Graphics&Geometric Computing Lab @USTC

Volumetric Meshing

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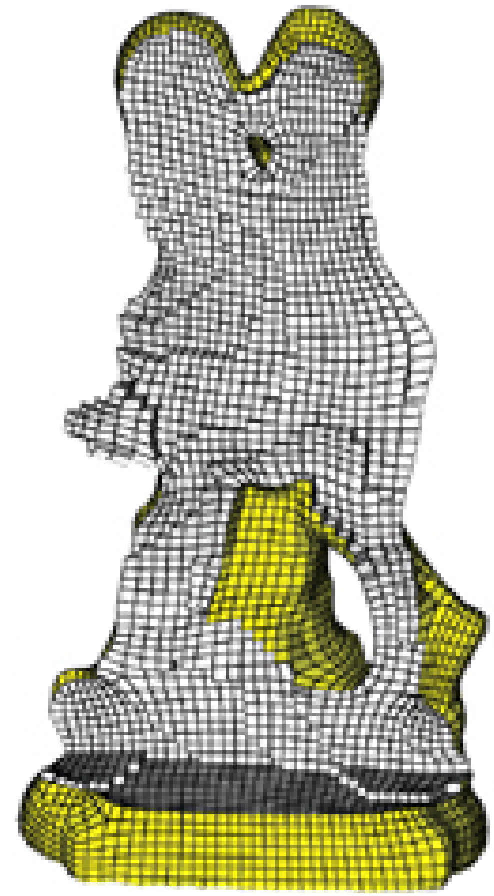
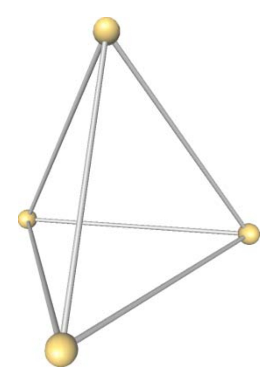
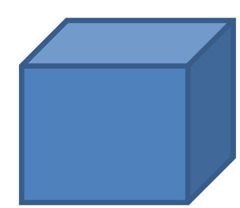
Why volumetric meshing?

- Interior of 3D shapes
  - FEM
  - Simulation
  - …
- Two typical types
  - Tetrahedral meshes
  - Hexahedral meshes

1. Tetrahedral Meshing

1.1 Variational Tetrahedral Meshing

[Alliez et al., Siggraph 2005]

Motivation

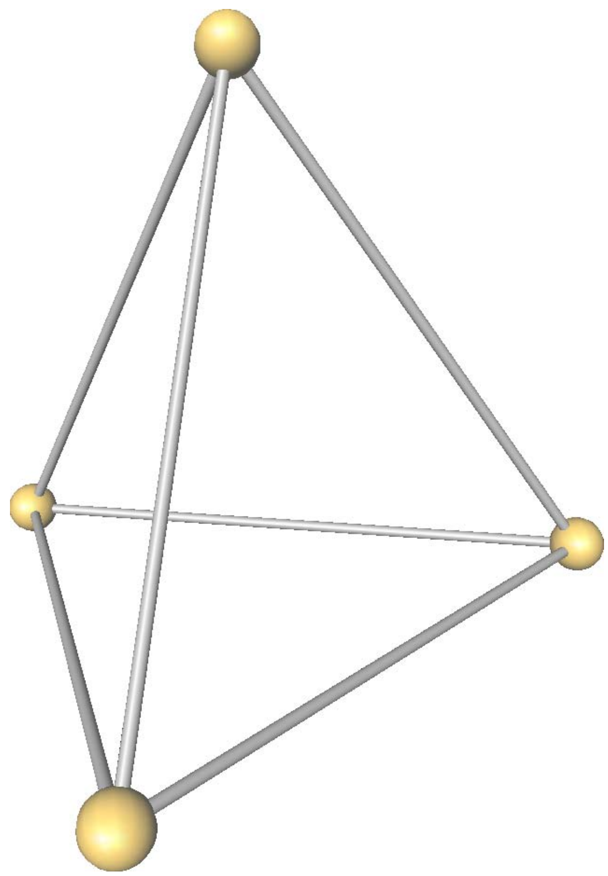
Meshing = essential preprocessing step

Simulation of physical phenomena:

- realistic animation in Computer Graphics, mechanics, fluids
- often modeled as PDE
  - domain discretization + finite elements

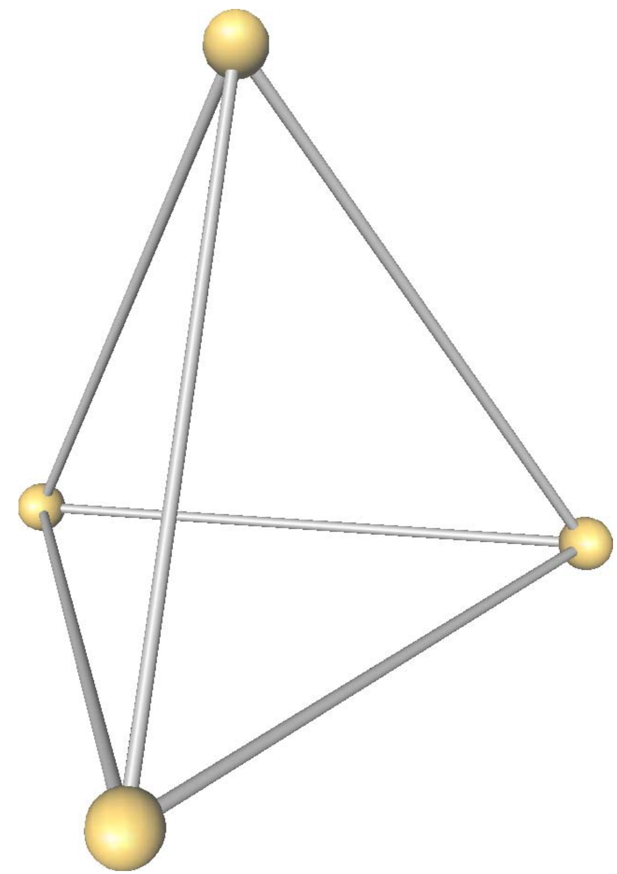
Goals

- Tetrahedral mesh generation



Goals

- Tetrahedral mesh generation
- Focus on:
  - **quality**: shape of elements
  - control over **sizing**
    - dictated by simulation
    - constrained by boundary
    - low number of elements



Popular Meshing Approaches

- Advancing front
- Specific subdivision
  - octree
  - crystalline lattice
- Delaunay
  - refinement
  - sphere packing

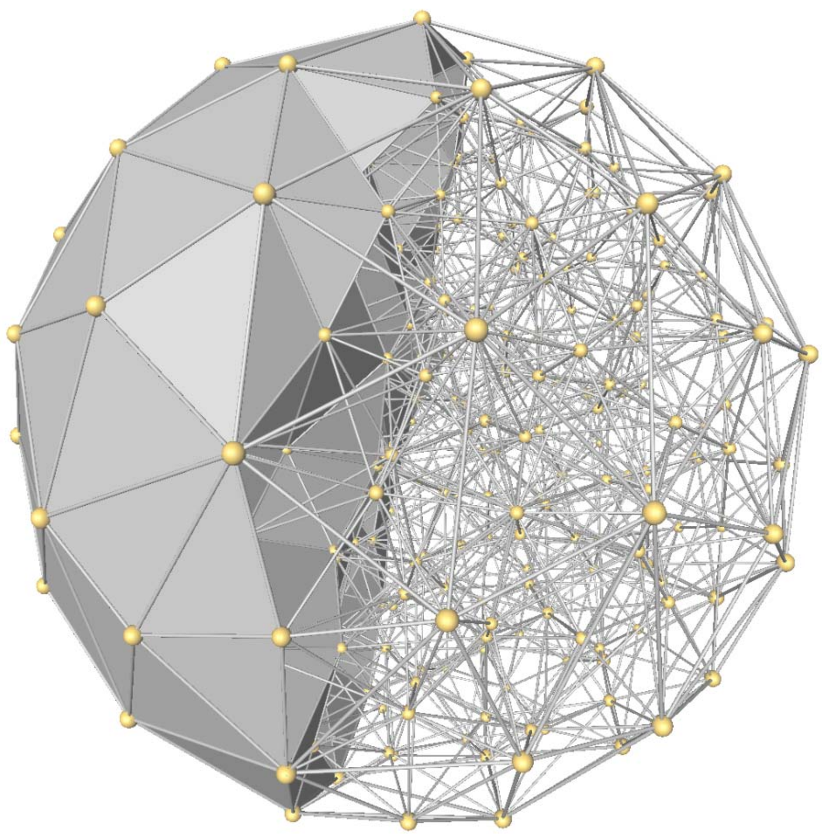
Optimization:

- spring energy
- aspect ratios
- dihedral angles
- solid angles
- volumes
- edge lengths
- containing sphere radii
- etc.

[Freitag et al.; Amenta et al.]

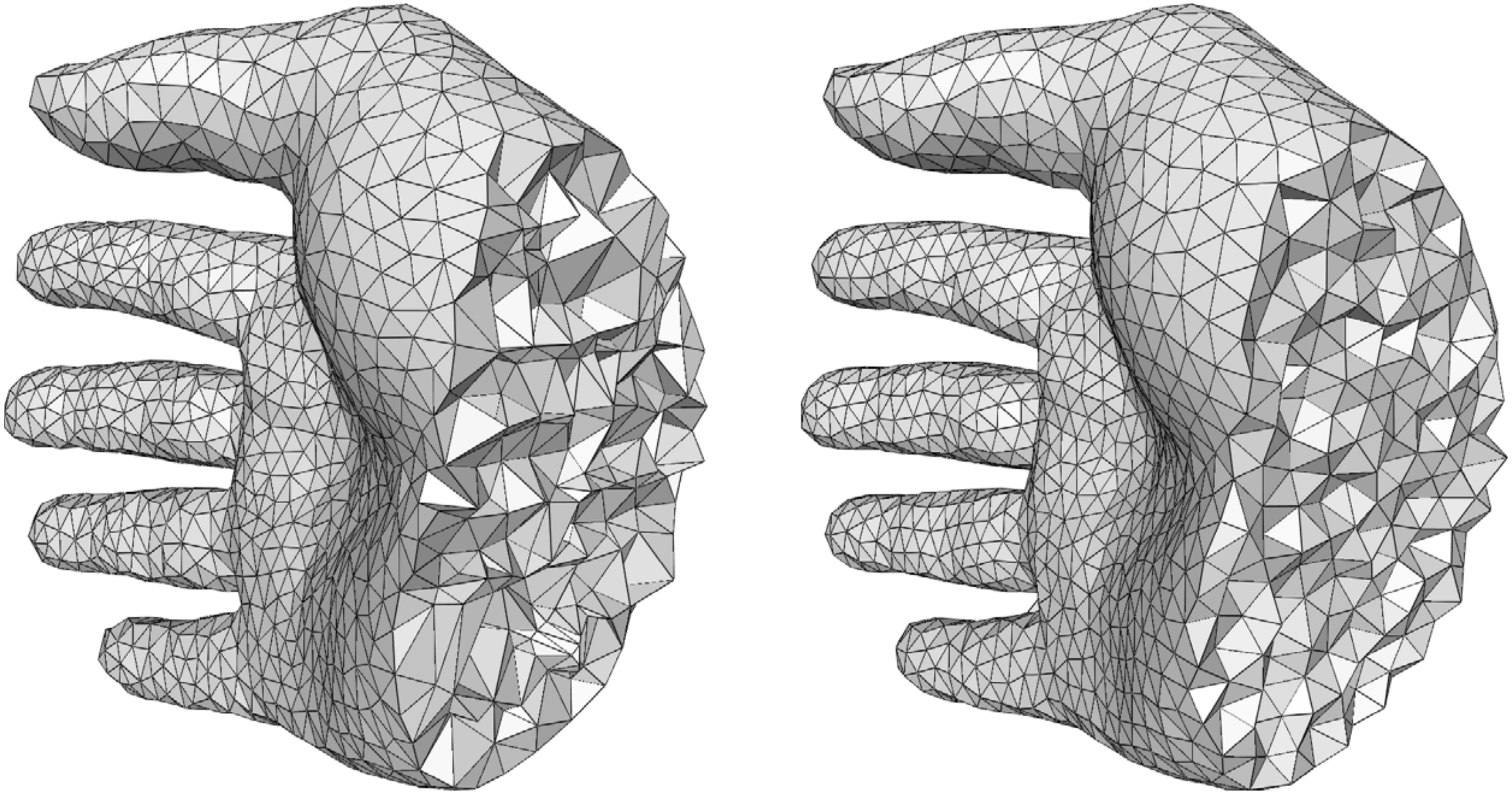
Delaunay Triangulation

Canonical, associated to any point set.



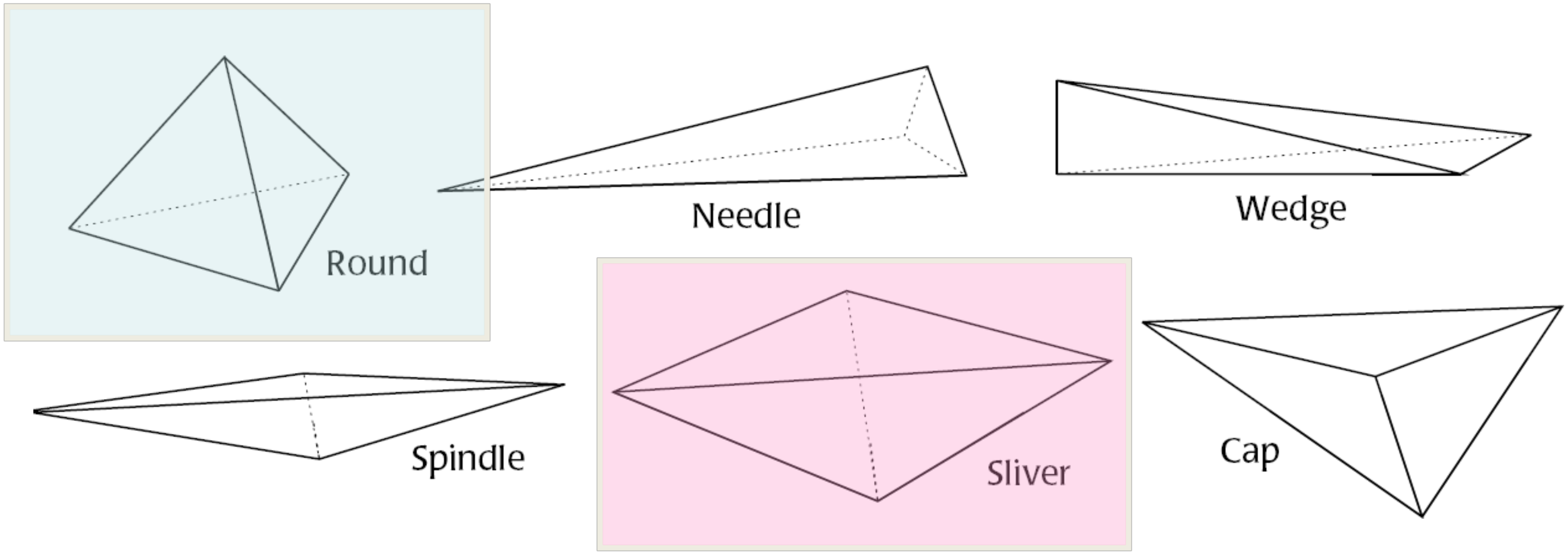
Delaunay Triangulation

Degree of freedom: vertex positions.

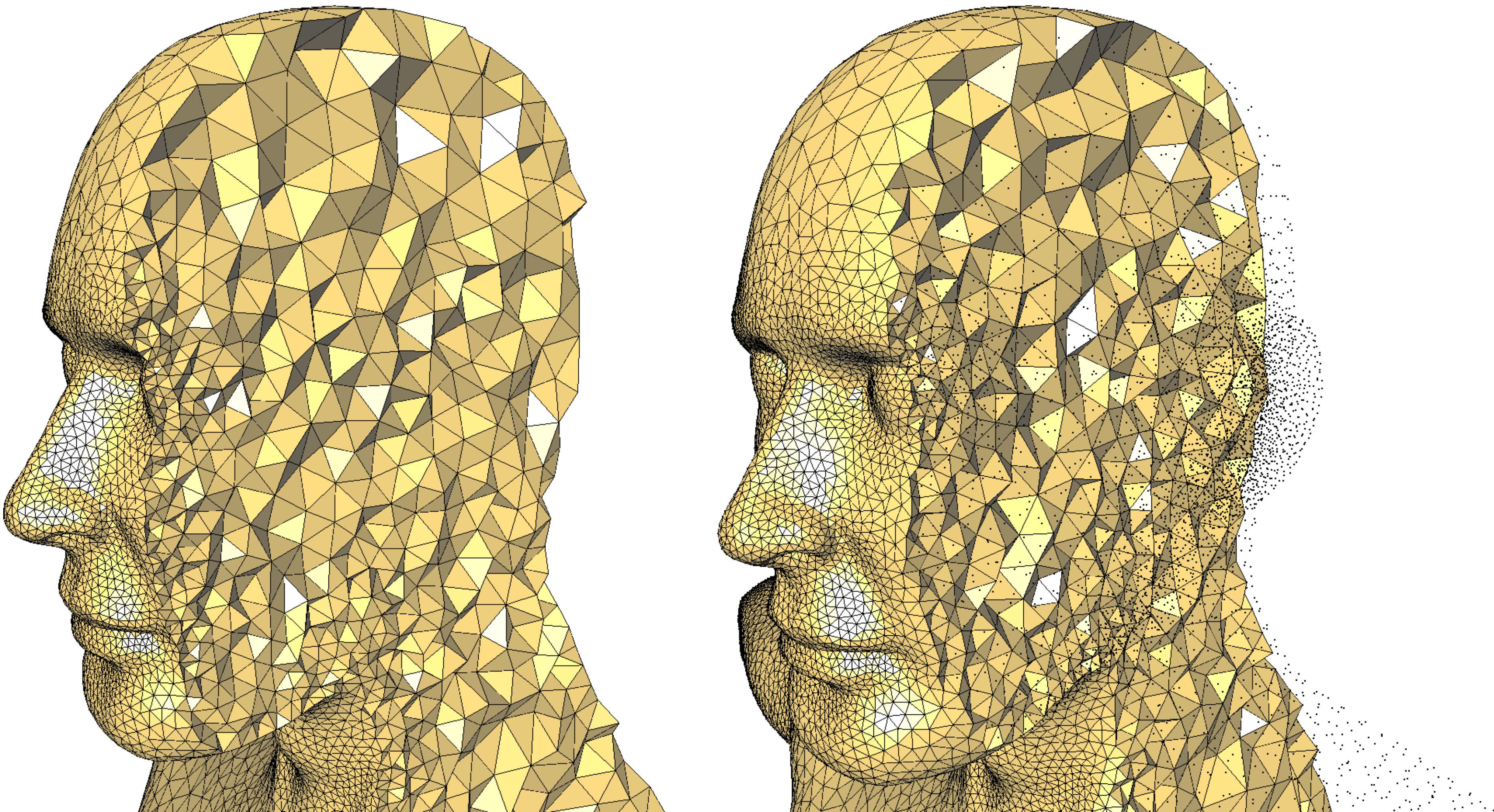


But, Harder in 3D...

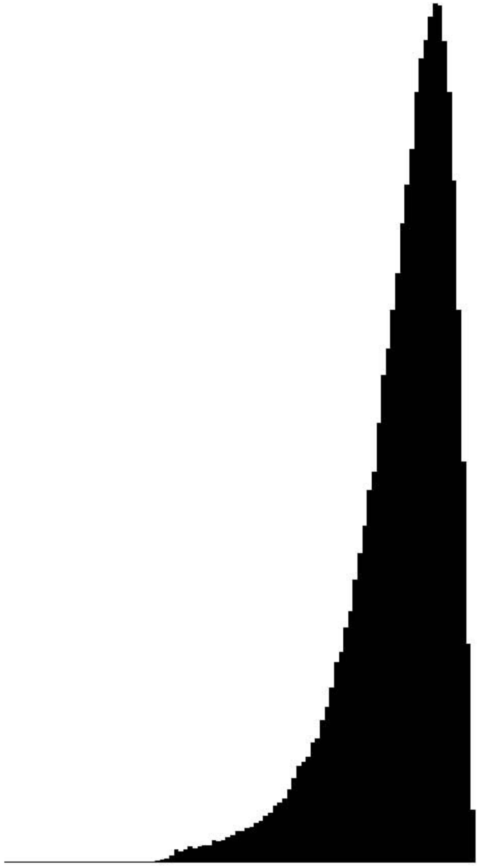
- well-spaced points generate only round or sliver Delaunay tetrahedra **[Eppstein 01]**

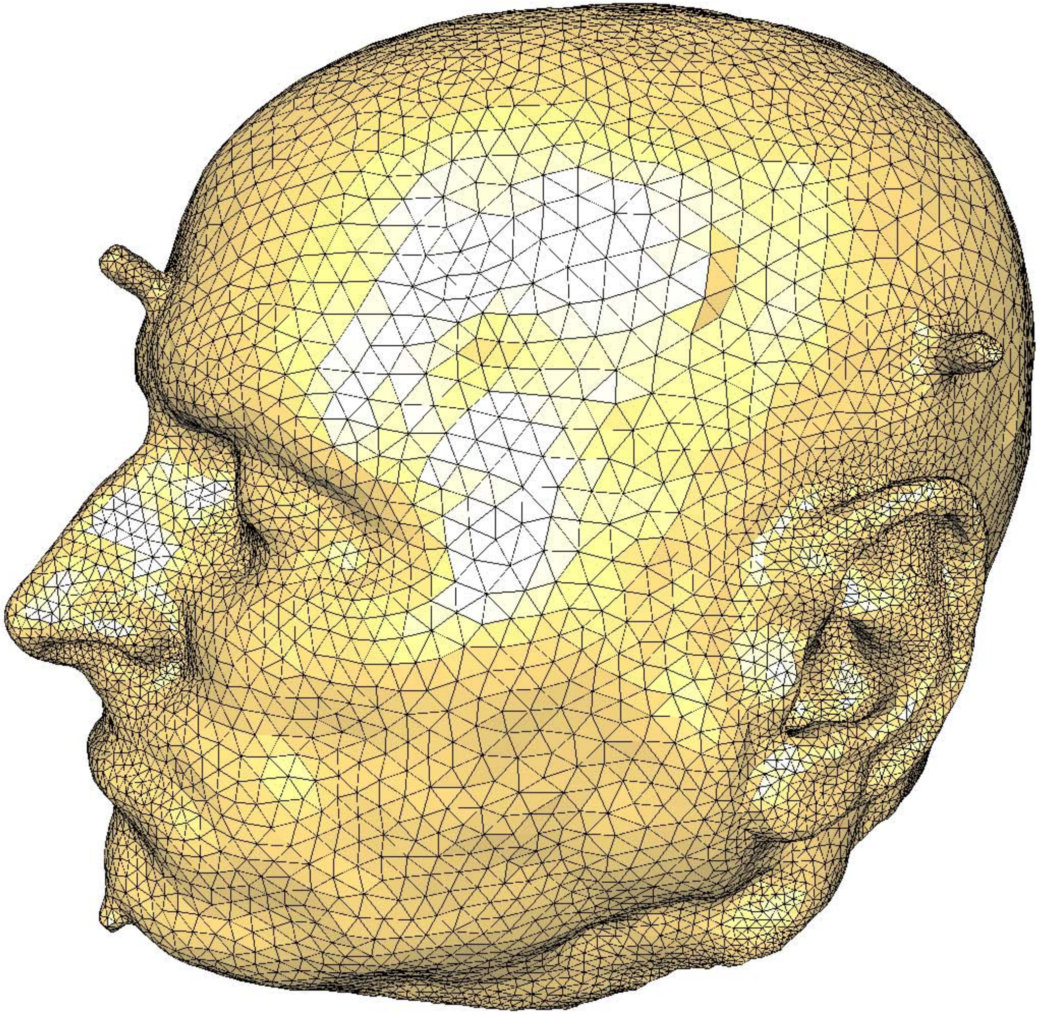


Torso

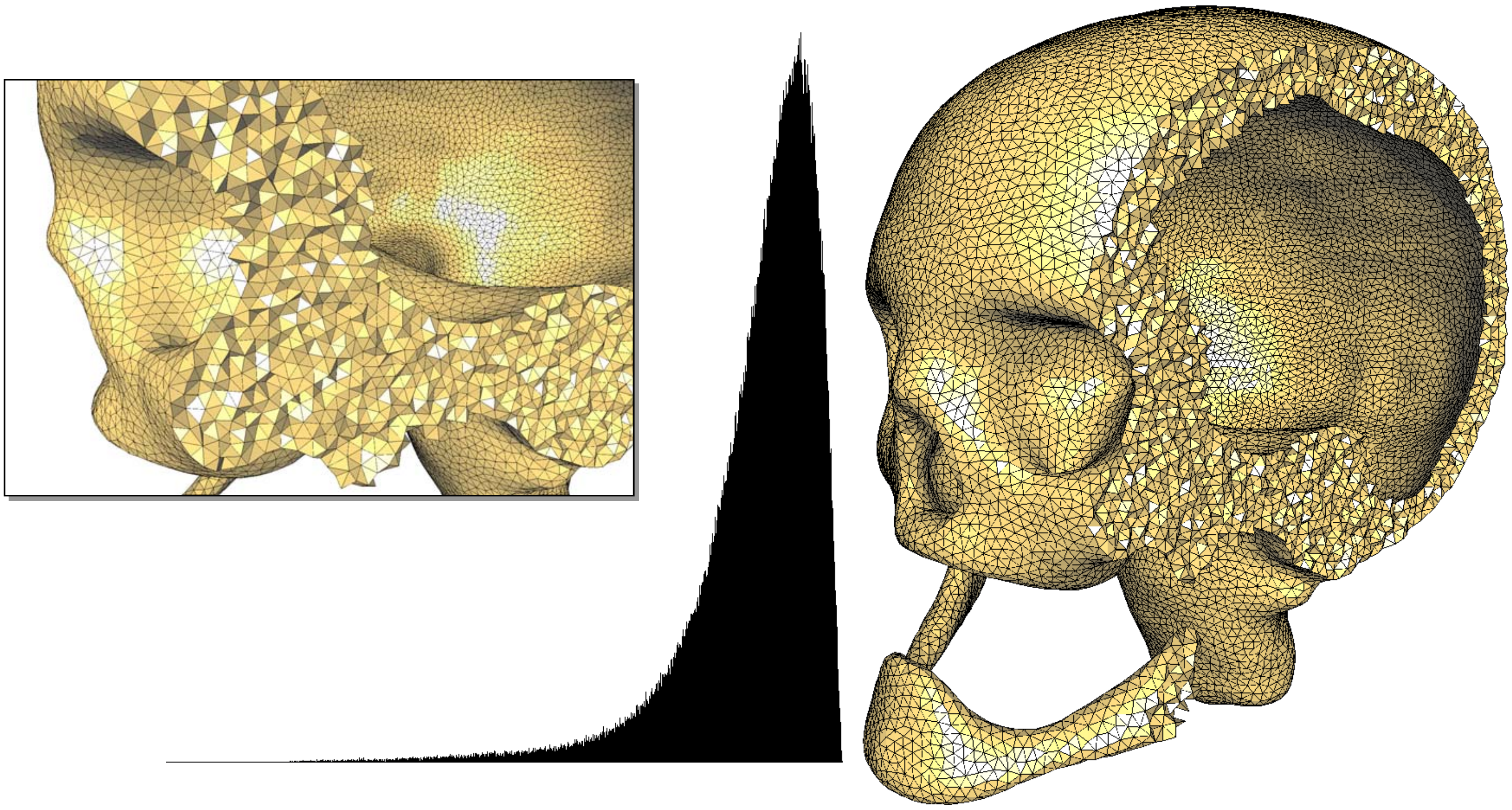


The Visible Human

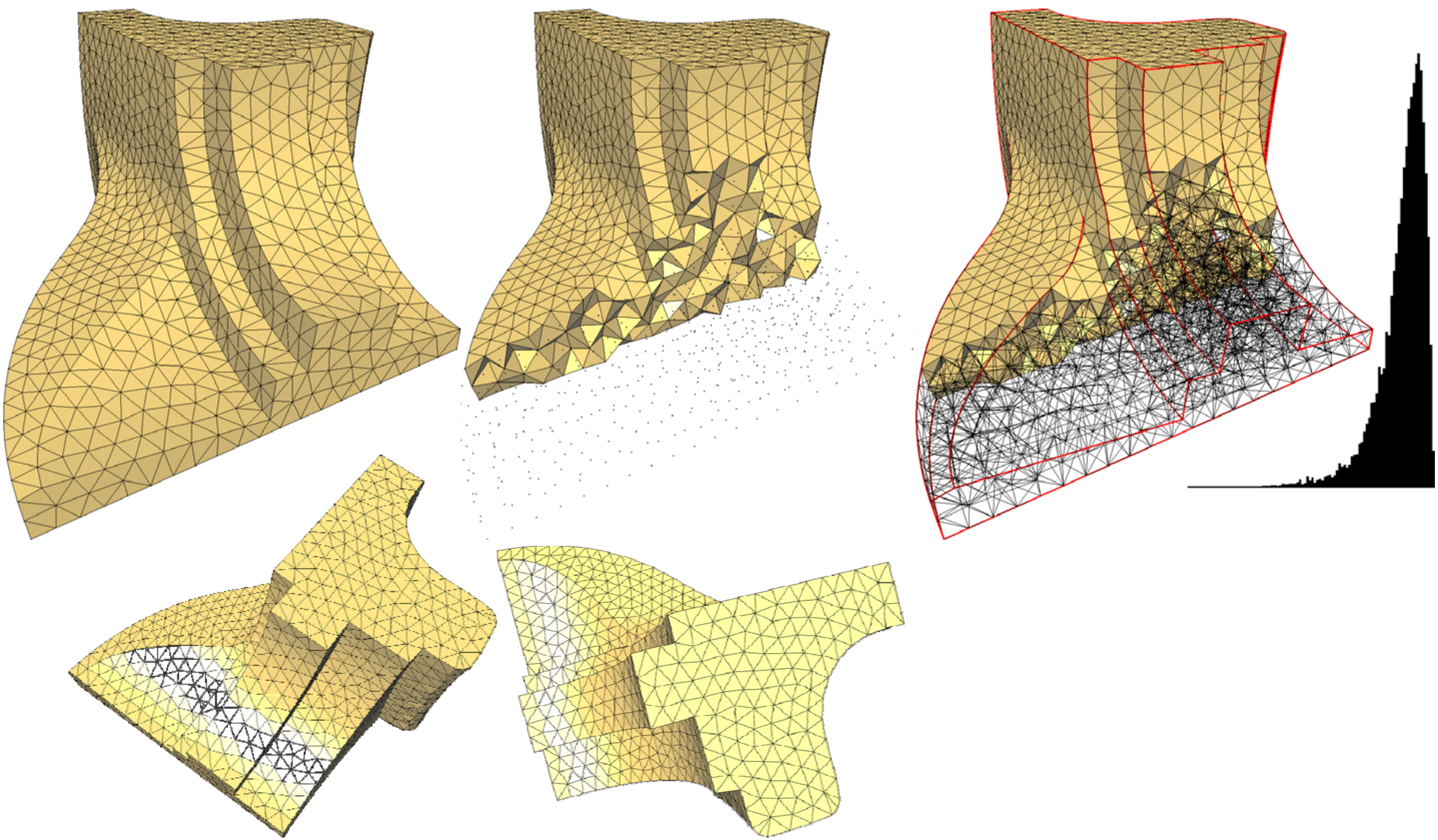




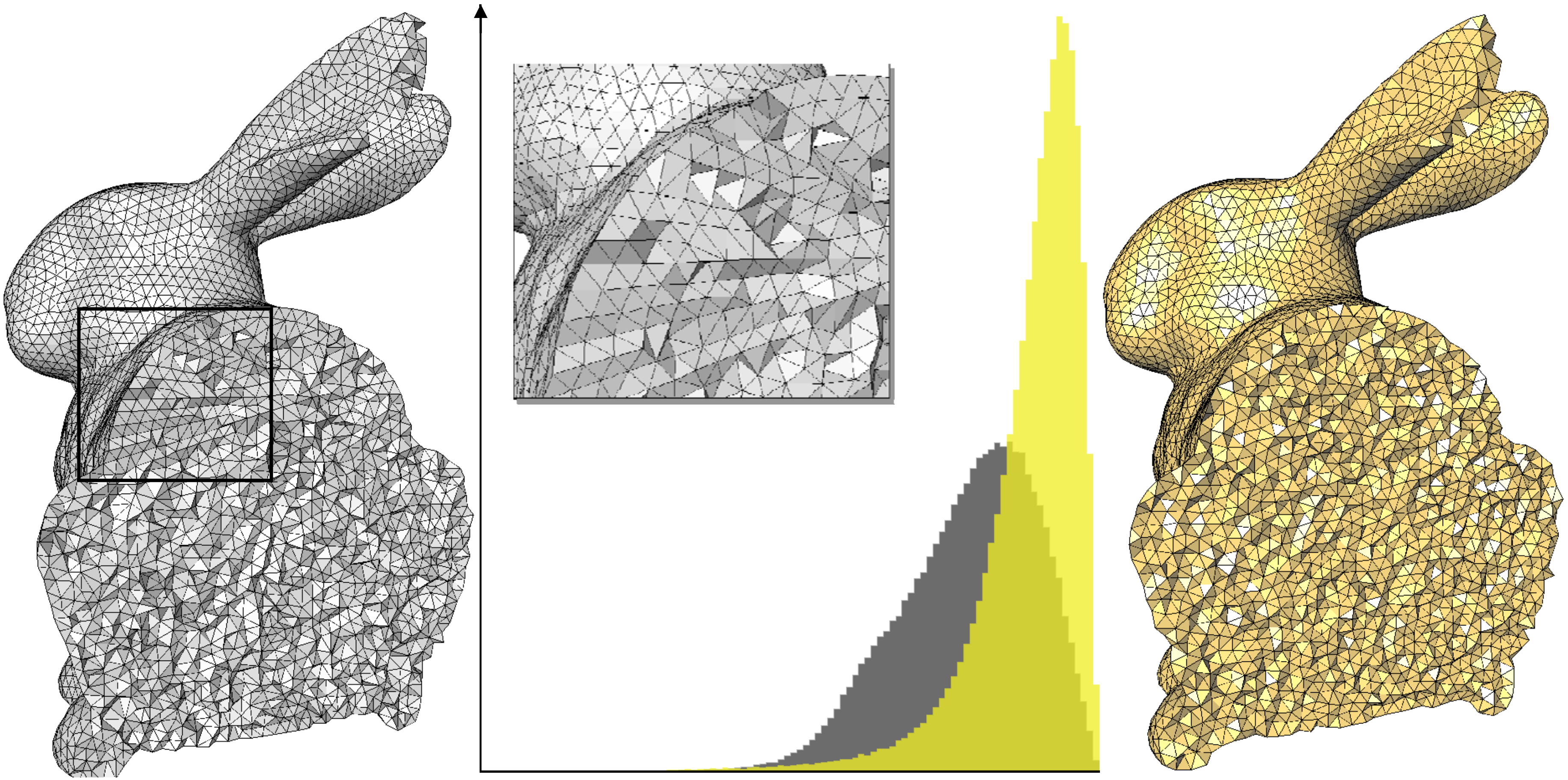
The Visible Human



Fandisk



Comparison with the Unit Mesh Approach



Conclusion

+ Generate high quality isotropic tetrahedral meshes (improved aspect ratios)
+ Simple alternated optimizations
  - connectivity: Delaunay
  - vertex positions: weighted circumcenters
- Theoretical guarantees to be developed

1.2 Interleaving Delaunay Refinement and Optimization for Practical Isotropic Tetrahedron Mesh Generation

Jane Tournois, Camille Wormser, Pierre Alliez, Mathieu Desbrun

Siggraph 2009

Contribution

- A practical system for isotropic tetrahedral meshing of 3D domains bounded by piecewise smooth surfaces
  – Delaunay refinement
  – Optimal Delaunay optimization

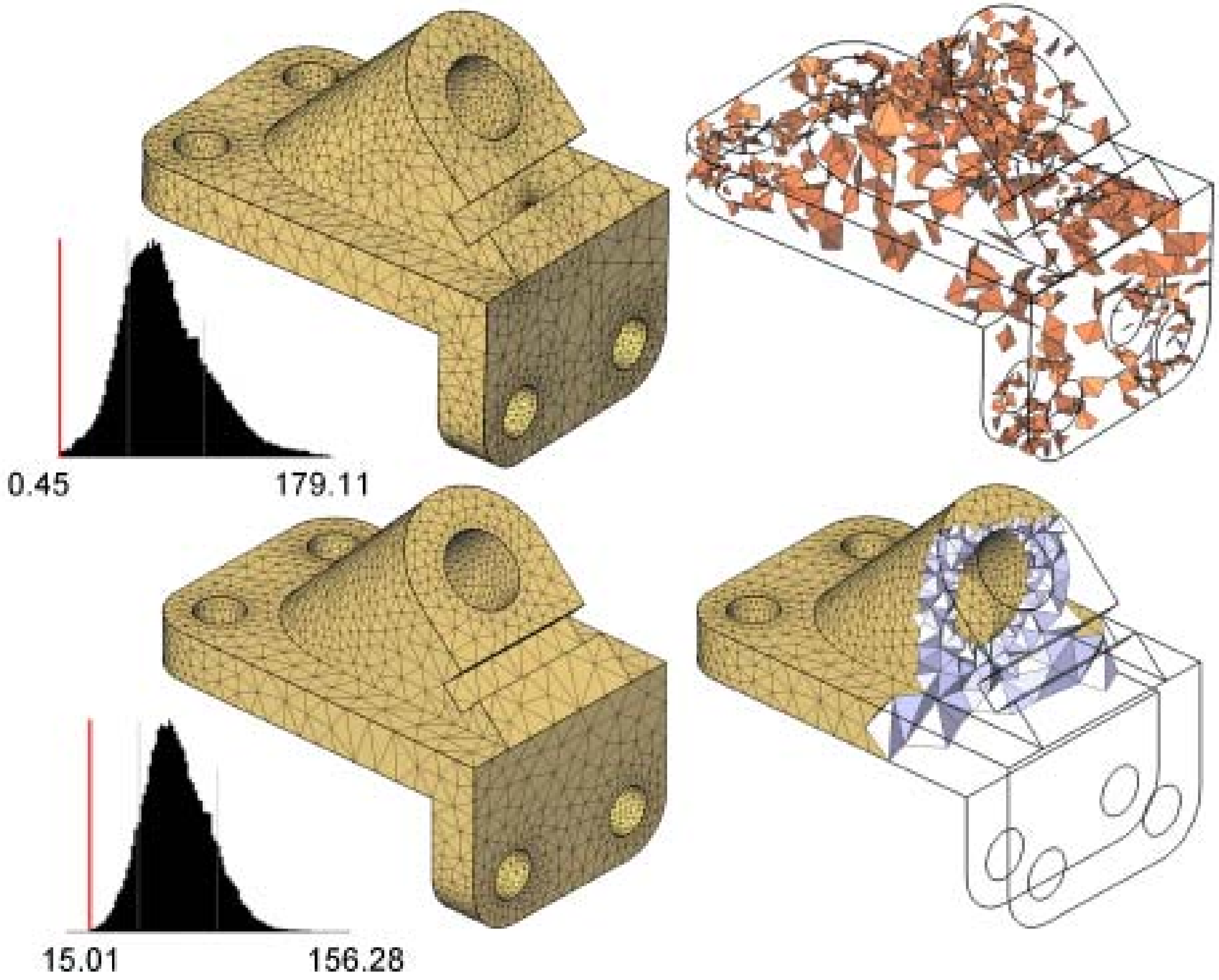


Figure 2: *Top: Mesh (5,499 vertices) generated by Delaunay refinement (shape and boundary approximation criteria activated). Notice the cluster in the middle of the armhole. Right image shows tetrahedra with dihedral angles smaller than 15 degrees. Bottom: Mesh (3,701 vertices) generated by interleaving Delaunay refinement and optimization so as to satisfy the same criteria. Distributions of dihedral angles are shown on the left.*

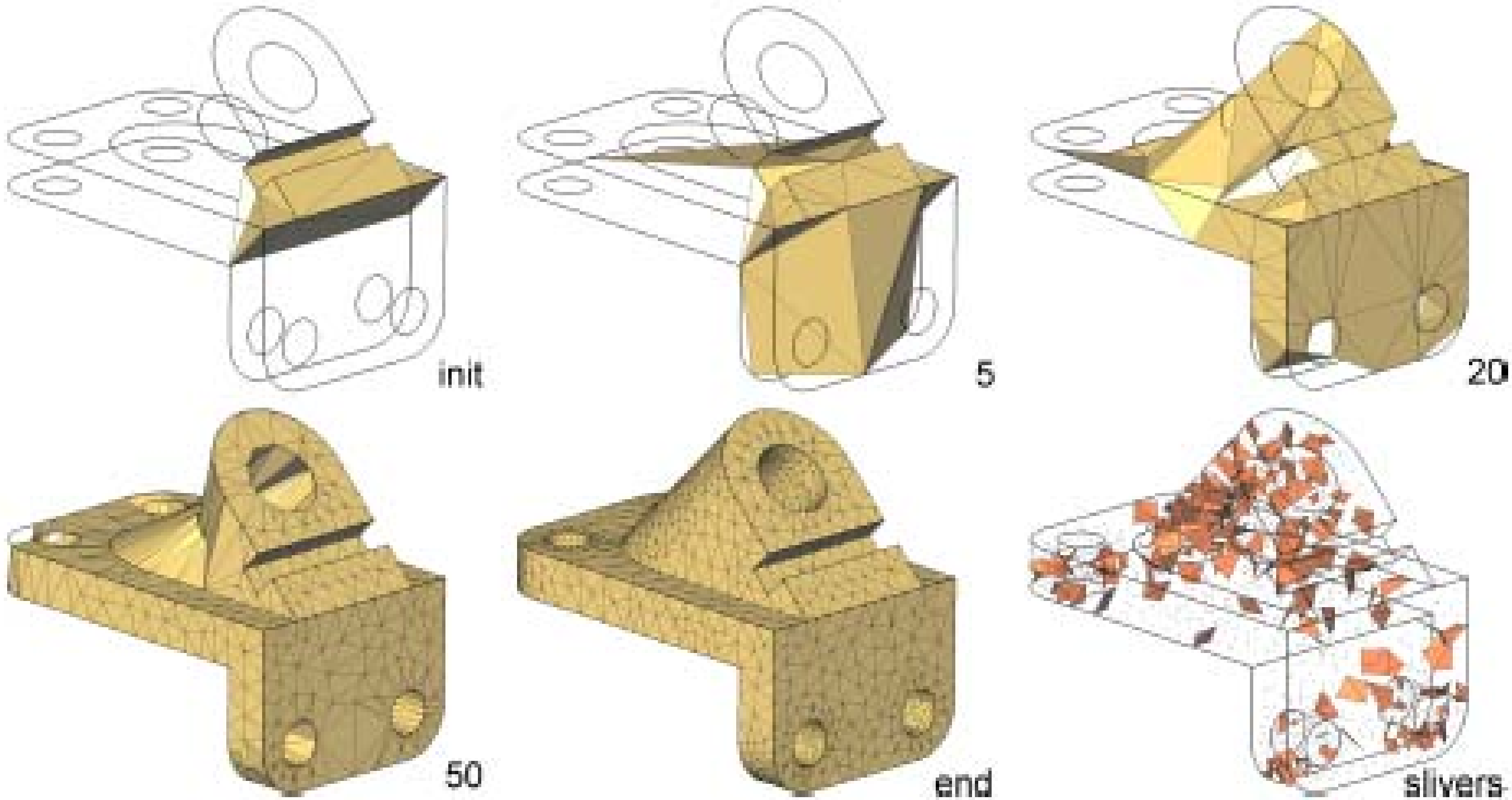
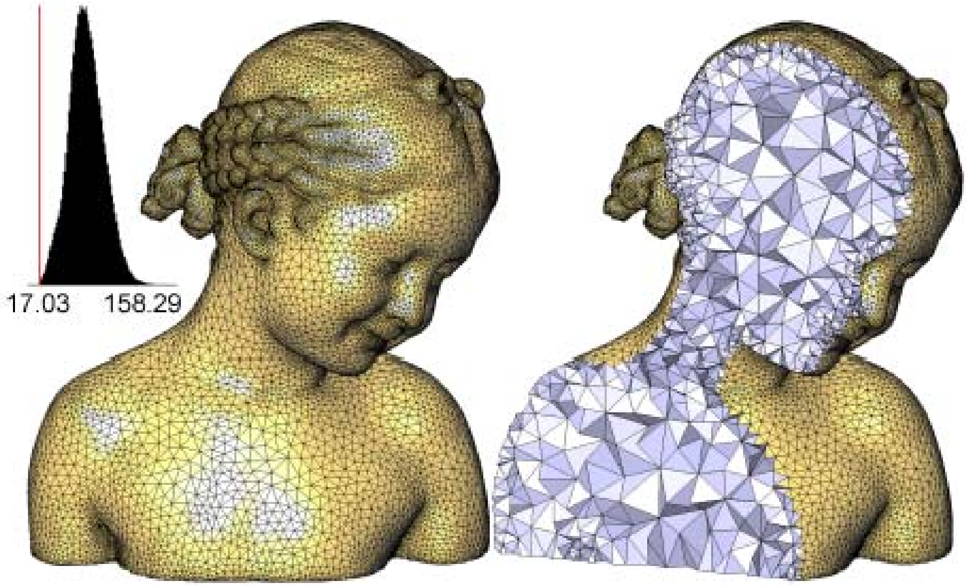
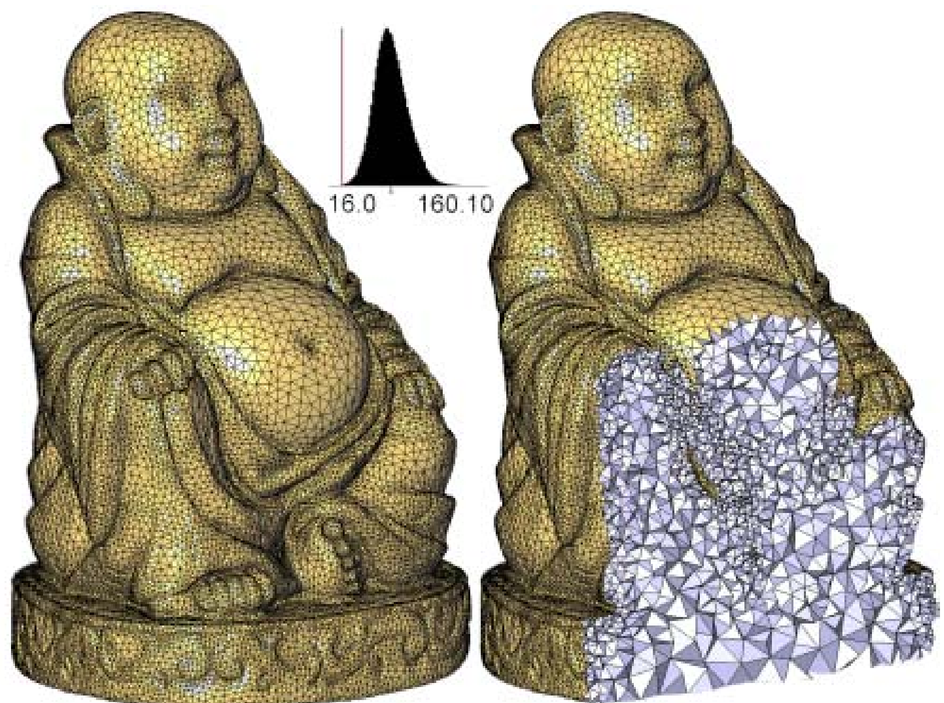


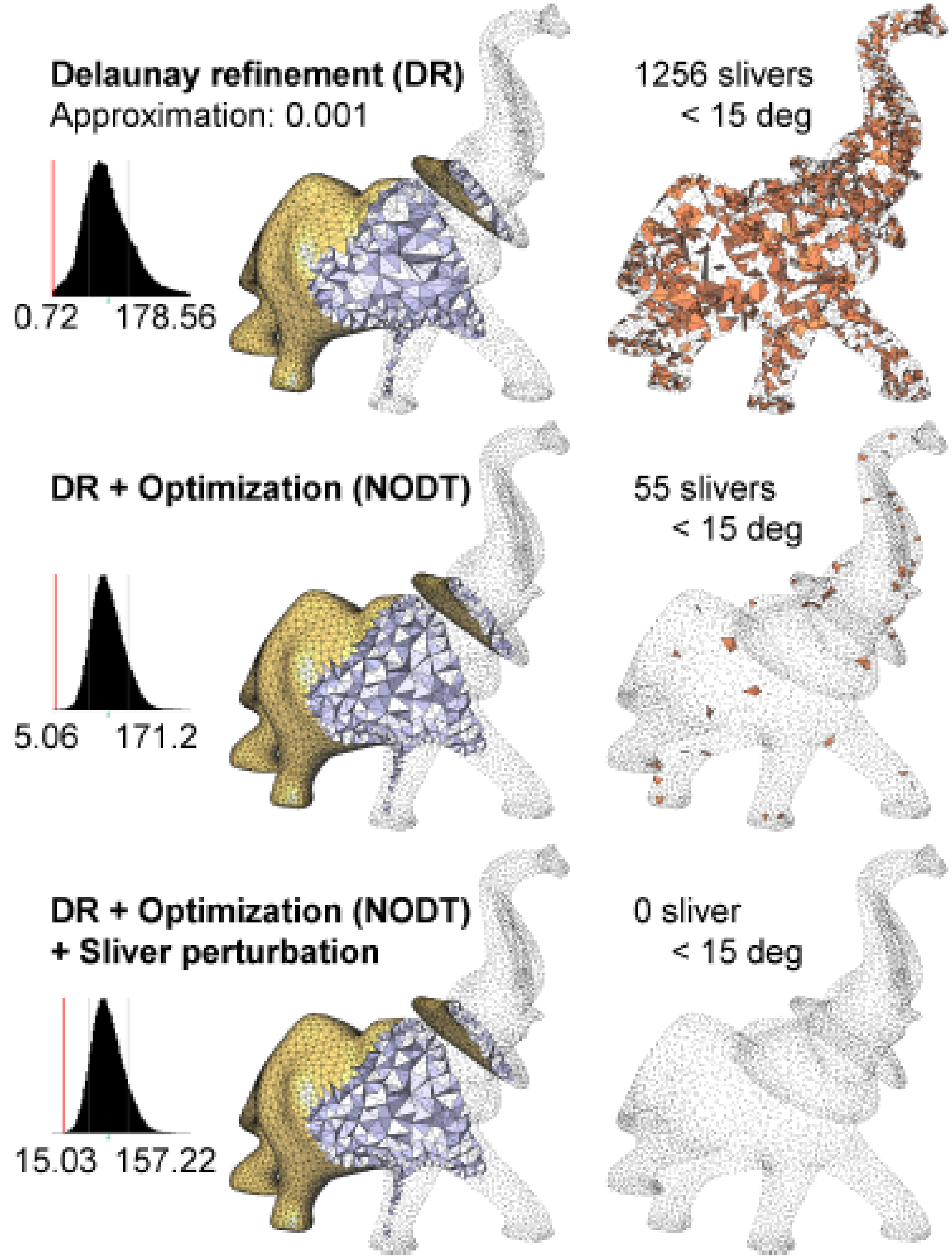
Figure 4: *Refinement steps without optimization. The mesh initialized with feature vertices; after a few batch refinement steps (from 5 to 50); the final refined mesh with shape and approximation criteria satisfied; and its 244 slivers (tetrahedra with dihedral angle smaller than 10 degrees).*

Results





Cc

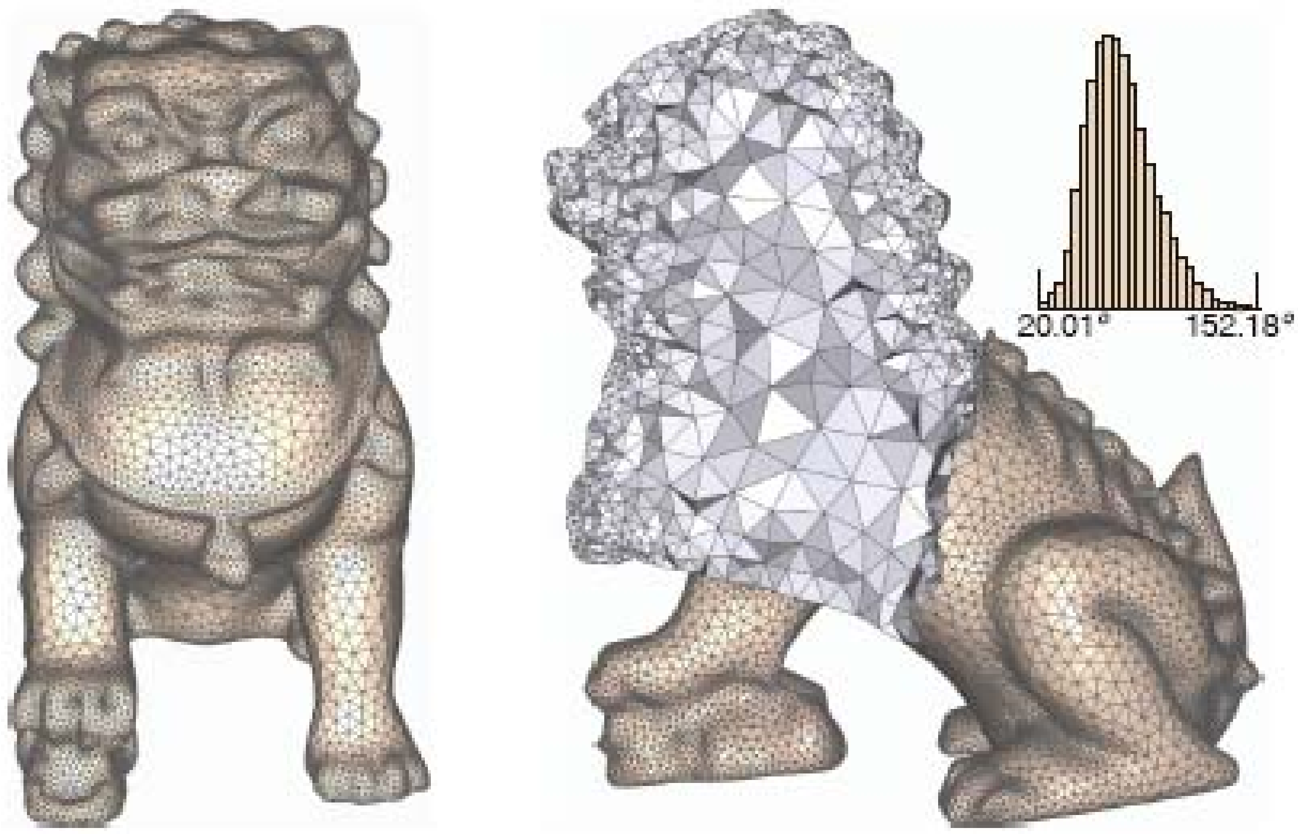


Delaunay refinement (DR)
Approximation: 0.001
0.72
178.56
1256 slivers
< 15 deg
DR + Optimization (NODT)
5.06
171.2
55 slivers
< 15 deg
DR + Optimization (NODT)
+ Sliver perturbation
15.03
157.22
0 sliver
< 15 deg

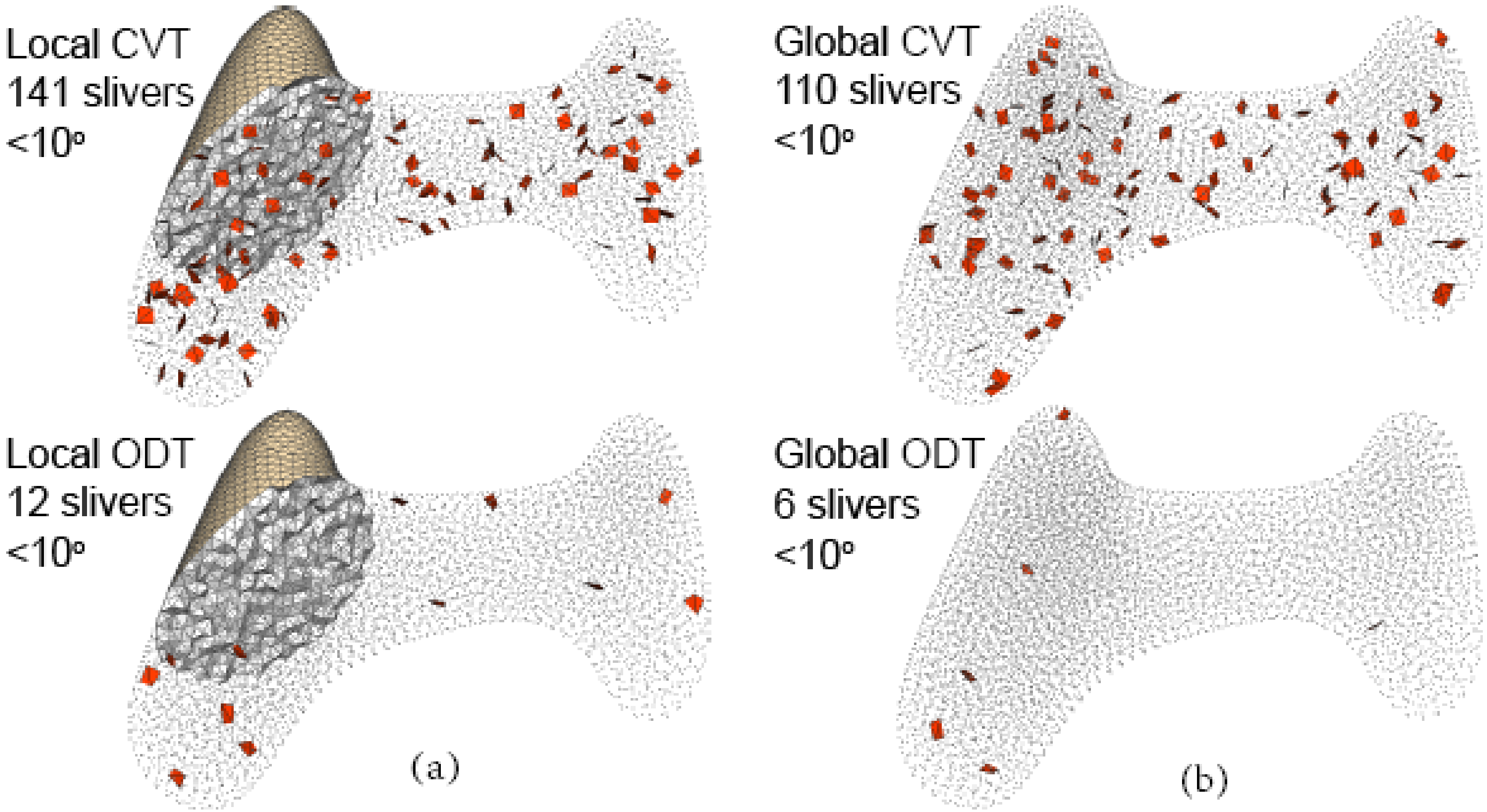
1.3 Graded mesh generation using ODT

[Chen et al., 2012]

Graded Tet-meshes



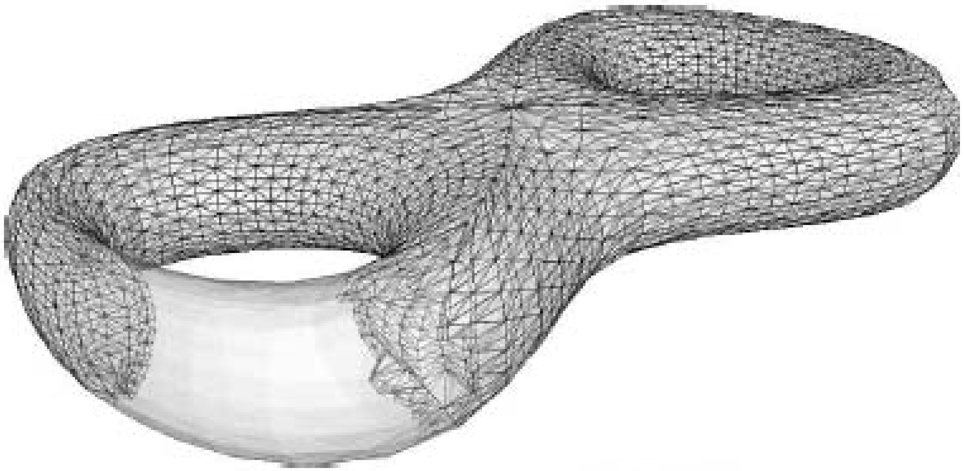
Less Slivers

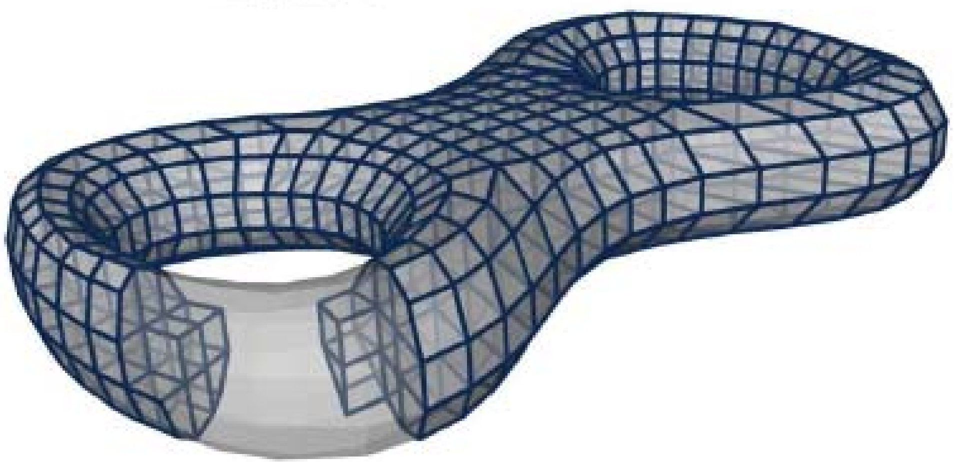


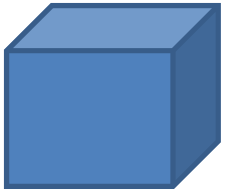
2. Hexahedral meshing

Hexahedral Meshes

- More popular in simulation and FEM





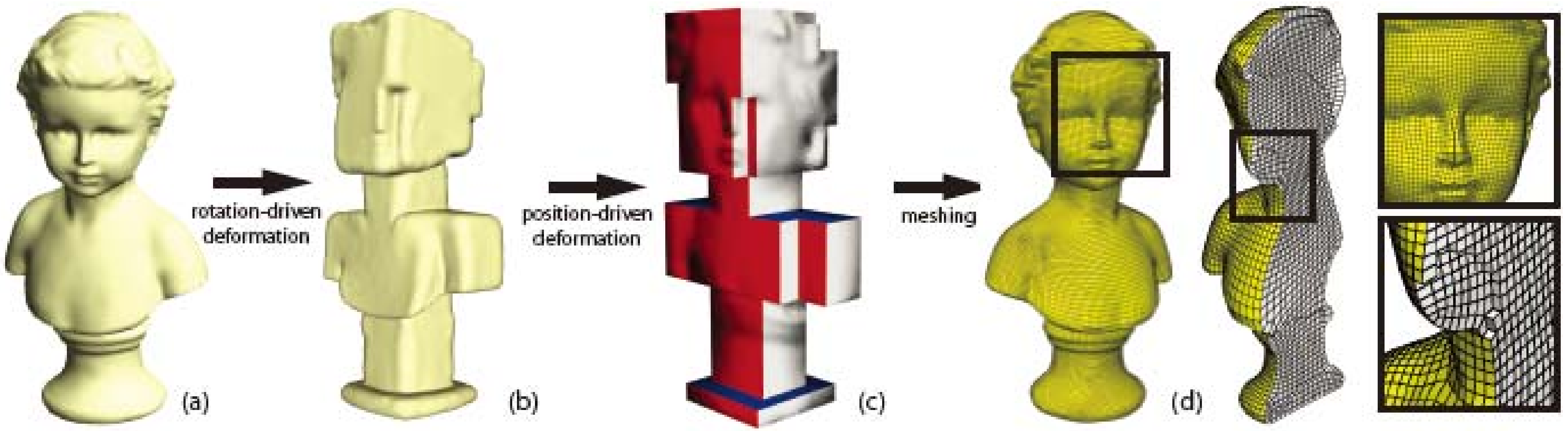


2.1 All-Hex Mesh Generation via Volumetric PolyCube Deformation

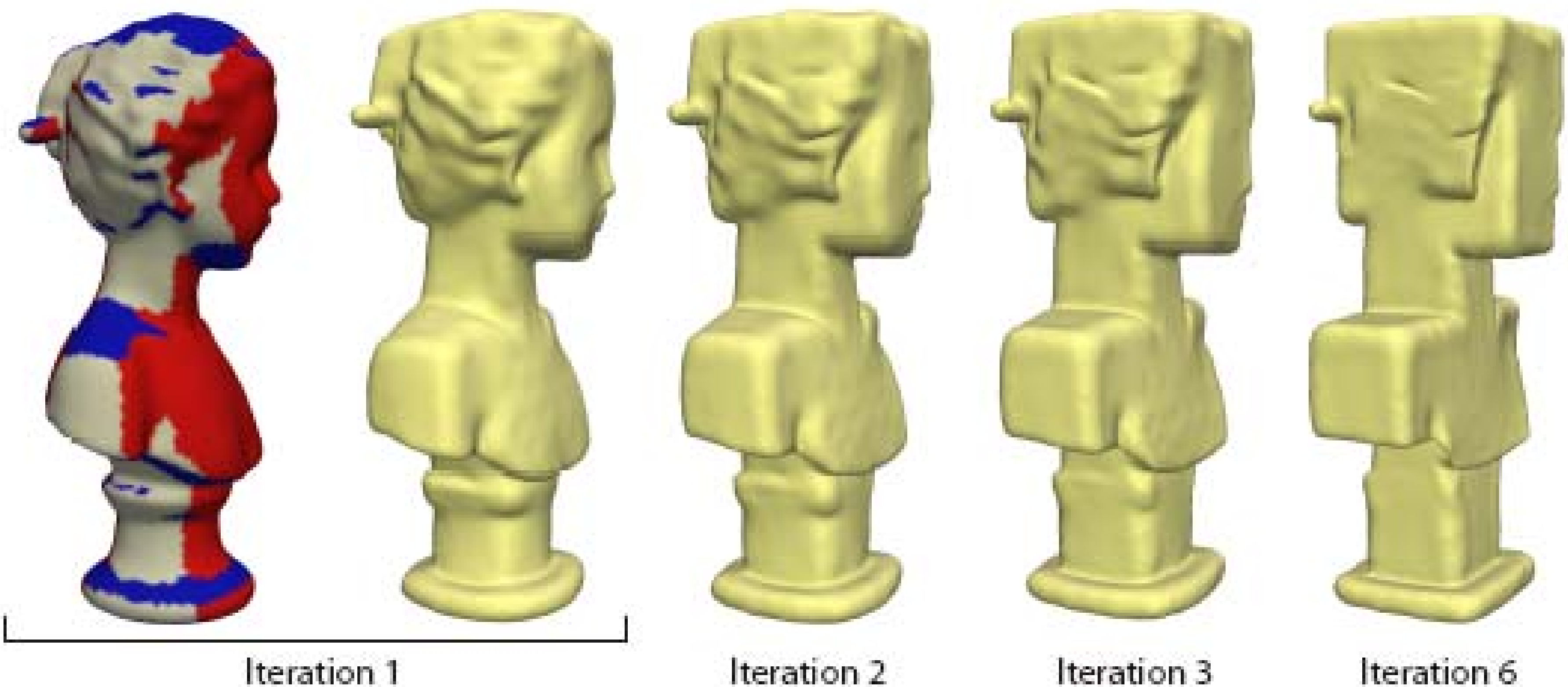
[Gregson et al., SGP 2011]

Pipeline

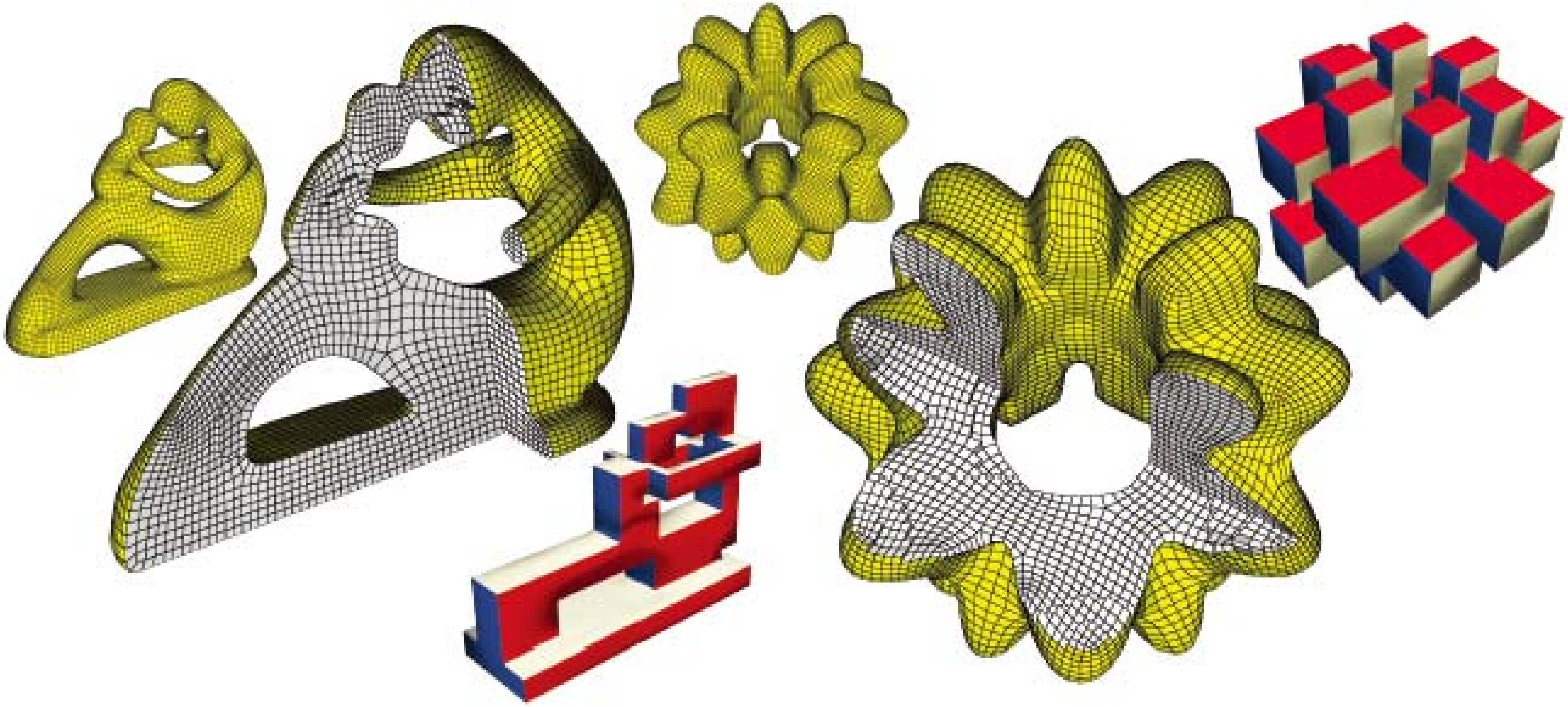
- Basic idea: PolyCube



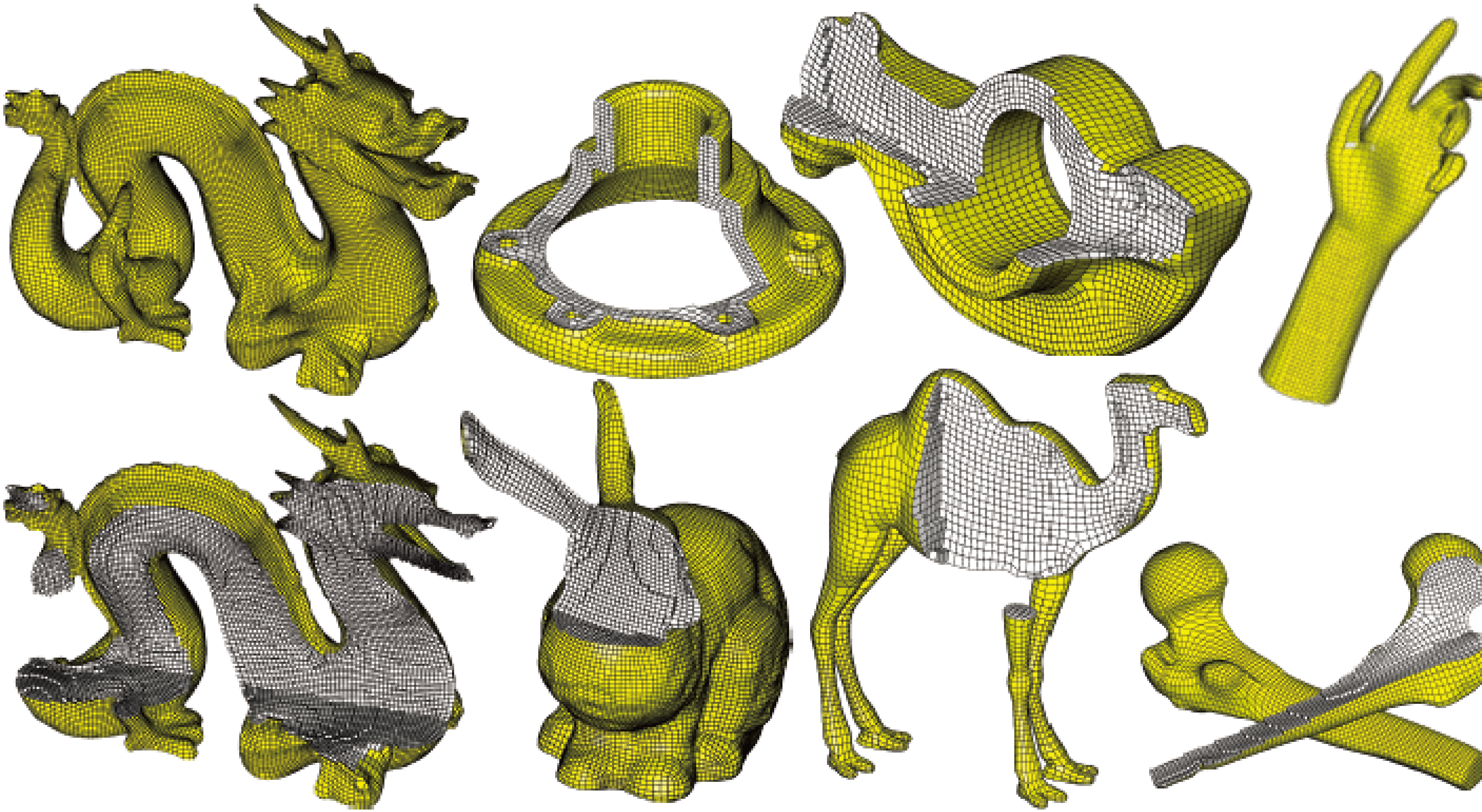
Deformation driven PolyCube



PolyCubes



Results

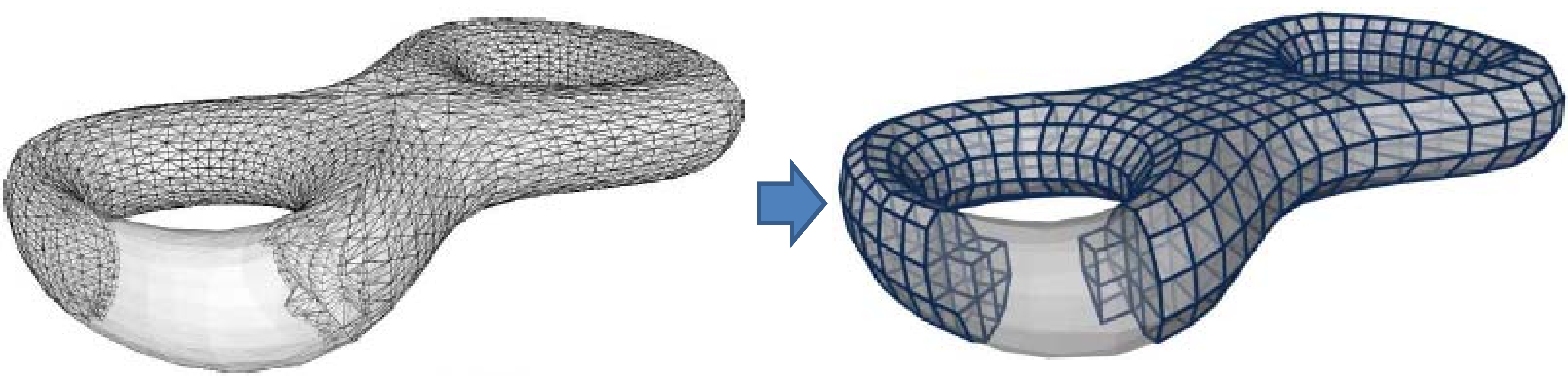


2.2 CUBECOVER – Parameterization of 3D Volumes

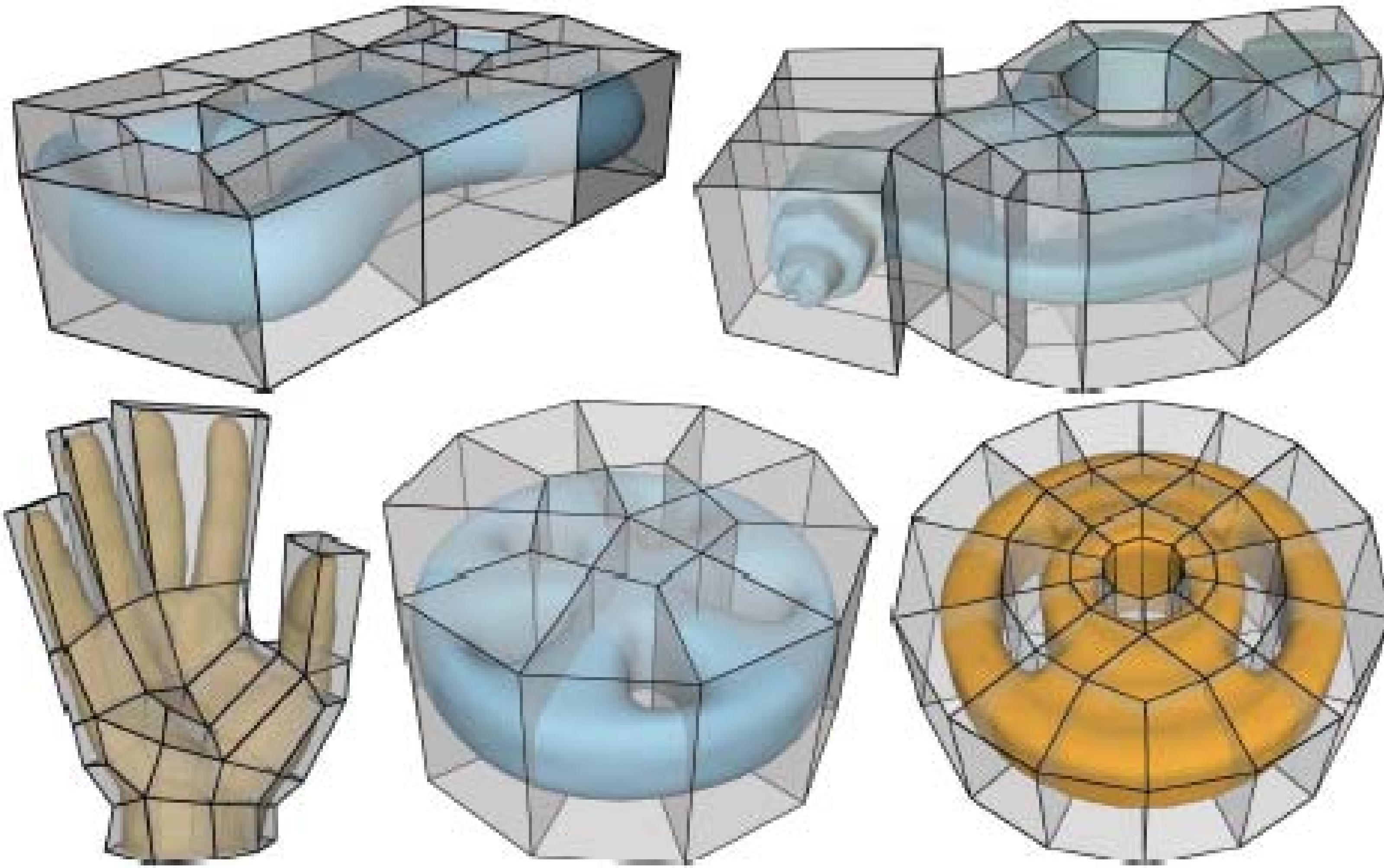
[Nieser et al., SGP 2011]

CubeCover

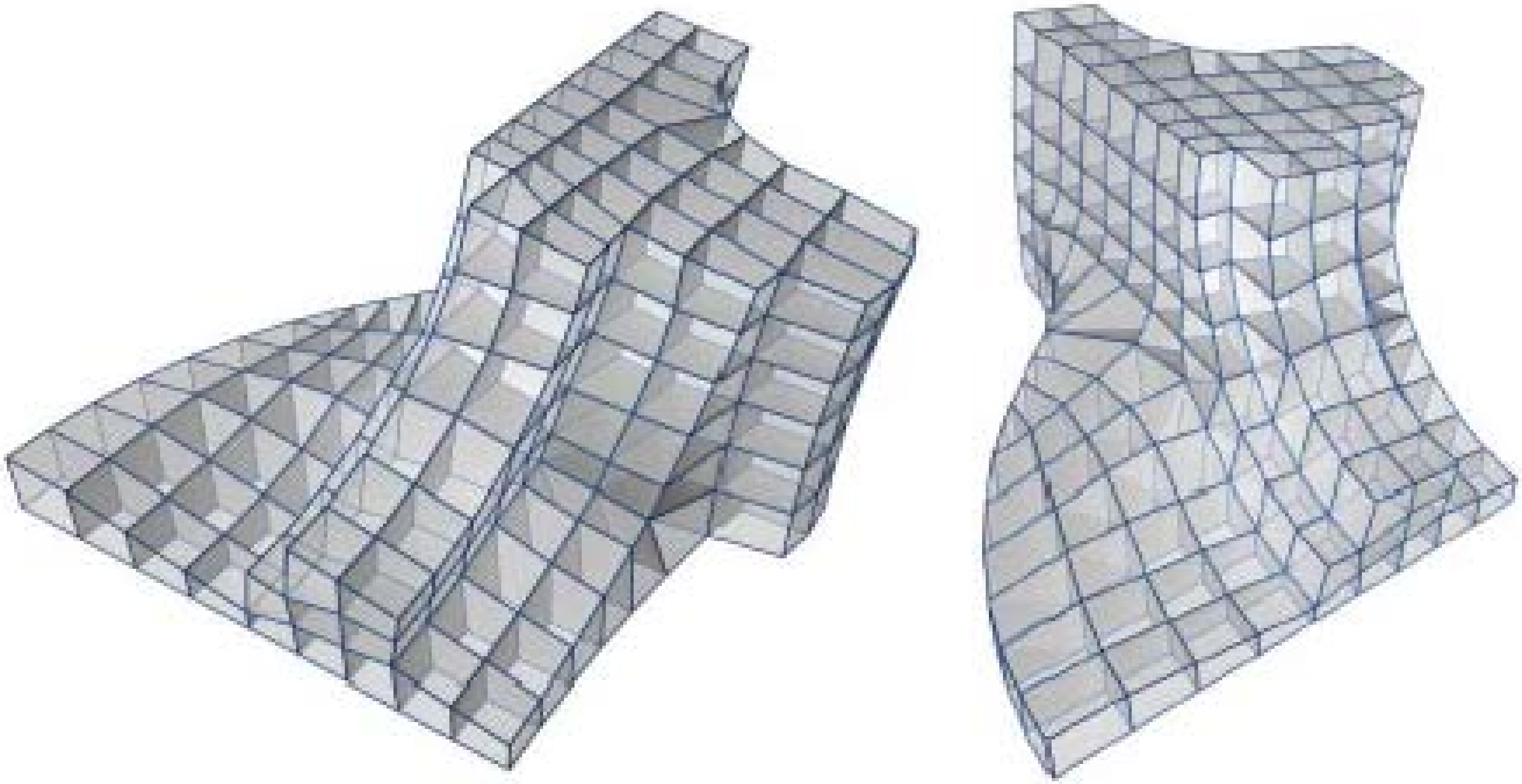
- Volumetric parameterization
  - Extend the idea of QuadCover
- Boundary aligned
- Guided by a frame field



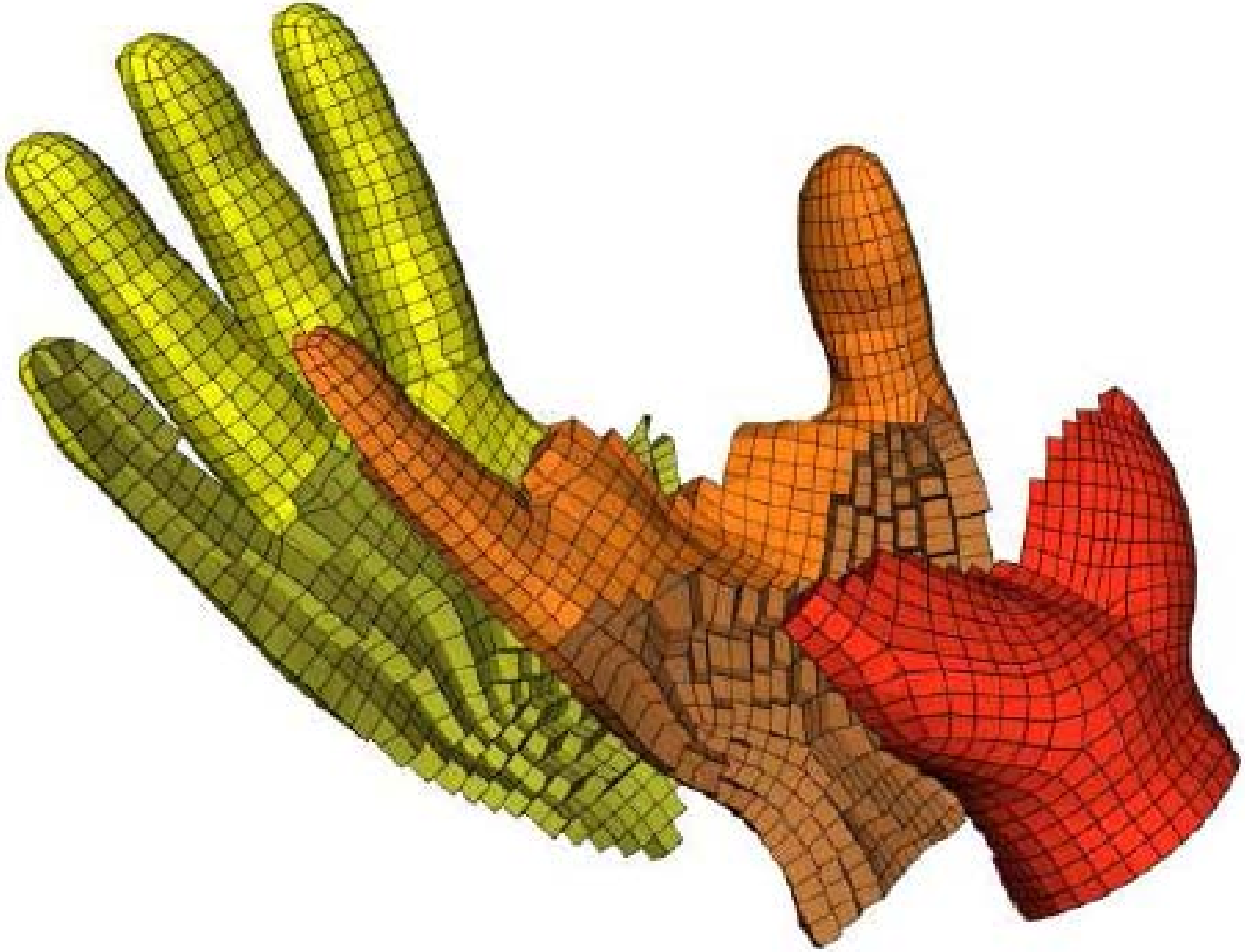
Volumetric Parameterization



Feature Alignment



Results

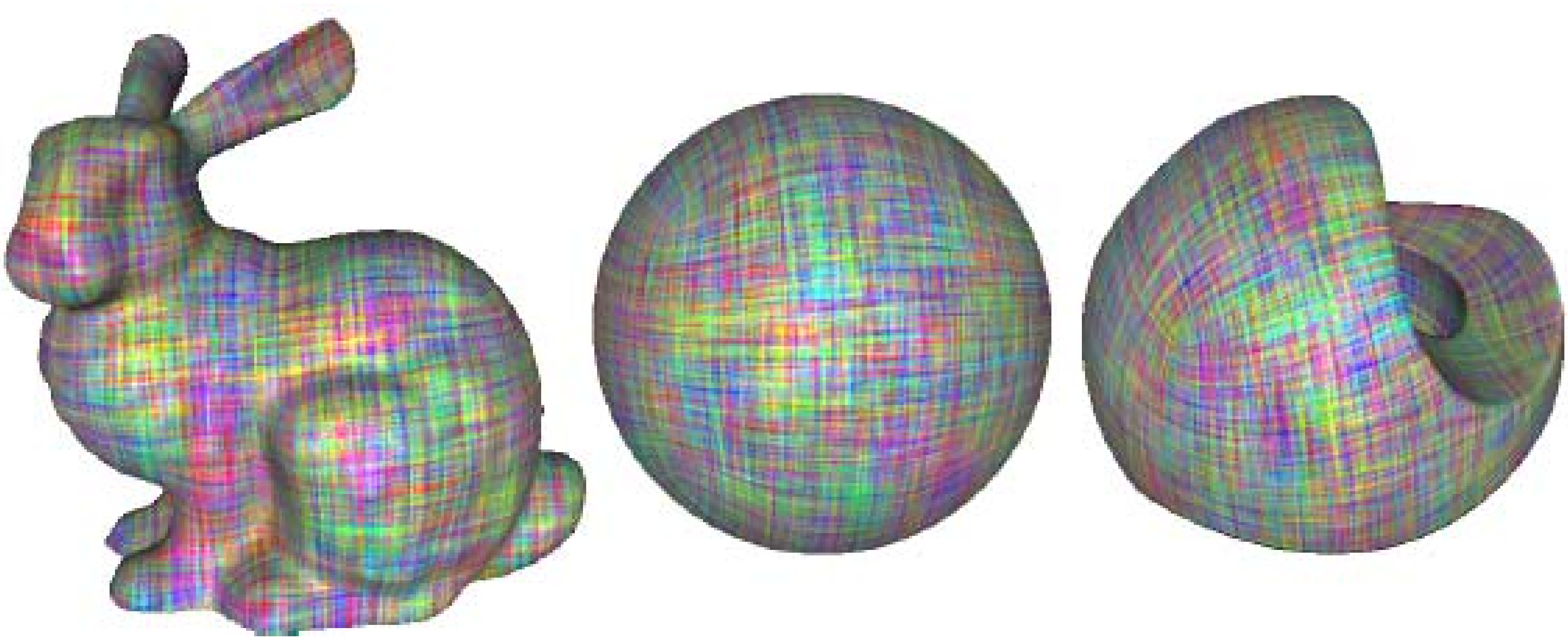


2.3 Boundary Aligned Smooth 3D Cross-Frame Field

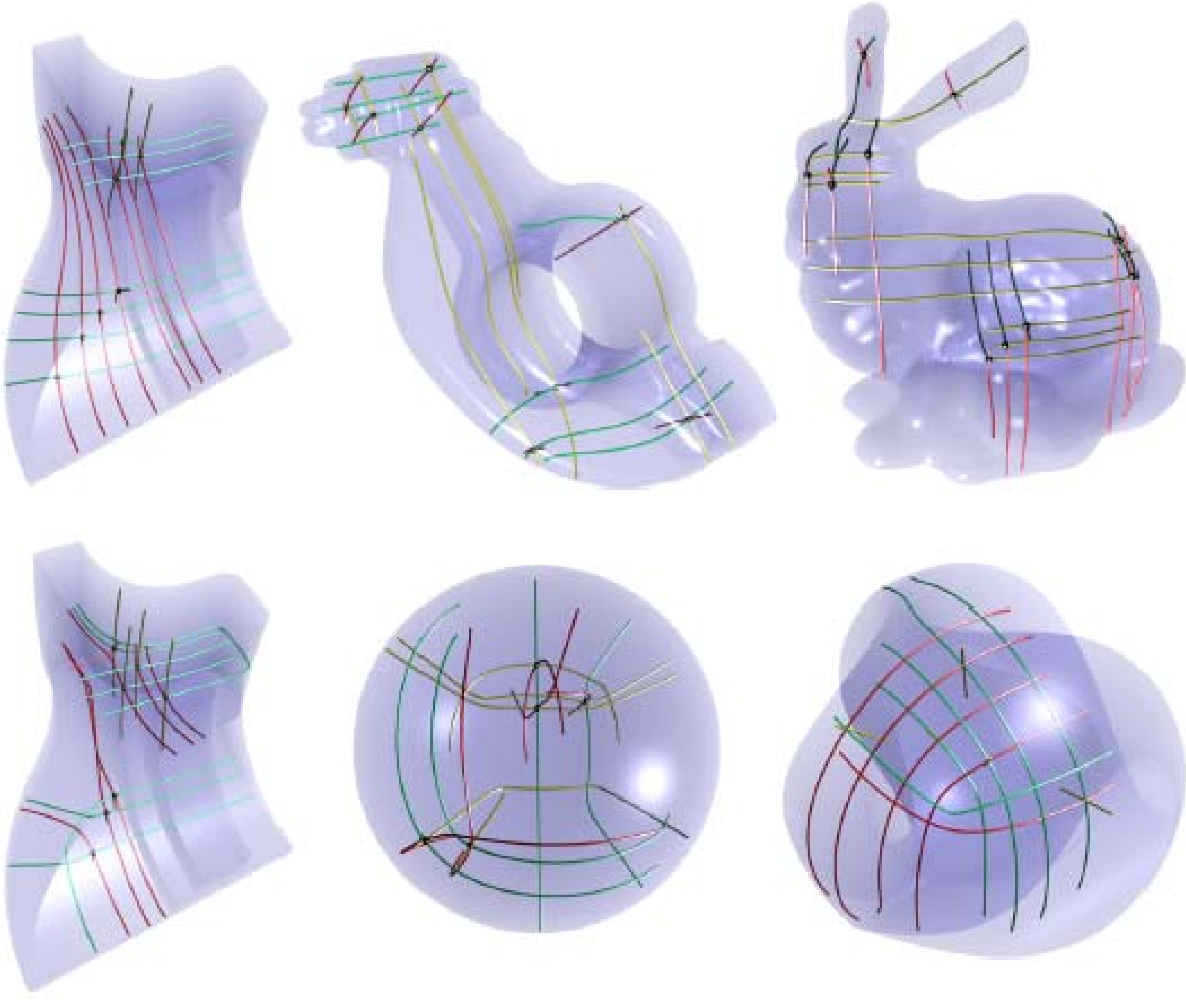
[Huang et al., Siggrah Asia 2011]

Boundary Aligned 3D Cross-frame Field

- 24-symmetric 3D cross-frame field



Stream Lines



Hex-Meshes

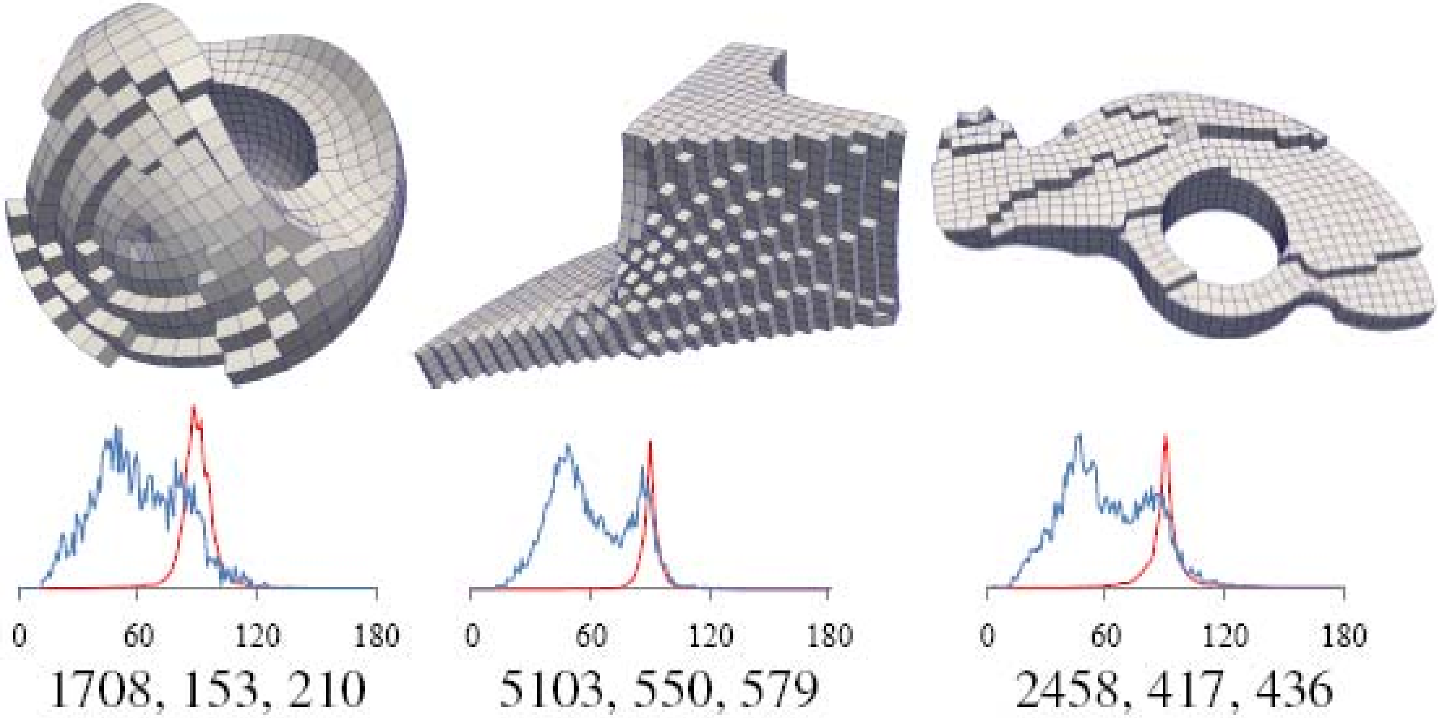


Figure 7: *The sectional views of the hexahedral dominant meshes generated from the 3D cross-frame field. The blue and red curves indicate the angle distributions for all the triangular and quadrilateral faces respectively. We list the numbers of hexahedra, pentahedra and tetrahedra separately at the bottom.*

2.4 More to come…

[Siggraph 2012]

Summary

- Volumetric meshing
  – Tetrahedral meshes
  – Hexahedral meshes
- Attractive in computer graphics
- More in engineering
  – FEM
  – Simulation

Discussions