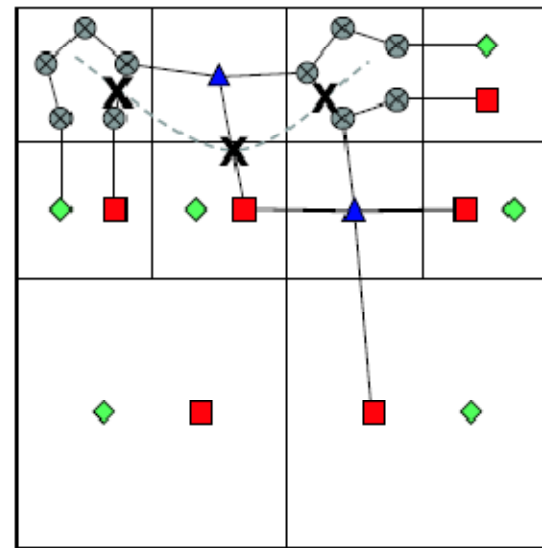
# Pipeline



# Pipeline

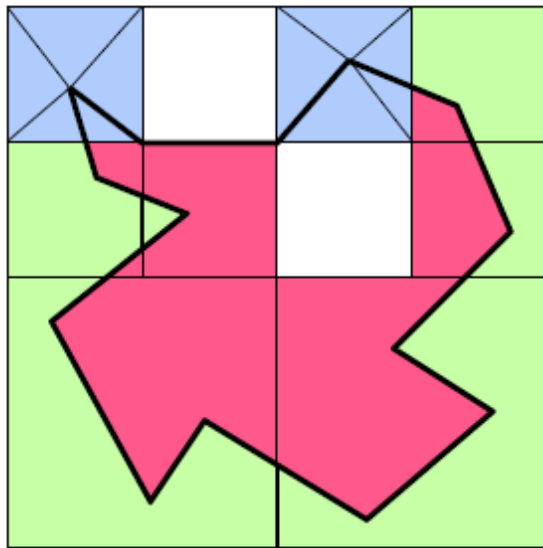


(c) Constraint Edges

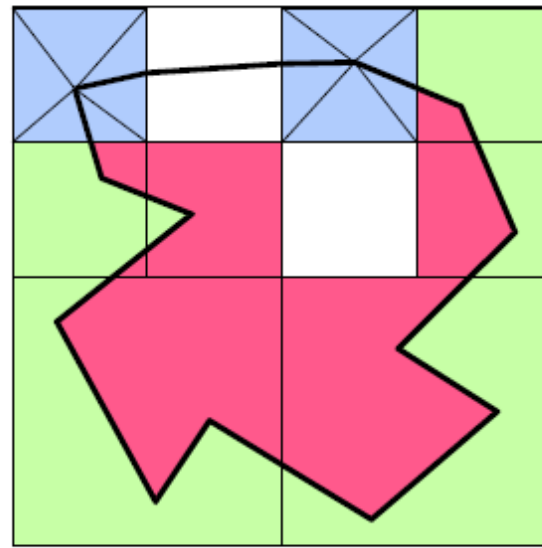


(d) Min Cut

# Pipeline

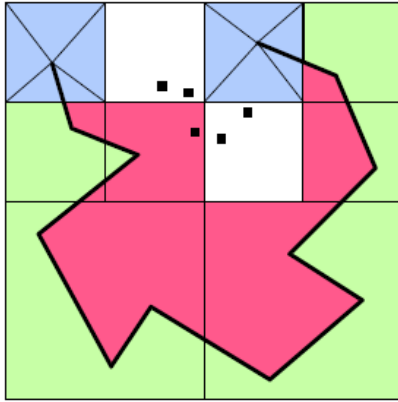


(e) Adding Faces

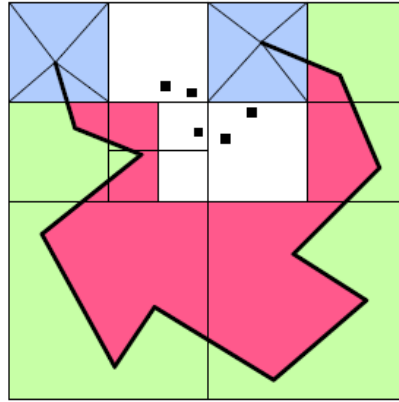


(f) Smoothing

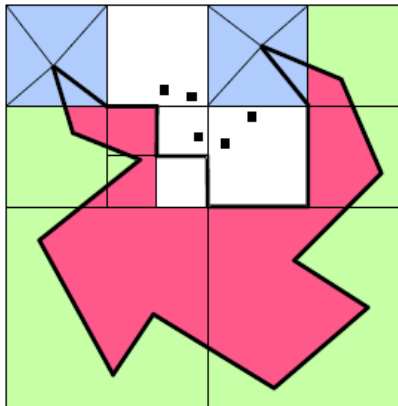
# User Constraints



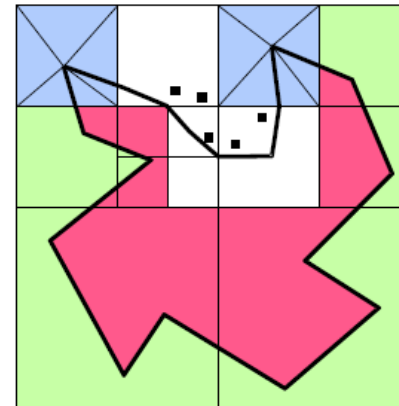
(a)



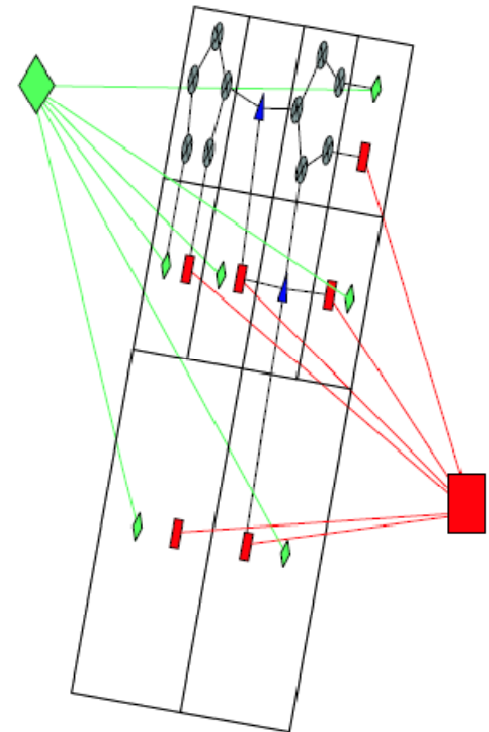
(b)



(c)

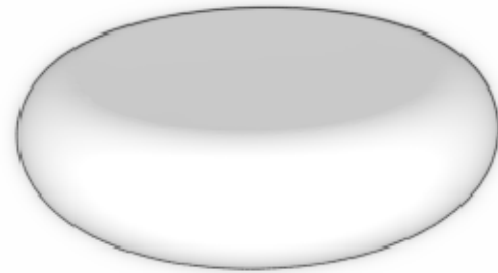
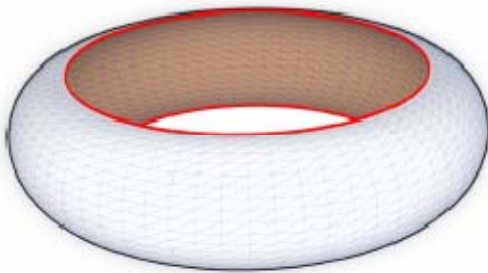


(d)



Constraint Edges

# Results





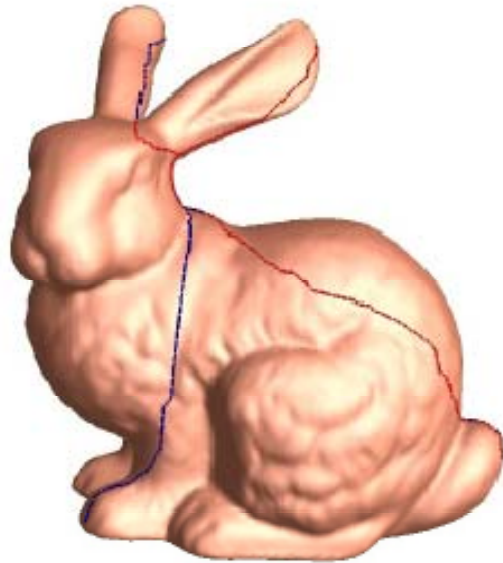
# Summary

- Avoid changing, approximating or re-sampling the original mesh data
- Incorporate user constraints
- Can't process holes with islands

## 2.7 Geometry Completion by Texture Synthesis

[Nguyen et al., PG 2005]

# Geometry Image

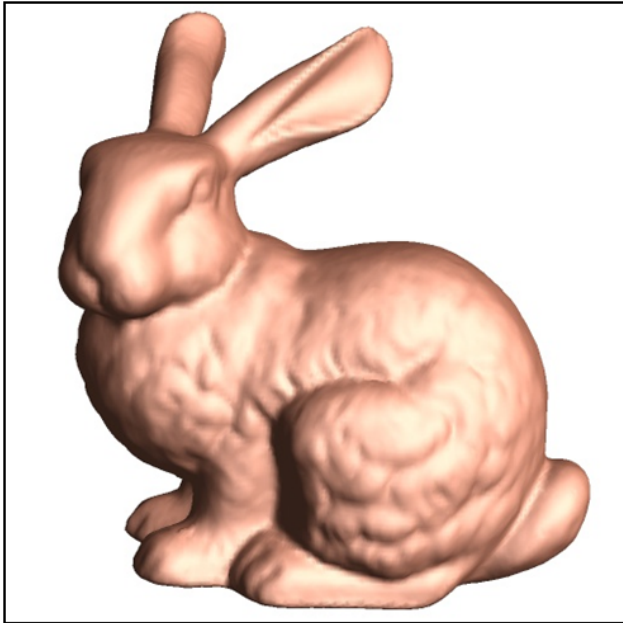


(a) Original mesh with cut

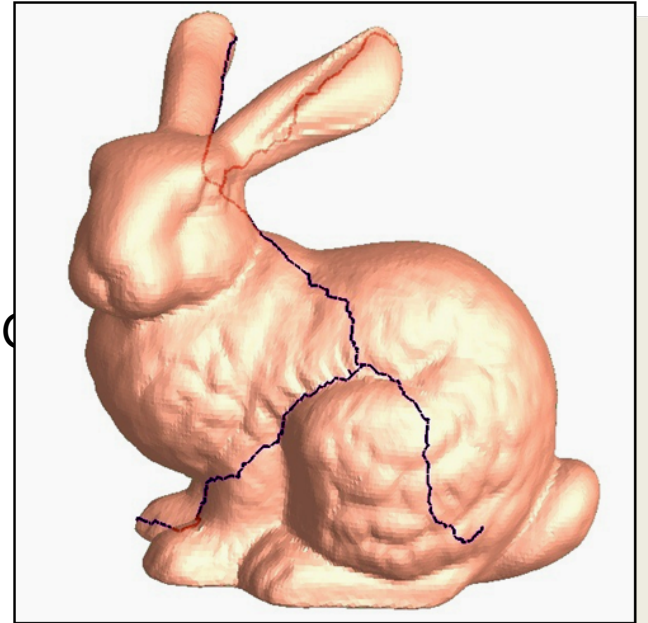


(b) Geometry image  $257 \times 257$

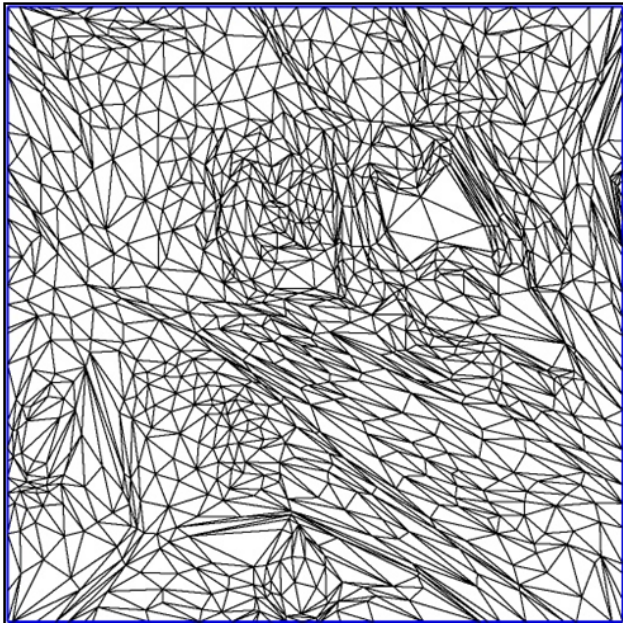
# Basic idea



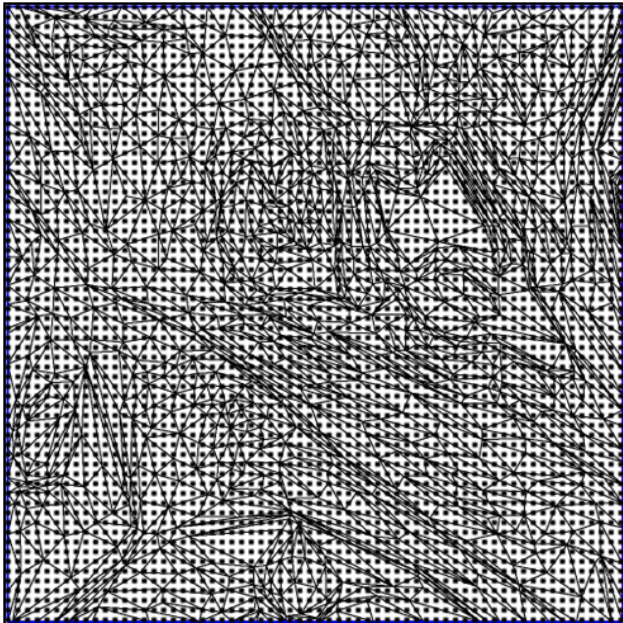
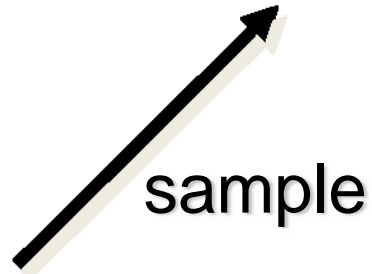
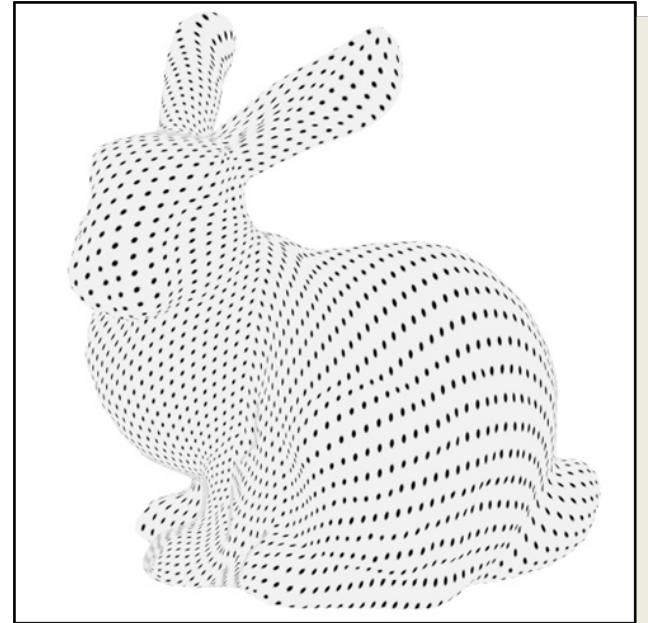
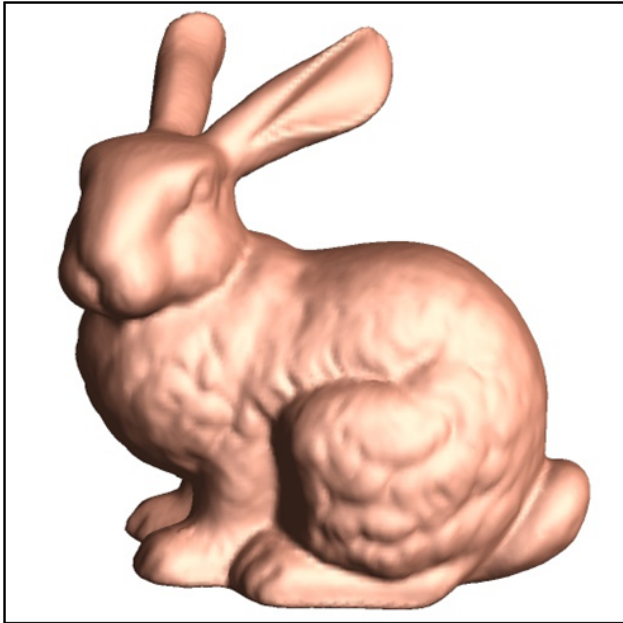
cut



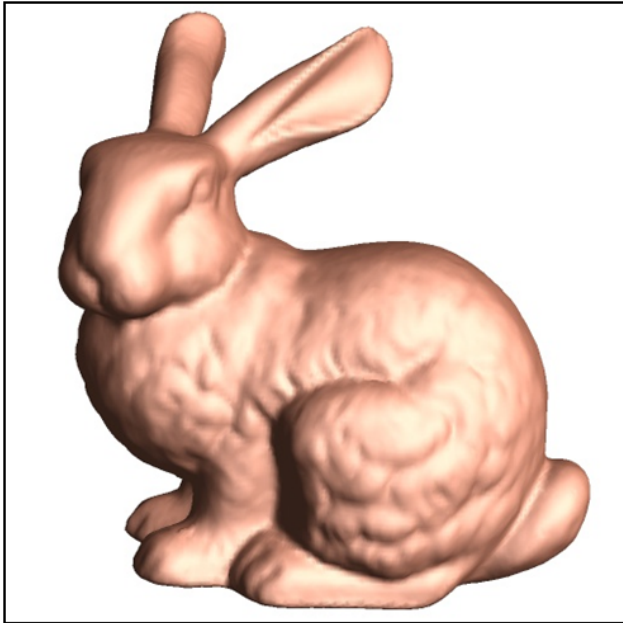
parametrize



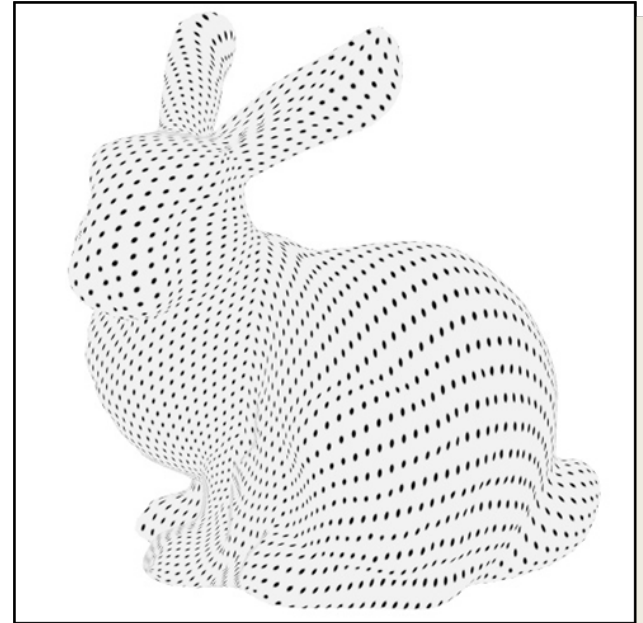
# Basic idea



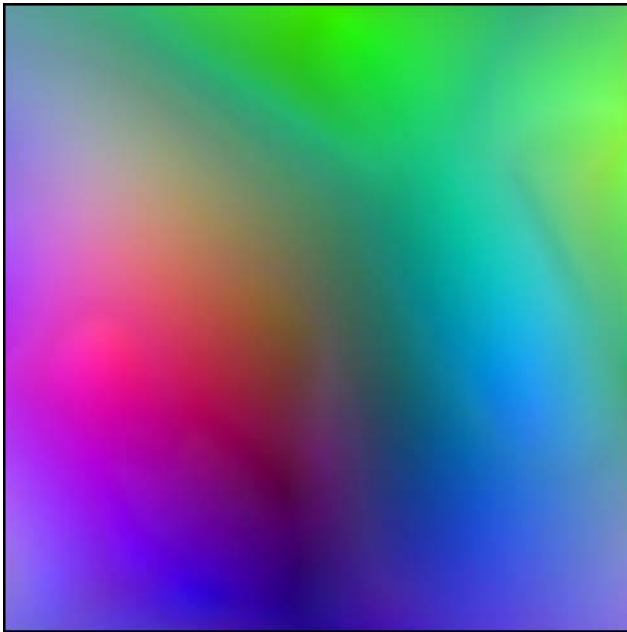
# Basic idea



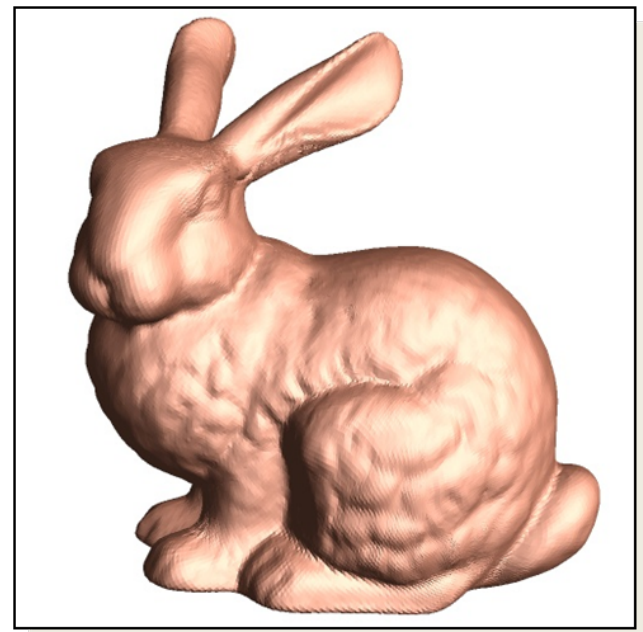
cut



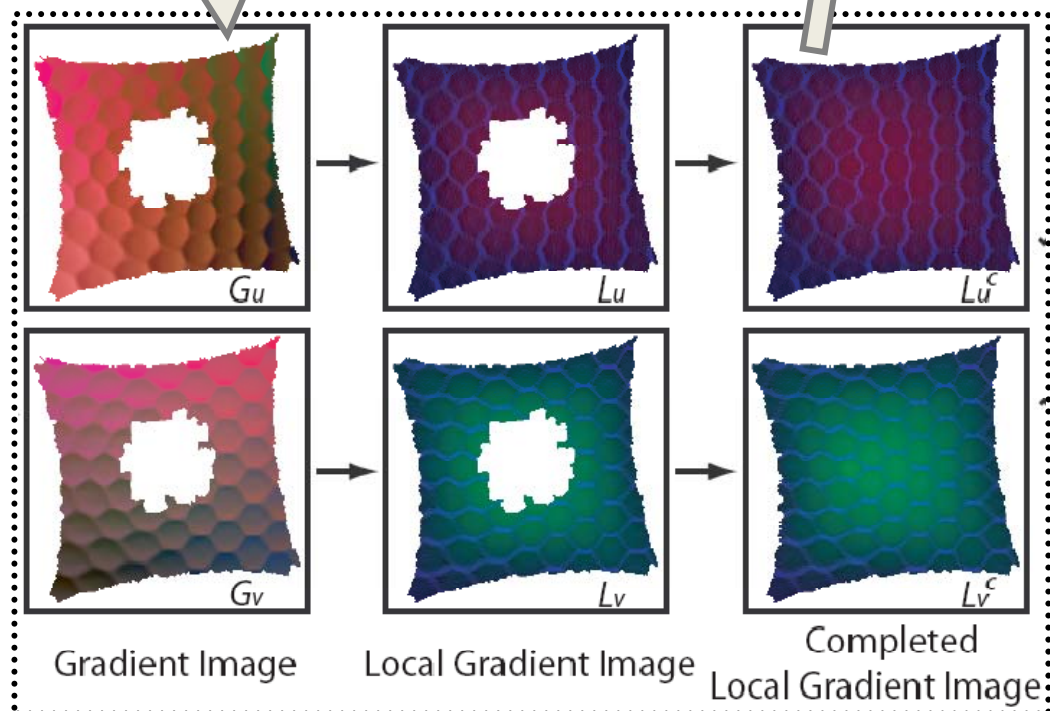
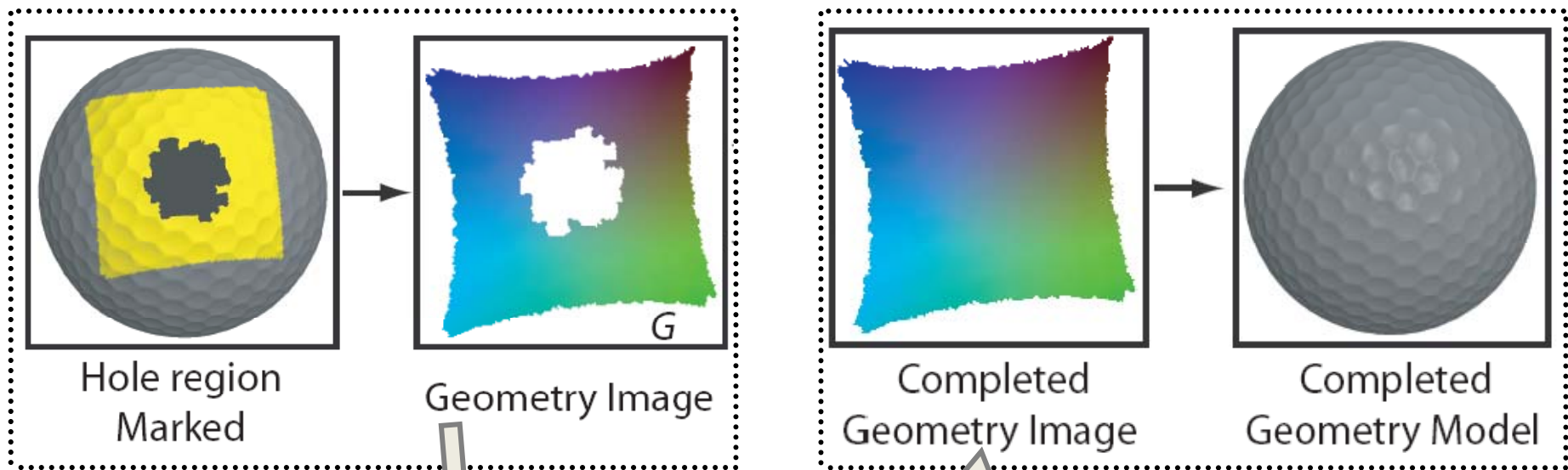
store



render



$[r,g,b] = [x,y,z]$

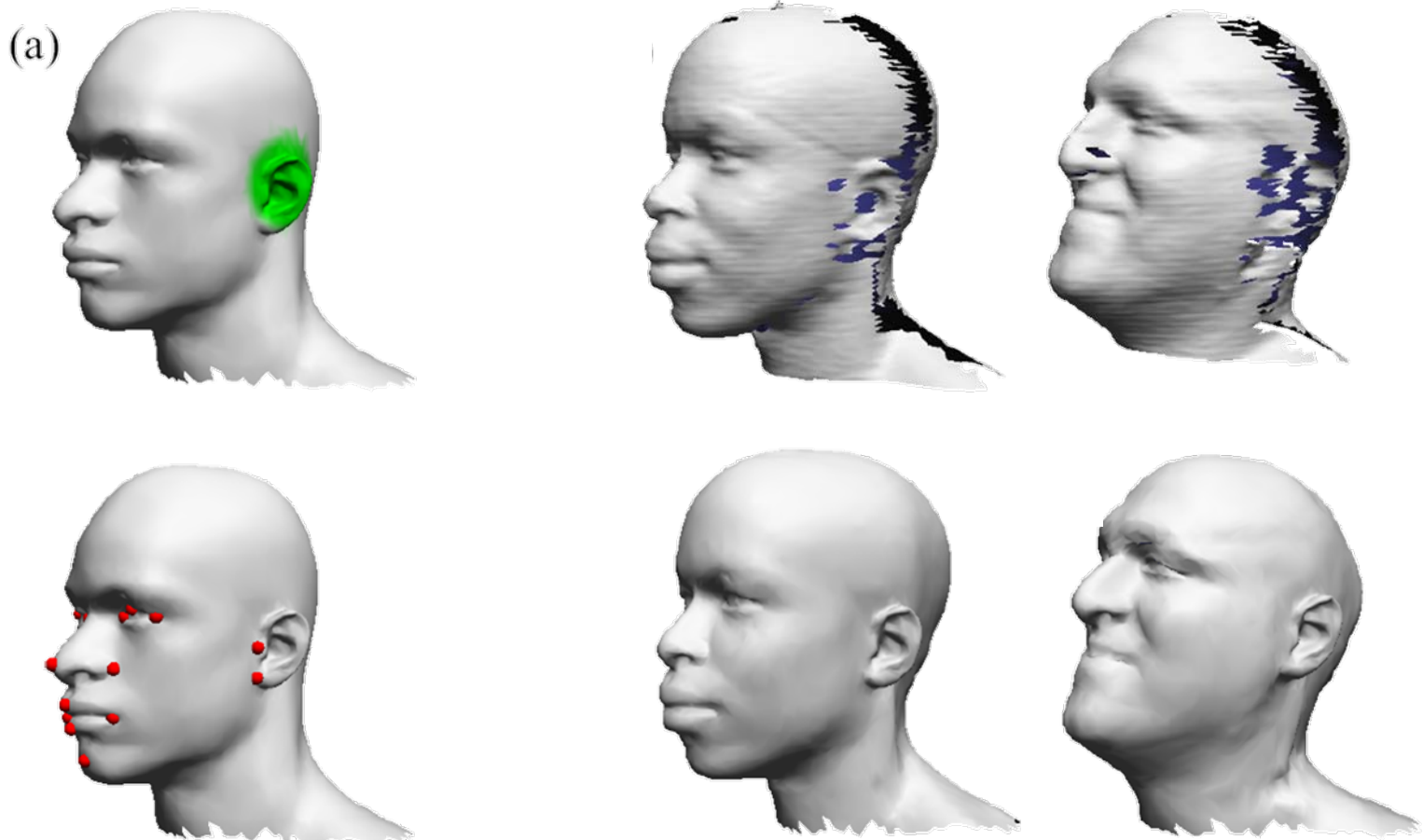


## 2.8 Others



# Template based Solution

(Allen, Curless, Popovic, 2003; Kraevoy and Sheffer, 2005)



# Discussion