Pseudo-one-dimensional surface order in tricritical Potts models

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- Introduction to surface criticality
  - 3D Ising model with surfaces
  - Theoretical predictions for 2D systems
- Numerical results
- Conclusions

Cooperate with

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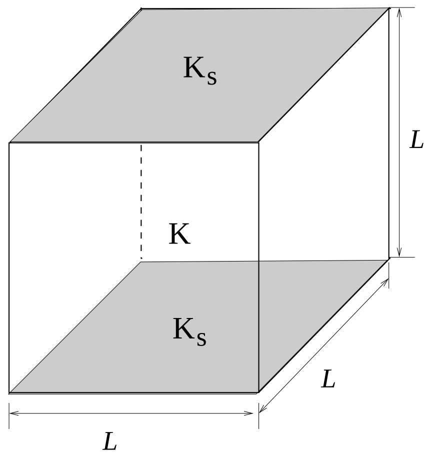
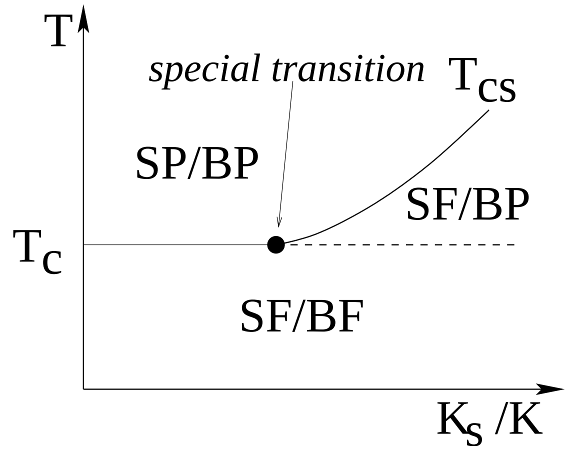
Introduction

- *3D Ising model*





Hamiltonian:

$$\mathcal{H}/k_{\mathrm{B}}T = \overbrace{-K\sum_{nn} s_i s_j - H\sum s_i}^{\text{bulk}} \underbrace{-K_s\sum_{nn} s_i s_j - H_s\sum s_i}_{\text{surface}}$$

$K-$ coupling constant; $H-$ magnetic field;

1. ordinary transition– $(2+1)$ universality:
   3D bulk: y_t, y_h + surface: y_{h1}
2. special transition – $(2+2)$ universality:
   3D bulk: y_t, y_h + surface: y_{t1}, y_{h1}
3. surface transition – 2D Ising universality:
   2D bulk: y_t, y_h
4. extraordinary transition
   3D bulk: y_t, y_h + $\langle s_0 s_r \rangle_{\mathrm{surf}} = c + ar^{-2X}$

- *2D Ising model*: surface– *one-dimensional* edge

  NO surface, special, extraordinary transitions

  ordinary transition: $y_{h1} = 1/2$

- *2D critical Potts model* (conformal field theory)

  $$\boxed{y_{h1} = (3 - 2y_t)/(3 - y_t)} \qquad \text{——— (1)}$$

- *2D tricritical Potts model* ($y_t > 3/2$)

  assume: Eq. (1) is still valid $\Rightarrow y_{h1} < 0$

  surface magnetic field: *irrelevant*

RESULTS

- *Tricritical $q = 1$ Potts model on square lattice*

1. periodic boundaries

$$\mathcal{H}/k_{\rm B}T = -J \sum_{nn} \sigma_i \sigma_j + D \sum \sigma_i \,, \quad (\sigma = 0, 1)$$

$$\Updownarrow \; s = 2\sigma - 1$$

Ising model

$$\mathcal{H}/k_{\rm B}T = -K \sum_{nn} s_i s_j - H \sum s_i \,, \quad (s = \pm 1)$$

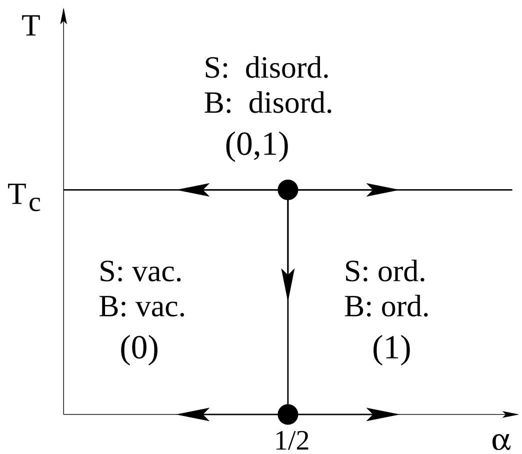
with $J = 4K$ and $D = -H + 8K$

2. free boundaries in y direction

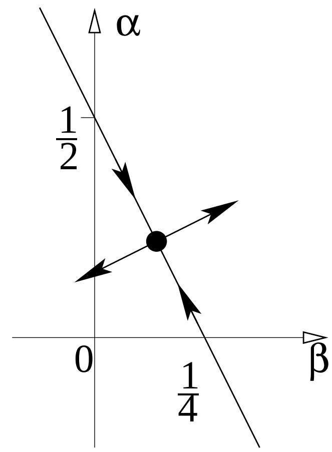
at surfaces: $\alpha = J_s/J - 1$, and $\beta = D_s/D - 1$

$\Updownarrow$ Ising model in surface magnetic field

$$\boxed{H_s = -(1 - 2\alpha - 4\beta)J/2}$$



(1) $\beta = 0$



(2) $T = T_c$

We have $y_{t1} = 1/2$

- *Tricritical Ising model*

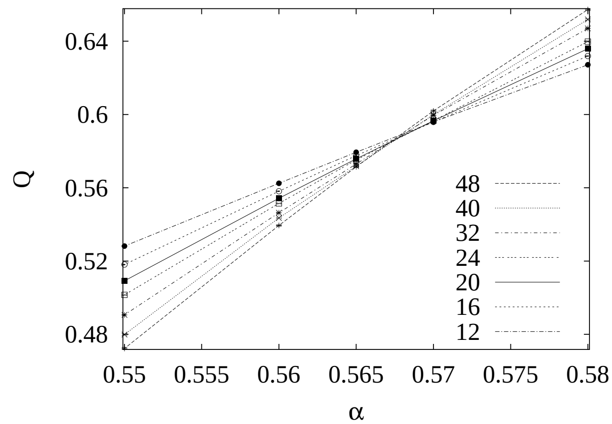
$$\mathcal{H}/k_{\mathrm{B}}T = -J\sum_{nn} s_i s_j + D\sum s_i^2\ , \quad (\sigma = 0, \pm 1)$$

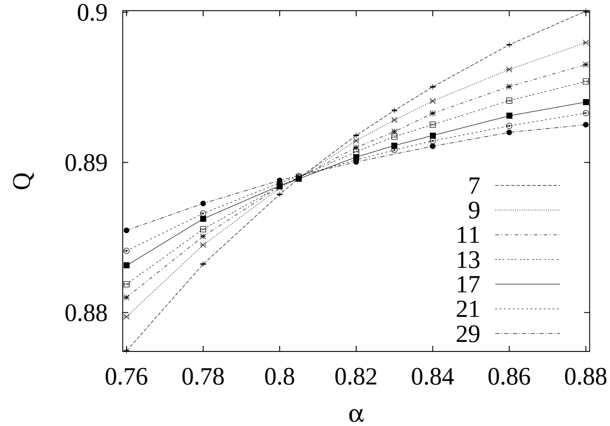
1. tricritical point: (transfer matrix):

   $K_{tc} = 1.64317590(1), D_{tc} = 3.23017970(2)$,

   $h_{tc} = 0$, and $V_{tc} = 0.4549506(2)$

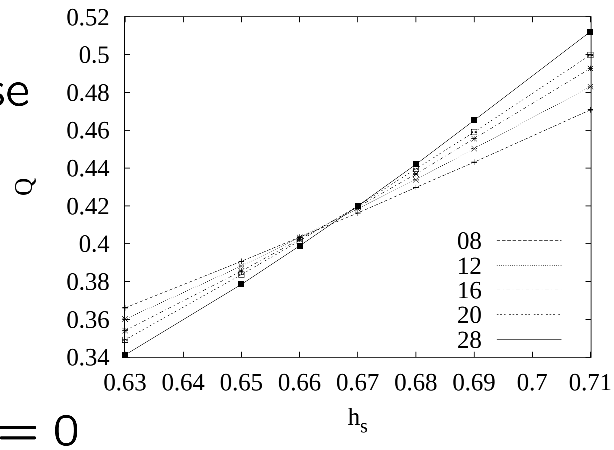
2. $K = K_{tc}, D = D_{tc}, h_s = 0$



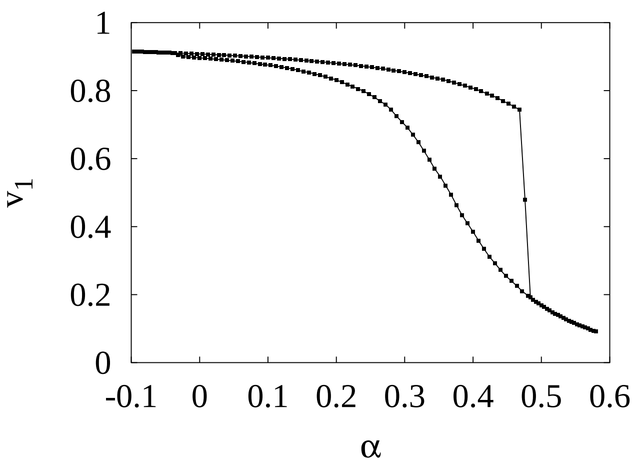


3. $K = K_{tc}, D = D_{tc}, h_s$

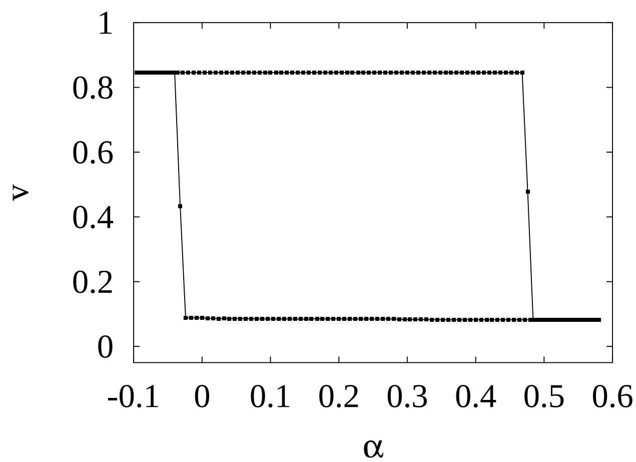
A second-order phase transition occurs at: $h_s = 0.6776(15)$



4. $K > K_{tc}, D > D_{tc}, h_s = 0$



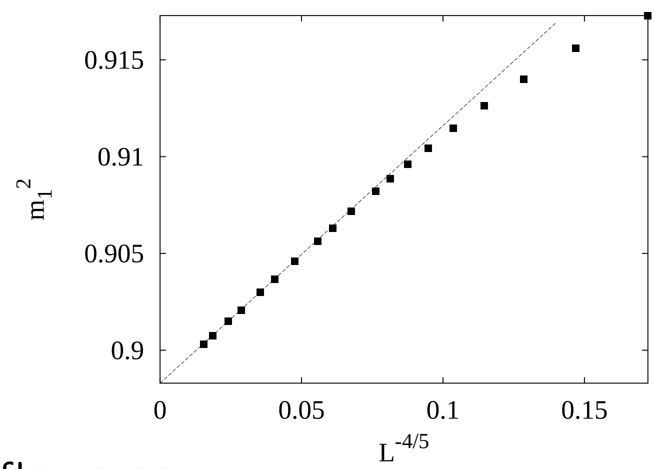
surface



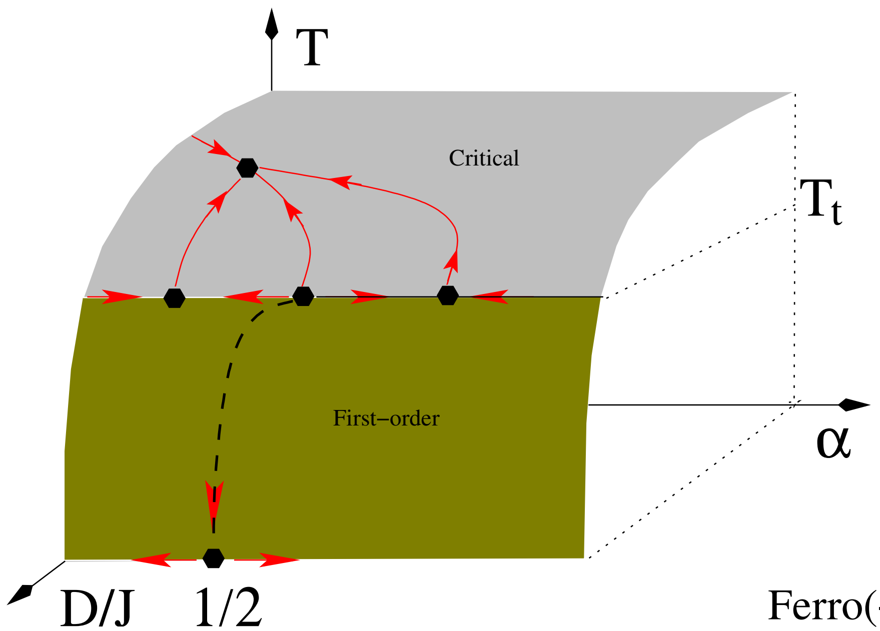
bulk

5. $K = K_{tc}, D = D_{tc}, h_s = 0, \alpha = 0.805$

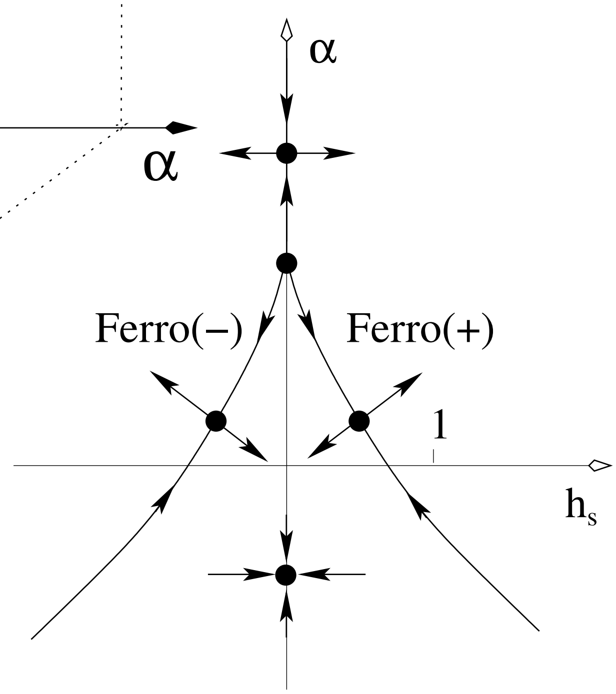
$y_{h1} = 0.601(2) \approx 3/5$



Thus, we assume RG flows as



(1) $h_s = 0$



(2) $K = K_{tc}, D = D_{tc}$

- *Tricritical $q = 3$ Potts model*

$$\mathcal{H}/k_\mathrm{B}T = \mathcal{H}_\mathrm{bulk}/k_\mathrm{B}T - K_s \sum_{\langle i,j \rangle} \delta_{\sigma_i,\sigma_j} - D_s \sum_i \delta_{\sigma_i,0}$$

$$-h_s^{(1)} \sum_i \delta_{\sigma_i,1} + \frac{1}{2} h_s^{(1)} \sum_i (\delta_{\sigma_i,2} + \delta_{\sigma_i,3})$$

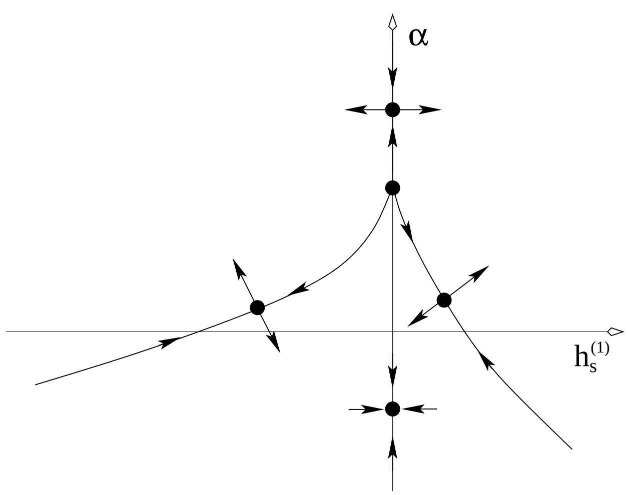
$$(\sigma_i = 0, 1, 2, 3)$$

1. tricritical point: (transfer matrix):

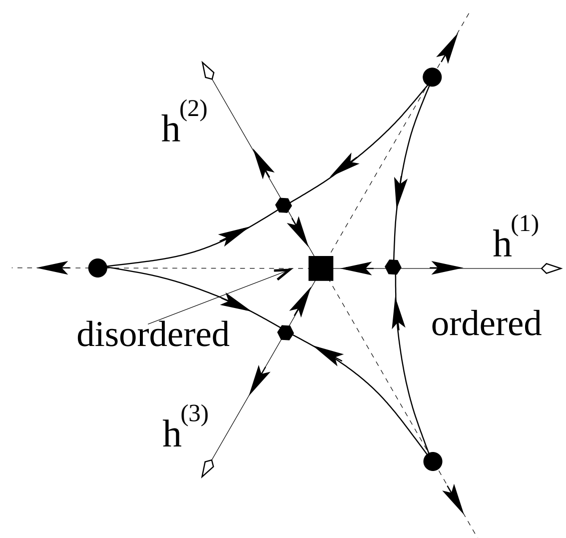
   $K_{tc} = 1.649913(5), D_{tc} = 3.152173(10)$,

   $h_{tc} = 0$, and $V_{tc} = 0.34572(5)$

2. $K = K_{tc}, D = D_{tc}$



(1) $h_s^{(2)} = h_s^{(3)} = 0$



(2) $\alpha = 0$

Conclusions

- 2D systems can also have very rich surface phase transitions.

- This subject is largely unexplored, and further investigations are desirable.