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Shape Analysis Basics

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Outline

- Surface curvature
- Distance
- Saliency
- Feature points
- Feature lines
- Principal Component Analysis (PCA)

Surface Curvature

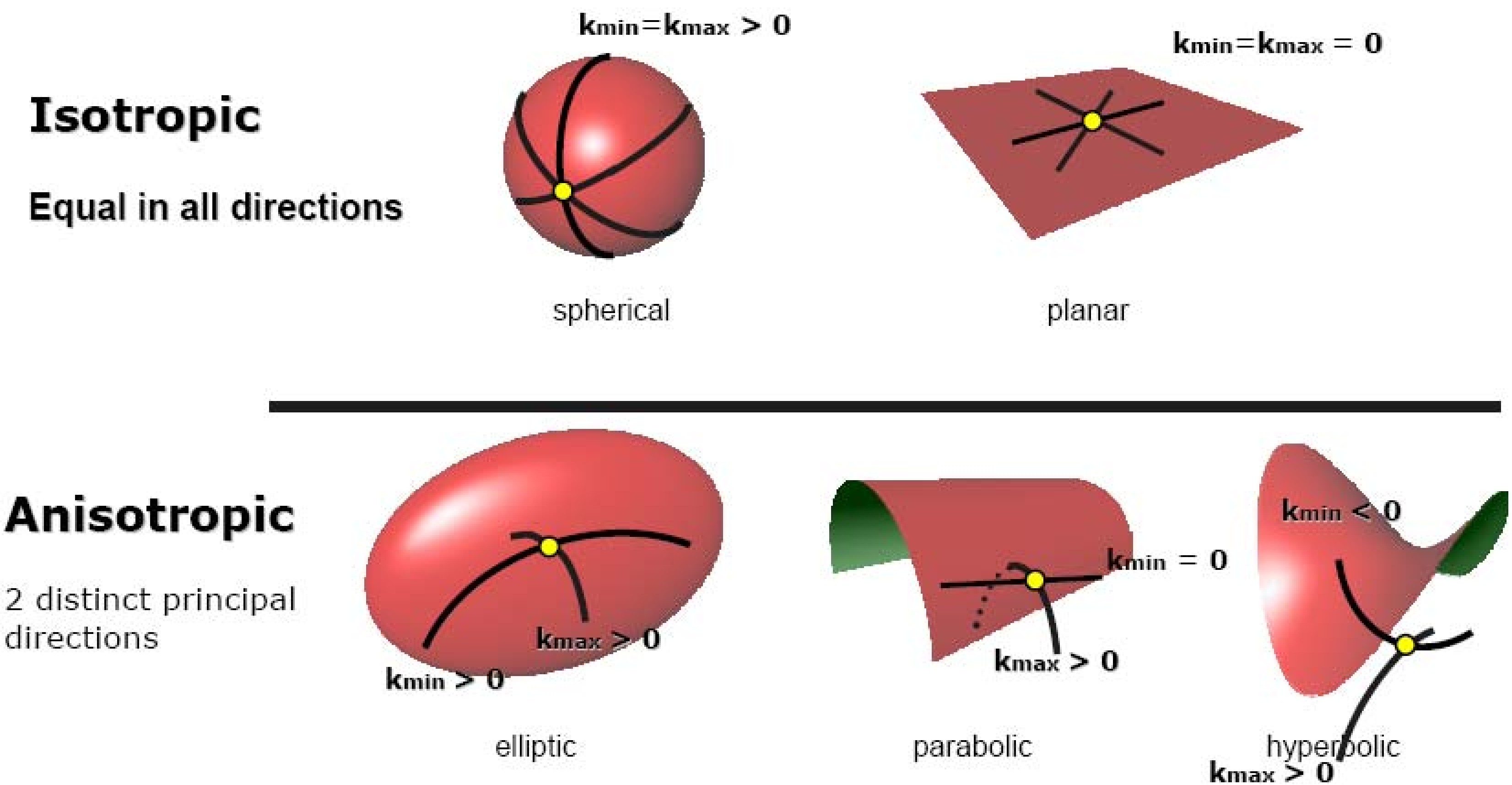
Surface Curvature

- ***Normal curvature*** of surface is defined for each tangential direction

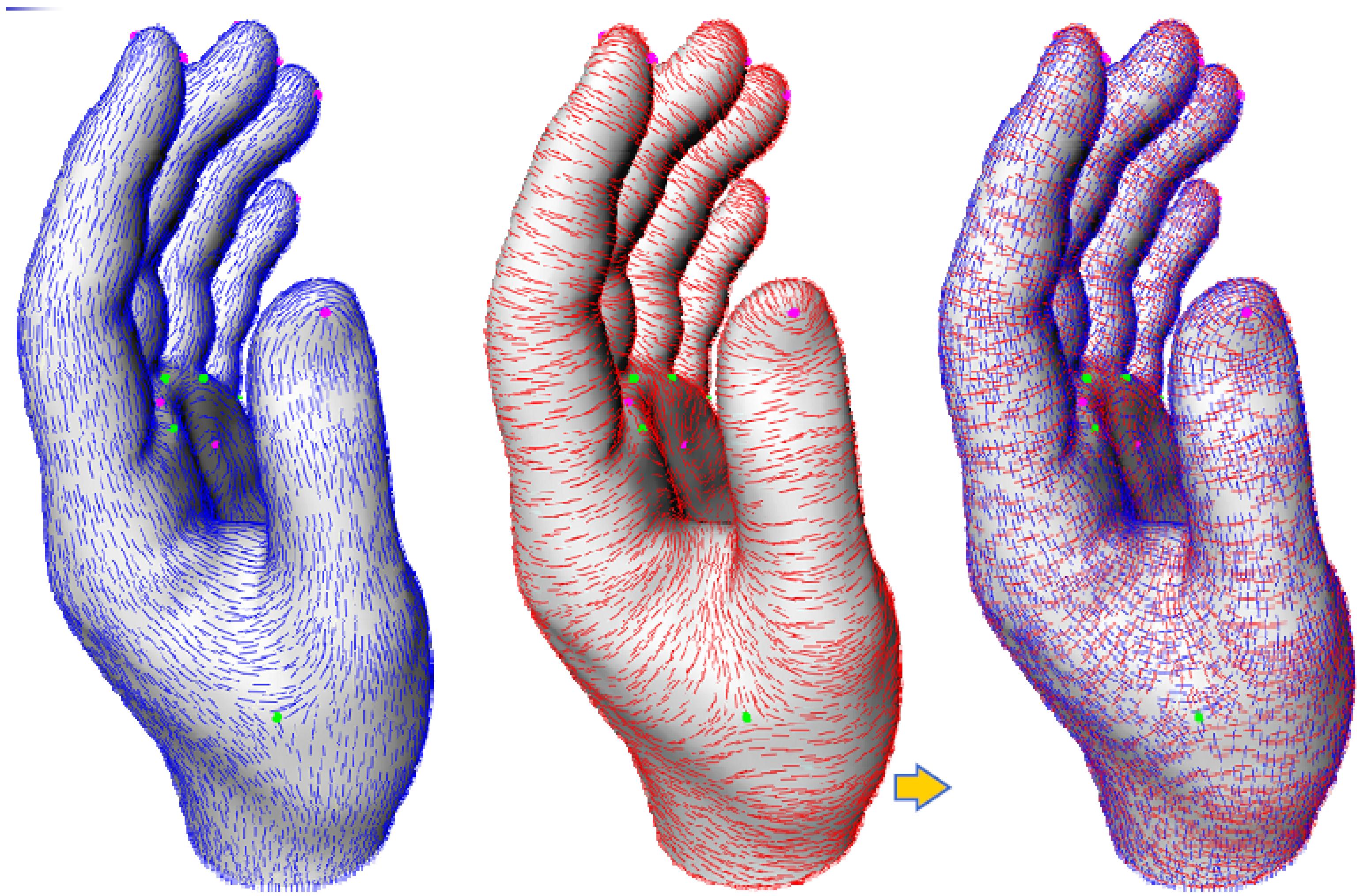
$$\kappa^N(\theta) = \kappa_1 cos^2(\theta) + \kappa_2 sin^2(\theta)$$

- ***Principal curvatures*** $Kmin$ & $Kmax$: maximum and minimum of normal curvature
  - Correspond to two **orthogonal** tangent directions
    - Principal directions
  - Not necessarily partial derivative directions
  - Independent of parameterization

Surface Curvature



Principal Directions



Min Curvature Max Curvature

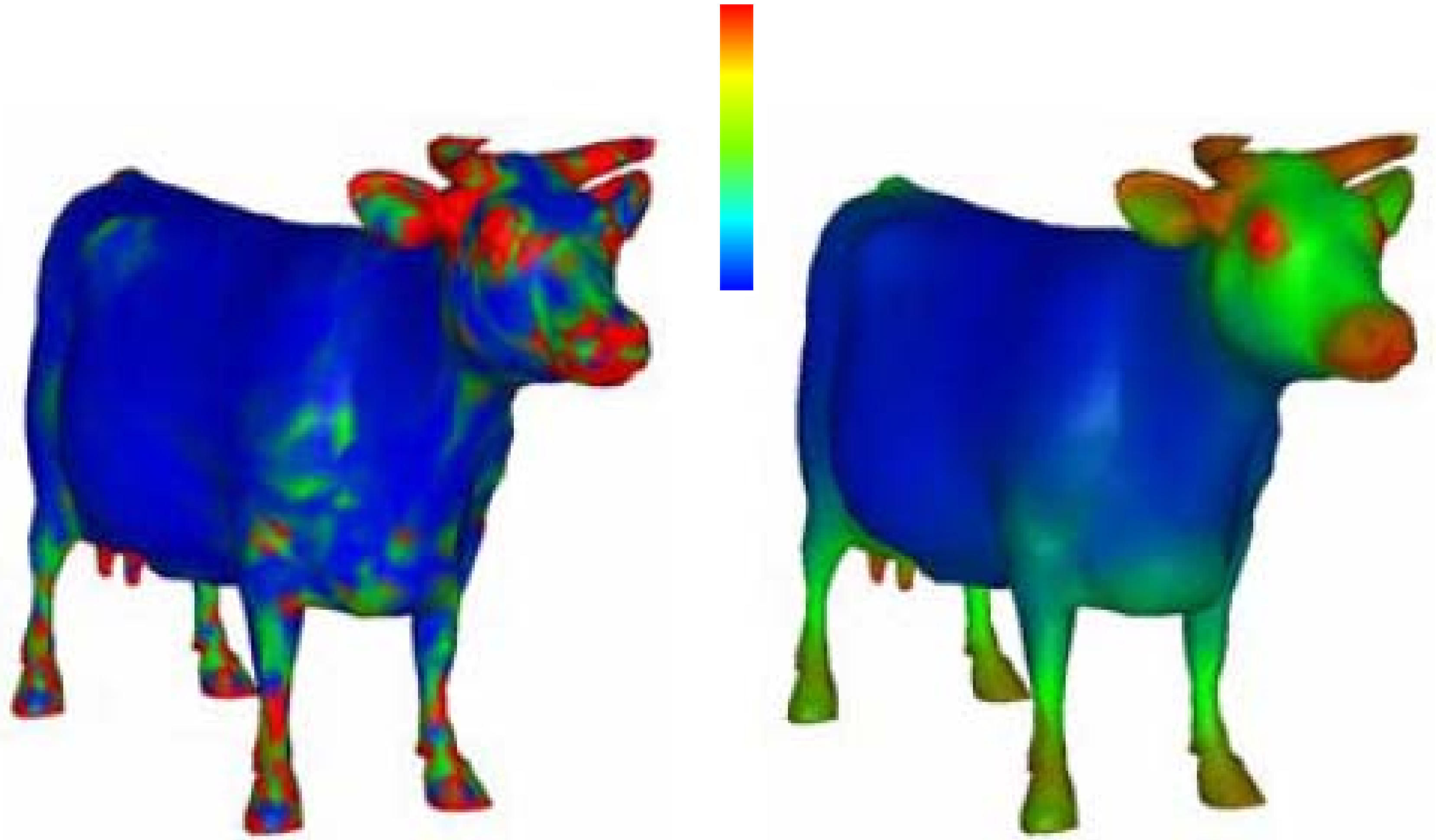
Surface Curvatures

- Typical measures:
  - ***Gaussian*** curvature

$$K = k_{\min} k_{\max}$$

  - ***Mean*** curvature

$$H = \frac{k_{\min} + k_{\max}}{2}$$



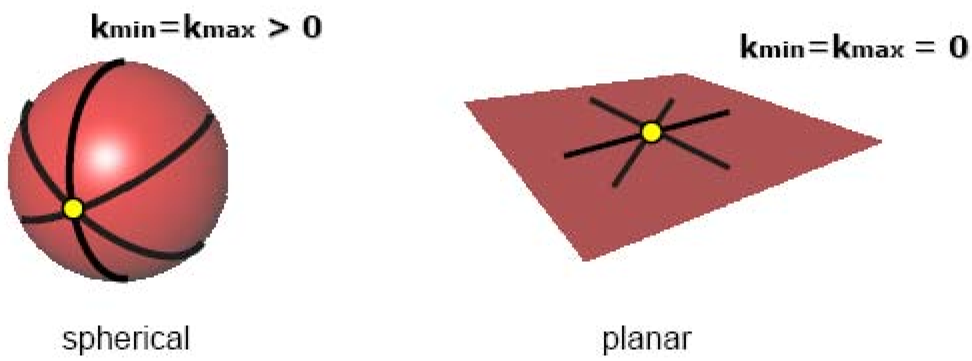
Gaussian curvature

Mean curvature

Surface Curvature

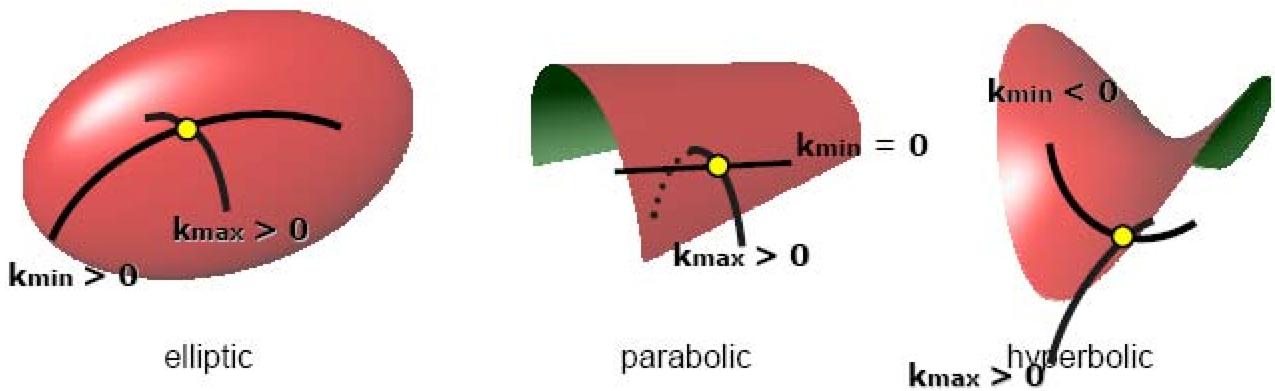
Isotropic

Equal in all directions



Anisotropic

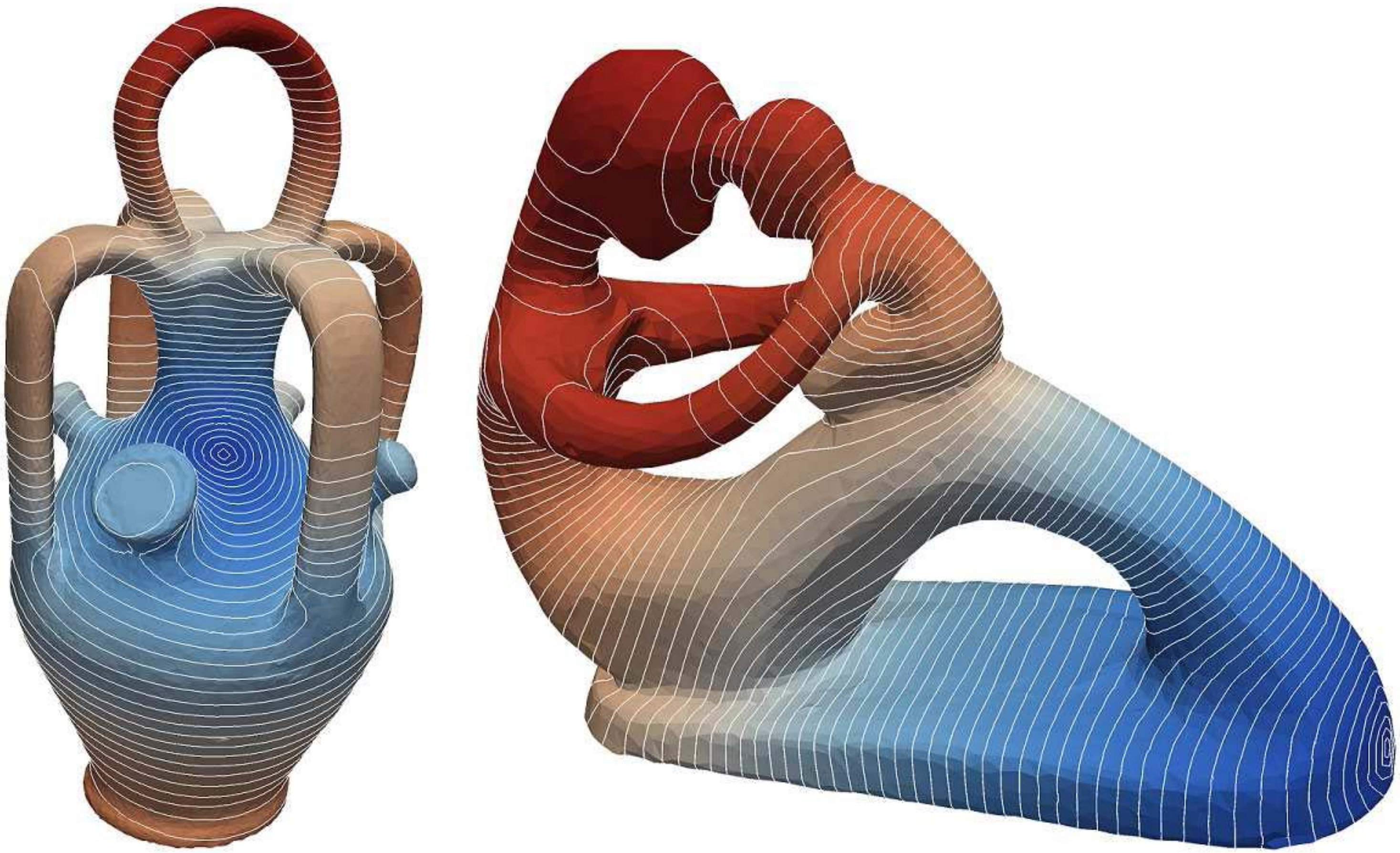
2 distinct principal directions



Distance

Distance on Meshes

- How close are two points?



Distance on Meshes

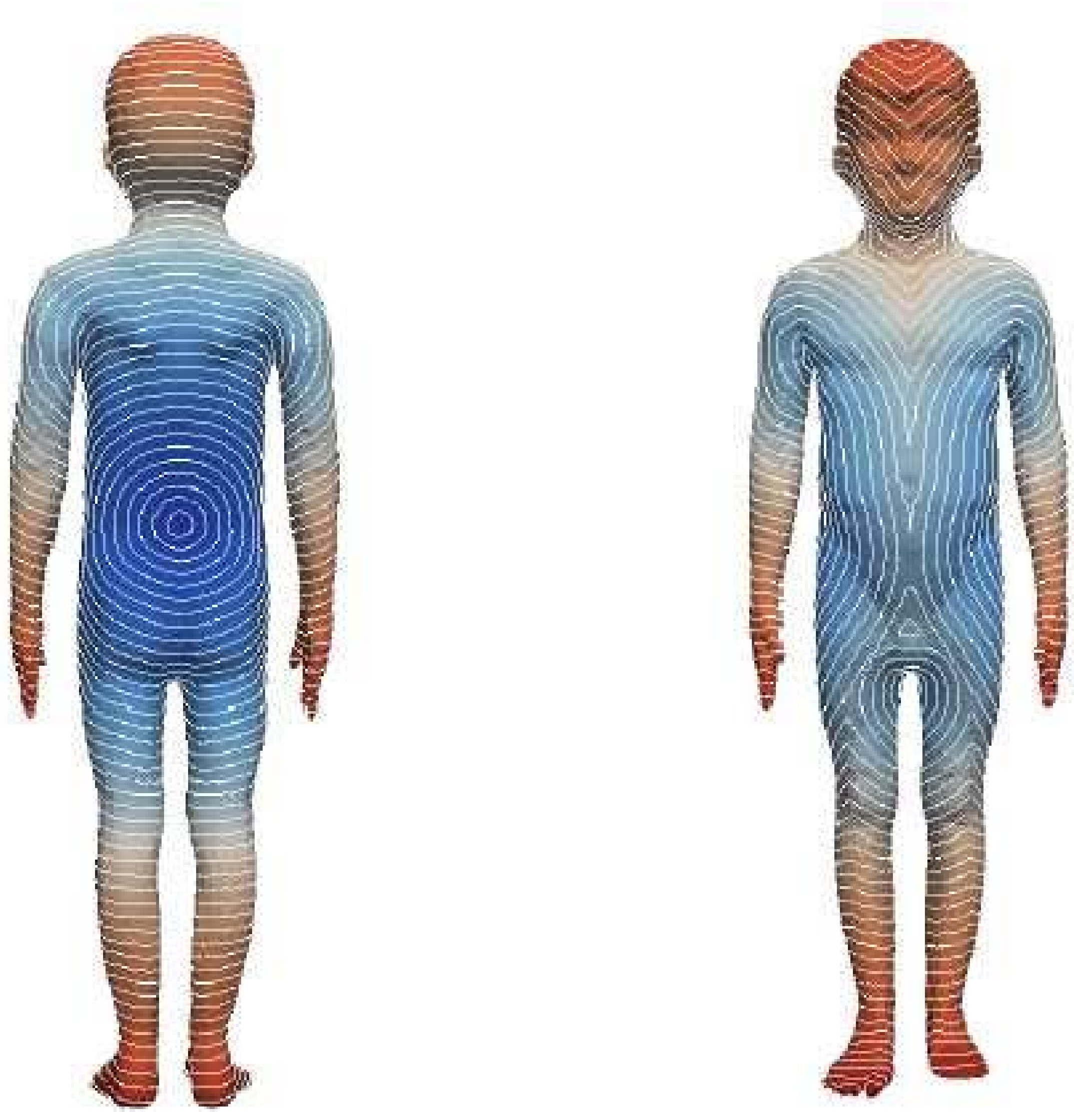
- Desirable properties:
  - – Parameter-free
  - – Is a metric
  - – Smooth
  - – Locally isotropic
  - – Fast to compute
  - – Shape aware
  - – Insensitive to noise
  - – Insensitive to topology changes

Geodesic Distance

- Length of shortest path between p and q on surface
- Can be computed exactly in O(n2logn) [Mitchell87]
- Often approximated with Dijkstra's algorithm on vertex graph

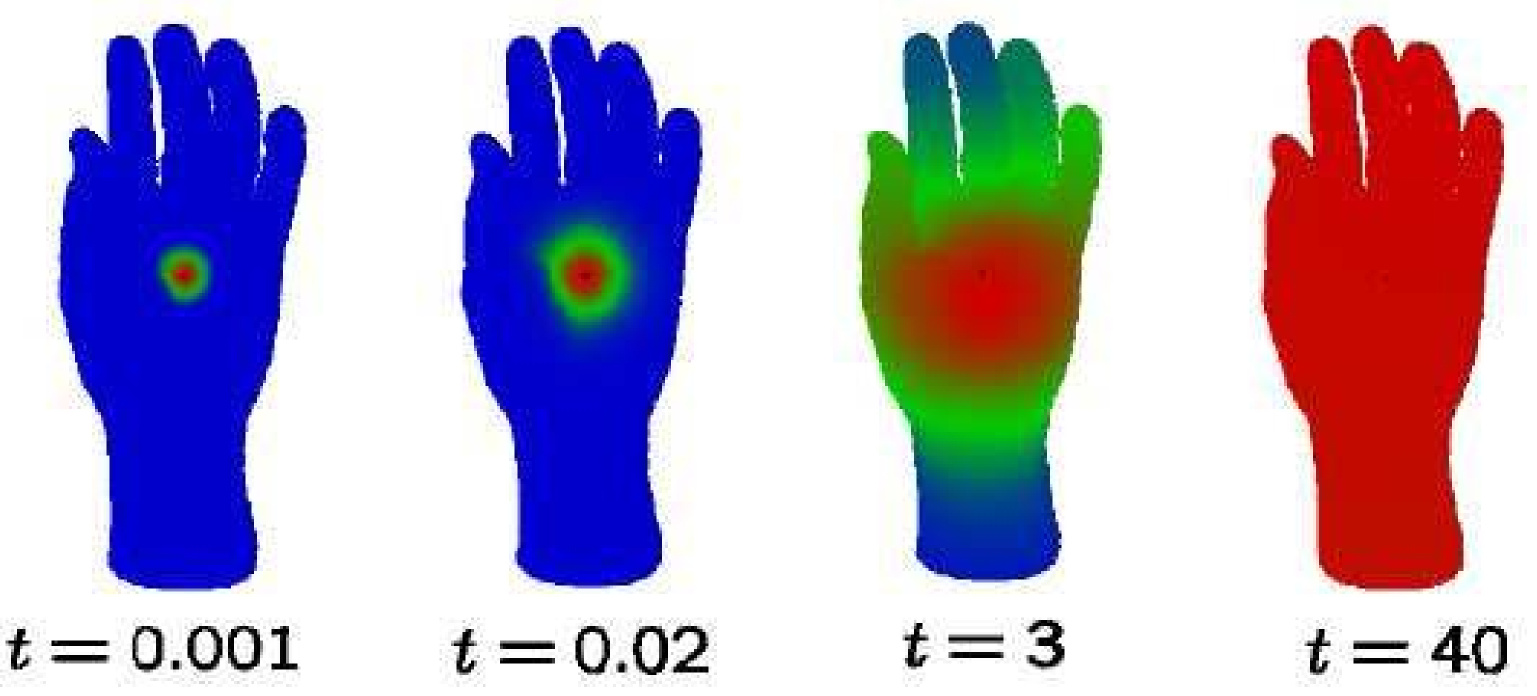
GMP 2012 tutorial “Geodesic distance and it’s applications”

Geodesic Distance



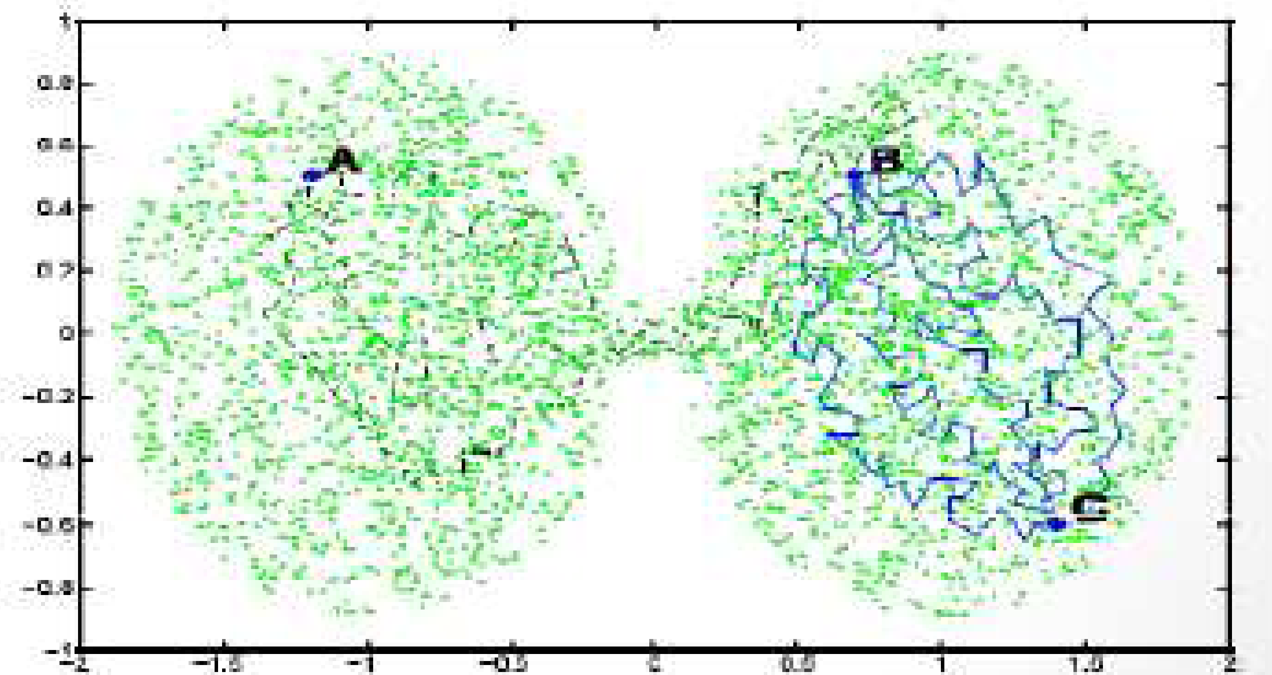
Diffusion Distance

- Amount of heat that diffuses from p to q in time t



Diffusion Distance

- Related to probability of random walk starting at p arriving at q after time t
  - Affected by all paths from p to q

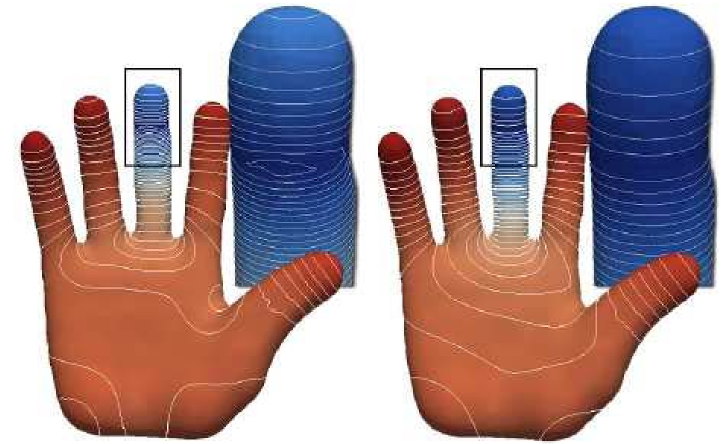


Diffusion Distance

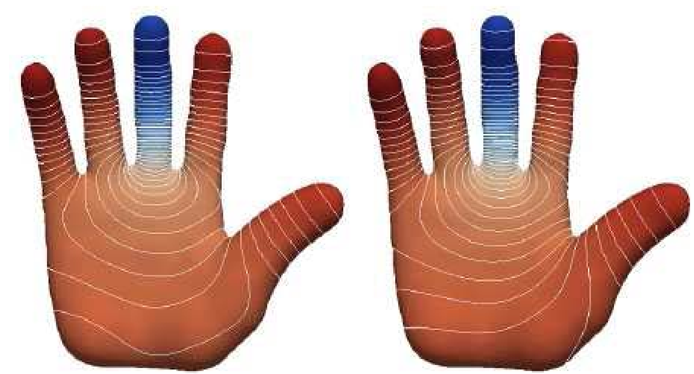
- Can be computed by eigenanalysis of Laplacian

$$d_D(x,y)^2 = \sum_{k=1}^{\infty} e^{-2t\lambda_k} \left(\phi_k(x) - \phi_k(y)\right)^2$$

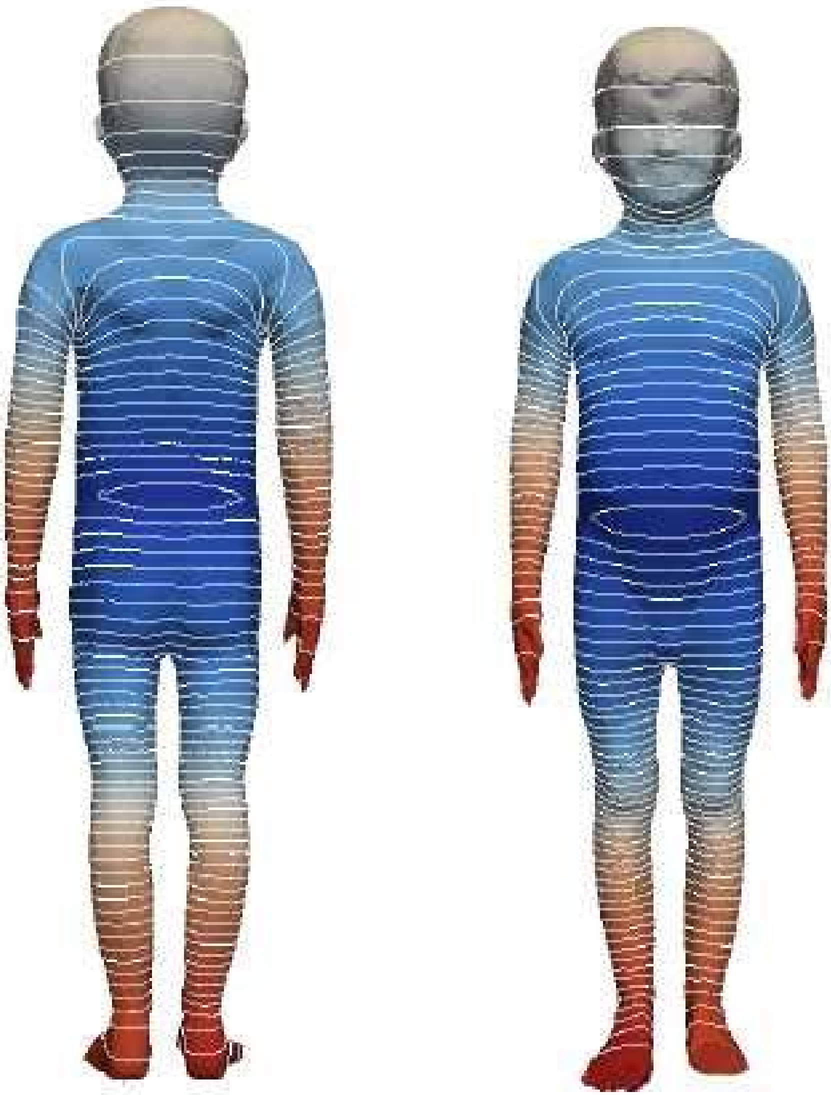
- Related to Euclidean distance in Spectral embedding
- Can be approximated for long times by first few eigenfunctionsn



(a) Diffusion ($t = 0.125$) (b) Diffusion ($t = 0.25$)



(c) Diffusion ($t = 0.5$) (d) Diffusion ($t = 1.0$)

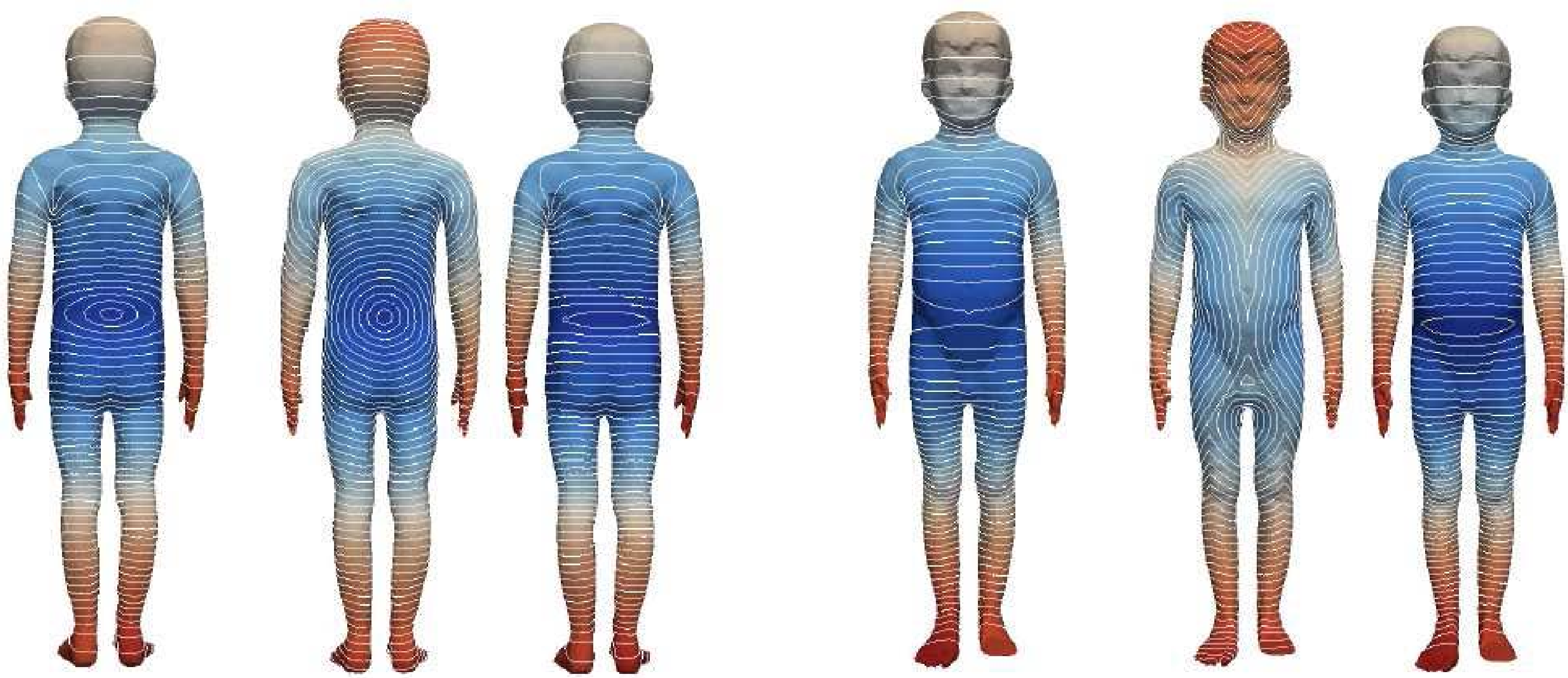


Biharmonic Distance

- Related to solution to biharmonic equations
- Can be computed by eigenanalysis of Laplacian

$$d_B(x, y)^2 = \sum_{k=1}^{\infty} \frac{(\phi_k(x) - \phi_k(y))^2}{\lambda_k^2}.$$

Distance Comparison



Biharmonic Geodesic Diffusion Biharmonic Geodesic Diffusion

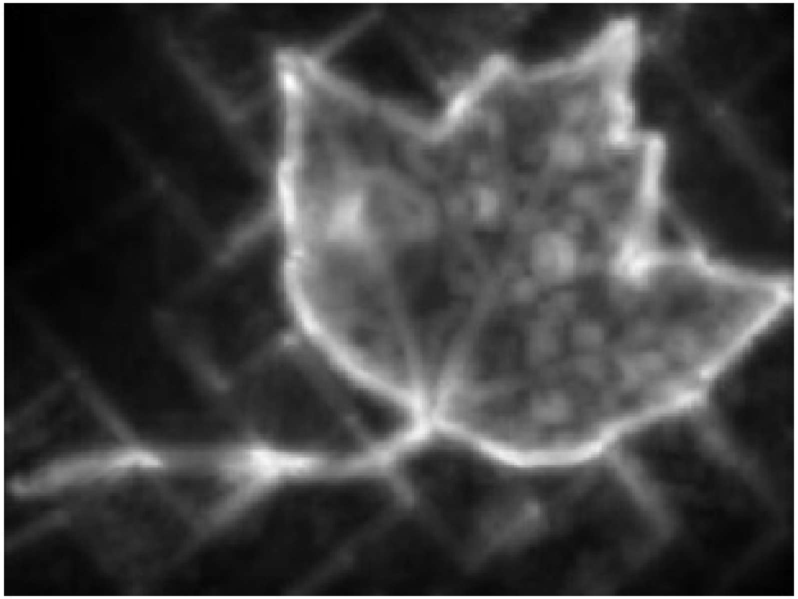
Distance Comparison

Biharmonic Geodesic Diffusion

Saliency in Images

What is saliency?

- Measure of conspicuity
- Likelihood of a location to attract attention of human



Applications

- Image classification
- Image resizing/retargeting
- Image/video compression
- Object recognition、tracking and detection
- ……

Papers

- **Itti and Koch**

  L. Itti, C. Koch, and E. Niebur. A Model of Saliency-Based Visual Attention for Rapid Scene Analysis. PAMI, pages 1254–1259, 1998

- **Harel et al**

  J. Harel, C. Koch, and P. Perona, Graph-based visual saliency, In Proc. Neural Information Processing Systems (NIPS) 19:545-552(2006)

- **Liu et al**

  T. Liu, J. Sun, N. Zheng, X. Tang, and H. Shum. Learning to Detect A Salient Object. In CVPR, 2007

- **Judd et al**

  Tilke Judd et al. learning to predict where humans look. ICCV 2009

- **Hou and Zhang**

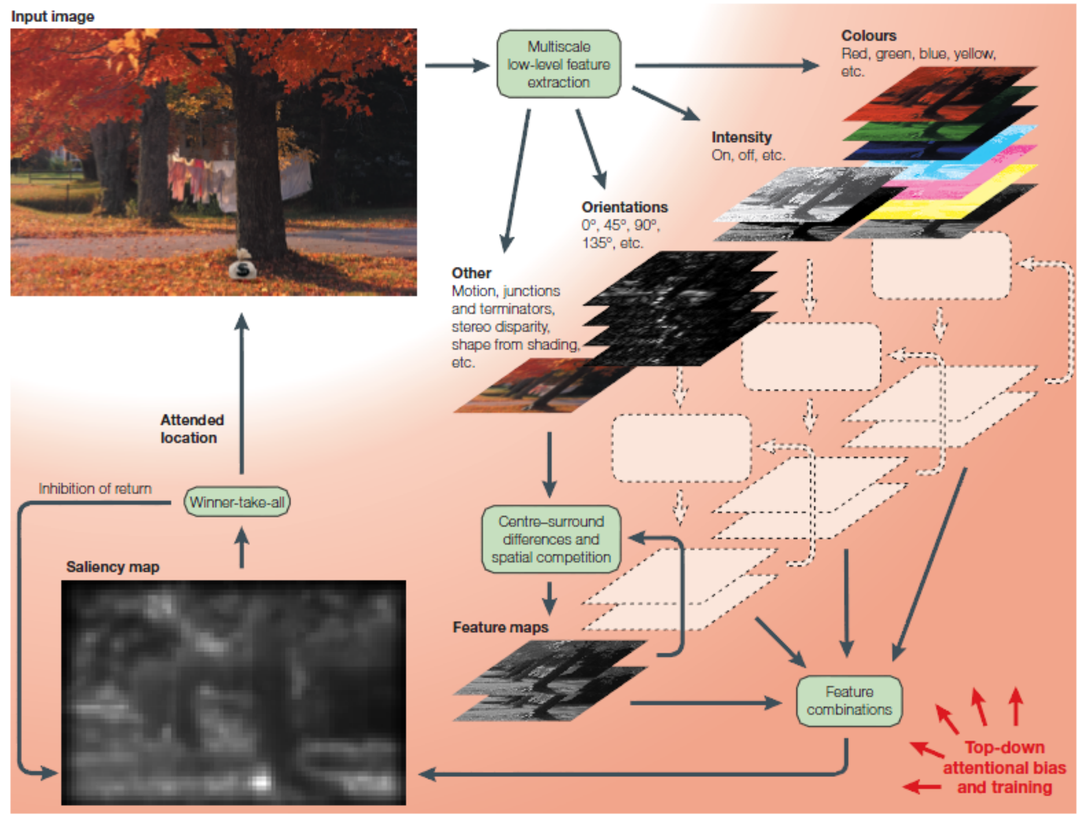
  X. Hou and L. Zhang. Saliency detection: A spectral residual approach. In CVPR, pages 1–8, 2007.

- **Goferman et al**

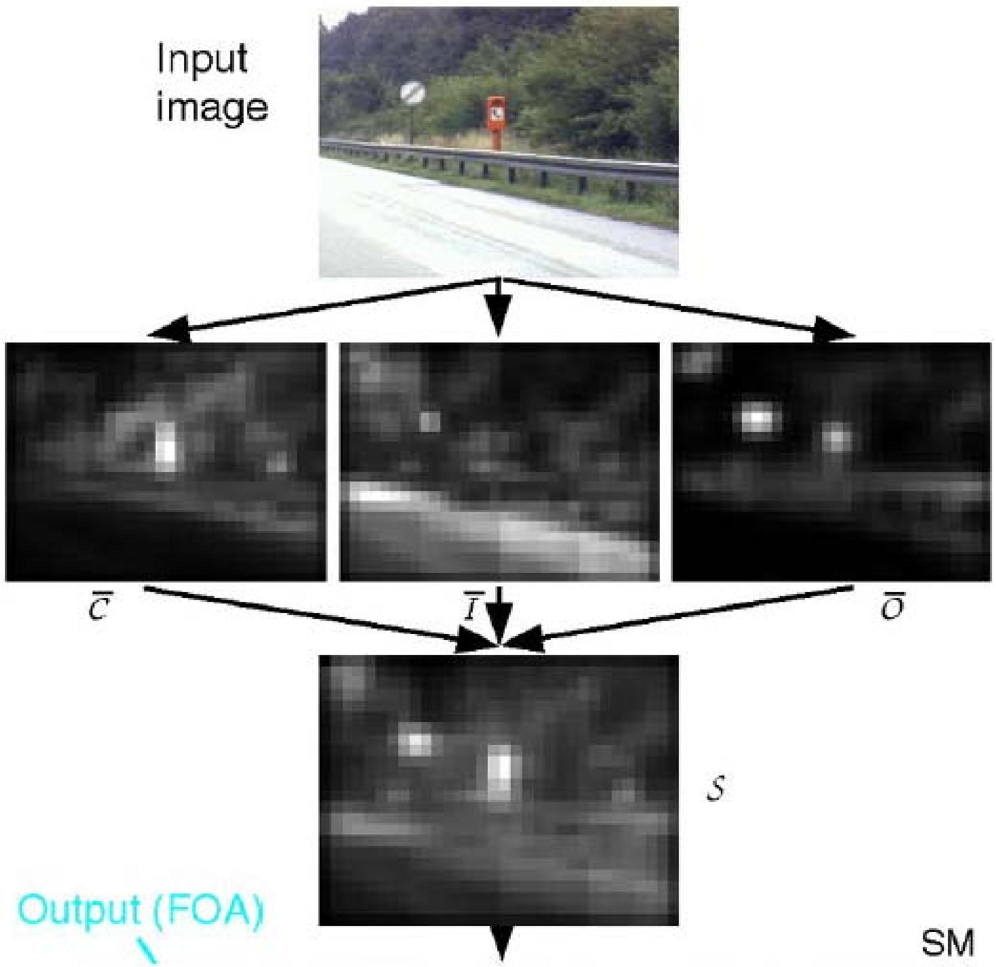
  Stas Goferman, Lihi Zelnik-Manor, and Ayellet Tal. Context-Aware Saliency Detection, CVPR 2010 (ORAL)

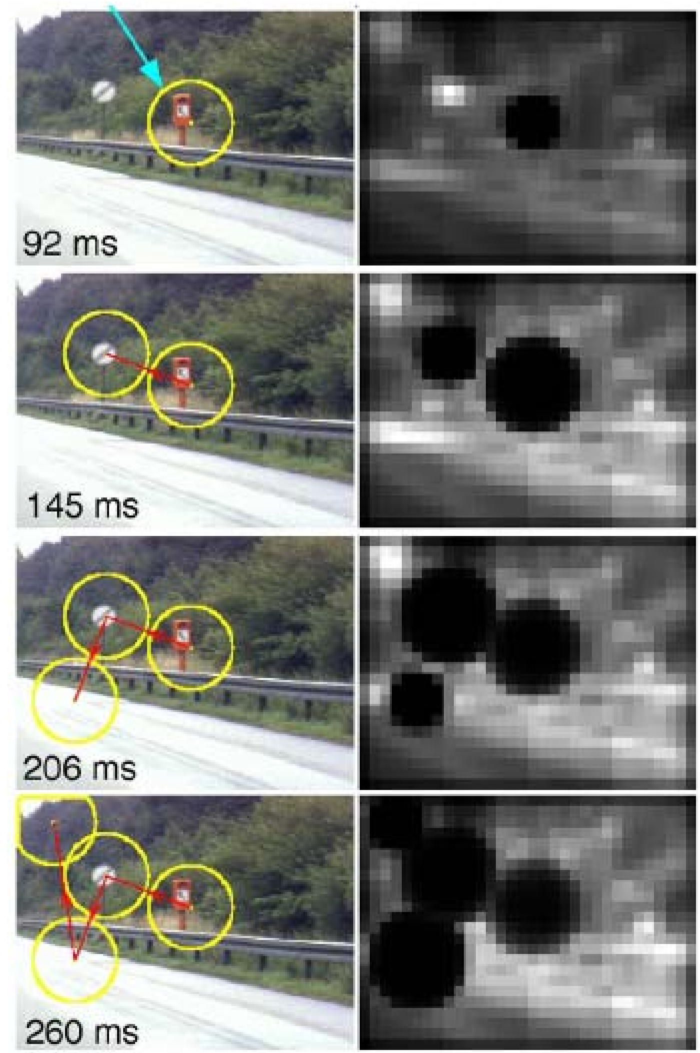
Typical model

- (S1) Extraction
- (S2) Activation
- (S3) Normalization /combination



Itti and Koch 98





Itti and Koch 98

- **S1:**
- **S2: dyadic Gaussian pyramids**

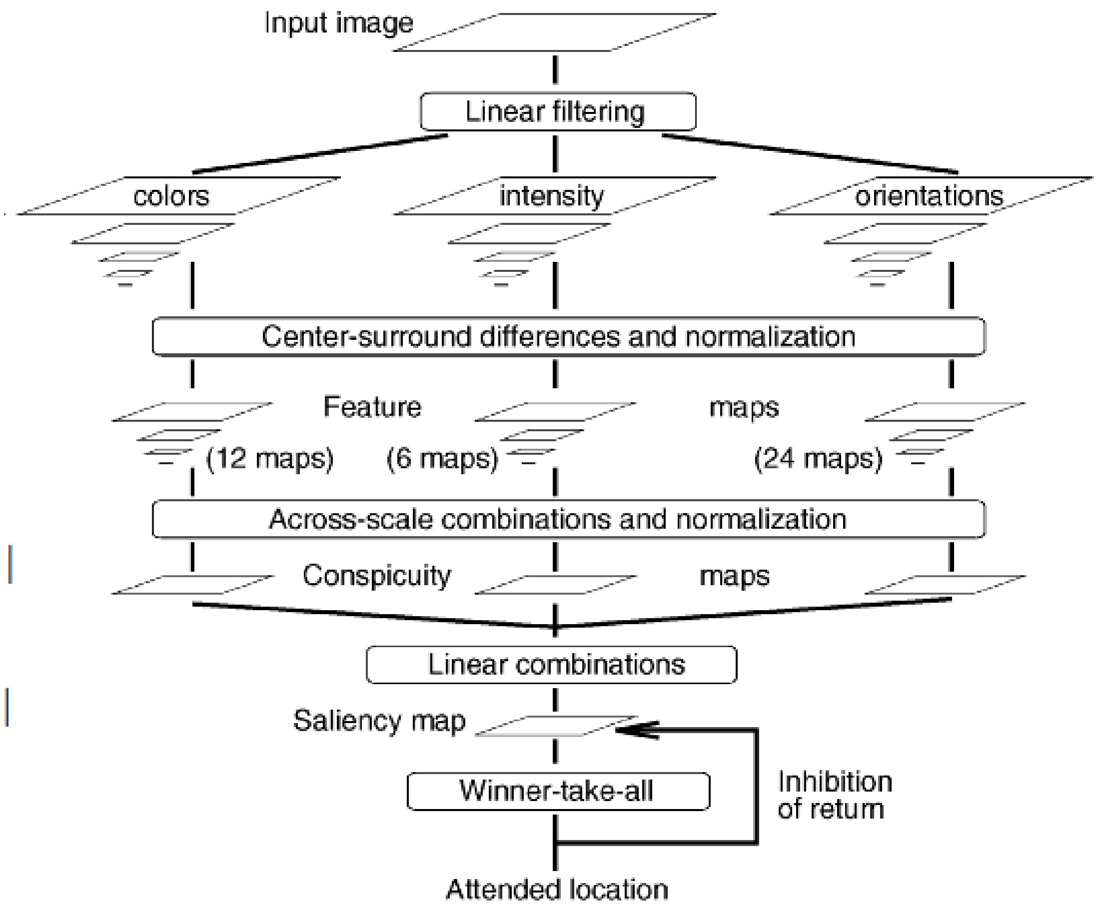
$$\mathcal{I}(c, s) = |I(c) \ominus I(s)|$$

$$\mathcal{RG}(c, s) = |(R(c) - G(c)) \ominus (G(s) - R(s))|$$

$$\mathcal{BY}(c, s) = |(B(c) - Y(c)) \ominus (Y(s) - B(s))|$$

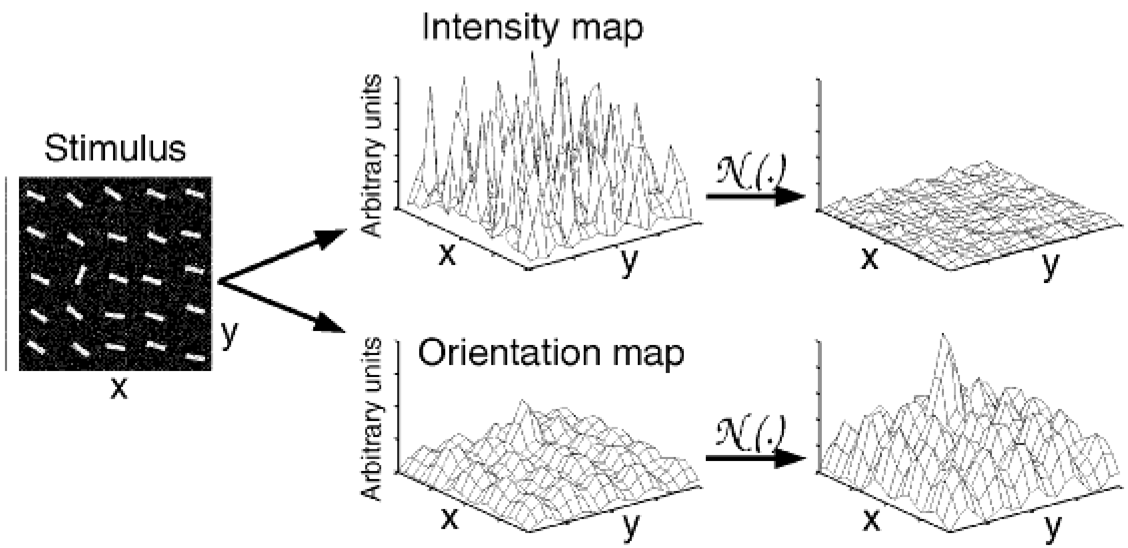
$$\mathcal{O}(c, s, \theta) = |O(c, \theta) \ominus O(s, \theta)|$$

with $c \in \{2, 3, 4\}$ and $s = c + \delta$, $\delta \in \{3, 4\}$



Itti and Koch 98

- **S3:**



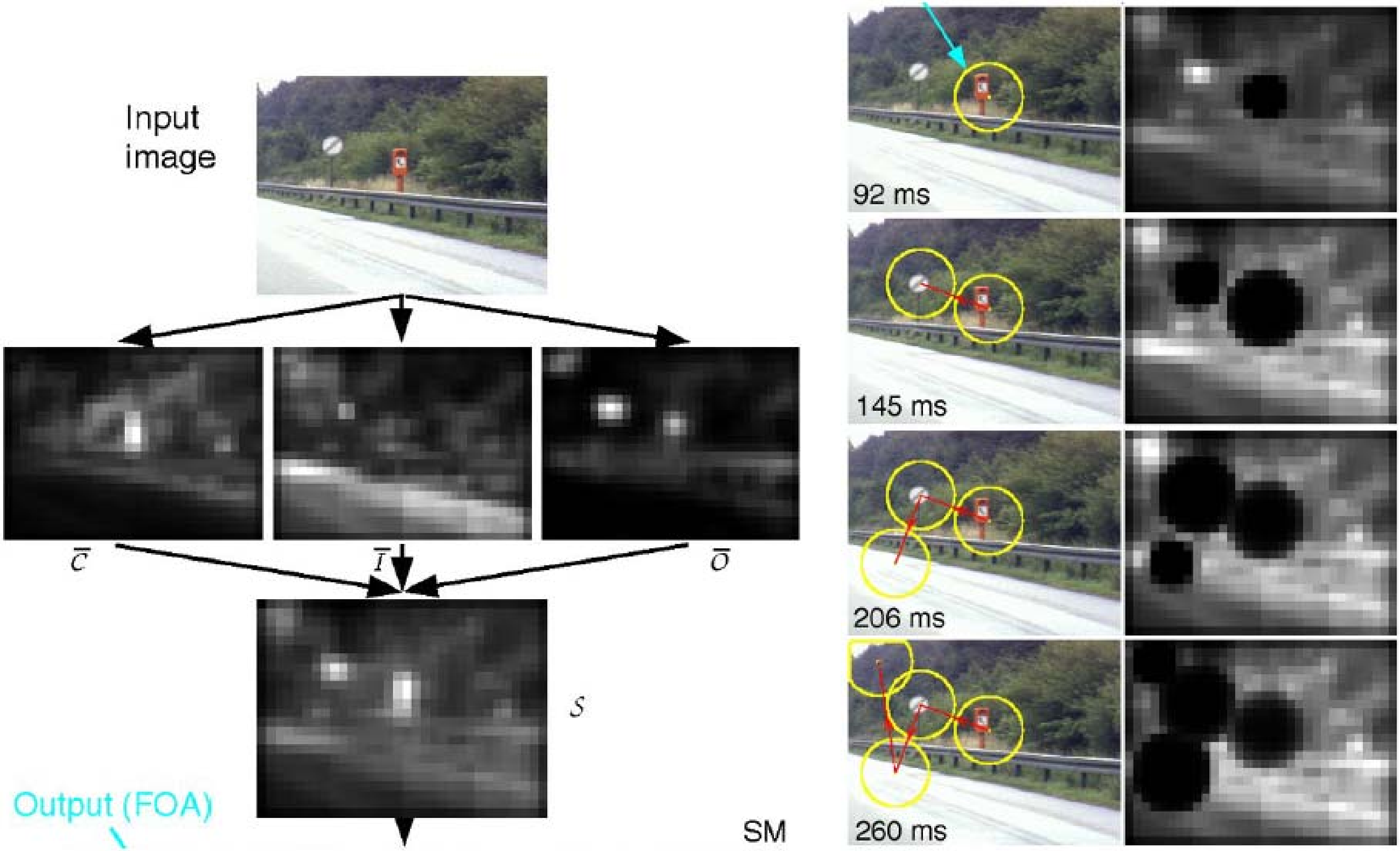
$$\bar{I} = \bigoplus_{c=2}^{4} \bigoplus_{s=c+3}^{c=4} \mathcal{N}\left(I(c,s)\right)$$

$$\bar{C} = \bigoplus_{c=2}^{4} \bigoplus_{s=c+3}^{c+4} \left[\mathcal{N}\left(\mathcal{RG}(c,s)\right) + \mathcal{N}\left(\mathcal{BY}(c,s)\right)\right]$$

$$S = \frac{1}{3}\left(\mathcal{N}(\bar{I}) + \mathcal{N}(\bar{C}) + \mathcal{N}(\bar{O})\right)$$

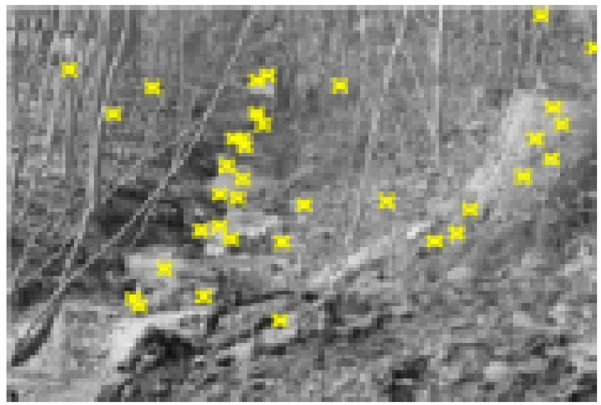
$$\bar{O} = \sum_{\theta \in \{0°, 45°, 90°, 135°\}} \mathcal{N}\left(\bigoplus_{c=2}^{4} \bigoplus_{s=c+3}^{c+4} \mathcal{N}\left(O(c,s,\theta)\right)\right)$$

Itti and Koch 98



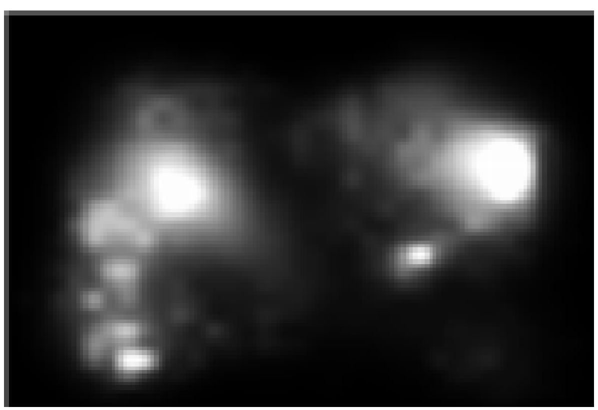
Harel et al. NIPS 2006

(a) Sample Picture With Fixation



(b) Graph-Based Saliency Map

ROC area = 0.74



(c) Traditional Saliency Map

ROC area = 0.57

Harel et al. NIPS 2006

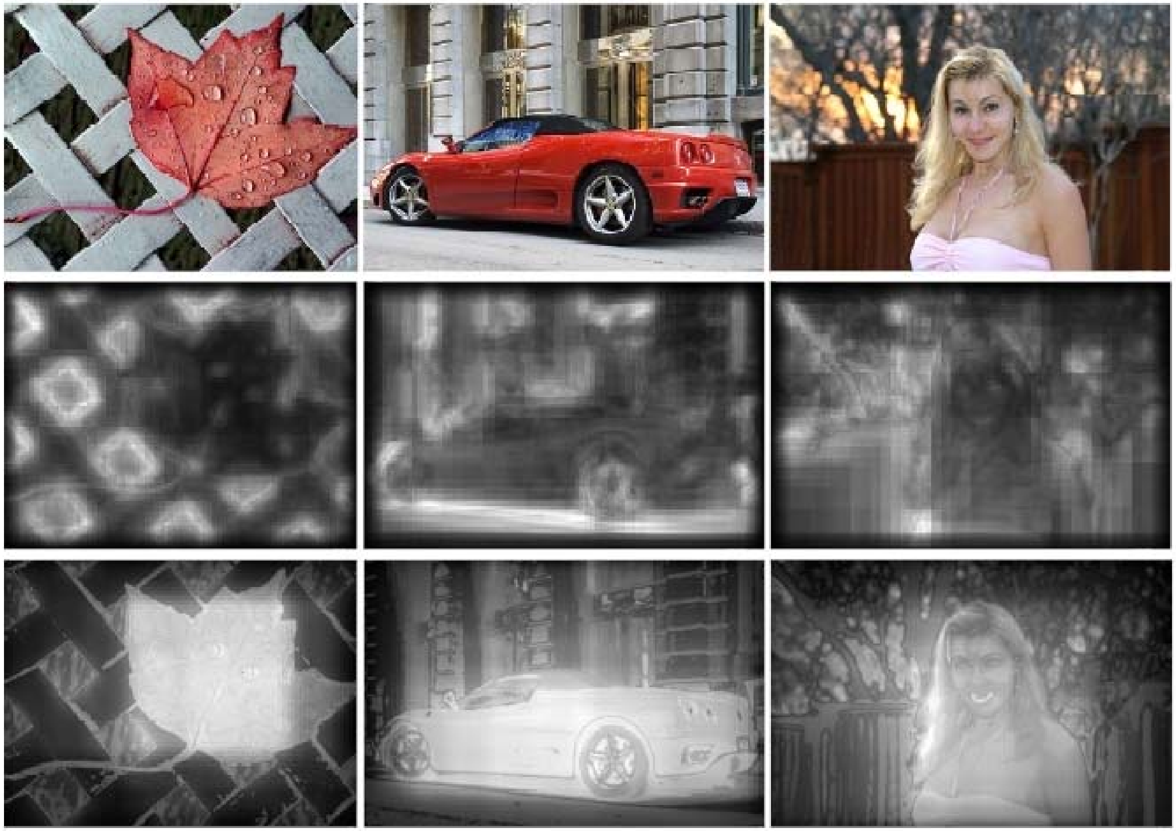
- **S1:**
- **S2:**
  - given a feature map: $M : [n]^2 \rightarrow \mathbb{R}.$
  - define the dissimilarity: $d((i,j)||(p,q)) \triangleq \left| \log \frac{M(i,j)}{M(p,q)} \right|.$
  - **Markov chain**:
    - full-connected directed graph
    - nodes = states, edges weights = transition probabilities

$$w_1((i,j),(p,q)) \quad \triangleq \quad d((i,j)||(p,q)) \cdot F(i-p, j-q), \text{ where}$$

$$F(a,b) \quad \triangleq \quad \exp\left(-\frac{a^2+b^2}{2\sigma^2}\right)$$

- **S3:** $w_2((i,j),(p,q)) \triangleq A(p,q) \cdot F(i-p, j-q)$

Liu et al. CVPR 07



(a) (b) (c)

Figure 1. Salient map. From top to bottom: input image, salient map computed by Itti's algorithm (http://www.saliencytoolbox.net), and salient map computed by our approach.

Liu et al. CVPR 07

- Incorporate the high level concept of **salient object**
- Formulate salient object detection as **a binary labeling problem**
- A large database: 20,000+ well labeled images--**top-down information**
- **Local, regional, global** features
- **Condition Random Field** (CRF) learning

Liu et al. CVPR 07

- the binary mask labeled by the *m*th user:

$$A^m = \{a_x^m\}$$

- the saliency probability map:

$$G = \{g_x | g_x \in [0, 1]\}$$

$$g_x = \frac{1}{M} \sum_{m=1}^{M} a_x^m$$

- **Consistency:**

$$C_t = \frac{\sum_{x \in \{g_x > t\}} g_x}{\sum_x g_x}$$

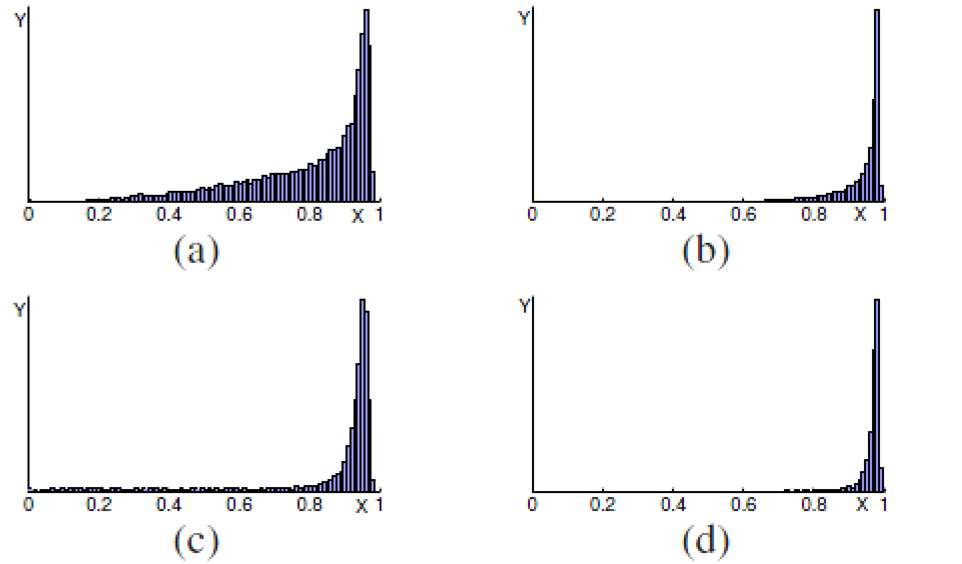


Figure 4. Labeling consistency. (a) (b) $C_{0.9}$ (agreed by all 3 users) and $C_{0.5}$ on image set $\mathcal{A}$. (c) (d) $C_{0.9}$ (agreed by at least 8 of 9 users) and $C_{0.5}$ on image set $\mathcal{B}$.

Liu et al. CVPR 07

- **Evaluation:** $$F_\alpha = \frac{(1+\alpha)\times Precision \times Recall}{\alpha \times Precision + Recall}$$
  - $$Precision = \sum_x g_x a_x / \sum_x a_x$$
  - $$Recall = \sum_x g_x a_x / \sum_x g_x$$
- **CRF:**
  - conditional distribution: $P(A|I) = \frac{1}{Z}\exp(-E(A|I))$
  - energy: $$E(A|I) = \sum_x \sum_{k=1}^{K} \lambda_k F_k(a_x, I) + \sum_{x,x'} S(a_x, a_{x'}, I)$$

$$F_k(a_x, I) = \begin{cases} f_k(x, I) & a_x = 0 \\ 1 - f_k(x, I) & a_x = 1 \end{cases} \qquad S(a_x, a_{x'}, I) = |a_x - a_{x'}| \cdot \exp(-\beta d_{x,x'})$$

  - optimal : $$\overrightarrow{\lambda}^* = \arg\max_{\overrightarrow{\lambda}} \sum_n \log P(A^n | I^n; \overrightarrow{\lambda})$$

Liu et al. CVPR 07

- **Salient object feature**:
  - (**local**) Multi-scale contract: $f_c(x, I) = \sum_{l=1}^{L} \sum_{x' \in N(x)} ||I^l(x) - I^l(x')||^2$

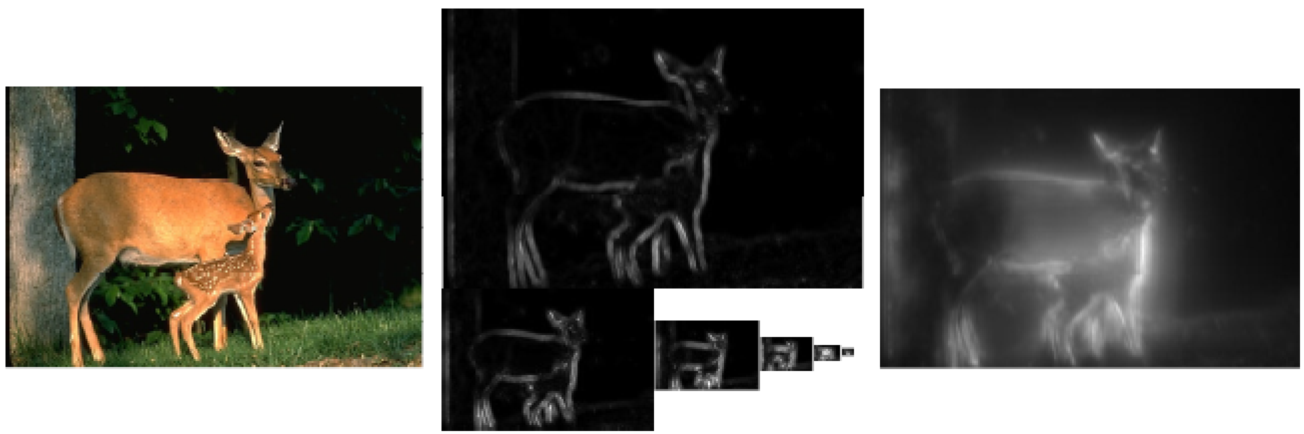


Figure 5. Multi-scale contrast. From left to right: input image, contrast maps at multiple scales, and the feature map from linearly combining the contrasts at multiple scales.

Liu et al. CVPR 07

- **Salient object feature**:
  - (**regional**) Center-surround histogram:

$$\chi^2(R, R_S) = \frac{1}{2}\sum \frac{(R^i - R_S^i)^2}{R^i + R_S^i}$$

$$R^*(x) = \arg\max_{R(x)} \chi^2(R(x), R_S(x))$$

$$f_h(x, I) \propto \sum_{\{x' | x \in R^*(x')\}} w_{xx'} \chi^2(R^*(x'), R_S^*(x'))$$

Liu et al. CVPR 07

- **Salient object feature**:
  - **(global)** Color spatial-distribution:

Gaussian Mixture Models (GMMs) $\{w_c, \mu_c, \Sigma_c\}_{c=1}^{C}$

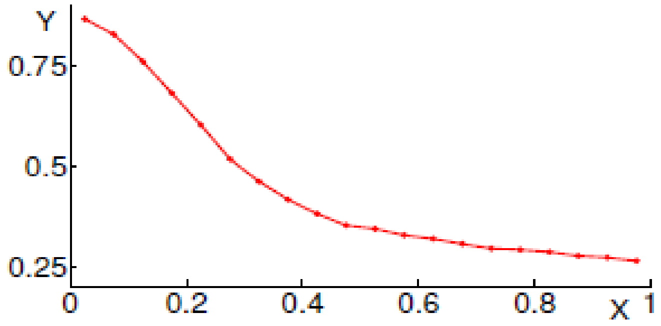


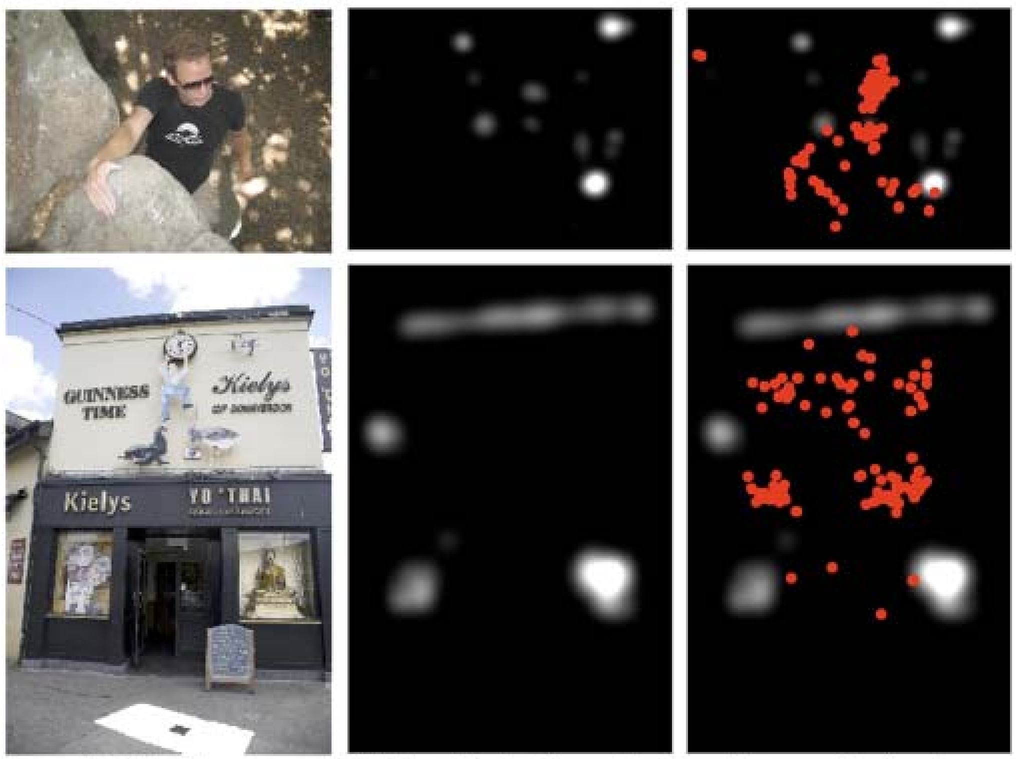
Figure 9. Color spatial variance (x-coordinate) v.s. average saliency probability (y-coordinate) on the image set $\mathcal{A}$. The saliency probability is computed from the "ground truth" labeling.

Liu et al. CVPR 07

Figure 14. Comparison of different algorithms. From left to right: FG, SM, our approach, and ground-truth.

Judd et al. ICCV 09

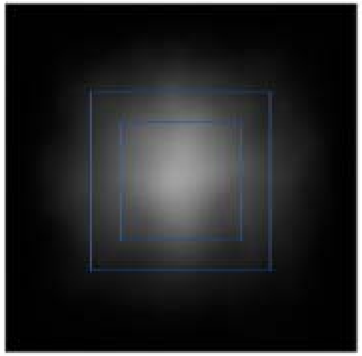
- Eyes tracking data of 15 viewers on 1003 images



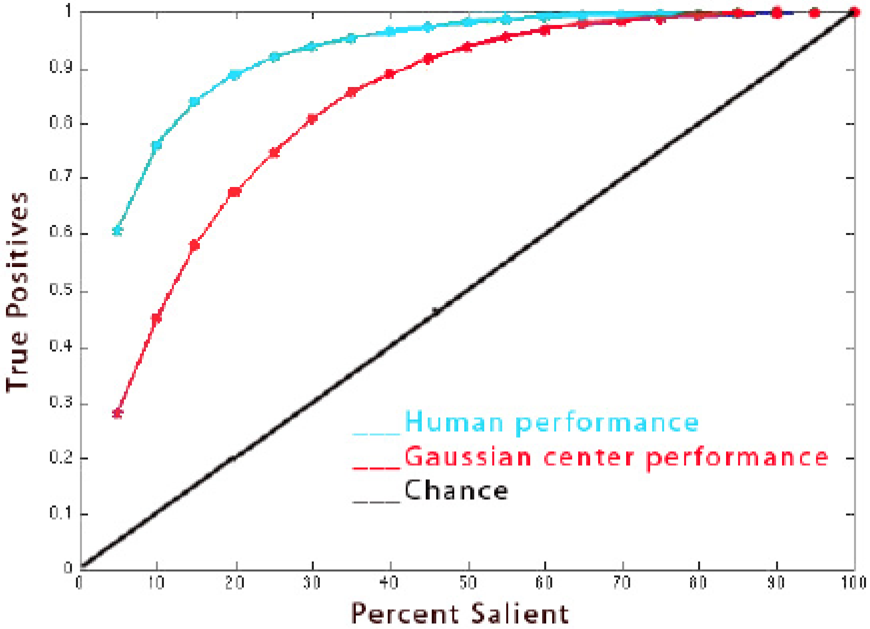
(a) Original image (b) Itti and Koch Saliency Map (c) eye tracking locations

Judd et al. ICCV 09

- **Central bias**:



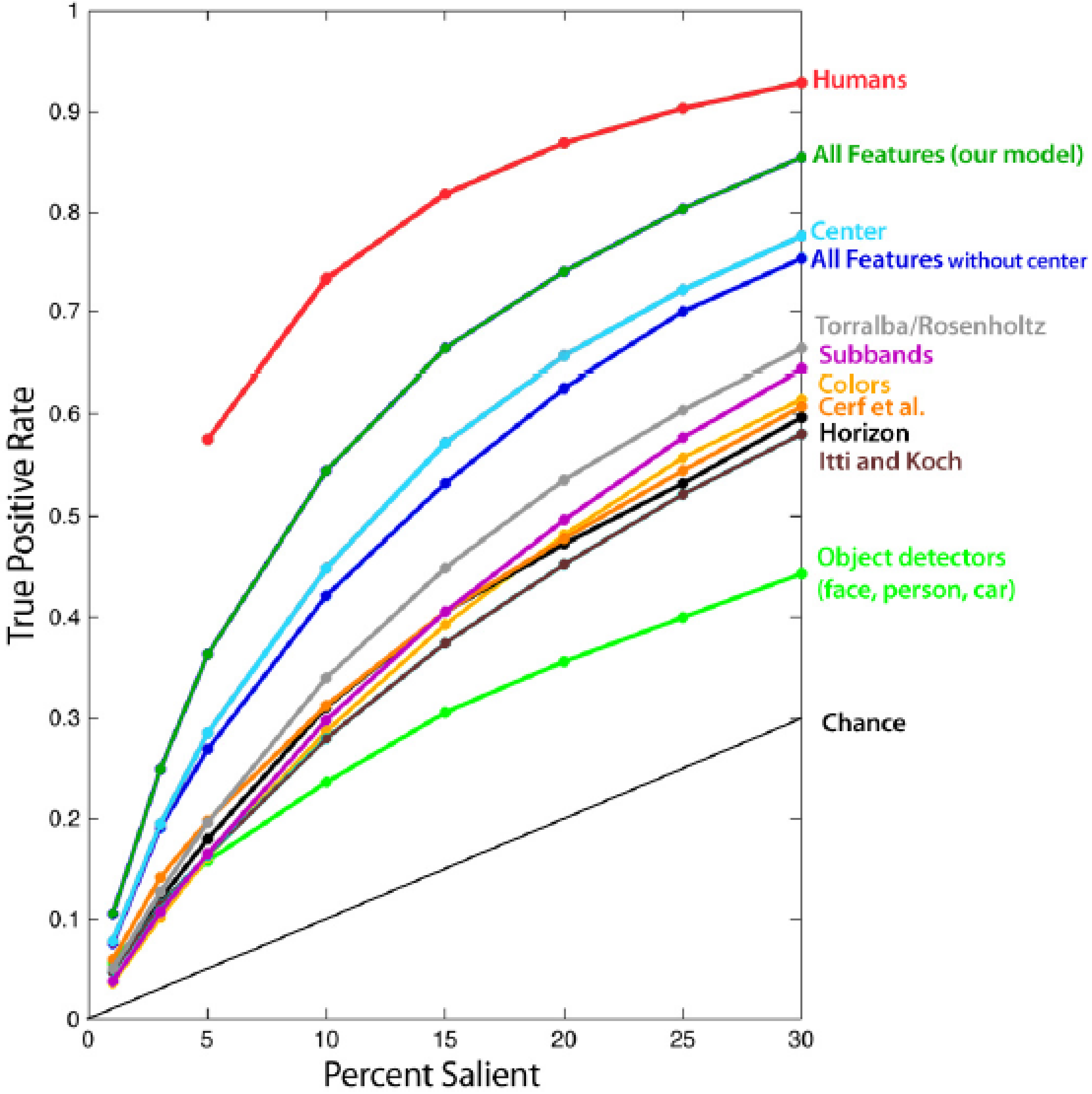
Avg of all saliency maps



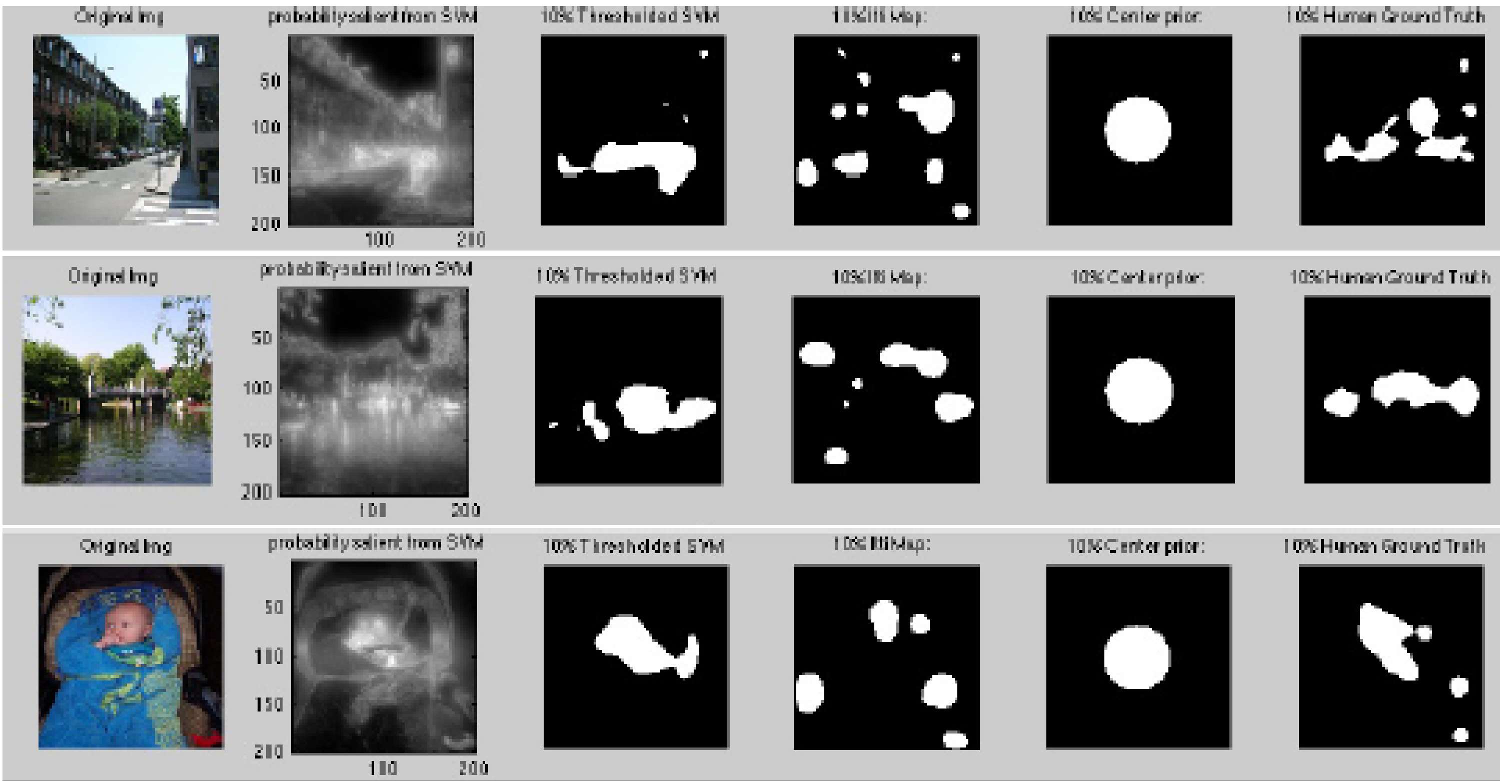
Judd et al. ICCV 09

- **Features used for machine learning**
  - **Low – level:** intensity, orientation, color contrast …
  - **Mid – level:** horizon line
  - **High – level:** face detector, person detector
  - **Center – prior:** the distance to the center for each pixel

Judd et al. ICCV 09



Judd et al. ICCV 09



Original Image | Saliency Map from SVM | 10% SVM map | 10% Itti Map | 10% Center | 10% Human Saliency Map

Judd et al. ICCV 09

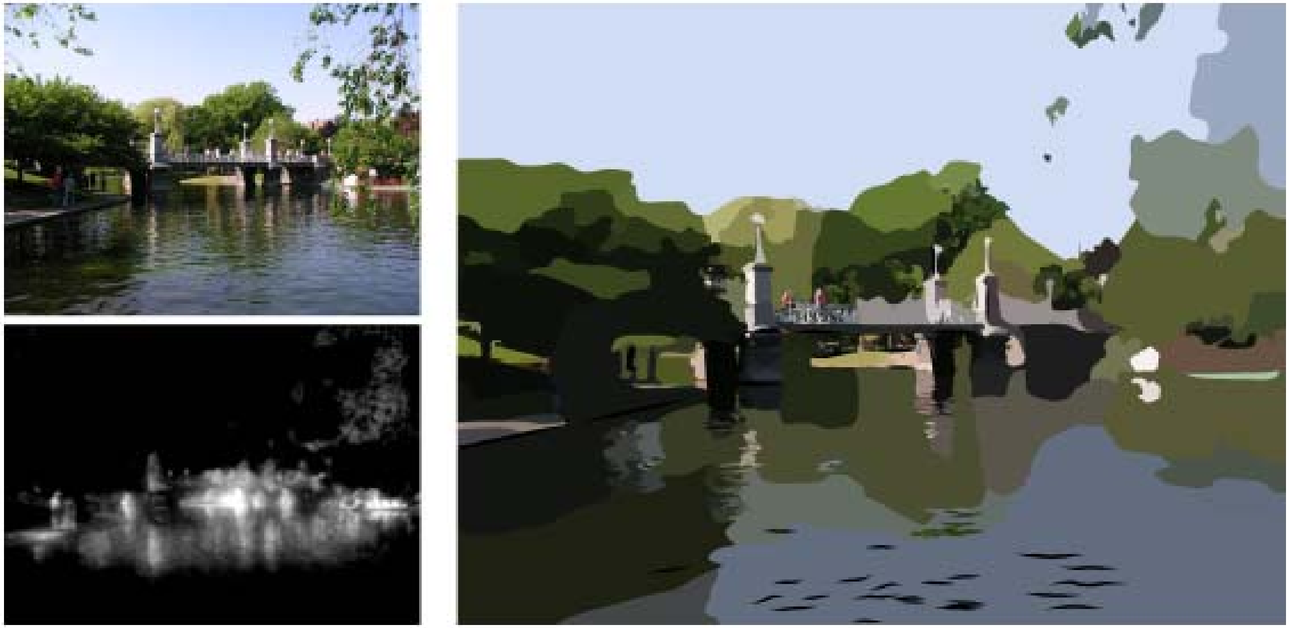
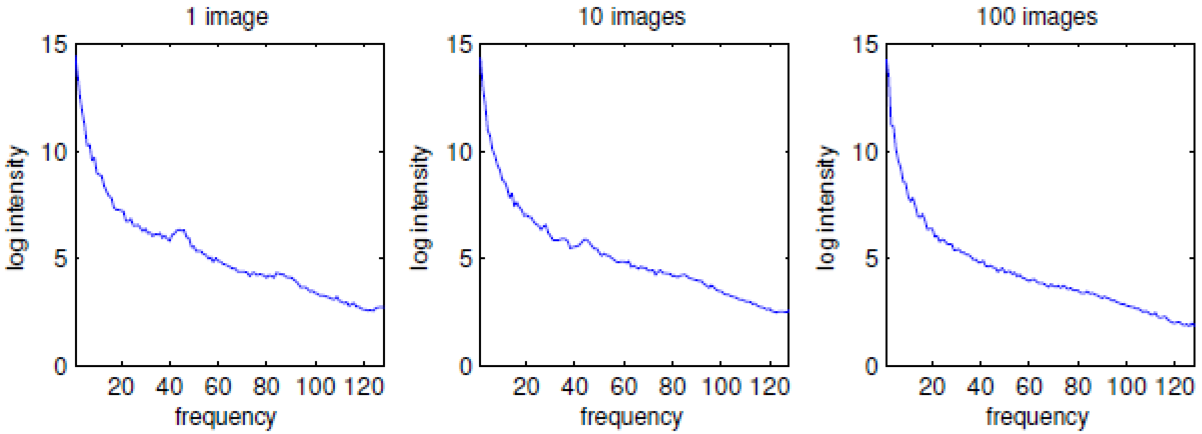


Figure 12. **Stylization and abstraction of photographs** *DeCarlo and Santella [4] use eye tracking data to decide how to render a photograph with differing levels of detail. We replicate this application without the need for eye tracking hardware.*

Hou and Zhang. CVPR 07

Hou and Zhang. CVPR 07

- Independent of feature
- Analyze the log-spectrum of an input image



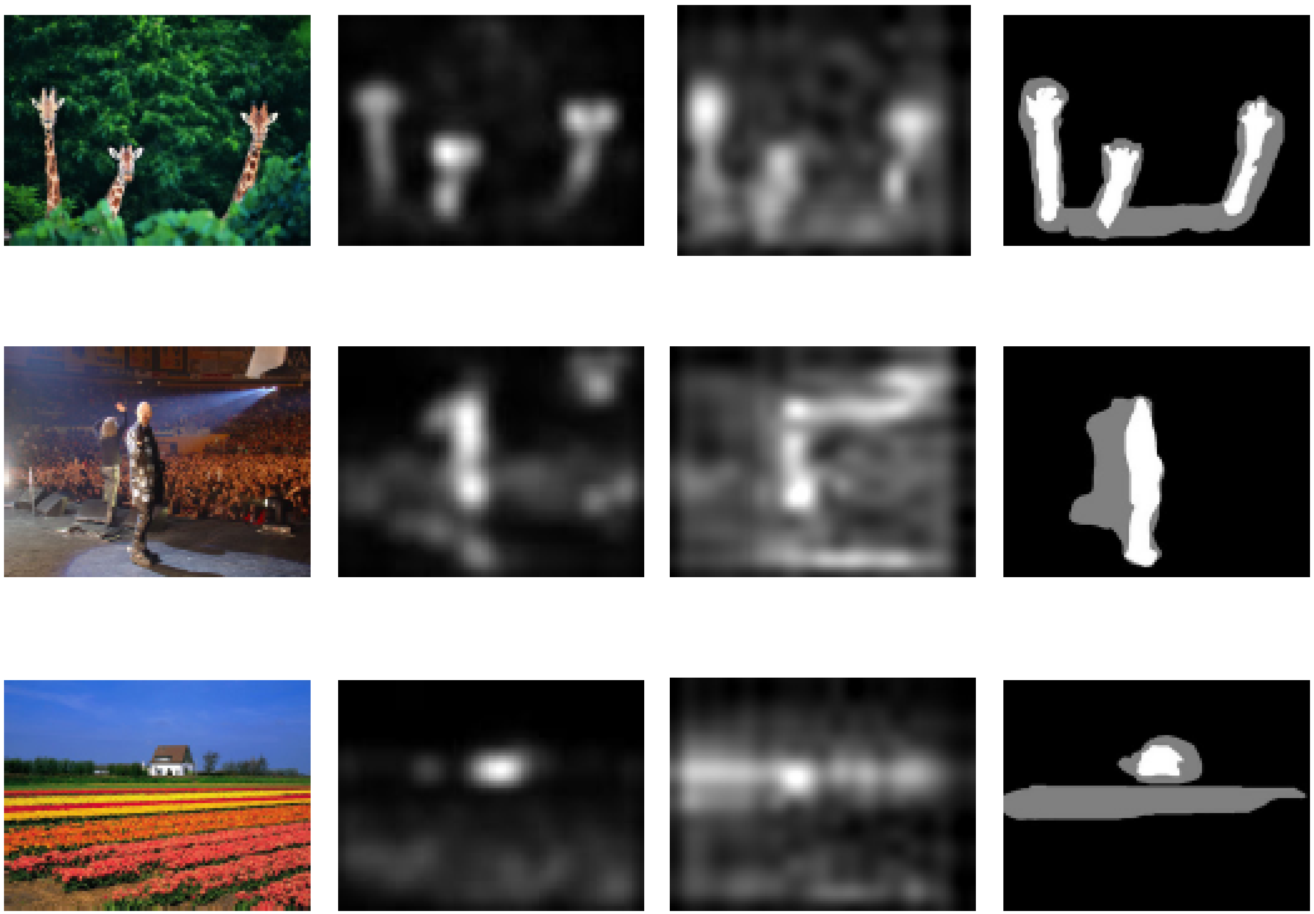
Similarities imply redundancies

Hou and Zhang. CVPR 07

- Image information: $H(\text{Image}) = H(\text{Innovation}) + H(\text{Prior Knowledge})$
- spectral residual: $\mathcal{R}(f) = \mathcal{L}(f) - \mathcal{A}(f)$
- given an image $\mathcal{I}(x)$

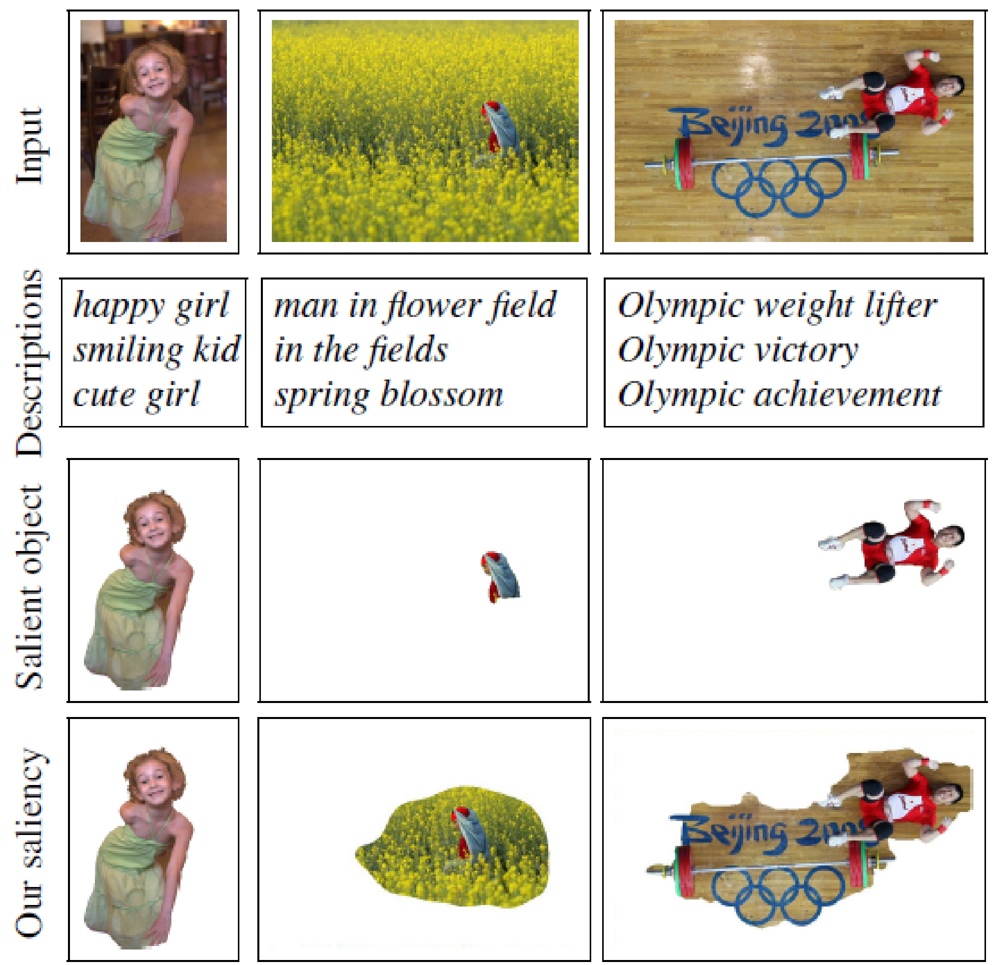
$$
\begin{aligned}
\mathcal{A}(f) &= \Re\Big(\mathfrak{F}\big[\mathcal{I}(x)\big]\Big), \\
\mathcal{P}(f) &= \Im\Big(\mathfrak{F}\big[\mathcal{I}(x)\big]\Big), \\
\mathcal{L}(f) &= \log\big(\mathcal{A}(f)\big), \\
\mathcal{R}(f) &= \mathcal{L}(f) - h_n(f) * \mathcal{L}(f), \\
\mathcal{S}(x) &= g(x) * \mathfrak{F}^{-1}\Big[\exp\big(\mathcal{R}(f) + \mathcal{P}(f)\big)\Big]^2
\end{aligned}
$$

Hou and Zhang. CVPR 07



Goferman et al. CVPR 2010

- Previous: identify **fixation points** or detect the **dominant object**
- New type of saliency -- -- detect **Image region representing the scene**



Goferman et al. CVPR 2010

- **Principles**:

  1. **Local low-level** consideration: contrast and color
  = areas that have distinctive colors or patterns should obtain high saliency

  2. **Global** considerations: suppress frequently-occurring features

  3. **Visual organization rules**: possess one or several centers of gravity
  = the salient pixels should be grouped together

  4. **High-level** factors: human faces

- **Challenges:**
  - How to define the distinctiveness both locally and globally?
  - How to incorporate positional information?

Goferman et al. CVPR 2010

- Local-global single-scale saliency:

$$S_i^r = 1 - exp\{-\frac{1}{K}\sum_{k=1}^{K} d(p_i^r, q_k^{r_k})\} \qquad d(p_i, p_j) = \frac{d_{color}(p_i, p_j)}{1 + c \cdot d_{position}(p_i, p_j)}$$

- Multi-scale saliency enhancement:

$$\bar{S}_i = \frac{1}{M}\sum_{r \in R} S_i^r \qquad S_i^r = 1 - exp\{-\frac{1}{K}\sum_{k=1}^{K} d(p_i^r, q_k^{r_k})\} \qquad R_q = \{r, \frac{1}{2}r, \frac{1}{4}r\}$$

- Including the immediate context:

$$\hat{S}_i = \bar{S}_i(1 - d_{foci}(i))$$

Let $d_{foci}(i)$ be the Euclidean positional distance between pixel i and the closest focus of attention pixel

- High-level factors

Goferman et al. CVPR 2010

- **Image retargeting:**

Input | Saliency of [19] | Our saliency | Results of [19] | Our result

Goferman et al. CVPR 2010

- **Summarization through collage creation:**

(a) The collage summarization

Mesh Saliency

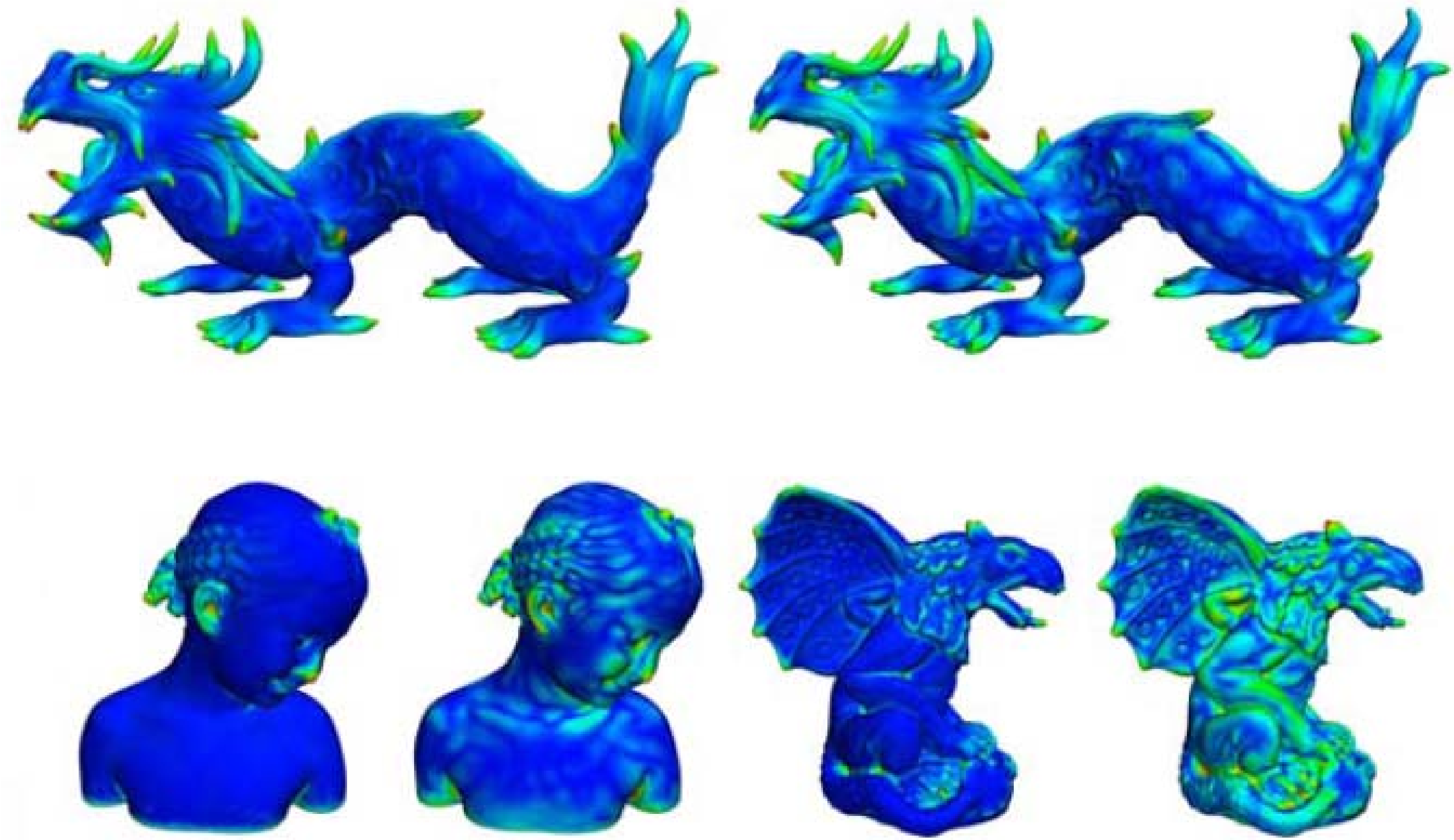
[Lee et al. Siggraph 2005]

- Motivated by models of perceptual salience
- Difference between mean curvature blurred with σ and blurred with 2σ

Mesh Saliency with Global Rarity

With global rarity [Lee et al. 2005]

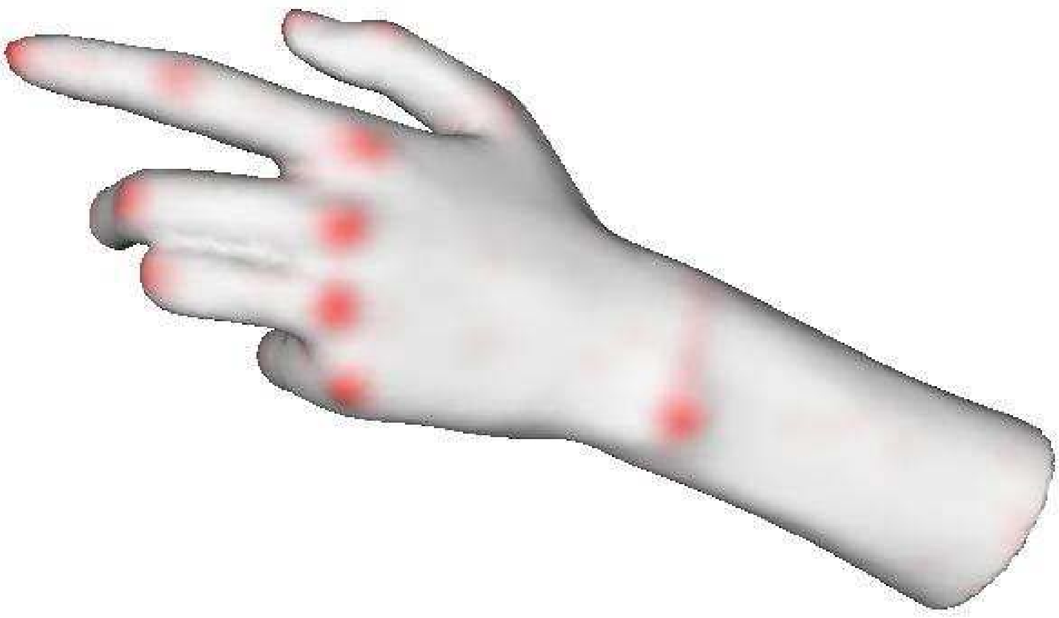
Comparisons

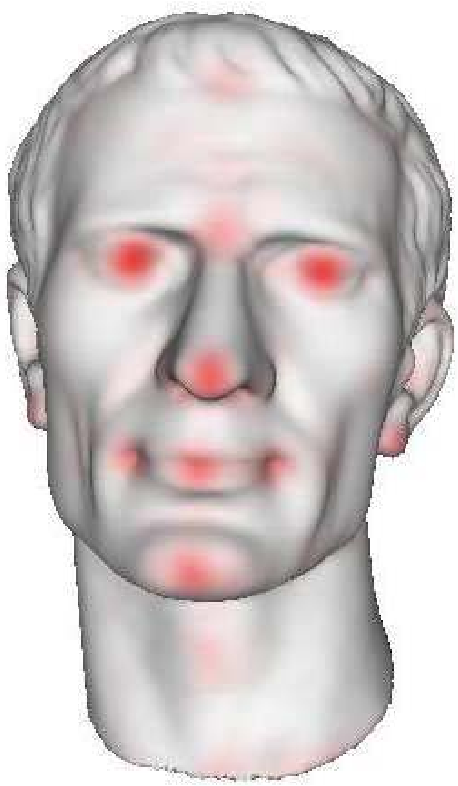


Feature Points

Feature Points

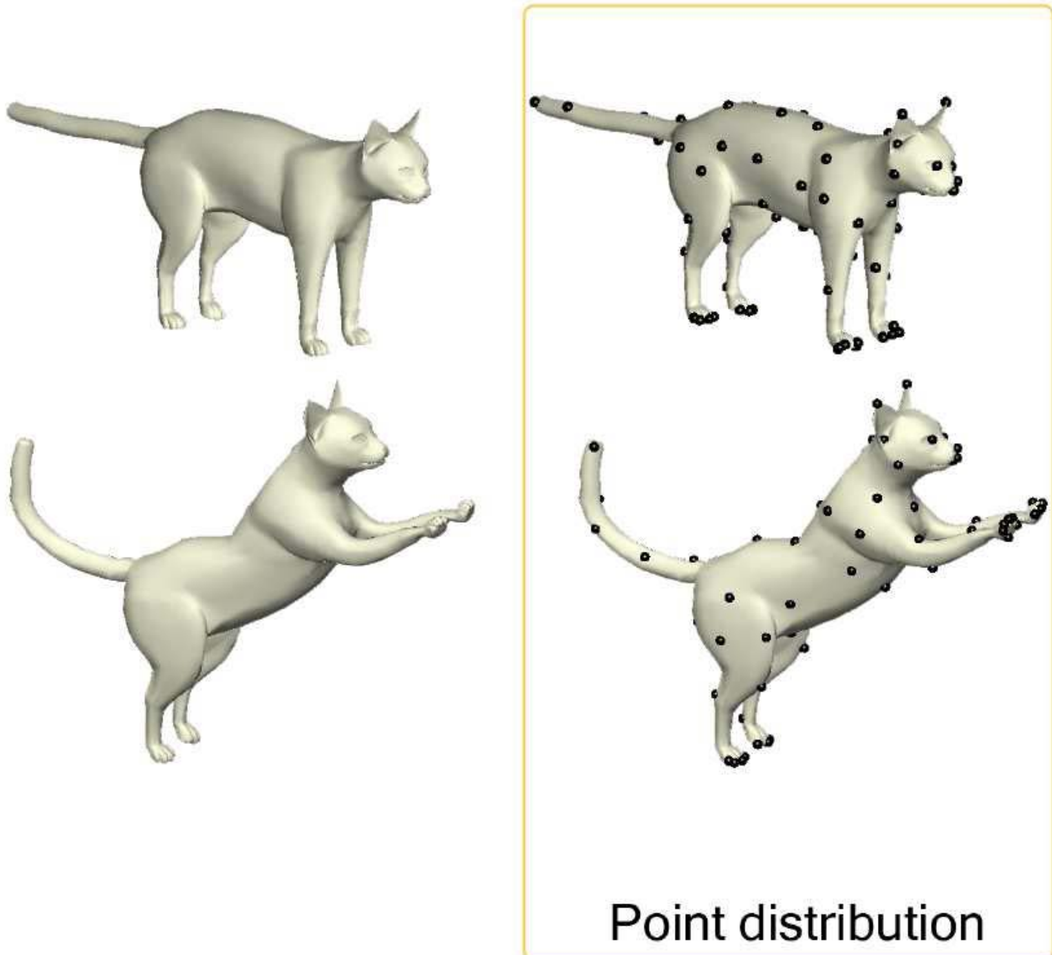
- A prominent part or characteristic





Point Feature Detection

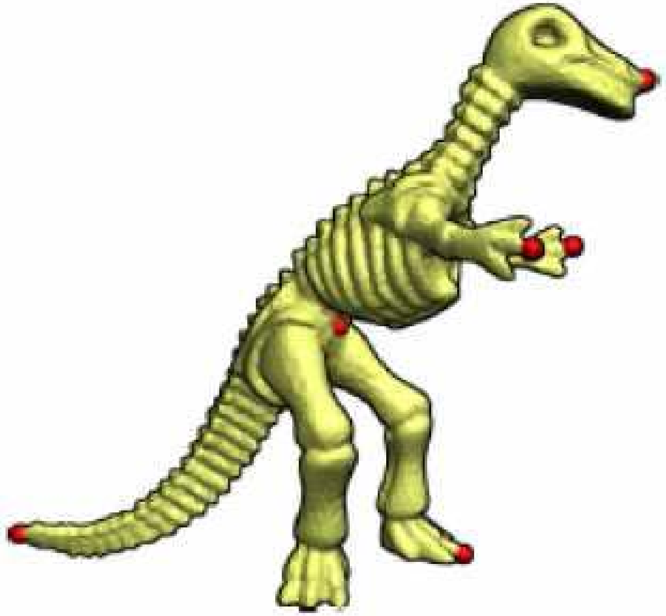
- Algorithmic methods
  - Iteratively choose furthest point
  - Others?

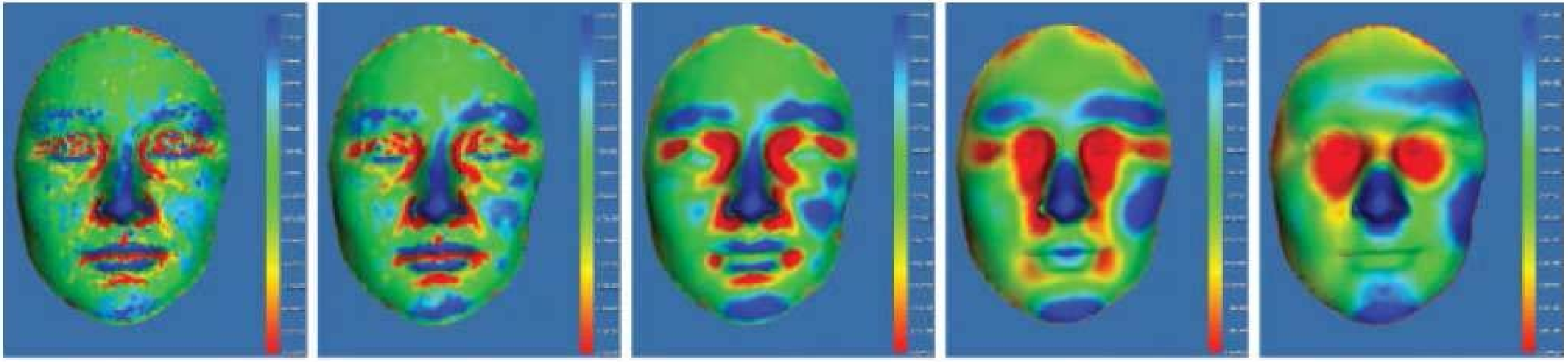


Point distribution

Point Feature Detection

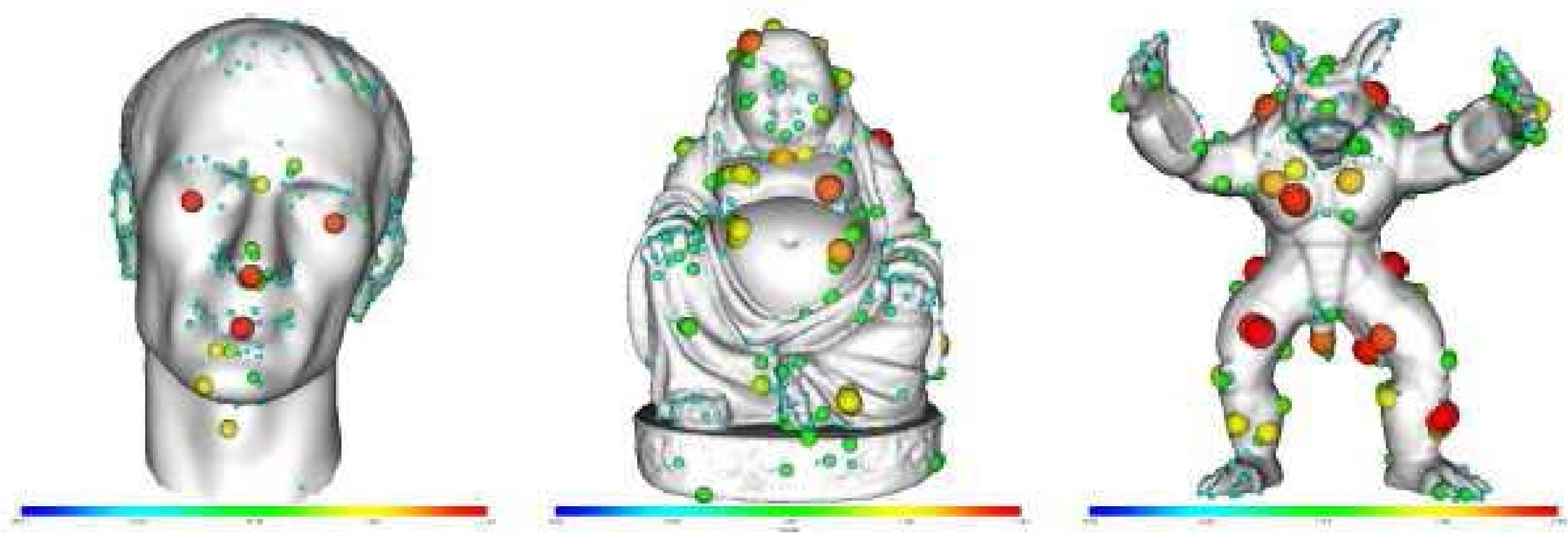
- Analytic methods
  - Extrema of DoG of curvature
  - Extrema of Gauss curvature
  - Extrema of HKS
  - Extrema of AGD
  - etc.





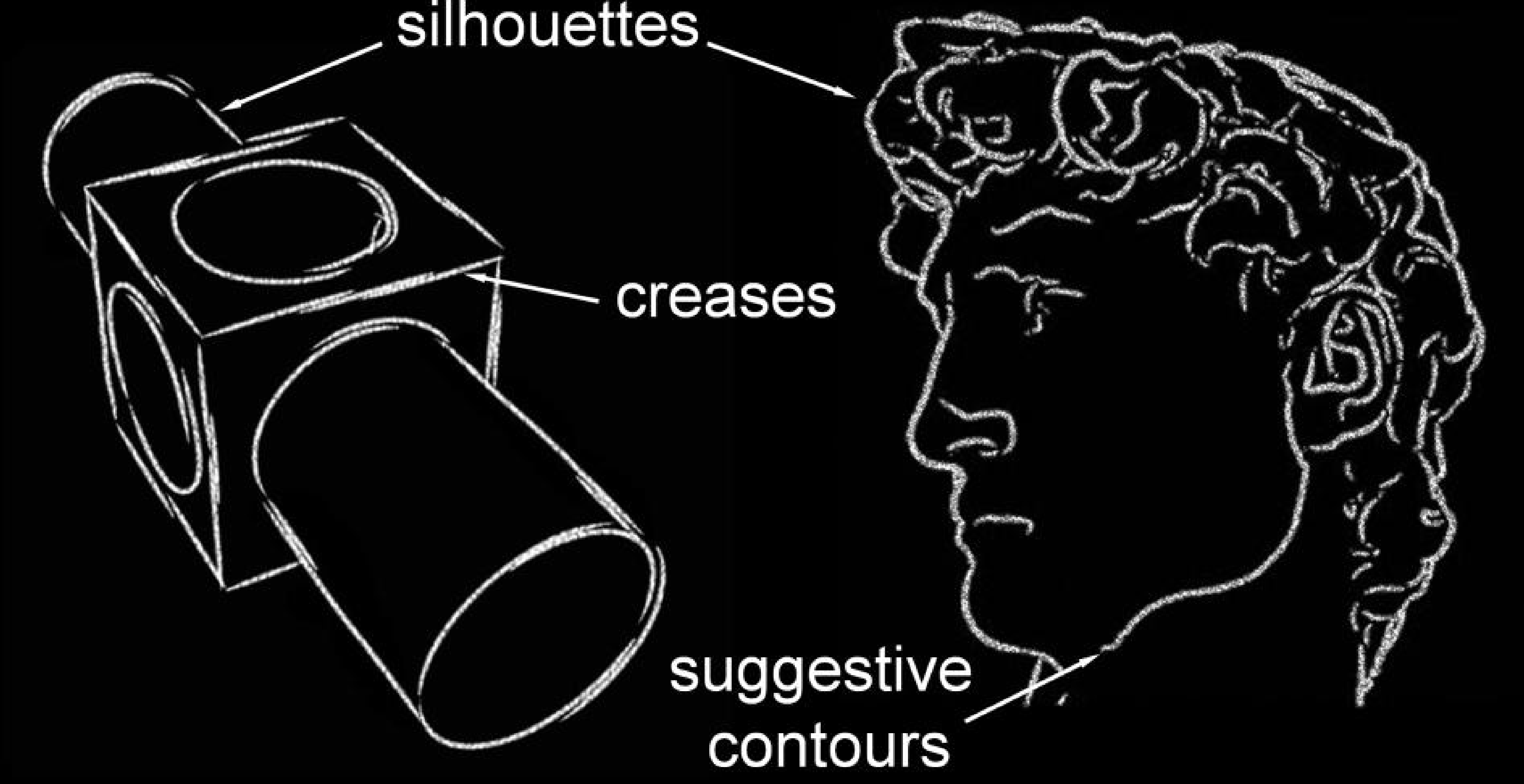
Point Feature Detection

- Still difficult to detect "semantic points"



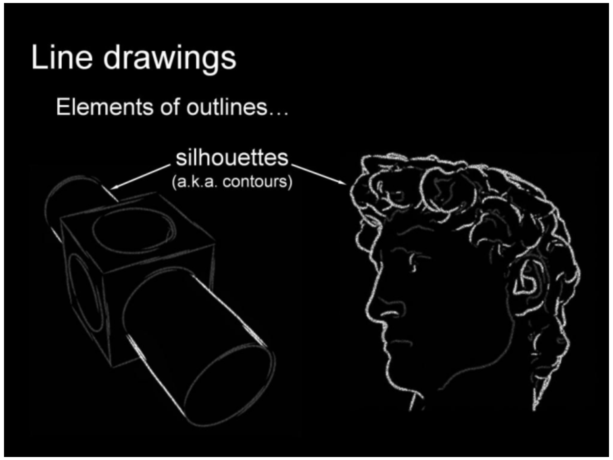
Feature Lines

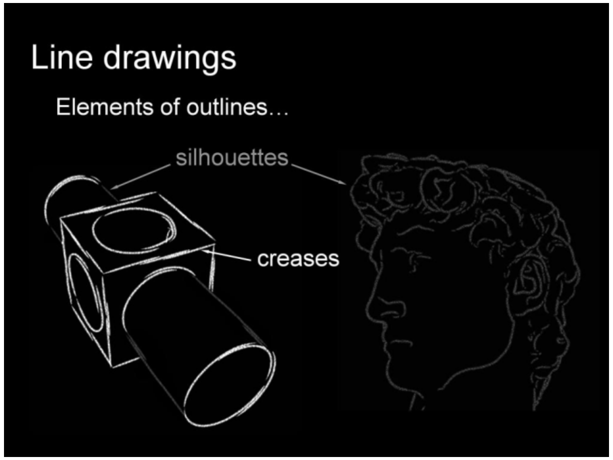
Line drawings

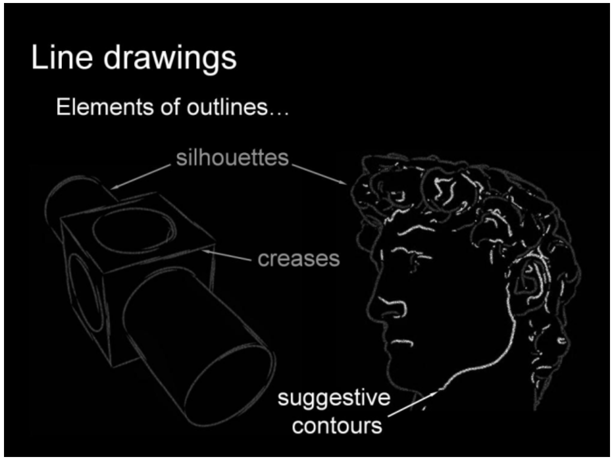


Main composition

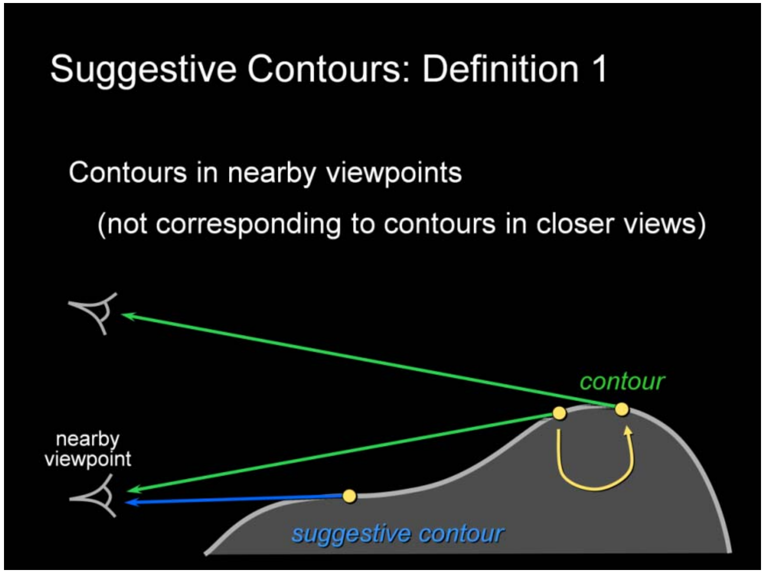
- **Silhouettes**
  - The boundary between the object and the background
- **Creases**
  - normal discontinuity, dihedral angle smaller than a threshold
- **Suggestive Contours**
  - Places that would be silhouettes from nearby views

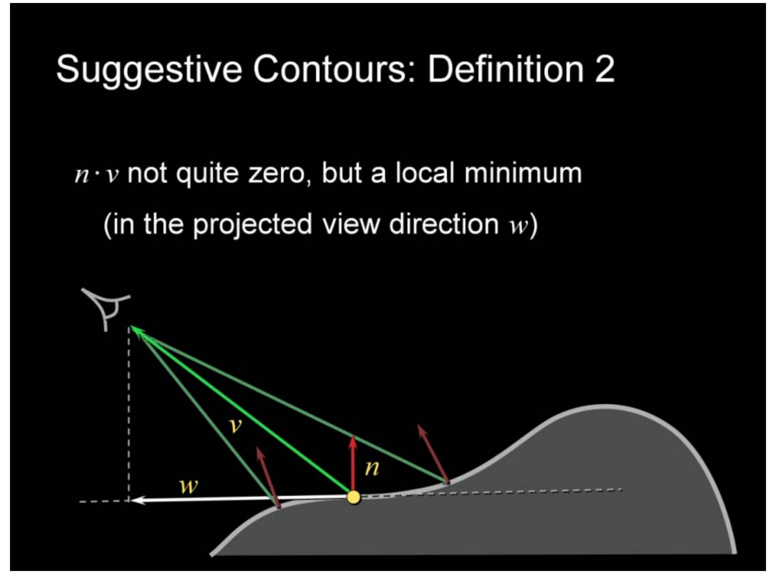


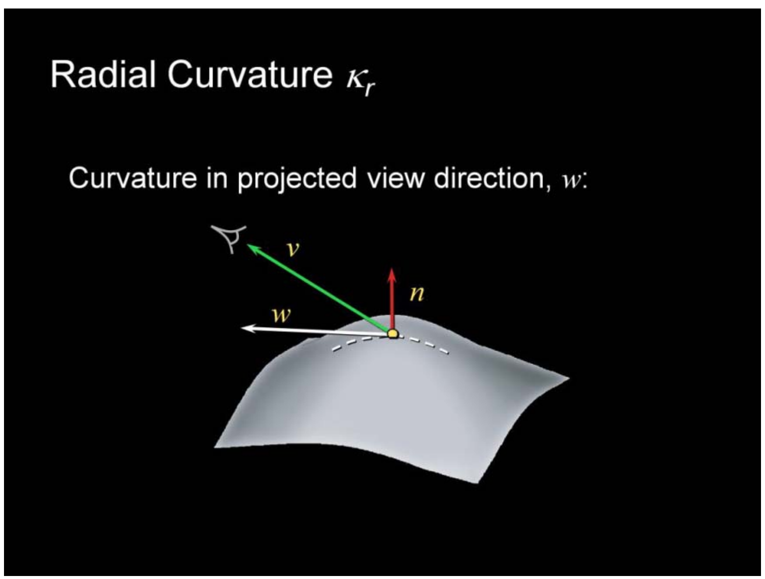


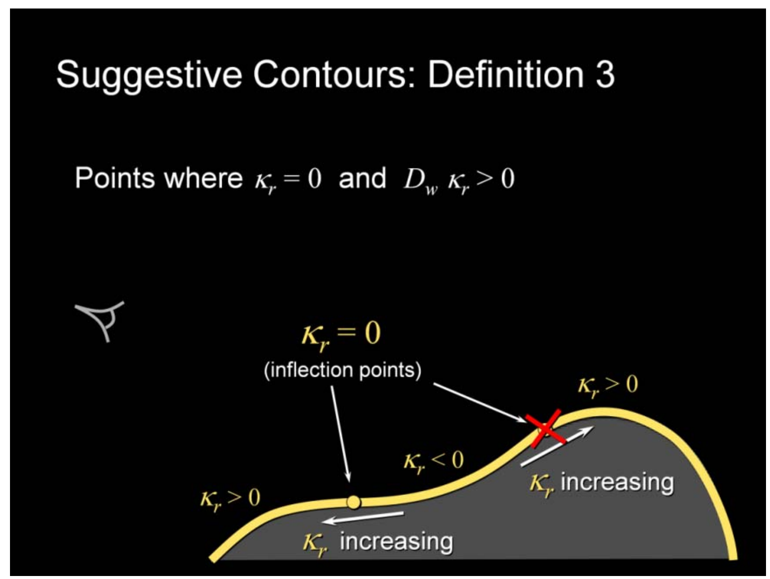


Suggestive Contours



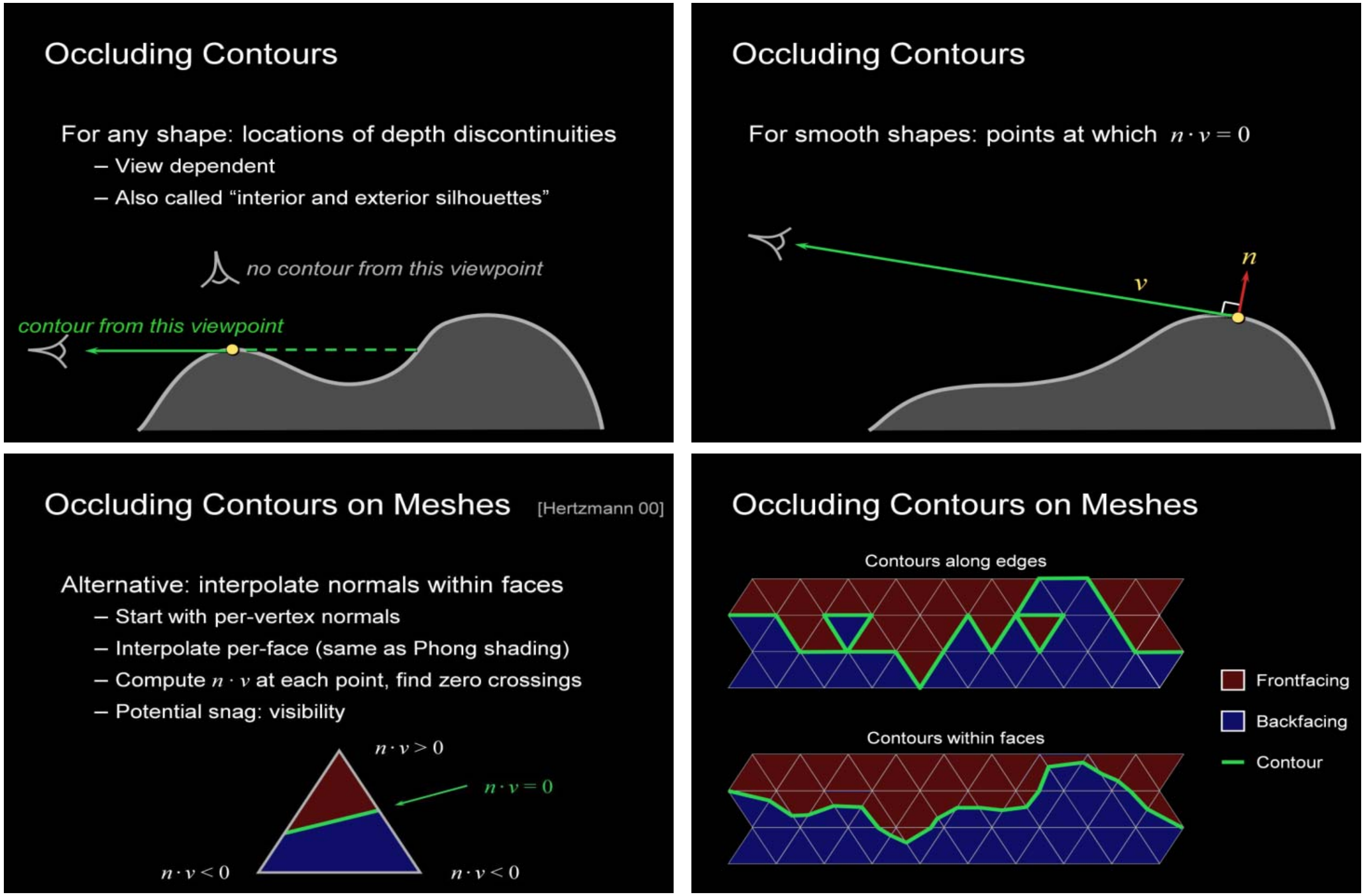




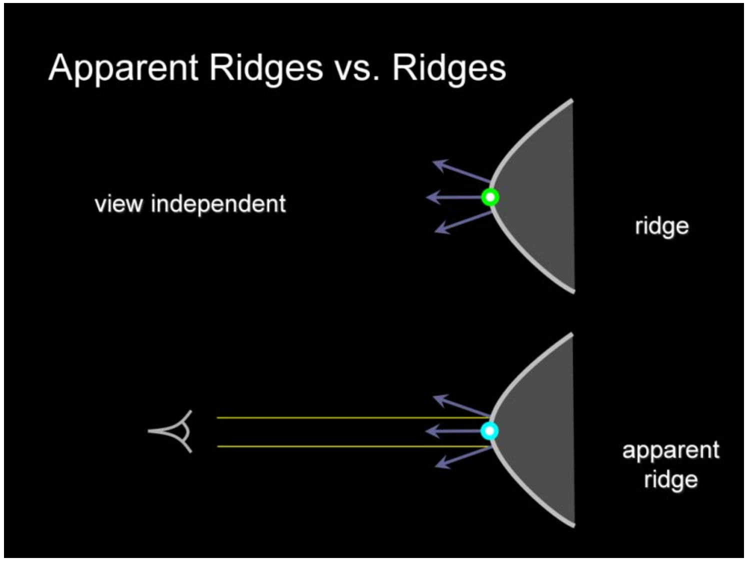


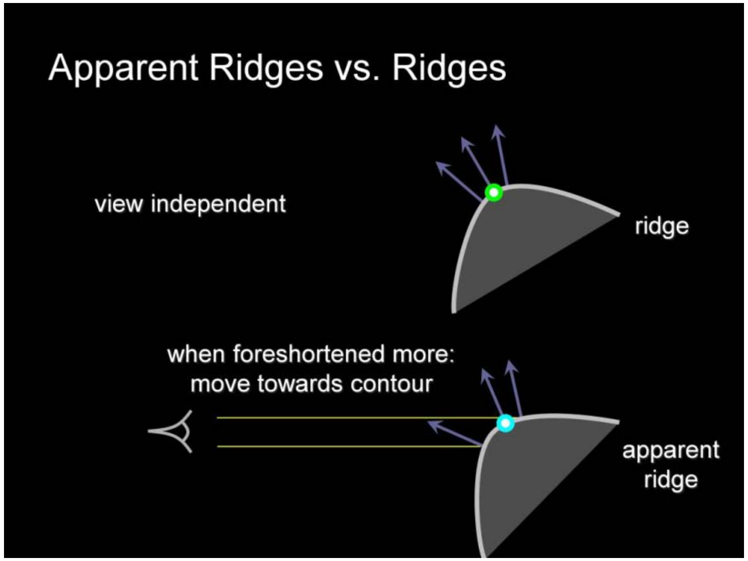
Occluding Contours

- Separate front-facing from back-facing region of the surface

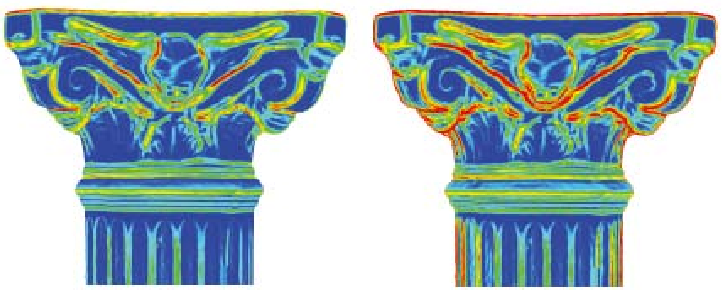


- Ridges and Valleys: a generalization of creases
  - Local maxima (minima) of curvature
- Apparent Ridges: a generalization of ridges and valleys
  - replace the use of standard surface curvature with a view-dependent quantity

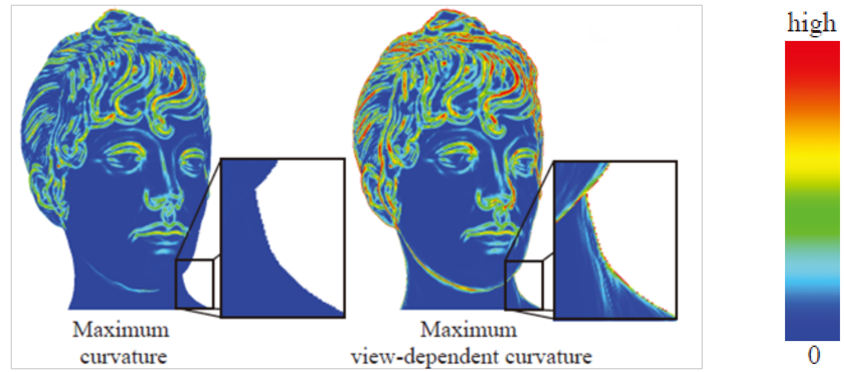




Apparent Ridges



Maximum curvature   Maximum view-dependent curvature

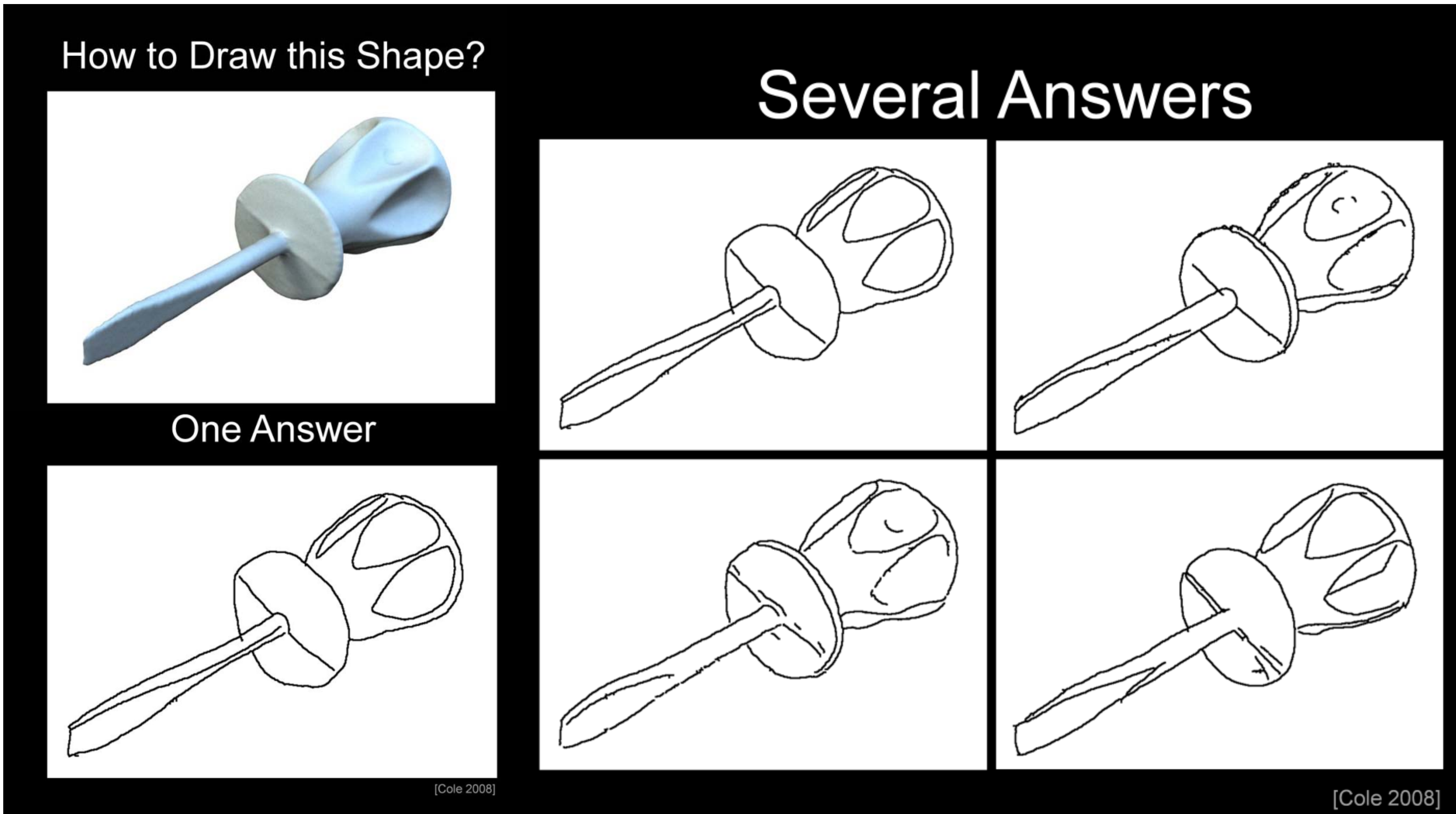


Shaded View   Contours   Suggestive Contours   Ridges & Valleys   Apparent Ridges

Lines Summary

Derivative Order	Image-Space	View-Independent Object-Space	View-Dependent Object-Space
0th	Isophotes	Topo-lines	Cutting planes
1st		Isophotes	Occluding contours
2nd	Edges, extremal lines	Parabolic lines	Suggestive contours, suggestive highlights, principal highlights
3rd		Crest lines (ridges and valleys)	Apparent ridges

- Where Do People Draw Lines?
  - [Cole et al. Siggraph 2008]
- How Well Do Line Drawings Depict Shape?
  - [Cole et al. Siggraph 2009]



Principal Component Analysis (PCA)

Principal Component Analysis (PCA)

- Neighborhood
- Covariance matrix
- Analyze eigenvalues and eigenvectors (SVD)
- Eigenvectors are Principal Axes

$$\mathbf{M} = \frac{1}{n}\sum_{i=1}^{n}\begin{bmatrix} q_i^x q_i^x & q_i^x q_i^y & q_i^x q_i^z \\ q_i^y q_i^x & q_i^y q_i^y & q_i^y q_i^z \\ q_i^z q_i^x & q_i^z q_i^y & q_i^z q_i^z \end{bmatrix}$$

Covariance matrix

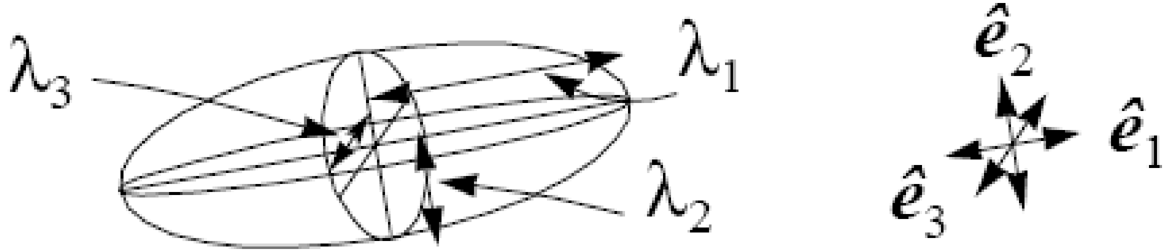
$$\mathbf{M} = \mathbf{U}\mathbf{S}\mathbf{U}^t$$

$$\mathbf{S} = \begin{bmatrix} \lambda_a & 0 & 0 \\ 0 & \lambda_b & 0 \\ 0 & 0 & \lambda_c \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{bmatrix}$$

Eigenvalues & eigenvectors

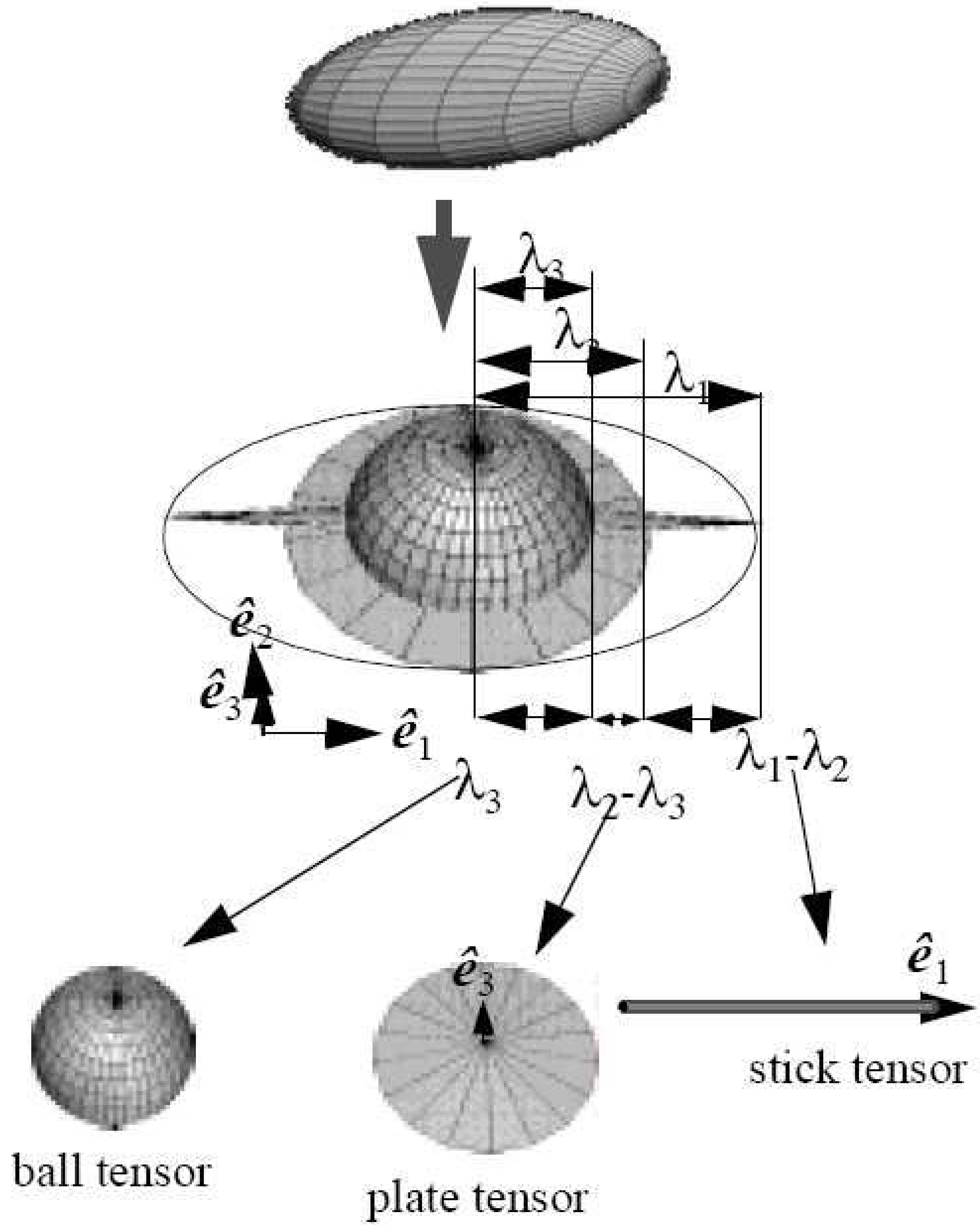
PCA

- Eigenvectors are "Principal Axes of Inertia"
- Eigenvalues are variances of the point distribution in those directions
  - Local frame at each point
  - Normal estimation



PCA

- Differentiate
  - Plane-like
  - Stick-like
  - Sphere-like
  - etc.



Discussion