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Construction of Manifolds via Compatible Sparse Representations

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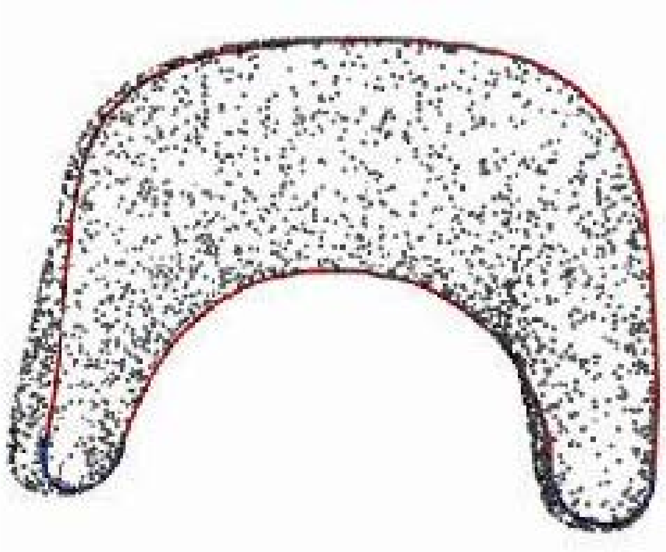
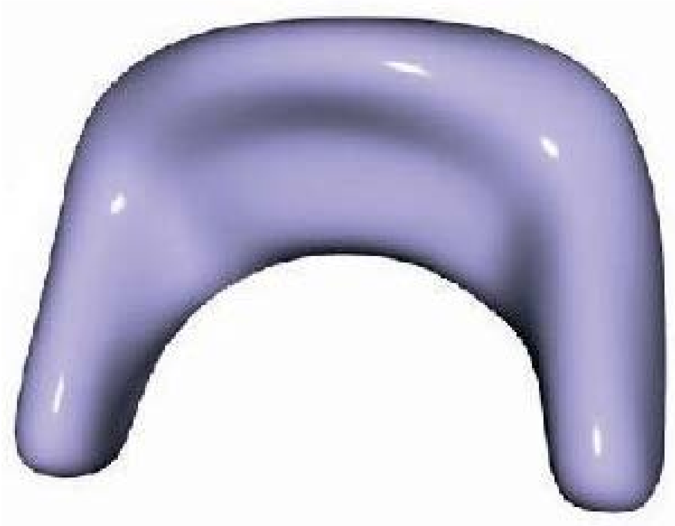
University of Science and Technology of China

Problem: Fitting Data with Smooth Surface



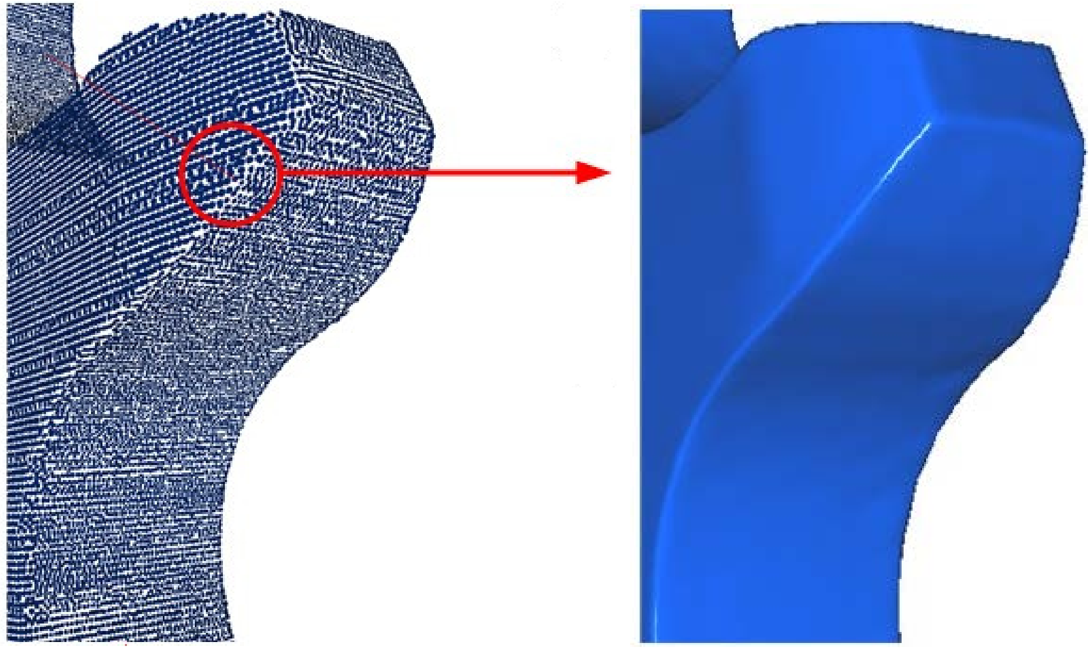


Point Cloud

A Smooth Surface

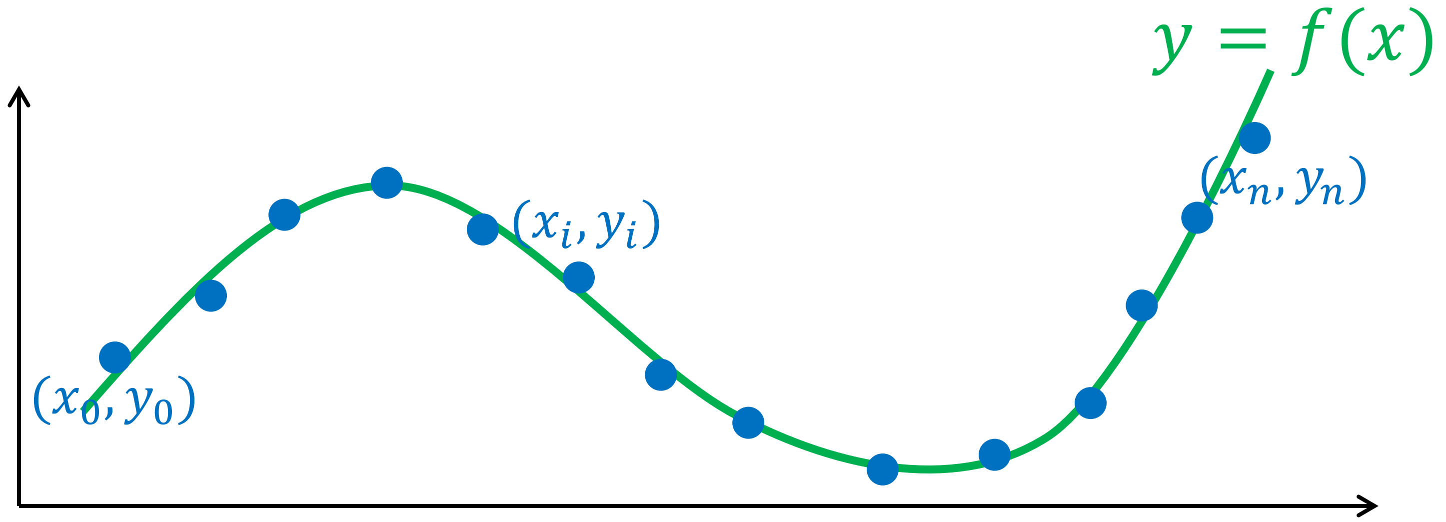
Problem: Fitting Data with Smooth Surface

- Challenging: capturing sharp features



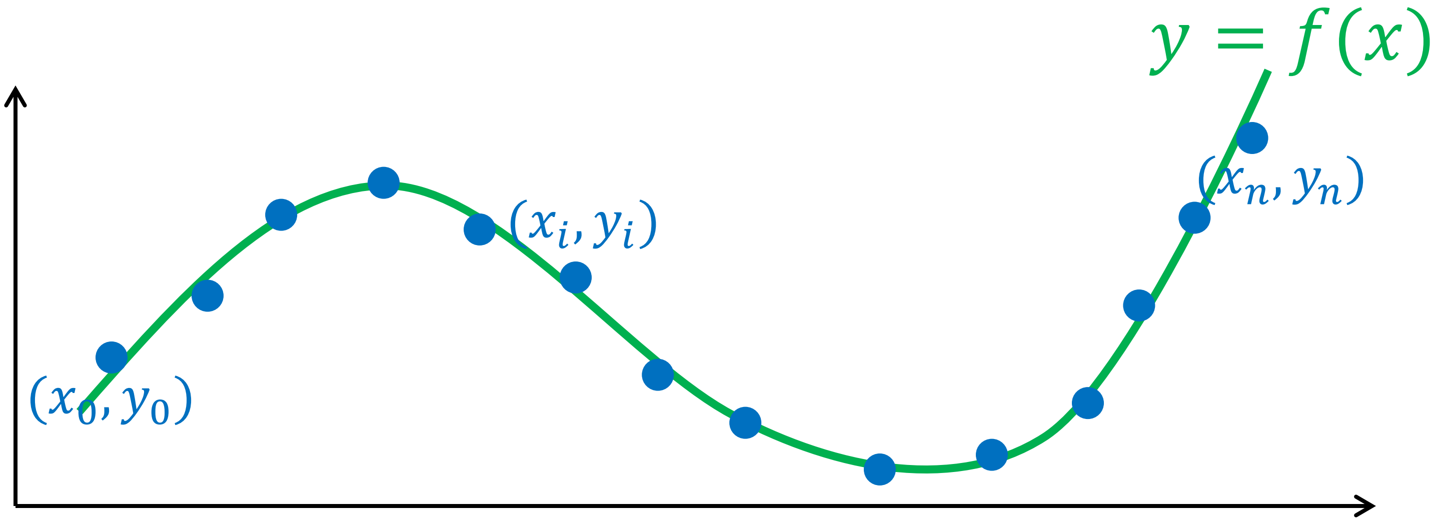
Problem: Data Fitting

- **Input:** A set of points $(x_i, y_i), i = 0, \dots, n$
- **Output:** A function which fits the point set



Data Fitting: What Function?

- What type of functions for $f(x)$?



Data Fitting: Function Space

- Assuming: basis functions $\{b_i(x), i = 0, \ldots, m\}$
- Finding a member in a family of functions:

$$f(x) = \sum_{k=0}^{m} \alpha_i b_i(x)$$

i.e., representing $f(x)$ as a (coefficient) point $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_m)$ in R^{m+1}

⟺ Finding optimal $(\alpha_0, \alpha_1, \ldots, \alpha_m)$ by minimizing the fitting error:

$$\min_{\alpha} (y_i - f(x_i))^2$$

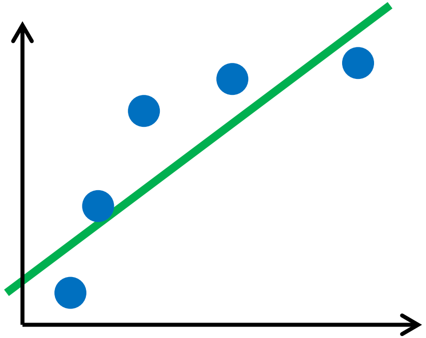
Data Fitting: Function Space

- Basis functions $\{b_i(x), i = 0, \dots, m\}$
  - Polynomial function basis $\{1, x, x^2, \dots, x^m\}$
  - Trigonometric function basis $\{1, \sin(x), \cos(x), \sin(2x), \cos(2x), \dots\}$
  - Exponential function basis $\{1, e^x, e^{2x}, \dots, e^{mx}\}$
  - …
- If we choose enough number of basis $(m = n)$, the fitting error can be 0!
  - the fitting function $f(x)$ is an interpolation

But…

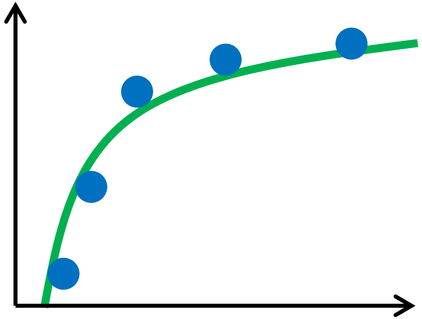
Overfitting Problem

- How to choose appropriate number of basis?



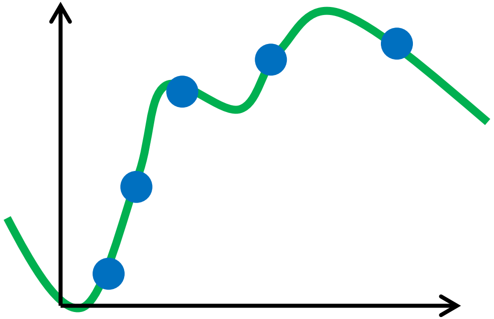
$\alpha_0 + \alpha_1 x$

High bias
(underfitting)



$\alpha_0 + \alpha_1 x + \alpha_2 x^2$

"Just right"



$\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \alpha_4 x^4$

High variance
(overfitting)

Sparse Representation

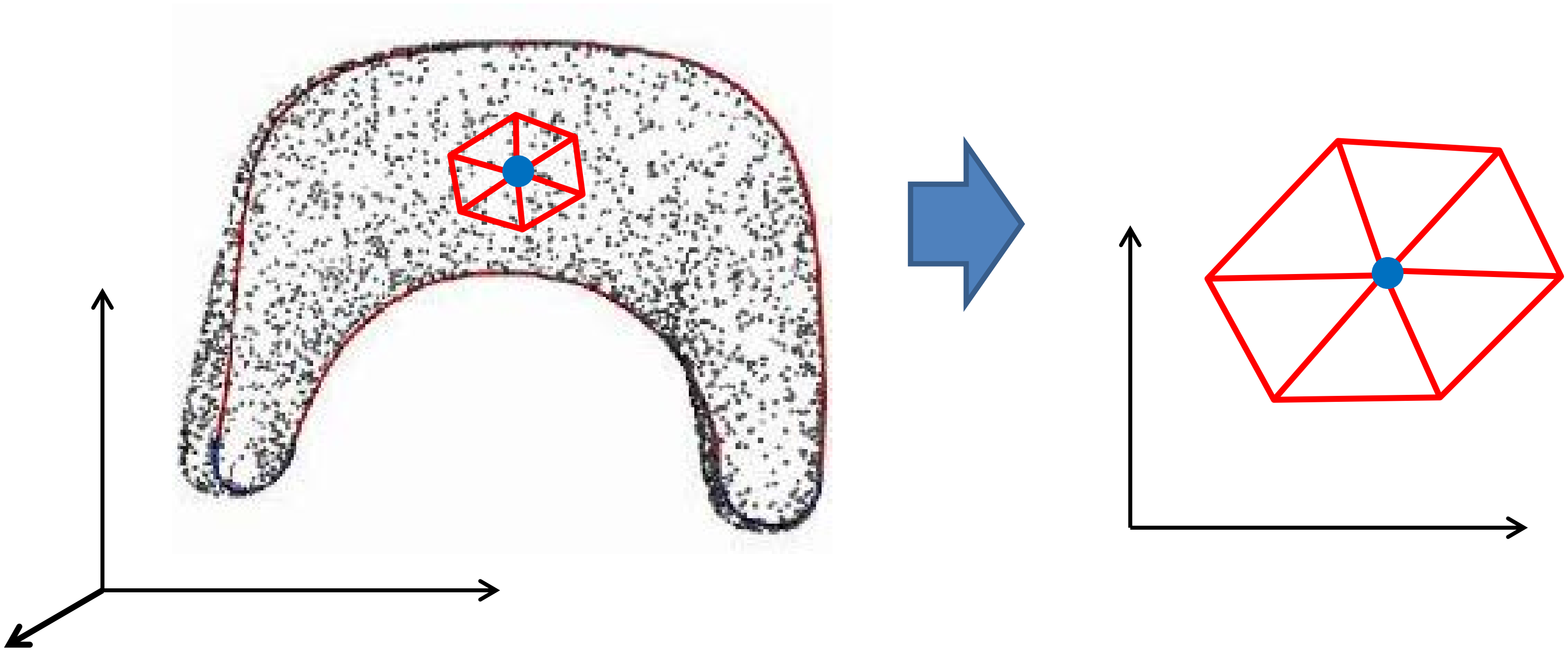
- An over-complete dictionary (atom functions)
  - Finding a 'best' fit from larger family of functions
- Choose as least number of basis as possible
  - most of the elements of $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_m)$ are 0
  - i.e., $\|\alpha\|_0$ (number of non-zero elements) is less than some threshold δ

$$\min_{\alpha}(y_i - f(x_i))^2$$

$$\min_{\alpha}(y_i - f(x_i))^2 \quad \text{s.t. } \|\alpha\|_0 \leq \delta$$

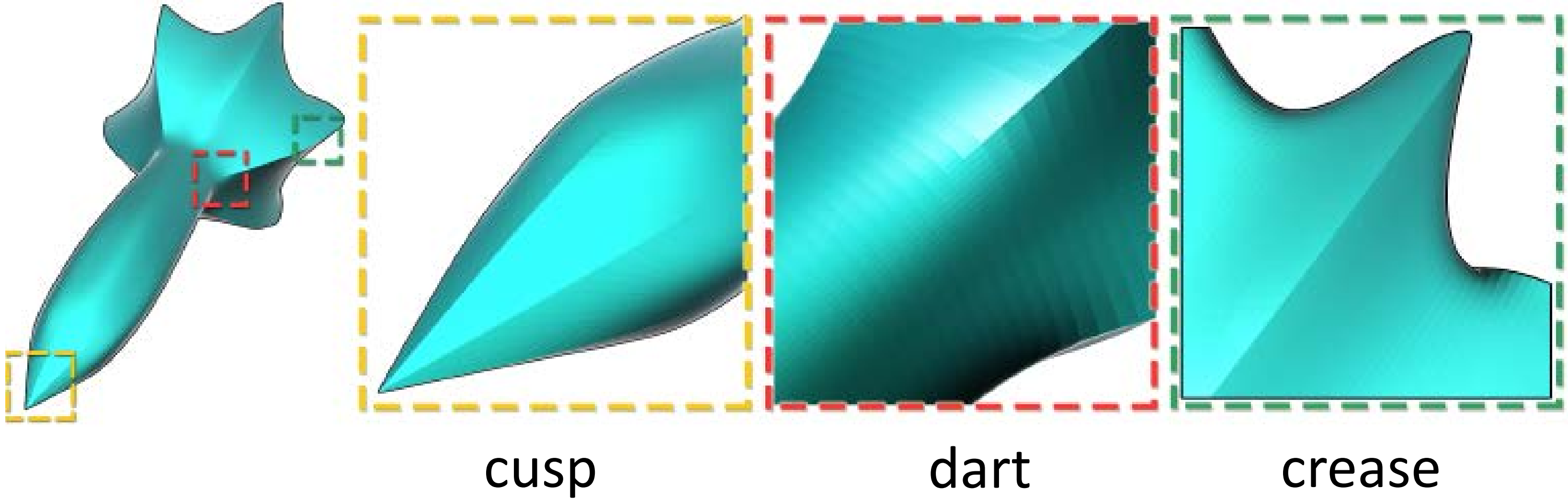
3D Surface Case

Parameterization of Local Patch



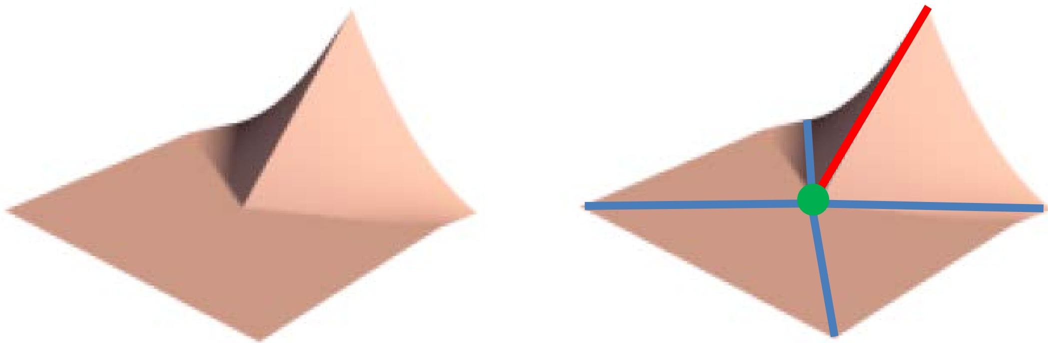
Representing Sharp Features?

- Smooth functions cannot represent C^0 sharp features



Idea: C^0 Atom Functions

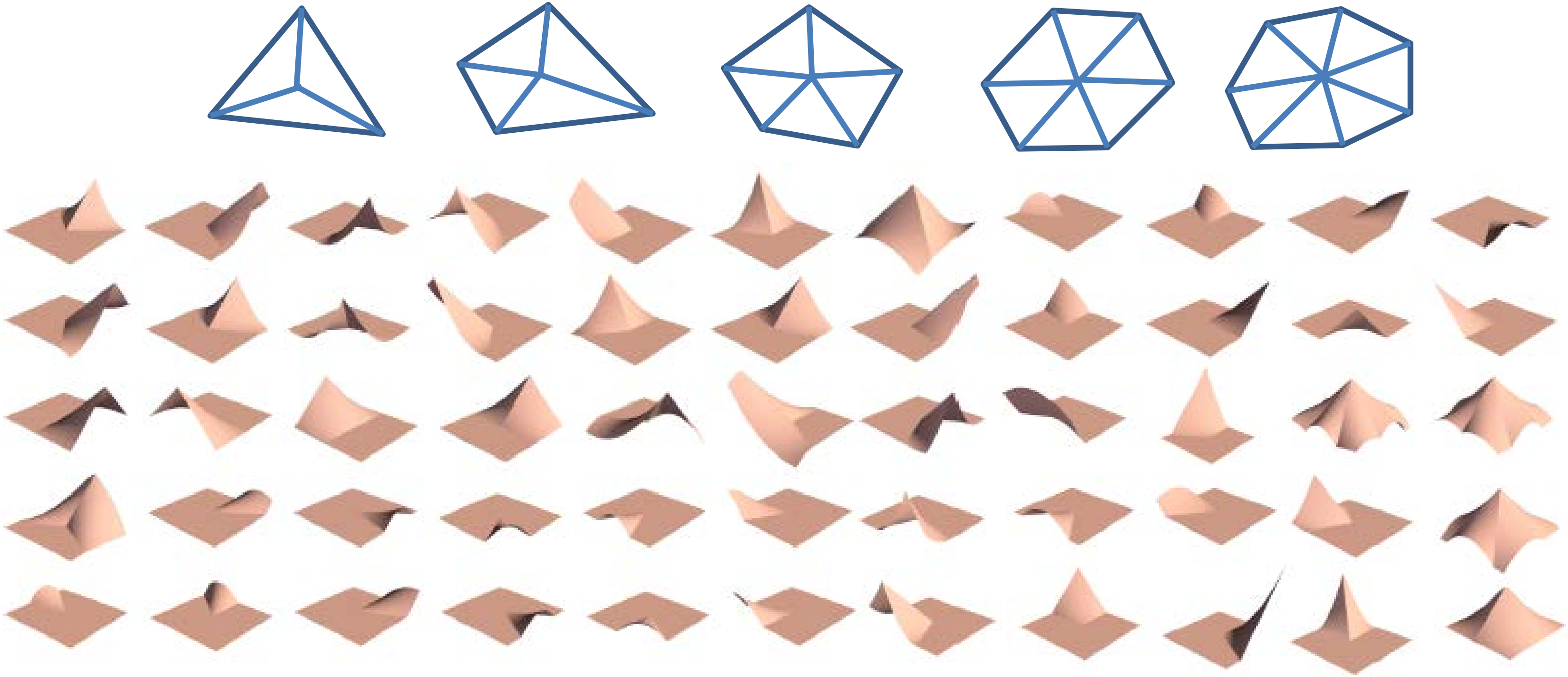
- Introduce C^0 atom functions in the dictionary
  - Shape functions representing non-smooth finite elements in FEM
- Each atom function
  - A bilinear quadrilateral element shape function defined on one edge



A C^0 atom function defined on the edge (in red) of a vertex (in green) with valence 5

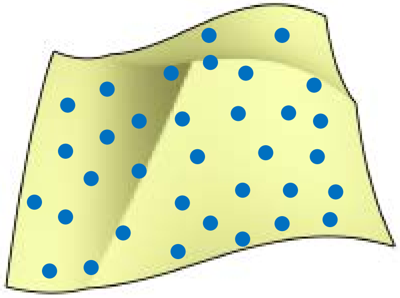
C^0 Atom Functions

- A total of 55 shape functions for vertices with valence 3-7
  - Add more atom functions for vertices with valence > 7

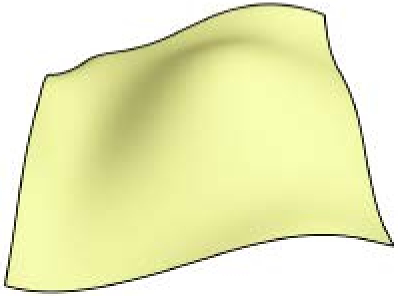


Dictionary: Total Atom Functions

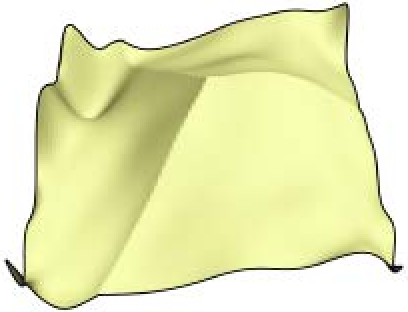
- 120 polynomial functions with degree up to 14
- 55 C^0 atom functions



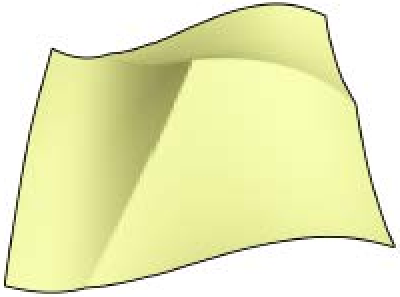
A patch with sharp features



Underfitting



Overfitting

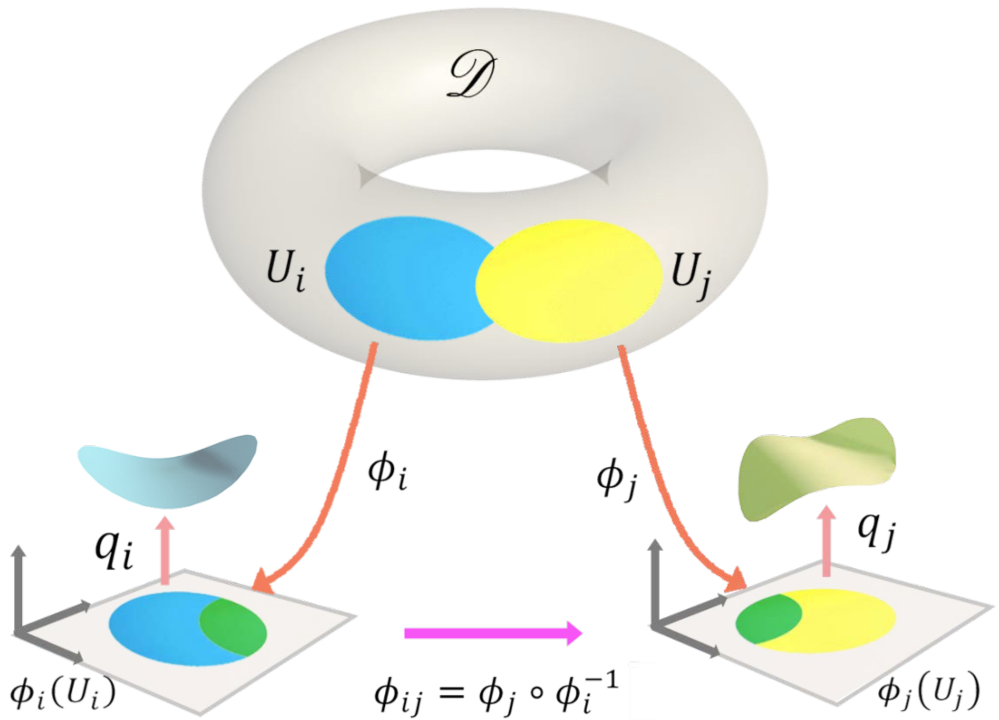


Result by sparse fitting

How to stitch local patches?

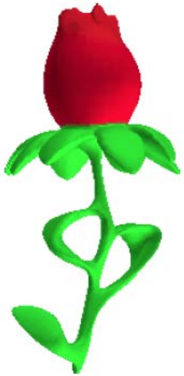
Manifold

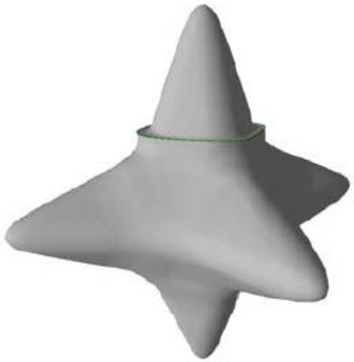
Manifold Representation



Previous Works on Manifold Construction

- [Grimm and Hughes 1995]
- [Ying and Zorin 2004]
- [Gu et al. 2006]
- [Wang et al. 2008]
- [Della Vecchia and Juettler 2009]
- [Tosun and Zorin 2011]
- …

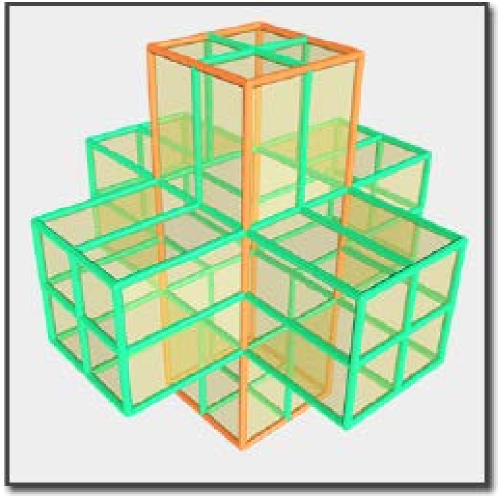




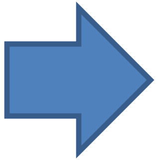
Application 1:
Approximating Subdivision Surface

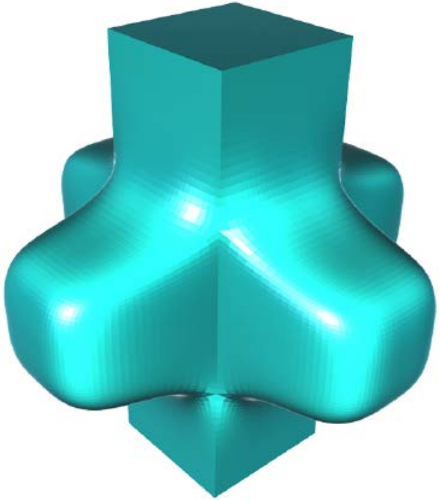
Problem

- Construct manifolds to approximate subdivision surfaces with sharp features
  - Orange lines are specified as sharp features



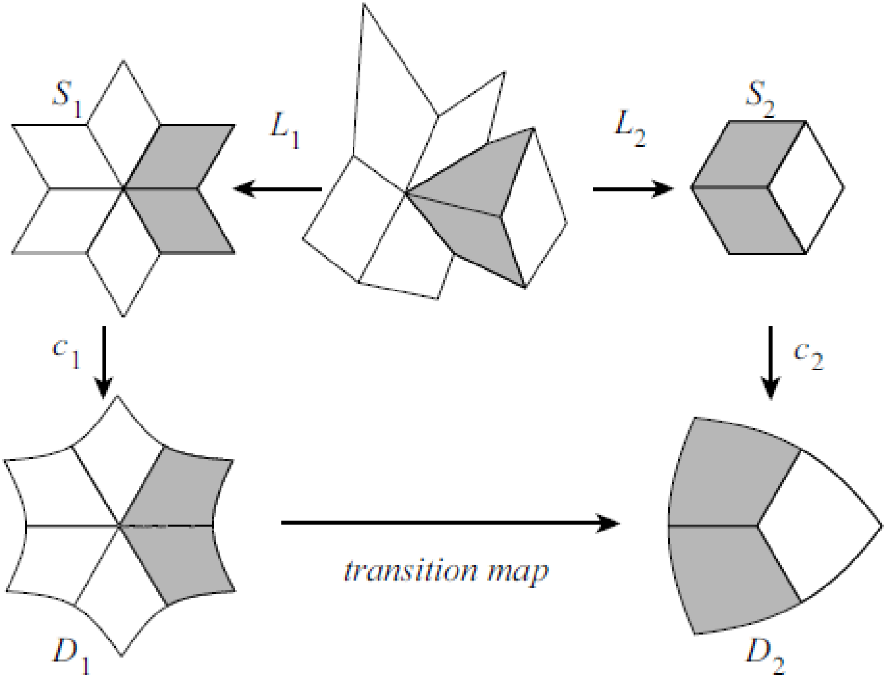
Input Mesh





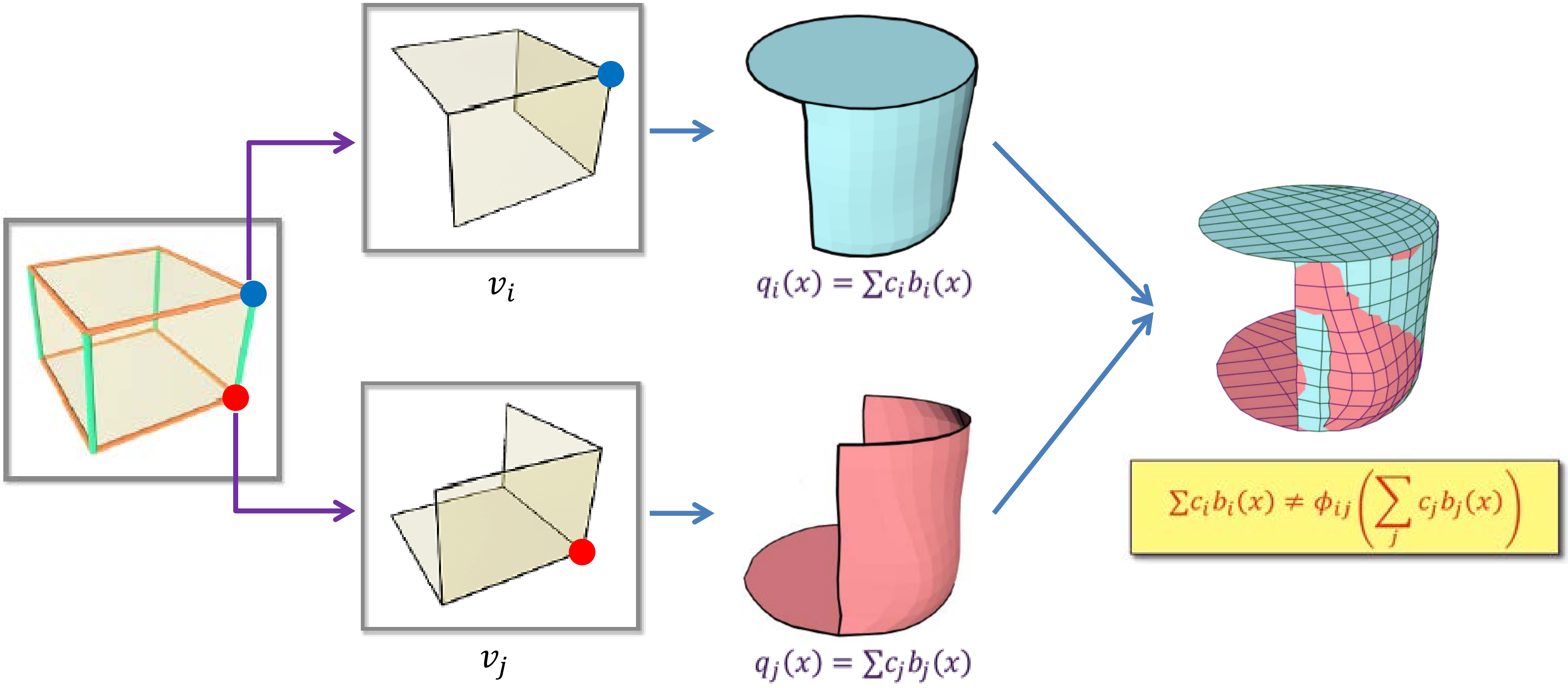
Manifold Surface

Construction of the Charts

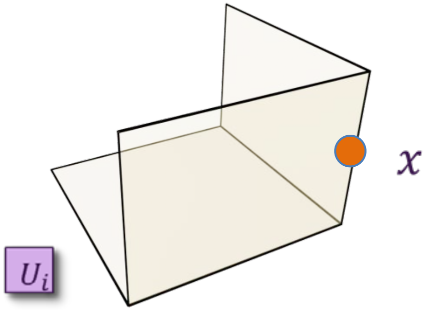


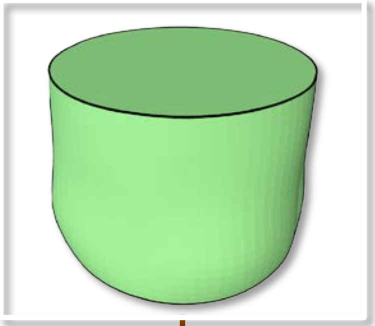
[Ying and Zorin 2004]

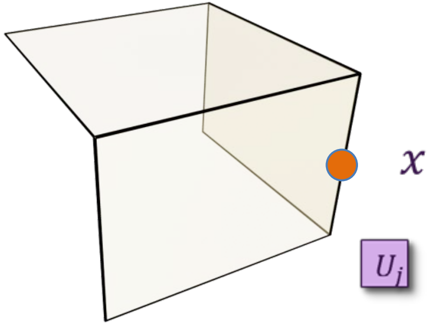
Incompatible Local Patches

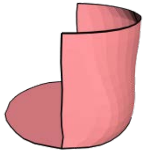


Global Fitting Error



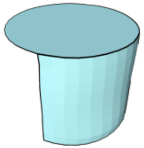




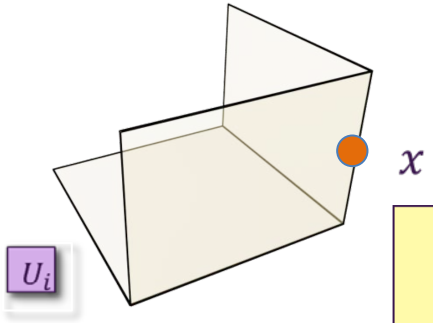


$$q_i(x) = \sum_{l=1}^{L} c_{li} b_l(\phi_i(x))$$

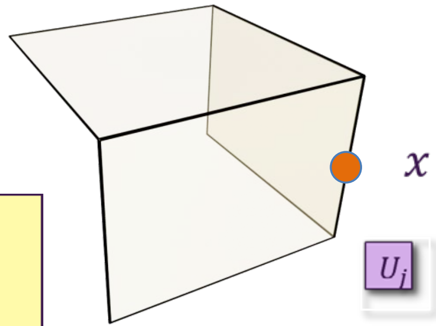
$$q_j(x) = \sum_{l=1}^{L} c_{lj} b_l(\phi_j(x))$$

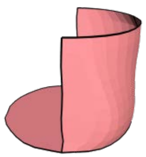


Global Fitting Error



$$E(x) = \left(\sum_{j:x \in U_j} \omega_j(\phi_j(x)) q_j(\phi_j(x)) - h_x \right)^2$$

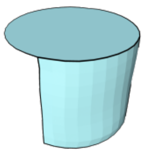




$$q_i(x) = \sum_{l=1}^{L} c_{li} b_l(\phi_i(x))$$

$\oplus$

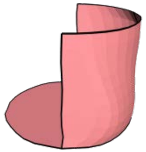
$$q_j(x) = \sum_{l=1}^{L} c_{lj} b_l(\phi_j(x))$$



Optimization Solver

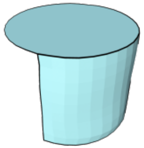
$$E(x) = \left(\sum_{j:x \in U_j} \omega_j(\phi_j(x)) q_j(\phi_j(x)) - h_x \right)^2$$

Forward error evaluation



$$q_i(x) = \sum_{l=1}^{L} c_{li} b_l(\phi_i(x))$$

$$q_j(x) = \sum_{l=1}^{L} c_{lj} b_l(\phi_j(x))$$

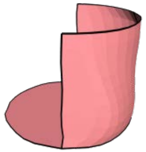


Optimization Solver

$$E(x) = \left(\sum_{j:x \in U_j} \omega_j(\phi_j(x)) q_j(\phi_j(x)) - h_x \right)^2$$

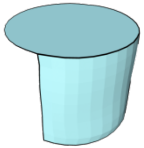
Forward error evaluation

Backward Update

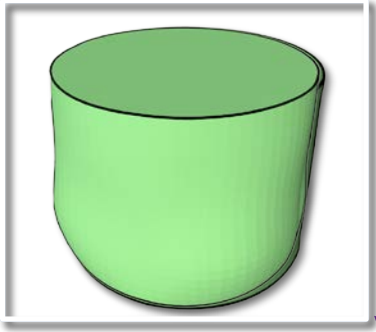


$$q_i(x) = \sum_{l=1}^{L} c_{li} b_l(\phi_i(x))$$

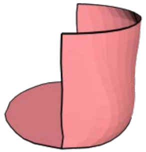
$$q_j(x) = \sum_{l=1}^{L} c_{lj} b_l(\phi_j(x))$$



Optimization Solver

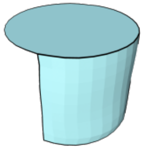


Final Result



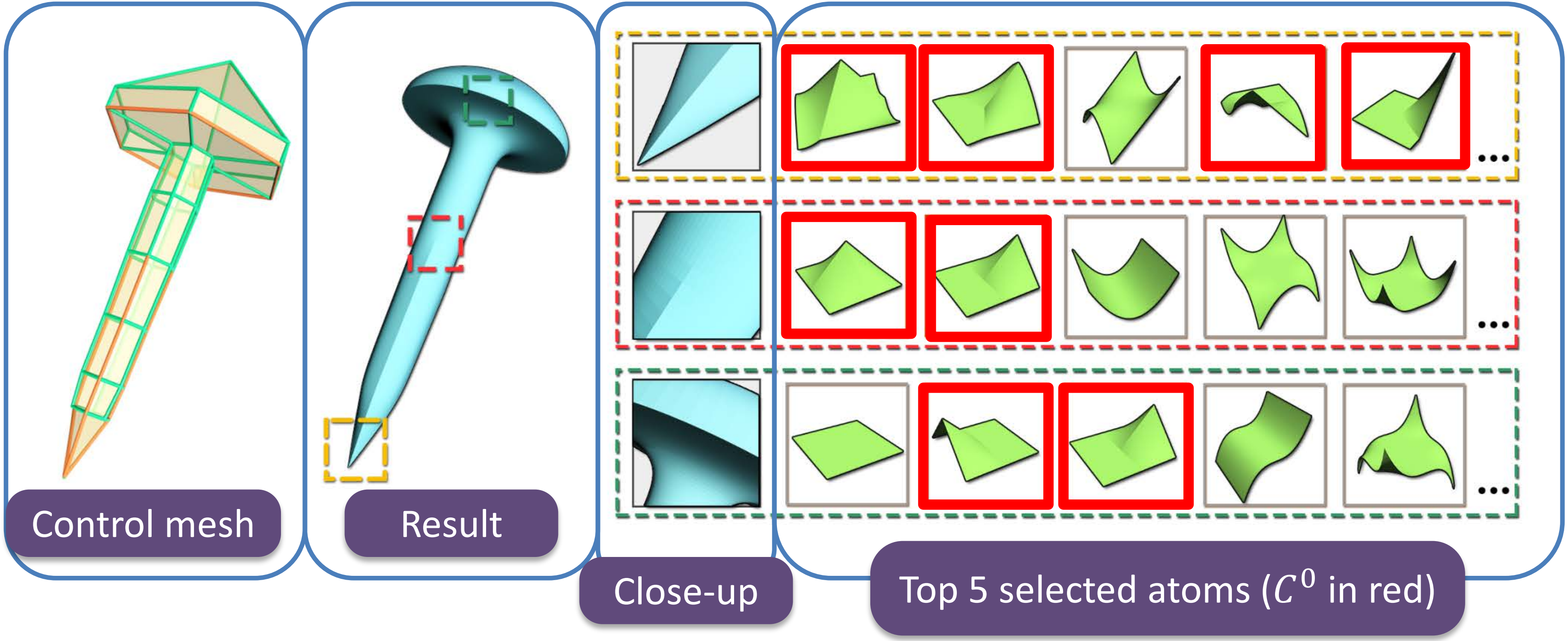
$$q_i(x) = \sum_{l=1}^{L} c_{li} b_l(\phi_i(x))$$

$$q_j(x) = \sum_{l=1}^{L} c_{lj} b_l(\phi_j(x))$$

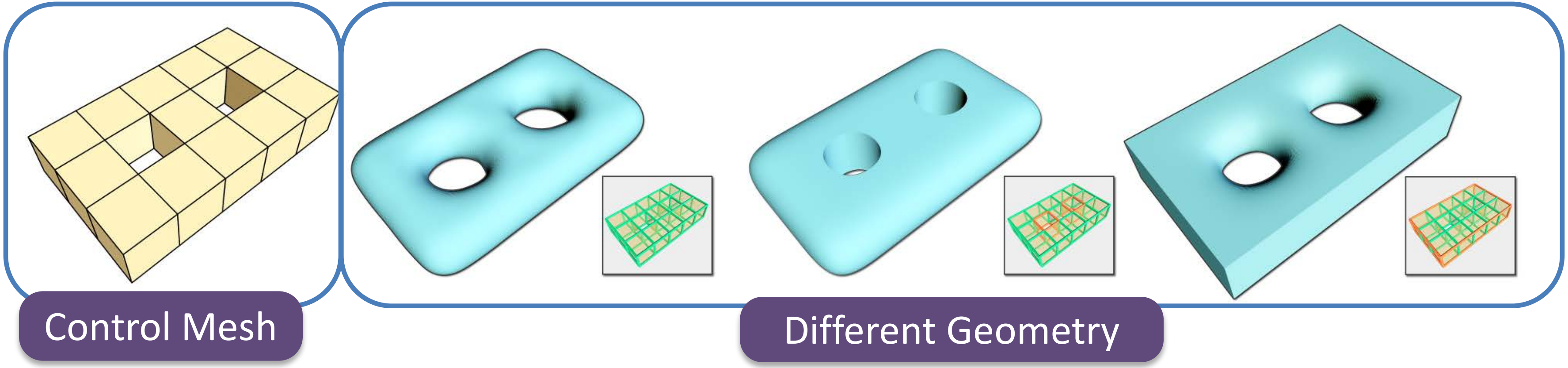


Local sparse optimization and
Global sparse optimization iteratively

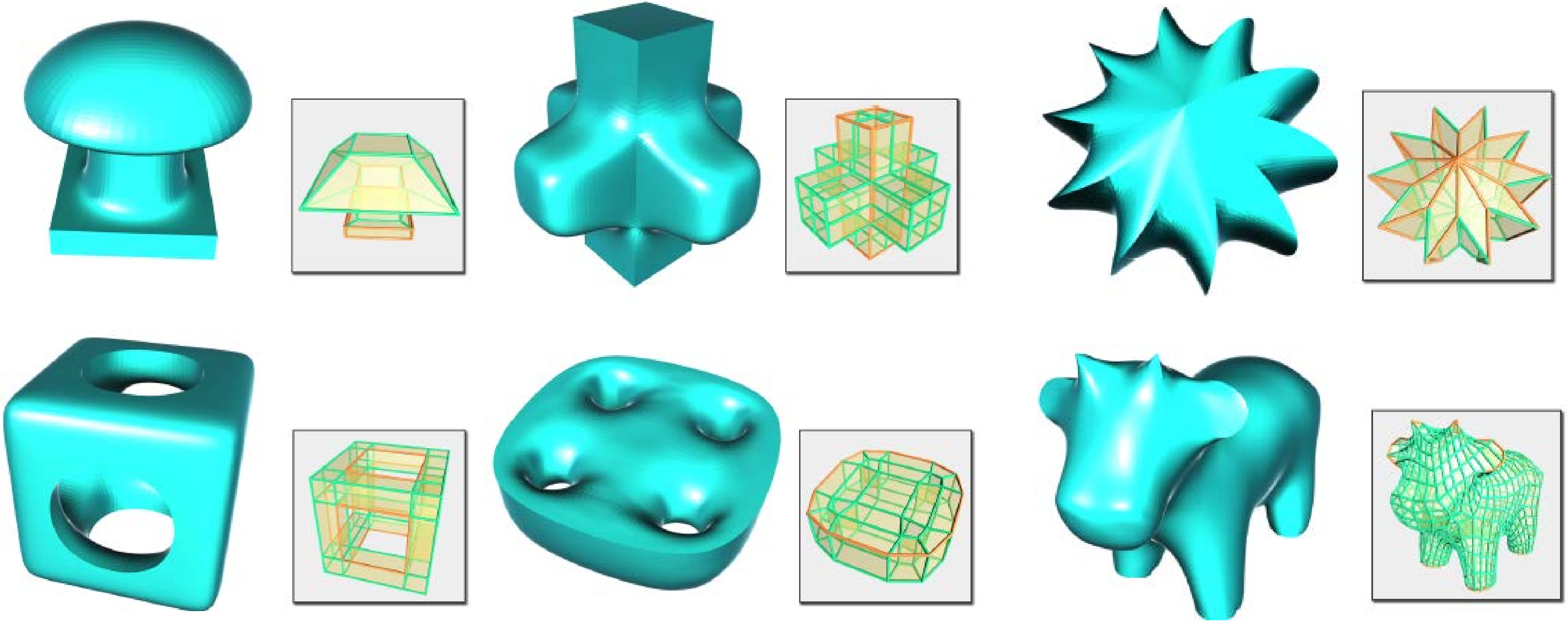
Example



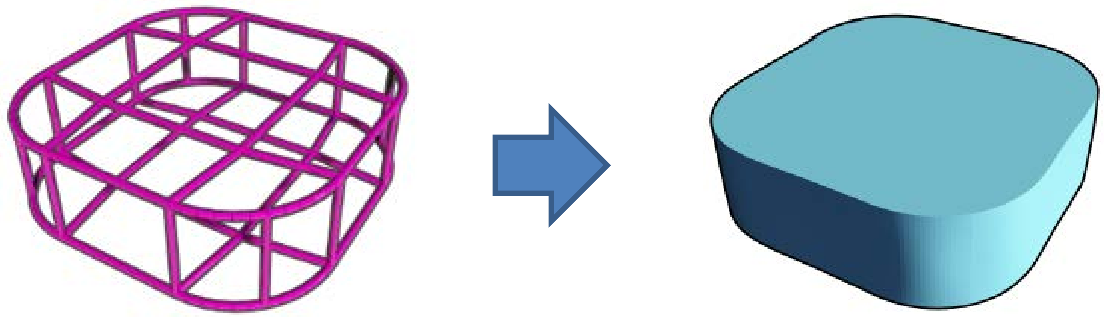
Different Subdivision Rules



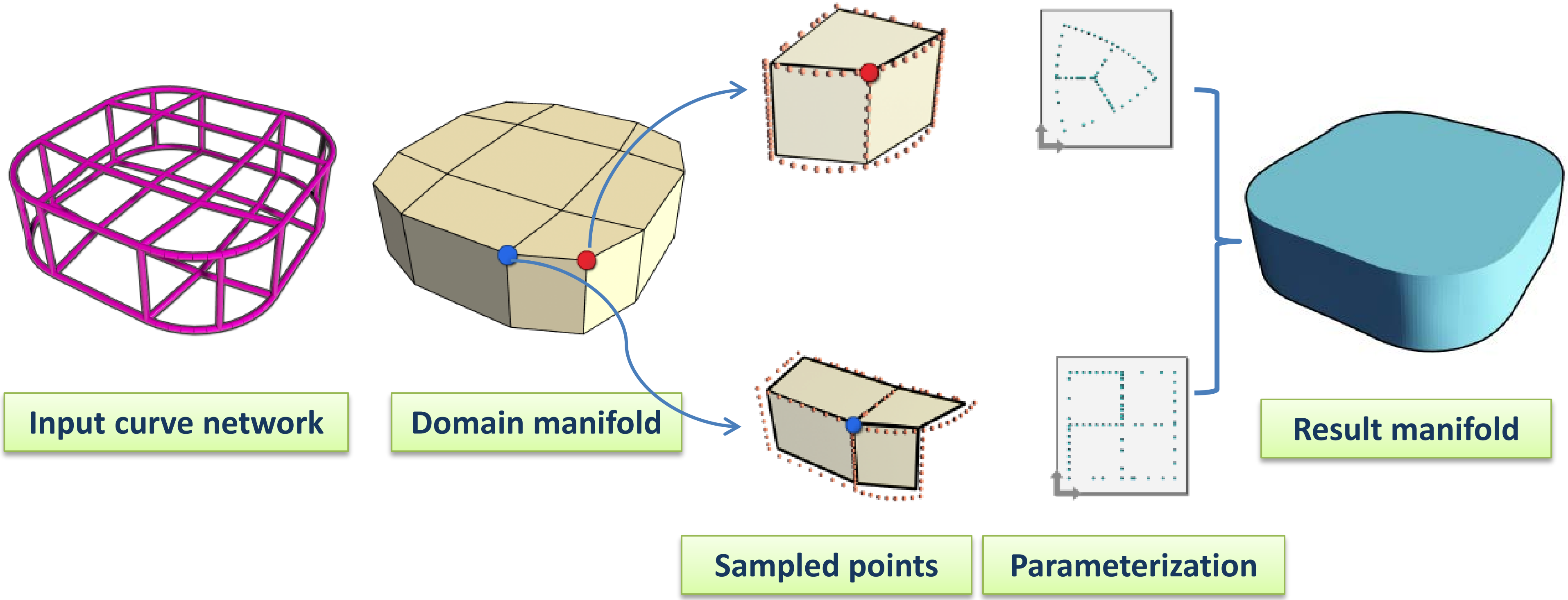
More Examples



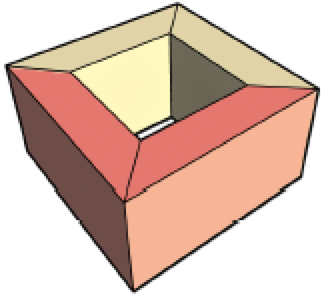
Application 2:
Manifold from Curve Network



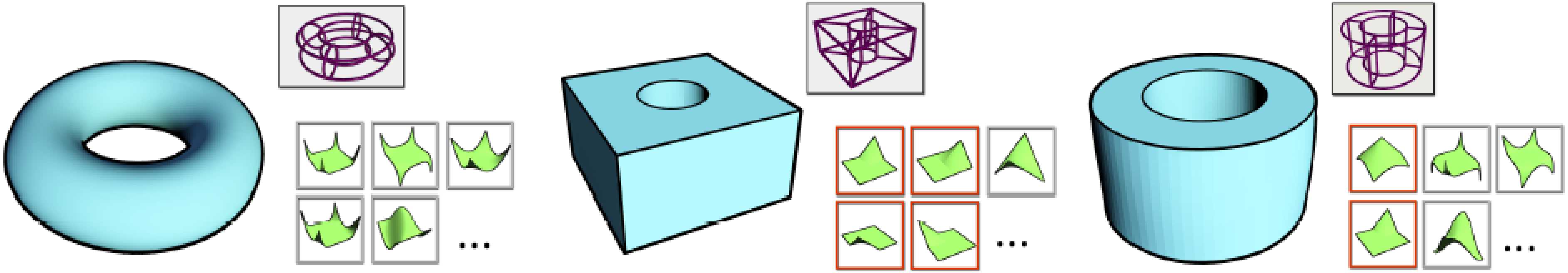
Sampling Points on Curves



Results

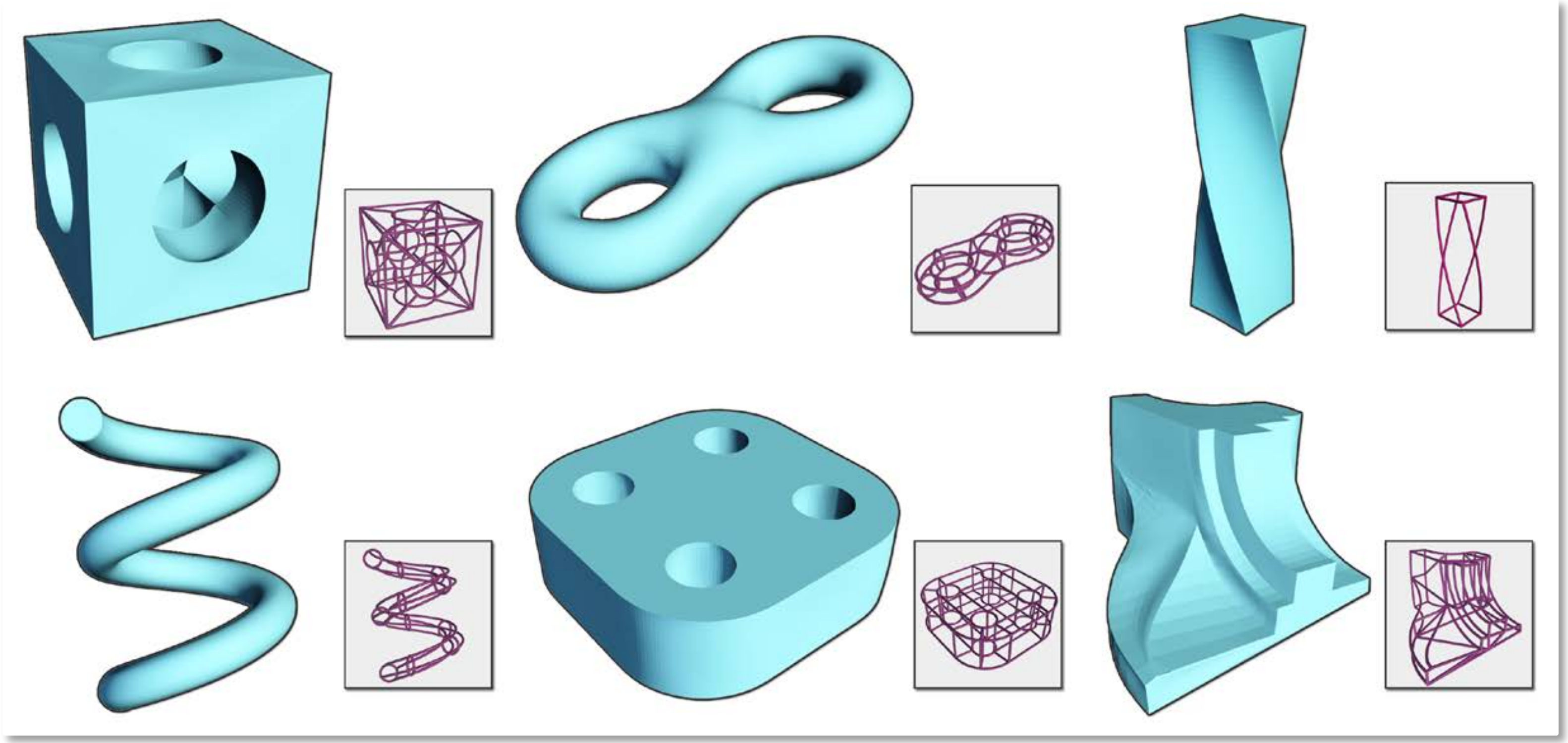


Domain mesh



Different manifold surfaces from different geometries

Results



Conclusions

- A novel manifold construction method
- Sparse representation for local geometry
- Global compatibility
- Representing sharp features

Future Work

- No guarantee to capture all geometric features
  - Learning geometry features
- Slow sparsity optimization
  - Speed up
- Other applications
  - Surface reconstruction, denoising, and compression

Thank you!

Ligang Liu, http://staff.ustc.edu.cn/~lgliu