Response to "Charge distributions and coefficients of potential for a system of conductors in electrostatic equilibrium"

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The authors appear to be objecting to writing the actual charge density in the form $\sigma_j = \langle \sigma_j \rangle f_j$, that is, as the product of the average charge density times a function of position that clearly must depend on the size, shape, orientation, distances, and charges of the other conductors. When $\langle \sigma_j \rangle$ is written as Q_j/S_j, one might then make the assumption that the actual charge density has been taken to be proportional to the total charge on the given conductor. Actually, f_j depends only on the charge ratios. If we set $Q_k = g_{kj}Q_j$, it is easy to show that $f_j/S_j = \Sigma_k \gamma_{jk} g_{kj}$ and that my Eq. (6-11) for the potential is again obtained with exactly the same expression for p_{ij} given by Eq. (6-12).

It turns out that Sommerfeld also discussed this problem in exactly the same way.[1] In terms of the present notation, he says that one can "... put $\sigma_j = (f_j/S_j)Q_j$, where (f_j/S_j) is the distribution of unit charge on S_j (in the presence of the remaining conductors!)." He then proceeds to obtain an expression for p_{ij} that is again the same as my Eq. (6-12).

If one's course organization allows it, an instructor can take the approach given by Jefimenko[2] in which these coefficients are introduced and shown to represent only geometrical relationships by means of using Laplace's equation and an appropriate uniqueness theorem. Recently, an elegant expression for the related coefficients of capacitance was given by Uehara[3]; however, since this involves Green's functions, it is probably inappropriate for an intermediate level class.

[1] A. Sommerfeld, *Electrodynamics* (Lectures on Theoretical Physics, Vol. III), translated by E. G. Ramberg (Academic, New York, 1952), p. 70.
[2] O. D. Jefimenko, *Electricity and Magnetism* (Appleton-Century-Crofts, New York, 1966), pp. 174–175.
[3] M. Uehara, Am. J. Phys. **54**, 184 (1986).