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Decoupling Noises and Features via Weighted ℓ_1-analysis Compressed Sensing

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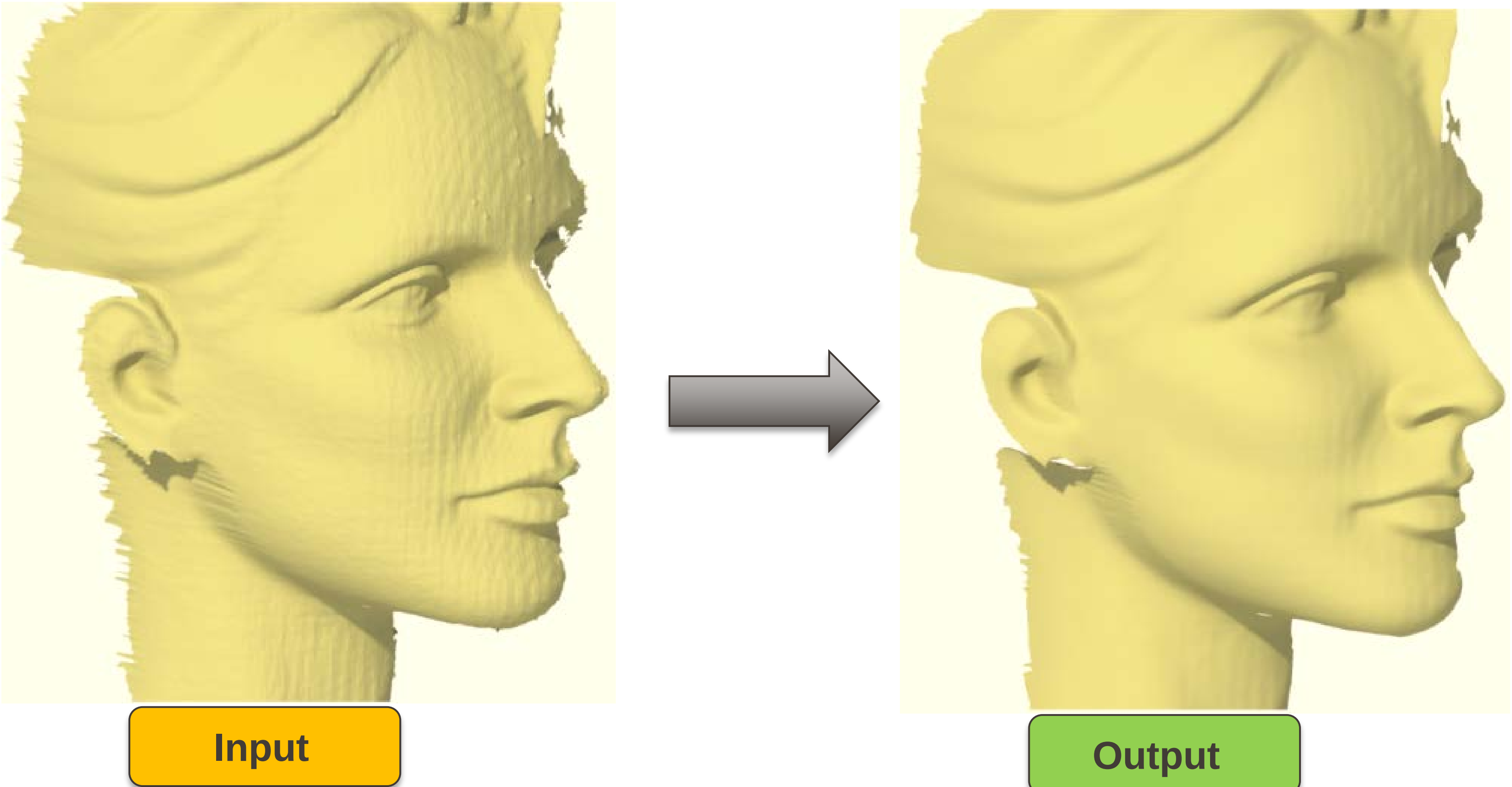
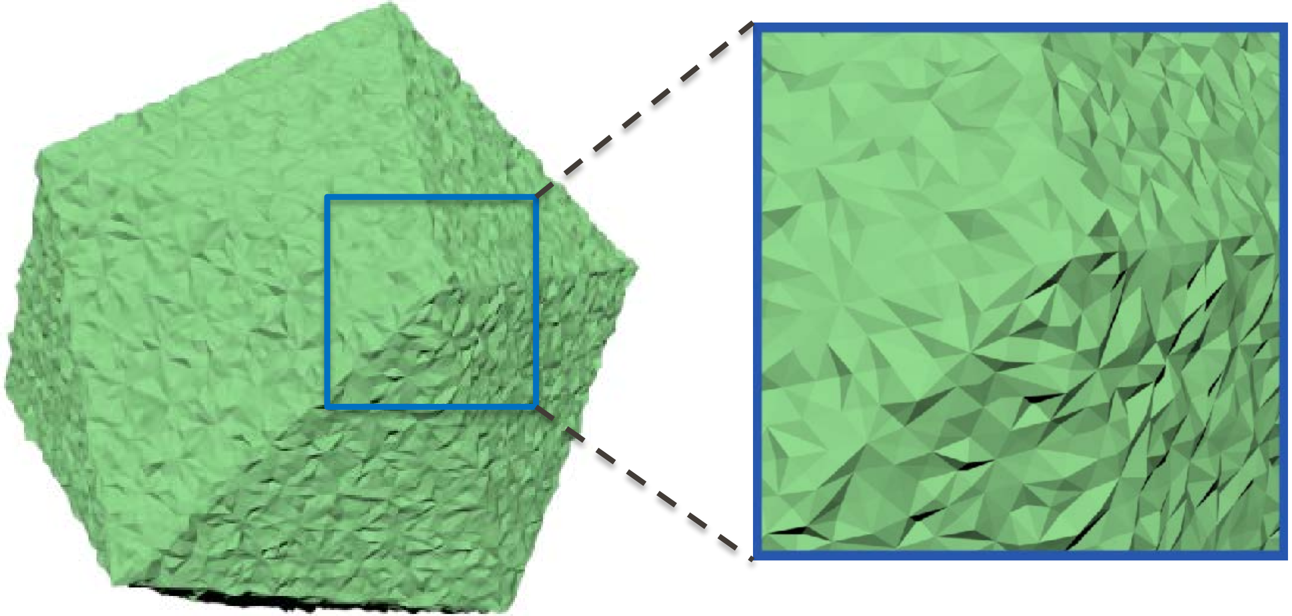
Denoising 3D Mesh Data



Challenging:
Denoising Objects with Sharp Features



Ambiguous: hard to distinguish

Challenging:
Denoising Objects with Sharp Features

- **Feature** detection is **unreliable** in the presence of noise
  - Feature measures (2nd derivatives) are **sensitive** to noise
- **Denoise** operations might **blur** features
  - Features are **vulnerable** to local filtering operations

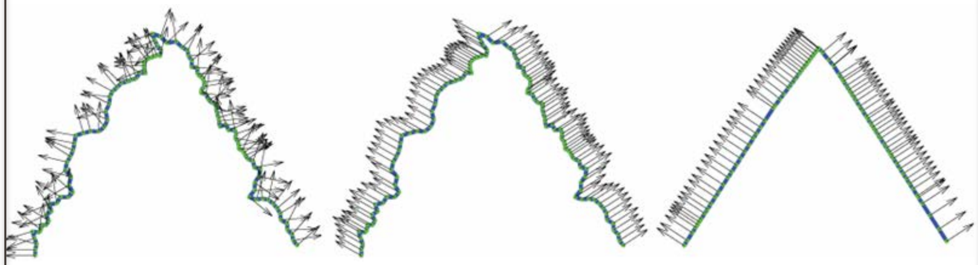
A chicken-and-egg problem!

Previous Works (1)

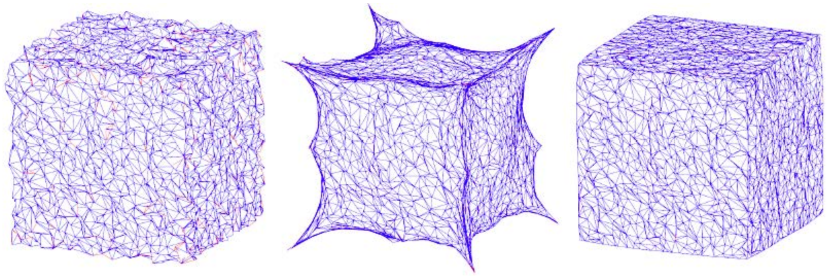
- Feature preserving/aware denoising
  - Laplacian filtering [Taubin 1995, Desbrun et al. 1999]
  - Higher order (e.g., bilateral) filtering [Fleishman et al. 2003, Jones et al. 2003, Duguet et al. 2004]
  - Normal filtering [Zheng et al. 2010, Fan et al. 2010]
  - Global methods [Nealen et al. 2006, Liu et al. 2007]

Previous Works (2)

- Sparsity optimization based denoising



ℓ_1-sparse reconstruction
[Avron et al. ACM ToG, 2010]



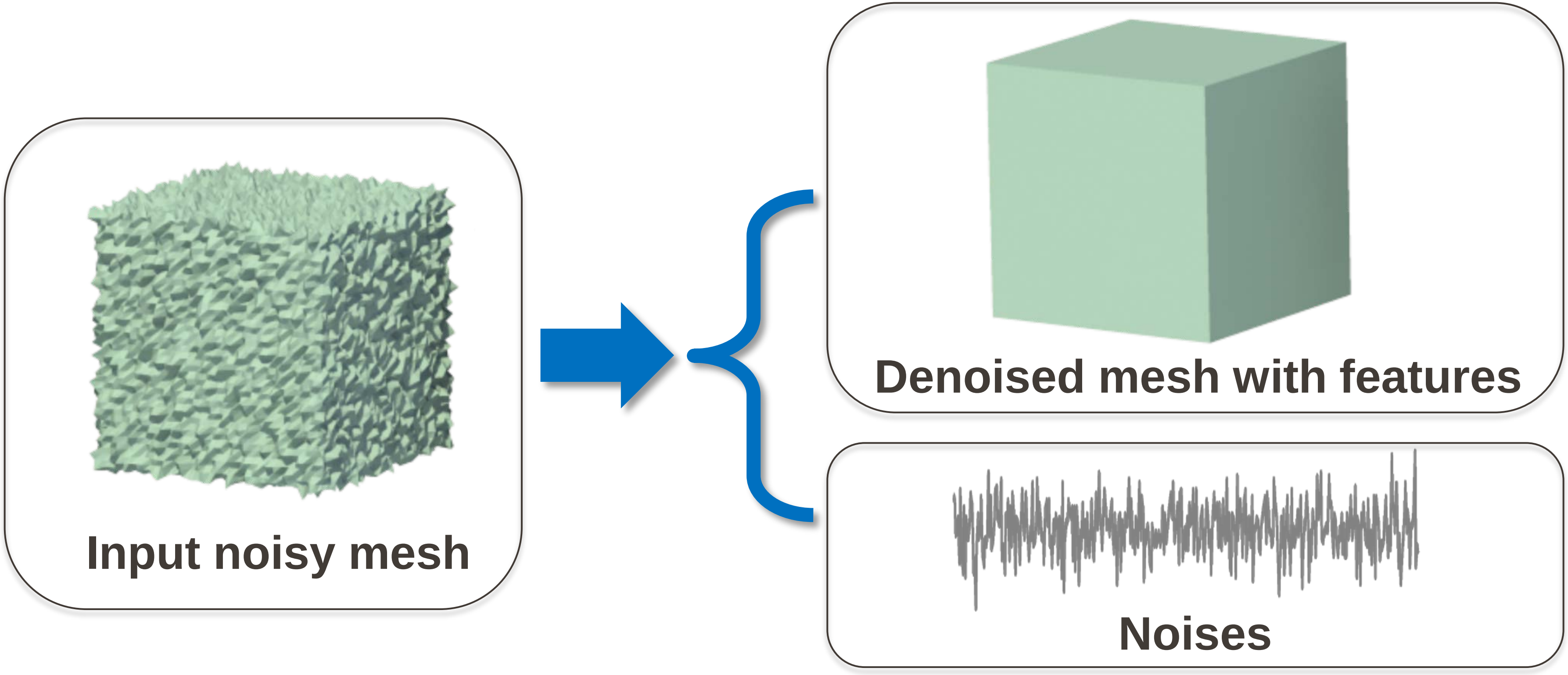
Mesh denoising via ℓ_0 minimization
[He and Schaefer, Siggraph 2013]

Normal gradients are sparse

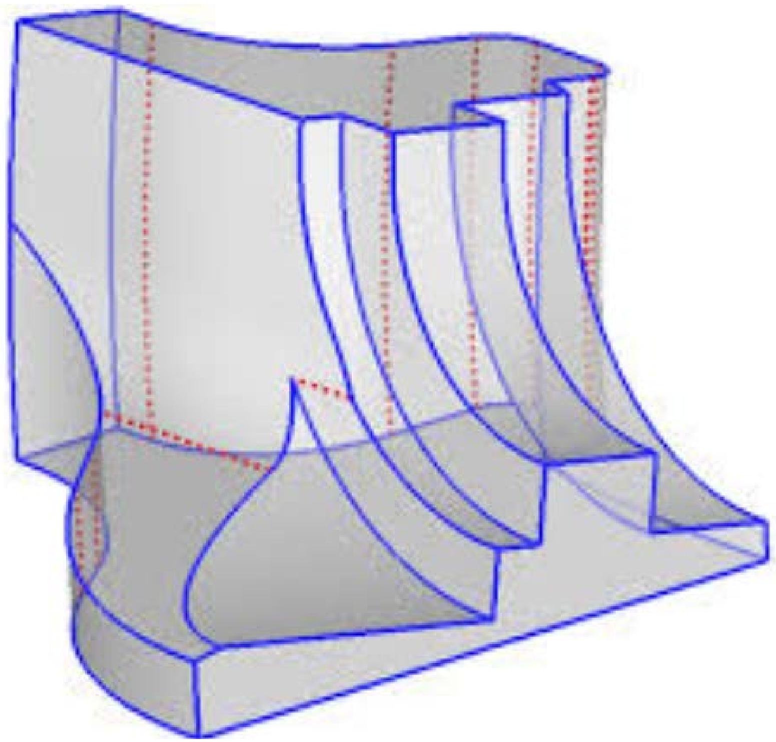
Edge operators are sparse

Our Method: Compressed Sensing

- Decouple **features** and **noises** **simultaneously**!



Sharp Features are Sparse on Meshes



A signal $x = \begin{bmatrix} \blacksquare \\ \blacksquare \\ \\ \blacksquare \\ \blacksquare \end{bmatrix} \in R^N$

is called ***sparse***
if $\|x\|_0$ is small.

number of non-zero elements

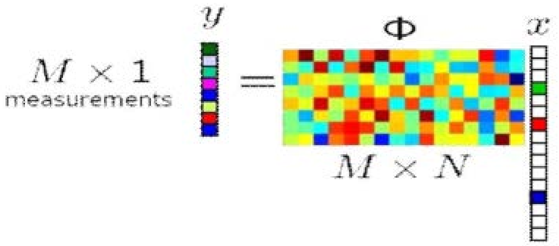
Short Overview on

Processing Sparse Signal via Compressed Sensing

Compressed Sensing

for **Sparse Signal x**

- Sensing: a small number ($M<<N$) of data $y = \Phi x$



- Recovering: via an ℓ_0 optimization (if Φ has R.I.P.)

$$\min \|x\|_0 \quad \text{s.t.} \quad \Phi x = y$$

or

$$\min \|x\|_0 \quad \text{s.t.} \quad \|y - \Phi x\|_2 \le \varepsilon$$

If x is corrupted by some noise

Analysis Compressed Sensing

for Non-sparse Signal x

If there exists a linear transformation L, such that

Lx is sparse

- Sensing: $y = \Phi x$

- Recovering:

$$\min \|Lx\|_0 \quad \text{s.t.} \quad \|y - \Phi x\|_2 \leq \varepsilon$$

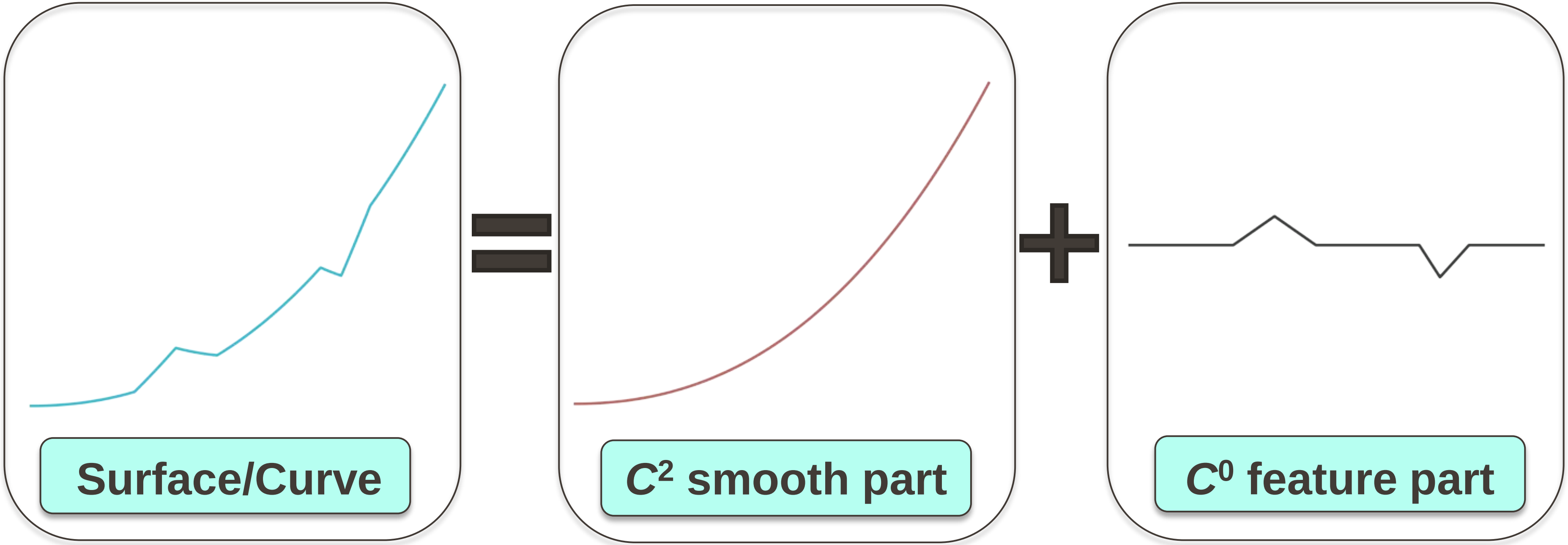
Compressed Sensing for Sparse Signal

The sparse signal can be robustly recovered
in the presence of noise
by solving some l_0 optimization!

Our Approach

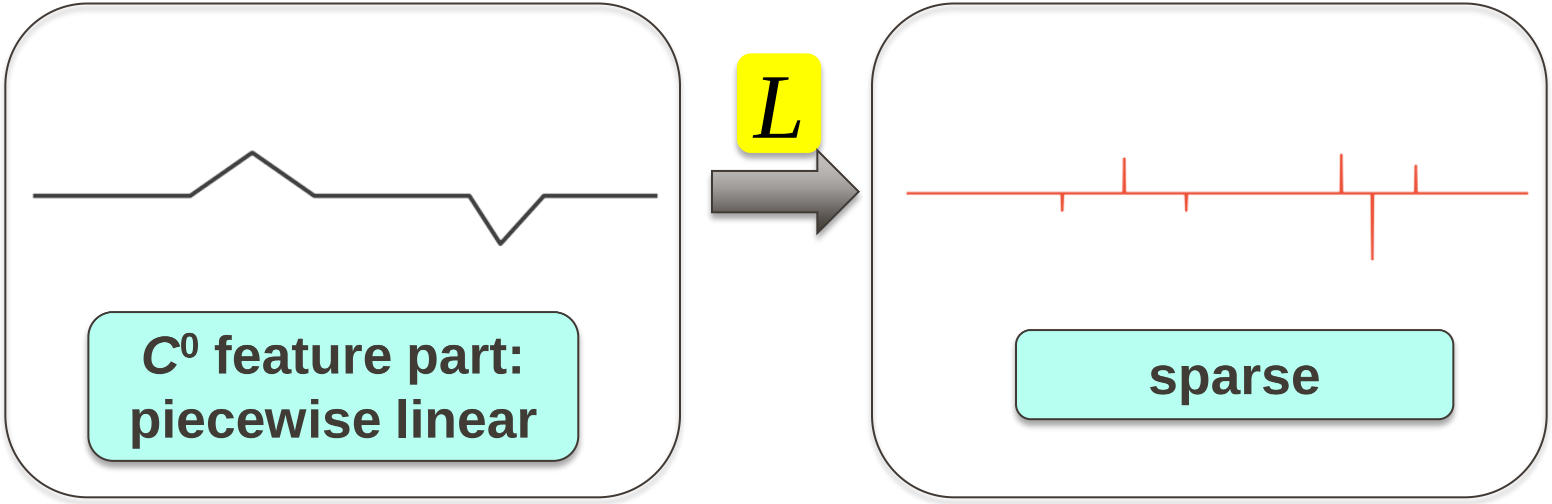
Our Key Observation

- Any surface is piecewise C^2
  - Sharp features are C^0 signal over smooth surface

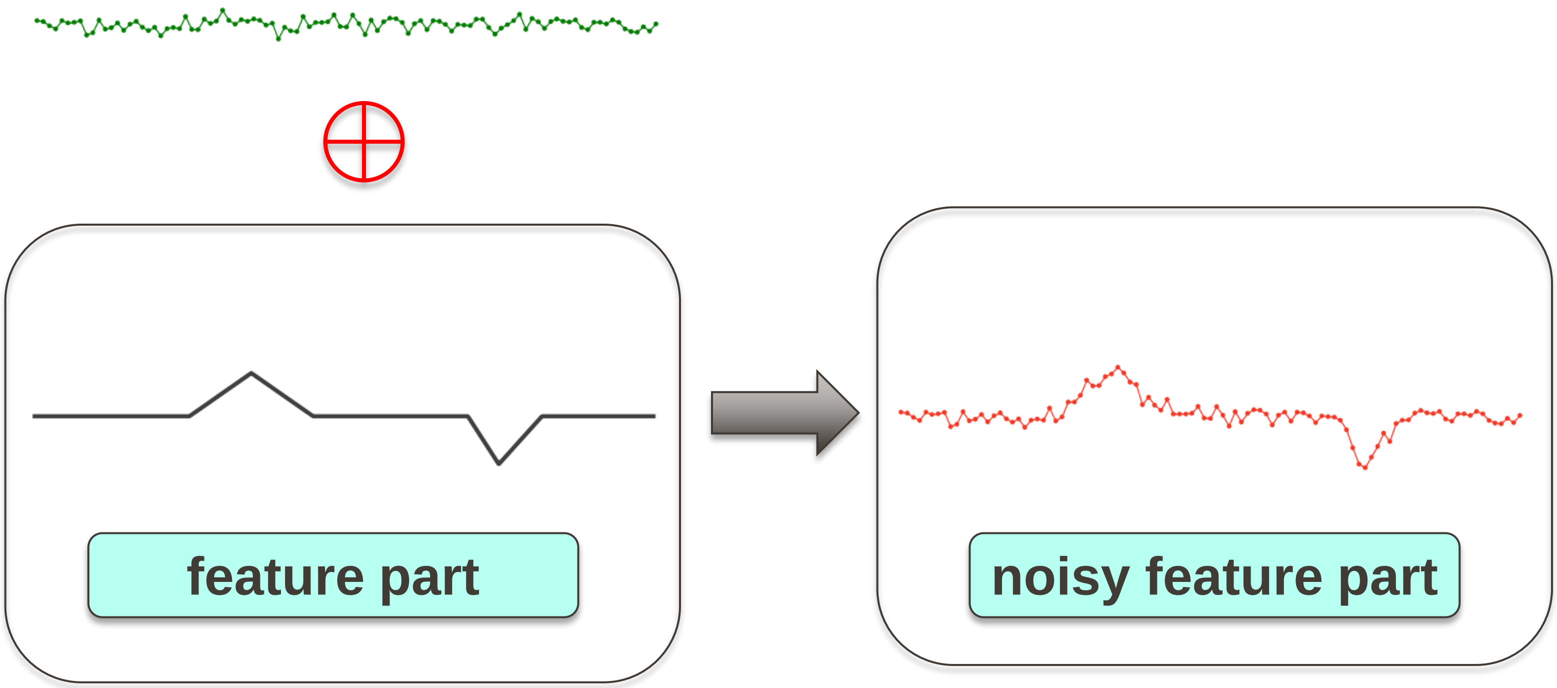


Our Key Observation

- Applying Laplacian operator *L* (2nd derivative operator) on feature part results in sparse signal



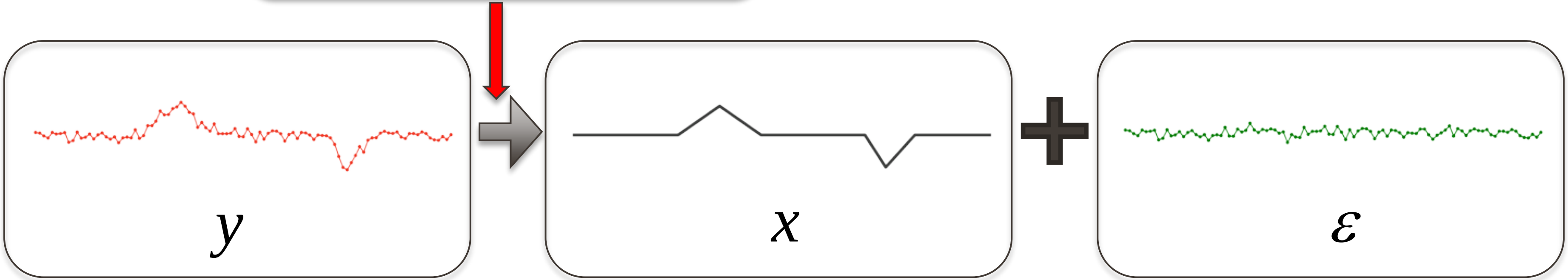
If feature part is corrupted by noise…



Our Solution

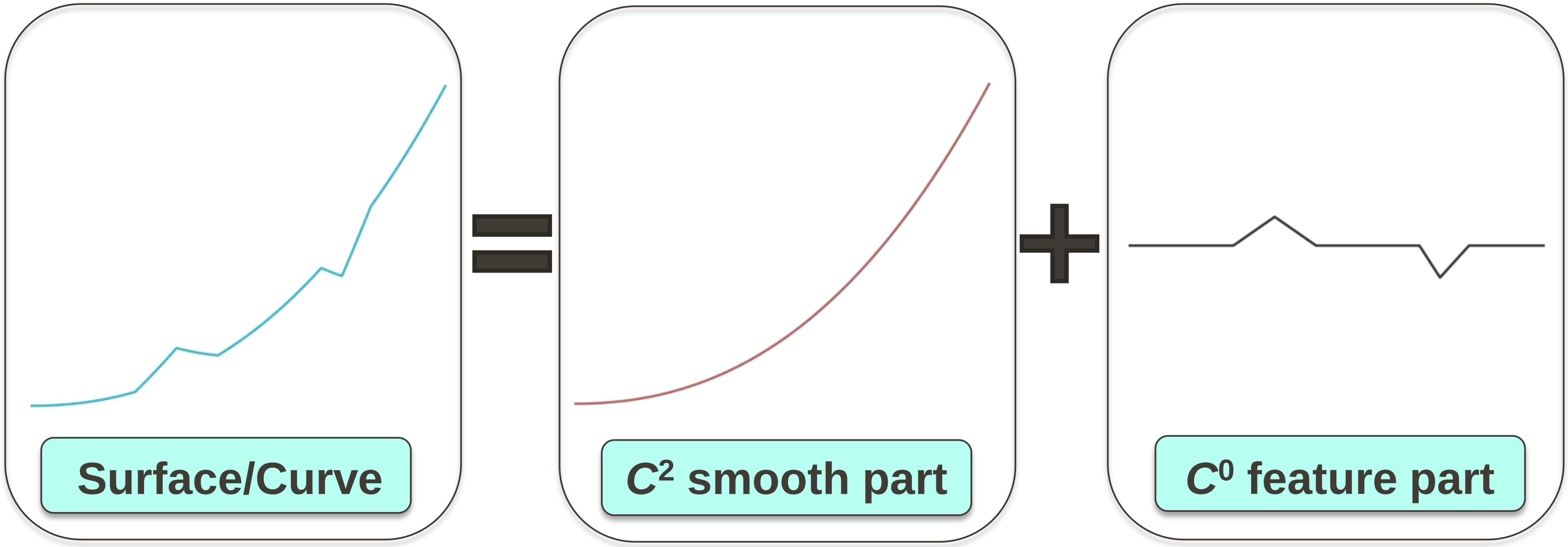
- Applying analysis compressed sensing on the noisy feature part

$$\min \|Lx\|_0 \quad \text{s.t.} \quad \|y - x\|_2 \le \varepsilon \qquad (\Phi = I)$$

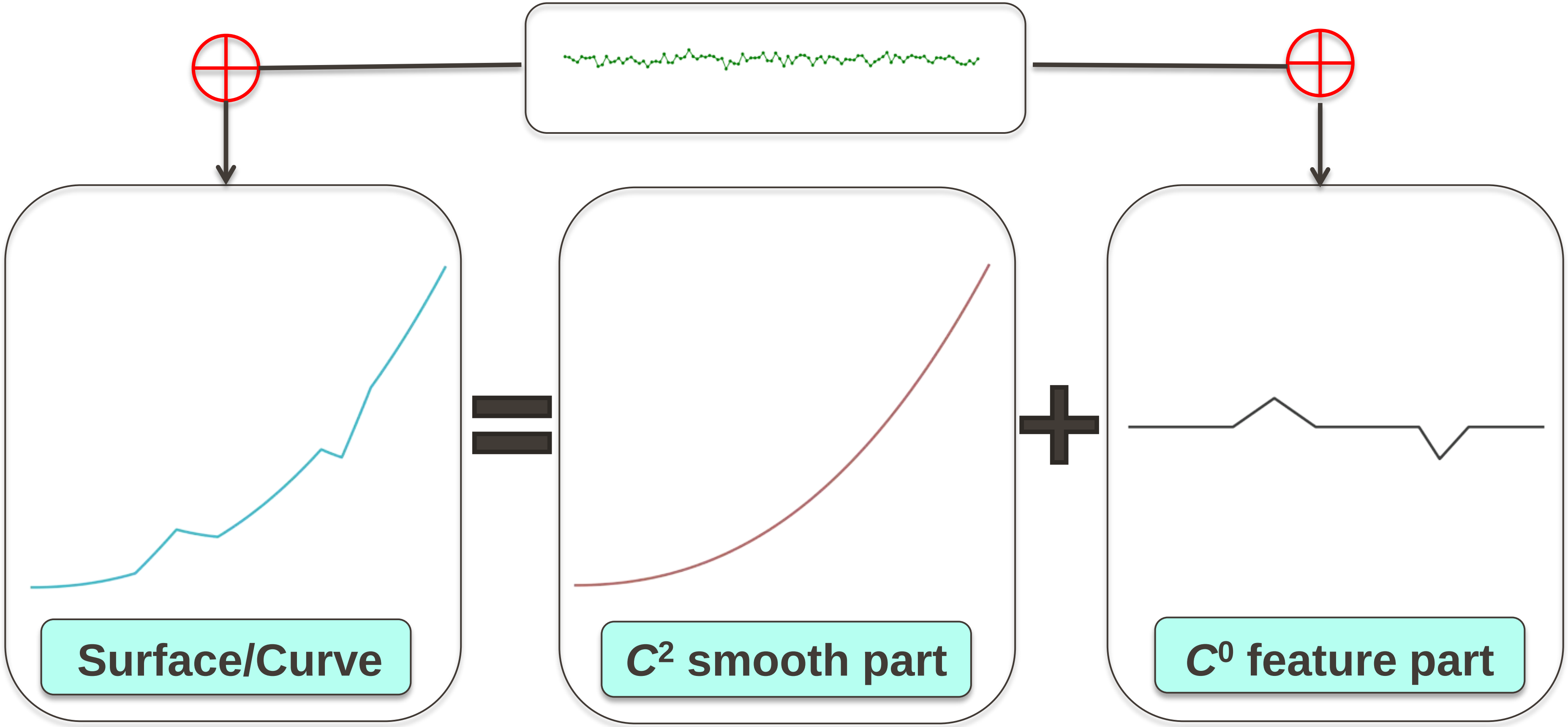


Decouple features and noises simultaneously!

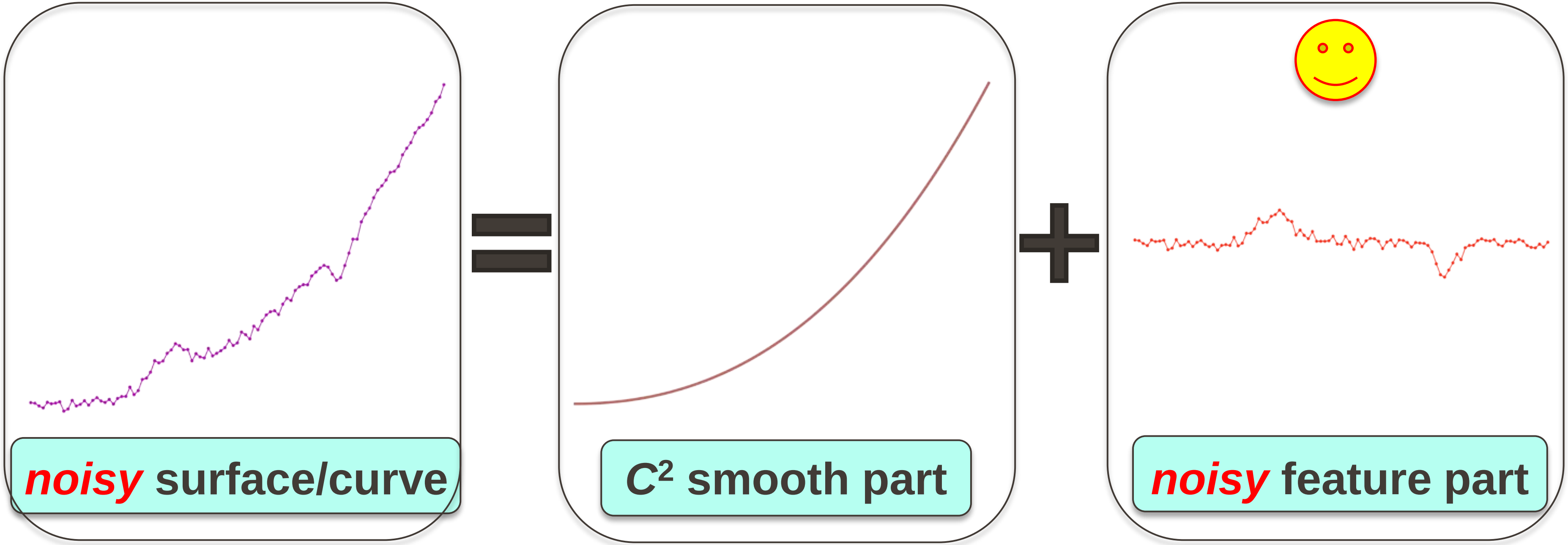
Remind: Smooth part + Feature part



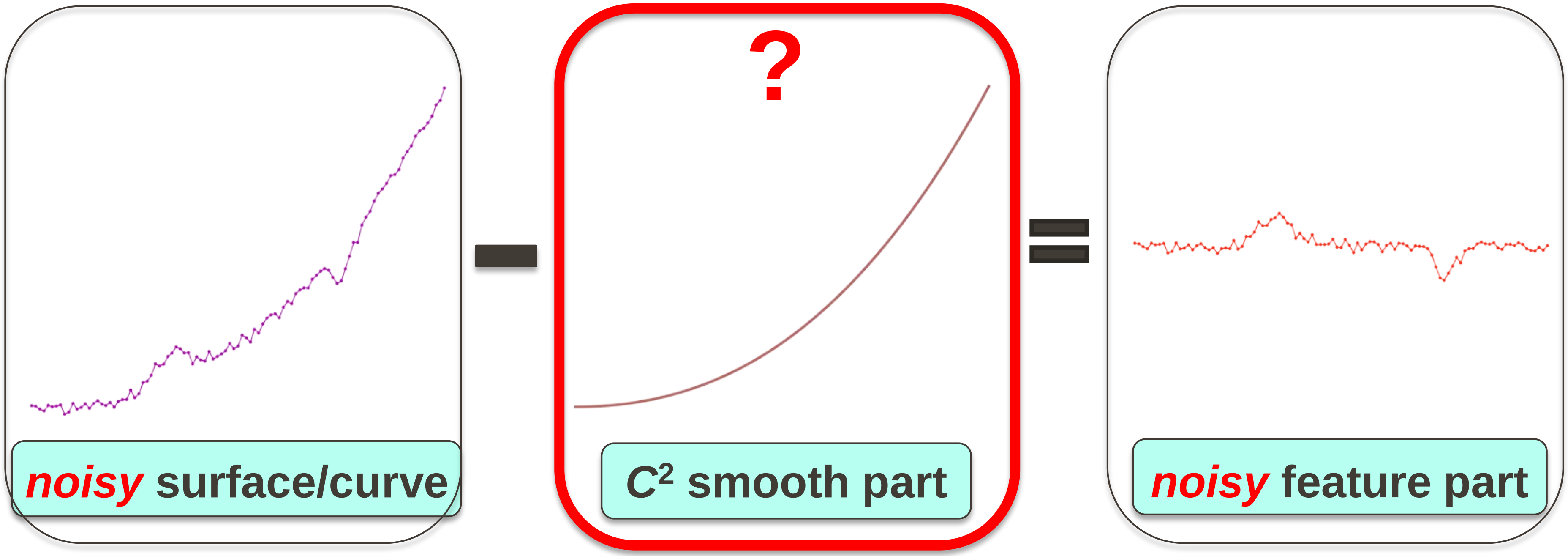
With noise



With noise

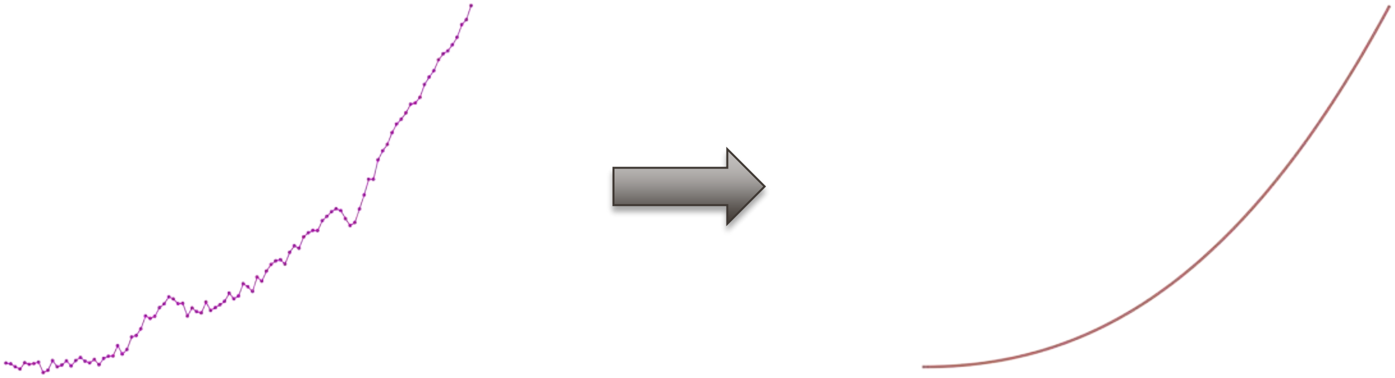


Key Problem:
How to extract the C^2 smooth part?



Problem:

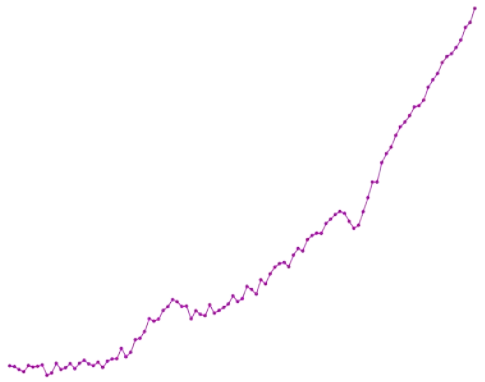
Estimation of the Smooth Part



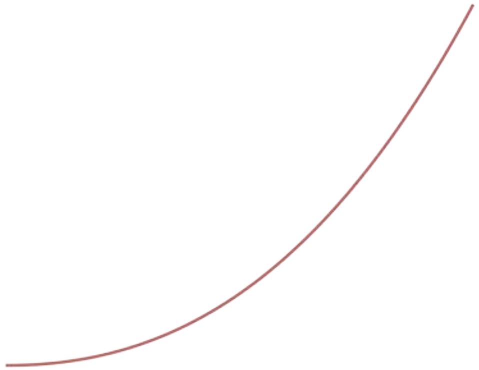
Input: noisy data $P = \{\mathbf{p}_i\}$

Output: smoothed data $S = \{\mathbf{s}_i\}$

Global Laplacian Smoothing



Input: noisy data $P = \{\mathbf{p}_i\}$



Output: smoothed data $S = \{\mathbf{s}_i\}$

$$\hat{S} = \arg\min_{S} \|S - P\|^2 + \lambda \|LS\|^2$$

L: Laplacian

Data term

Weight

Smoothness term

Solution

$$\hat{S} = \arg\min_{S} \|S - P\|^2 + \lambda \|LS\|^2$$

The minimization leads to a linear system

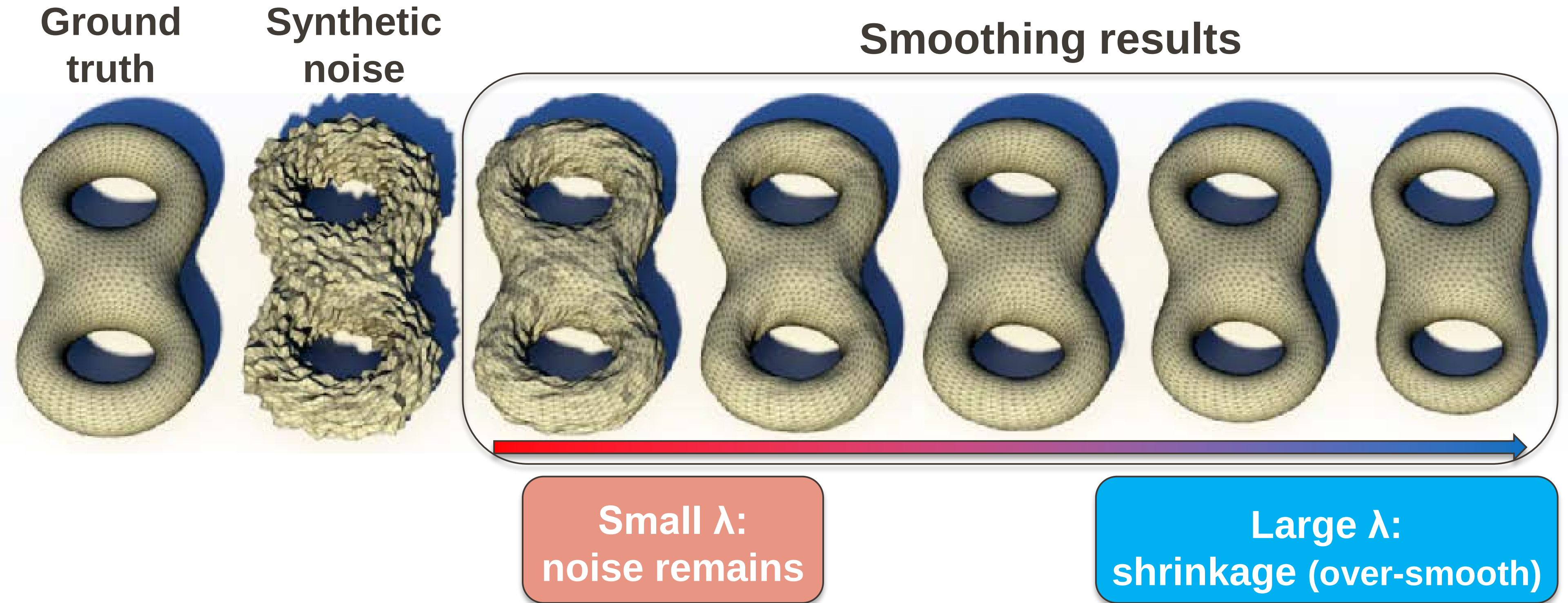
$$(I + \lambda L^T L)\hat{S} = P$$

Thus, the solution is

$$\hat{S} = (I + \lambda L^T L)^{-1} P$$

Choice of the Weight Parameter λ

$$\hat{S} = (I + \lambda L^T L)^{-1} P$$



Optimal Choice of the Weight Parameter λ

We define a Generalized Cross Validation (GCV) merit function (inspired from statistics)

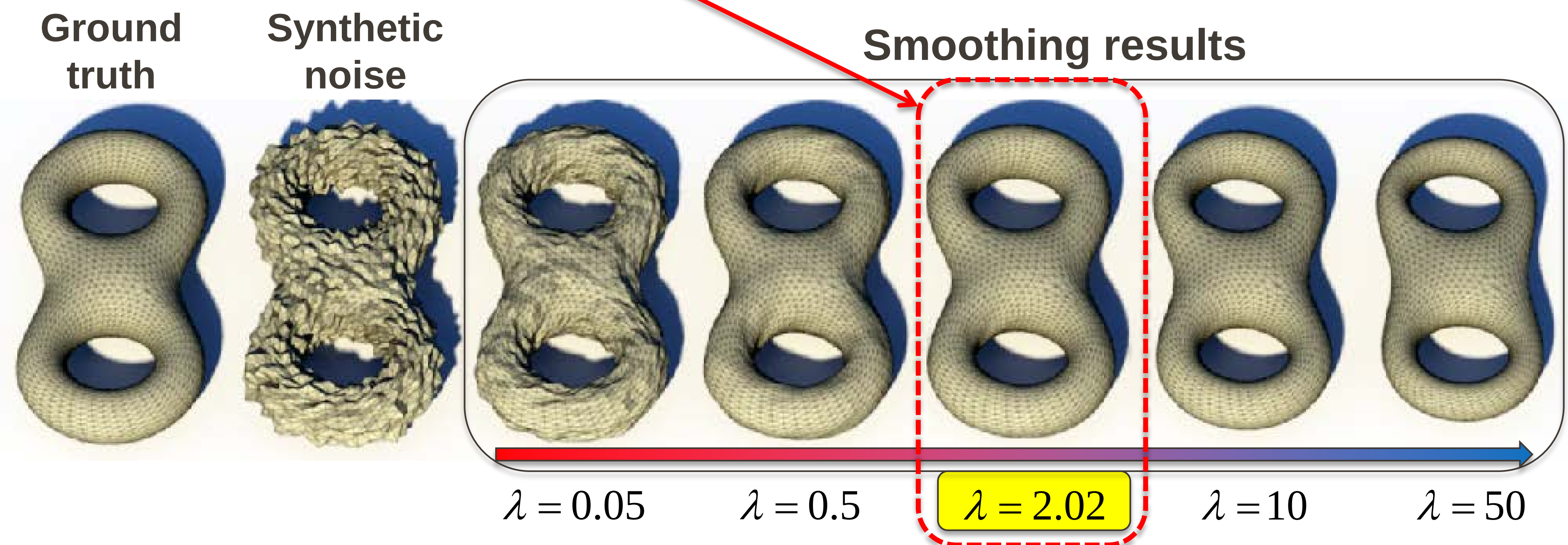
$$GCV(\lambda) = \frac{\frac{1}{n}\left\| P - \hat{S}(\lambda) \right\|_F^2}{\left(1 - \frac{1}{n} tr\left[(I + \lambda L^T L)^{-1} \right]\right)^2}$$

Then the optimal value of λ can be found:

$$\hat{\lambda} = \arg\min_{\lambda > 0} GCV(\lambda)$$

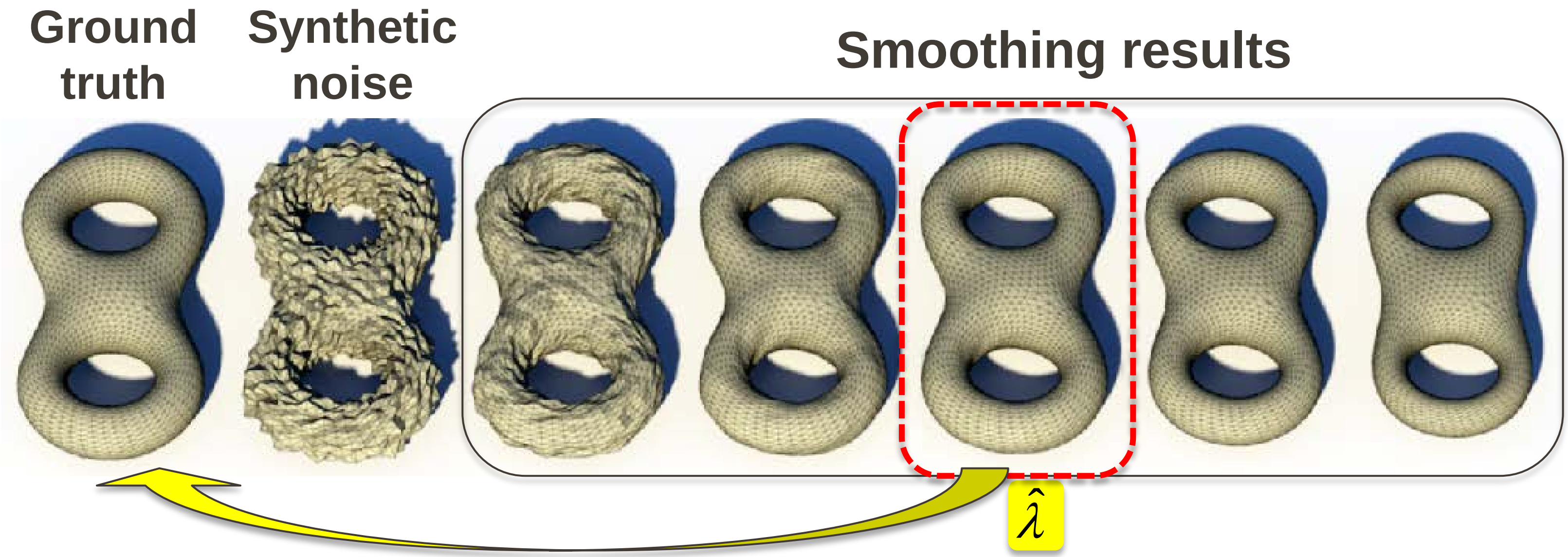
Optimal Weight Parameter λ

$$\hat{\lambda} = \arg\min_{\lambda>0} GCV(\lambda)$$

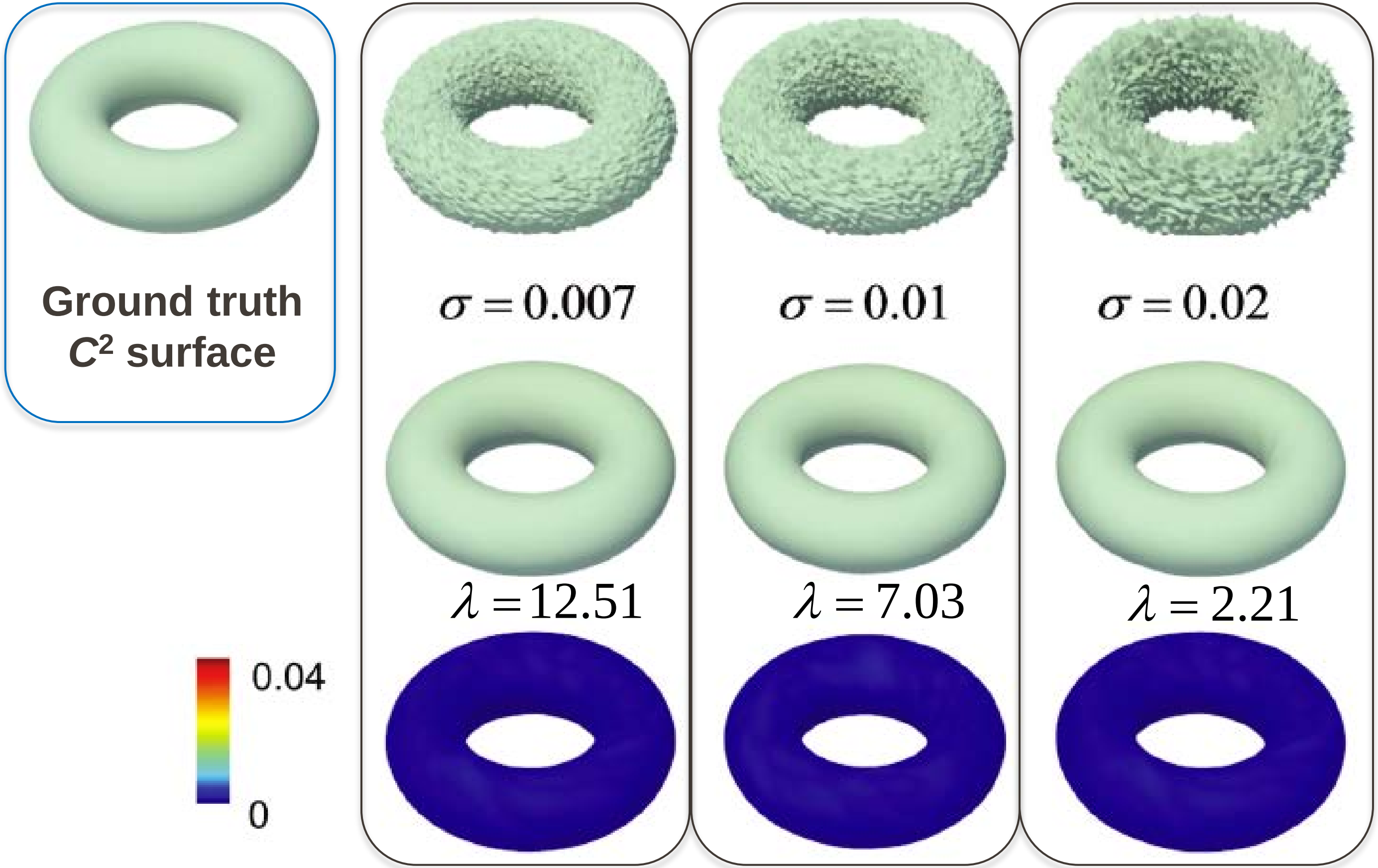


Theoretical Guarantee by Statistical Theory

[Theorem] The estimated surface asymptotically converges to the true underlying C^2 smooth surface with probability 1 as the sample number goes to infinity.



Smoothing Results

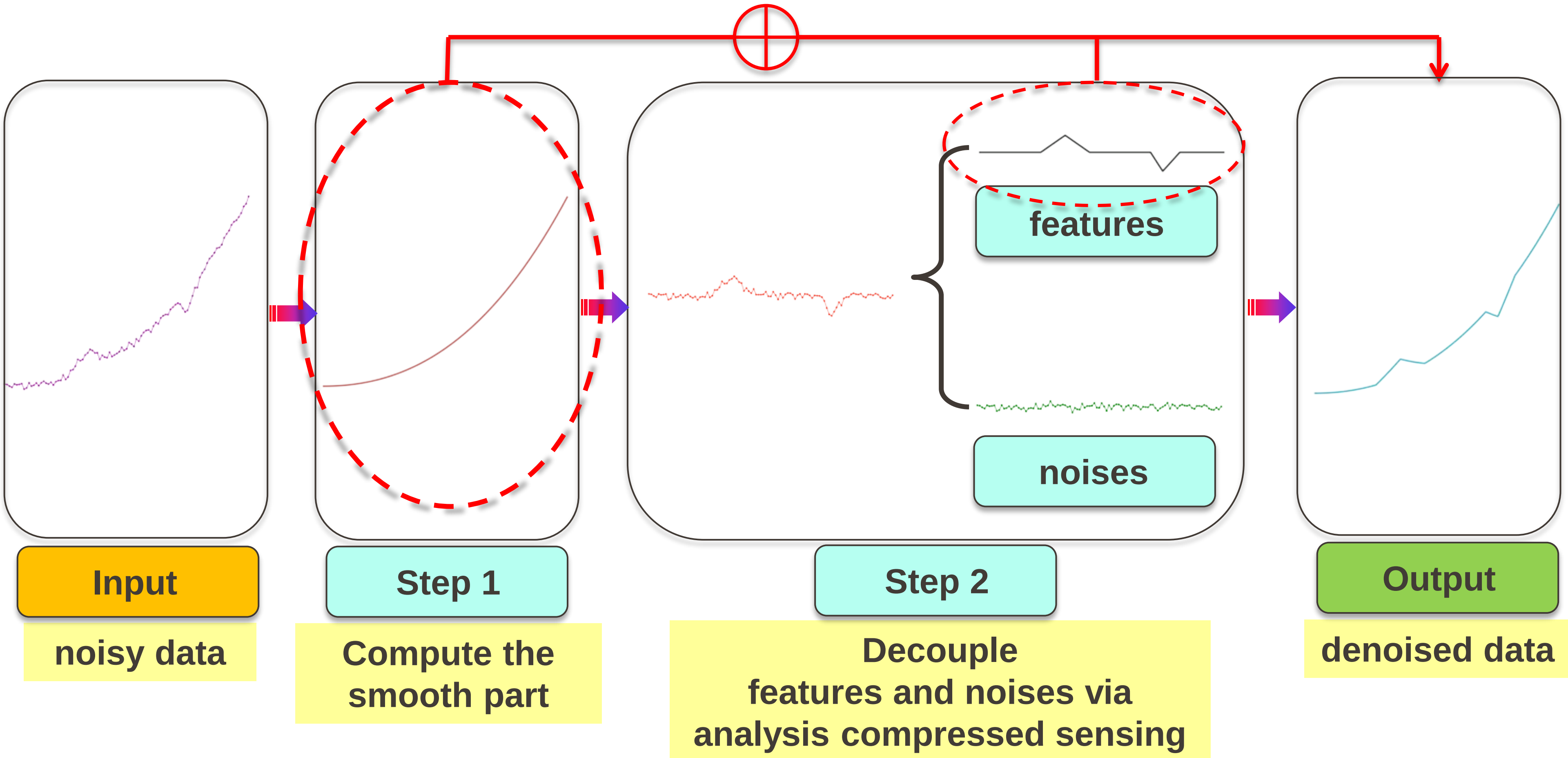


Synthetic noise with different deviation

Smoothing results with optimal weight

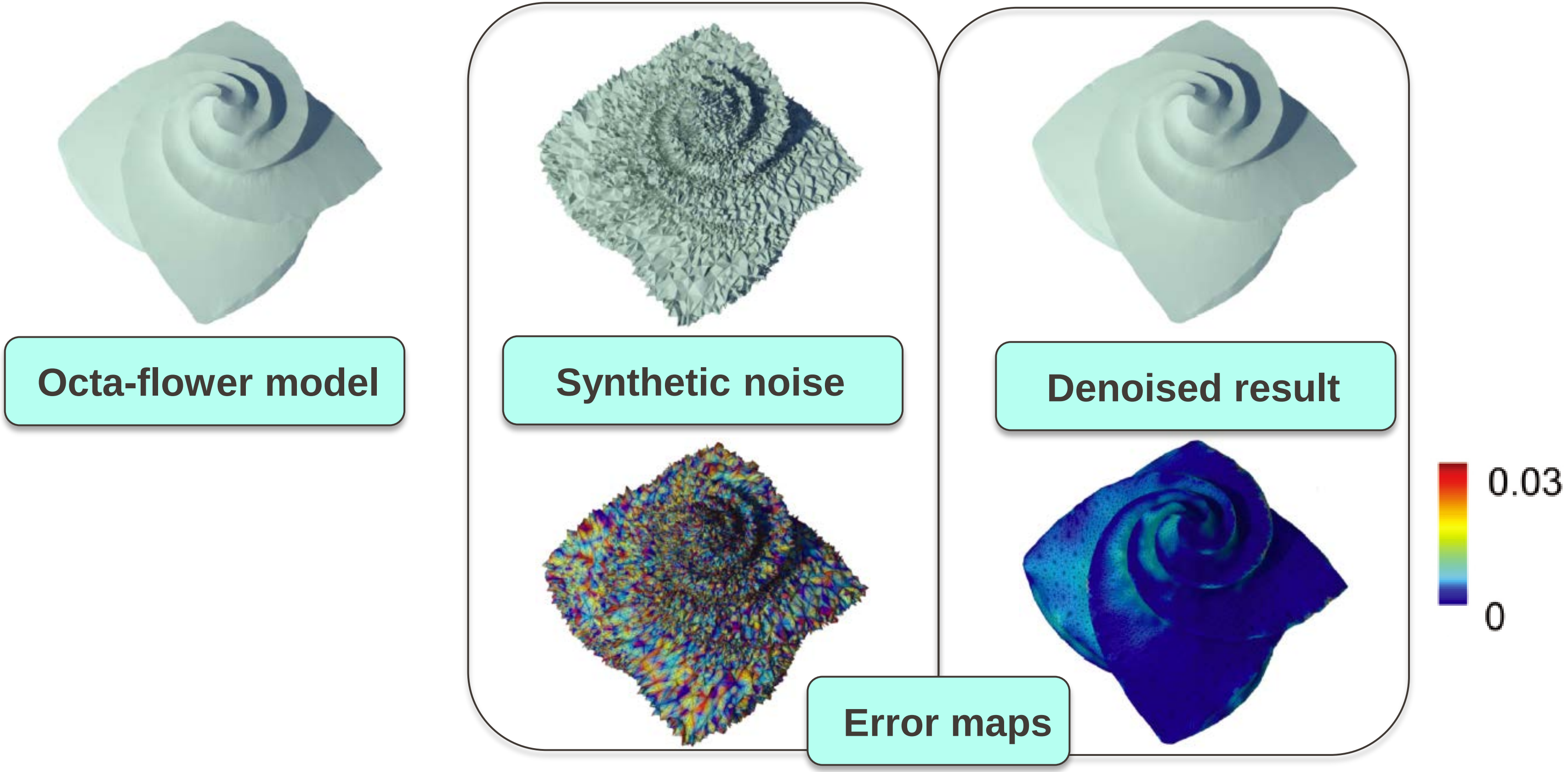
Error maps between smoothing results and ground truth

Recap: Algorithm

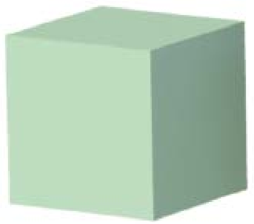


Experimental Results

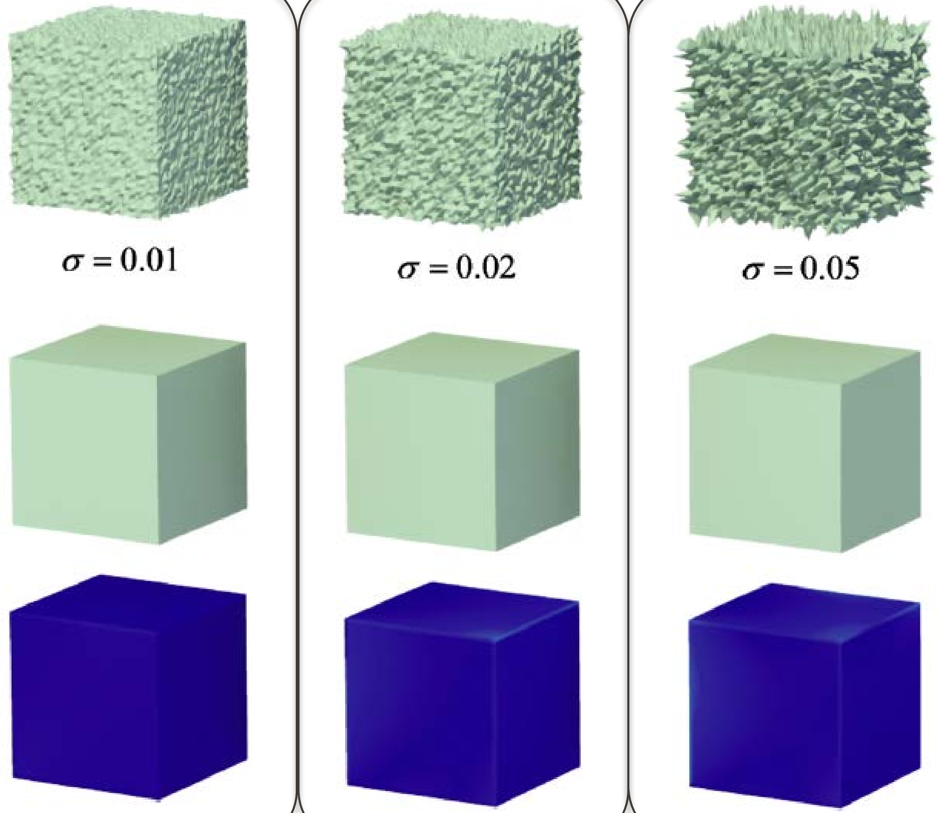
Result: Synthetic Example

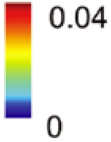


Result: Synthetic Examples



Cube model





Synthetic noise with different deviation

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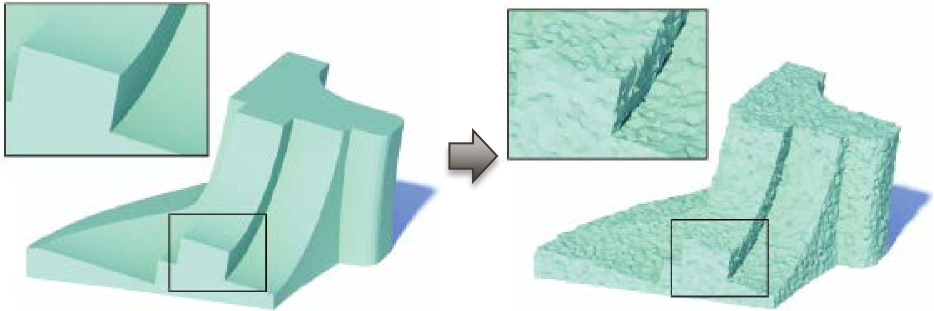
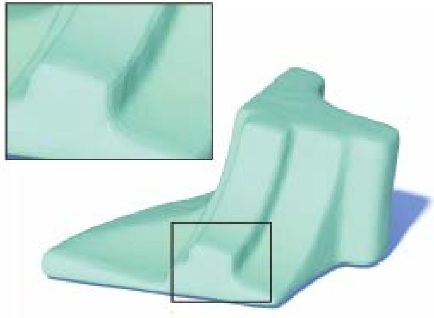
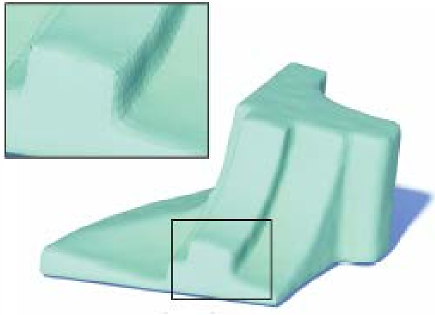
Comparisons

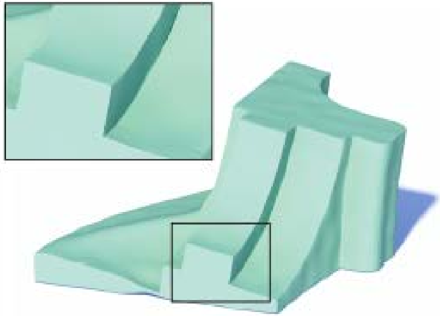




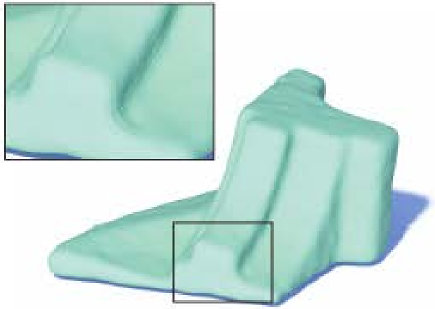
[Clarenz et al. 2000]



[Fleishman et al. 2003]



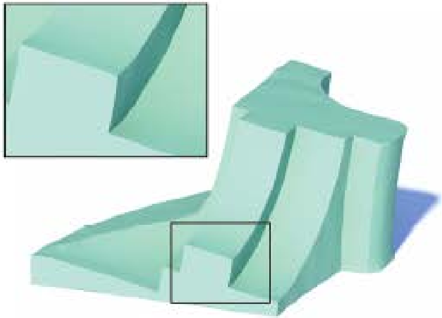
[Jones et al. 2003]



[Hildebrandt et al. 2004]

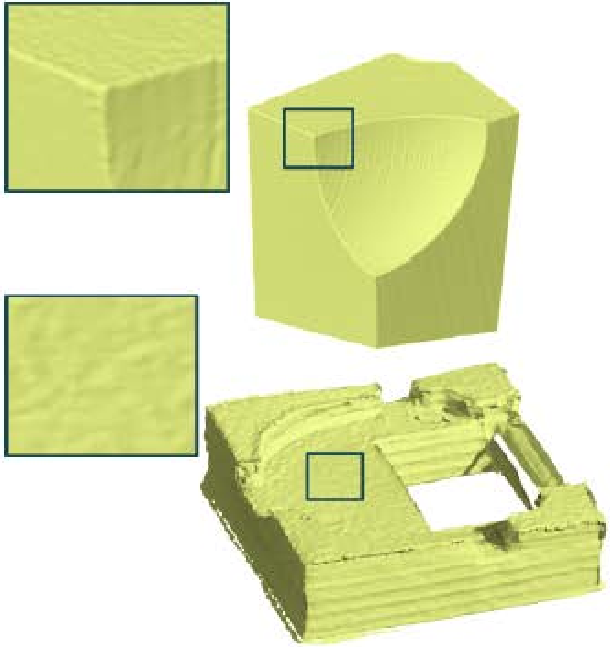
[Nealen et al. 2006]

[Zheng et al. 2010]

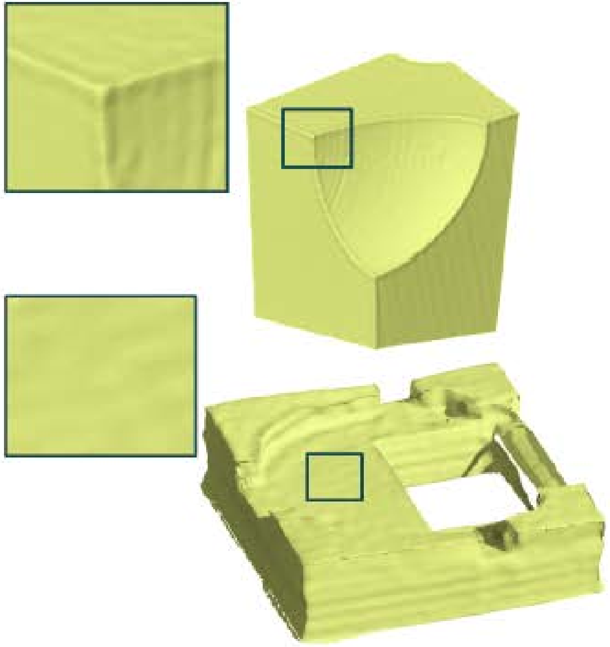


Our result

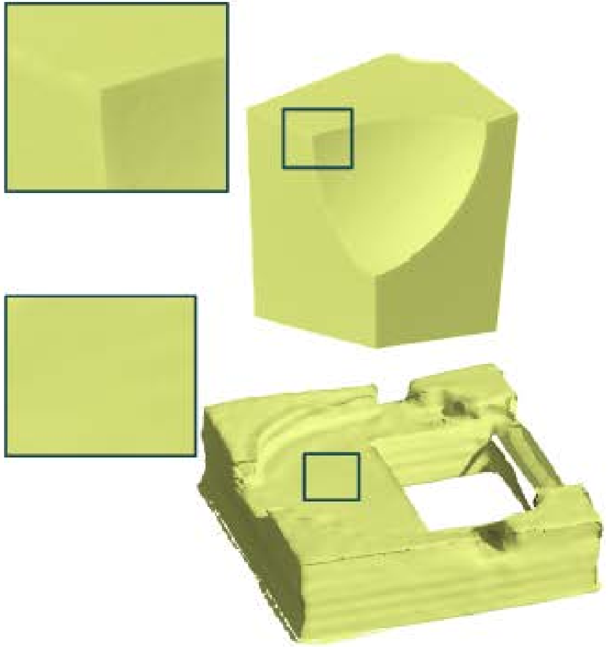
Real Data



Scanning raw data



[Jones et al. 2003]



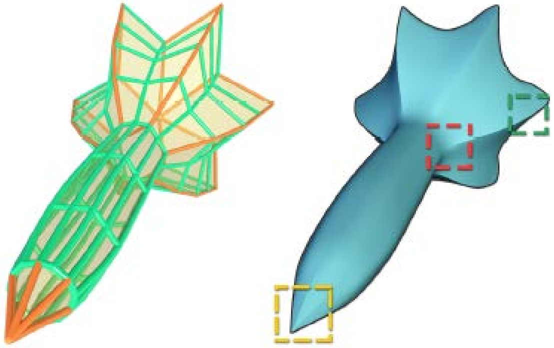
Our results

Limitations

- Noise should be *independent and identically distributed* (i.i.d.) random variables
  - Required in the proof of the convergence theorem
  - Practically useful
- Need correct connectivity information
  - Correct access to neighboring samples
  - Problem with point cloud

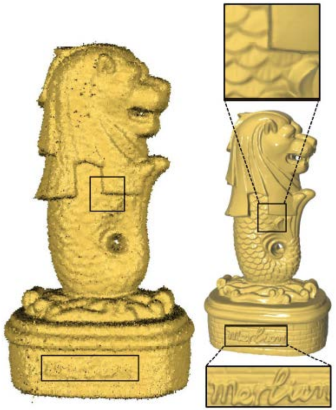
Future Work

- Compressed sensing (sparsity optimization) is a powerful tool for signal processing
- Apply it in other geometry processing problems



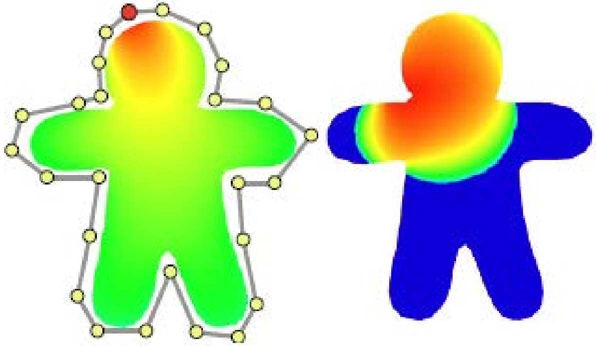
Manifold generation

To appear in ACM ToG



Surface reconstruction

To appear in SigAisa 14



Barycentric coordinates

To appear in SigAisa 14

Conclusions

- Asymptotically optimal smoothing
  - The generalized cross-validation scheme

- Decoupling features and noise simultaneously
  - The compressed sensing tool

Thank you!

Project page:
http://staff.ustc.edu.cn/~lgliu/Projects/2014_DecouplingNoise/default.htm

Google “Ligang Liu”