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Physical implications of Coulomb's Law

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Abstract
We examine the theoretical and experimental foundations of Coulomb's Law and review the various roles it plays not only in electromagnetism and electrodynamics, but also in quantum mechanics, cosmology, and thermodynamics. The many implications of Coulomb's Law draw attention to its fundamental importance within virtually all branches of physics and make this elementary yet profound law one of the most useful of all scientific tools.

1. Introductory historical outlook

Few investigations in physics have enjoyed as sustained an interest as have tests of Coulomb's Law. As has been the case with most of the fundamental laws of physics, it was discovered and elucidated through observations of basic phenomena. In his research, Coulomb was interested in the mutual interactions between electric charges, a topic that had been studied previously by Priestley [1], and in fact even earlier, in 1755, during the experimental work of Franklin [2].

Franklin (1706–1790) was an American printer, writer, politician, diplomat, and scientist. He is credited with the invention of such practical everyday items as bifocal eyeglasses and a free-standing, wood-burning heater called the 'Franklin stove'. His principal connection to electrical experimentation came via his investigations of the properties of Leyden Jars[4]. He is also commonly credited with giving the names 'positive' and 'negative' to the two opposite species of electrical charge, although his assignment convention was eventually reversed.

Also living in America at the time was Priestley (1733–1804)[5], an English chemist and amateur natural philosopher who had broad scientific interests in physics, electricity, magnetism, and optics, in addition to chemistry. He was a politically involved Unitarian preacher and a sympathizer with the French Revolution, and these aspects of his life forced him to move to America with his family in 1794. Priestley is credited with the discovery of oxygen in 1774, which he produced by focusing sunlight on mercuric oxide. During his studies of this 'dephlogisticated air', he noticed that it made him light-headed and that it had a similar effect on animals.

The background studies underpinning Coulomb's Law began when Franklin took a small sphere made of cork and placed it inside a charged metallic cup (see figure 1) and observed that it did not move, suggesting that there was no interaction between it and the cup. After Franklin communicated his finding to Priestley, the Englishman explained the phenomenon in 1767, providing a line of reasoning analogous to that used by Newton [3] to formulate and enunciate the law of universal gravitation.

Underlying Newtonian gravity was the observation that the gravitational field inside a spherical shell of homogeneous material is null if the field is inversely proportional to the square of the distance r, i.e. if its intensity goes as r^{-2}. By approximating Franklin's cup as a spherical shell, Priestley deduced that the observed phenomenon should be physically

[4] In 1752, he flew a kite attached to a silk string in a thunderstorm, and showed that a metal key tied to the thread would charge a Leyden jar. (Incidentally, the next two people who attempted the experiment were killed in the effort.) His experiments with Leyden jars showed that they discharged more easily if near a pointed surface. He thus suggested the use of lightning rods.

[5] The objects of his chemical studies included 'fixed air' (carbon dioxide), 'nitrous air' (nitric oxide), 'marine acid air' (hydrogen chloride), 'alkaline air' (ammonia), 'vitriolic air' (sulfur dioxide), 'phlogisticated nitrous air' (nitrous oxide, laughing gas), and 'dephlogisticated air' (oxygen). His chemical writings were published in the three-volume *Experiments and Observations on Different Kinds of Airs* (1774–1777) and in the three-volume *Experiments and Observations Relating to Various Branches of Natural Philosophie* (1779–1786). By dissolving fixed air in water, he invented carbonated water. He also noticed that the explosion of inflammable air with common air produced dew. Lavoisier repeated this experiment and took credit for it. Priestley believed in the phlogiston theory, and was convinced that his discovery of oxygen proved it to be correct.

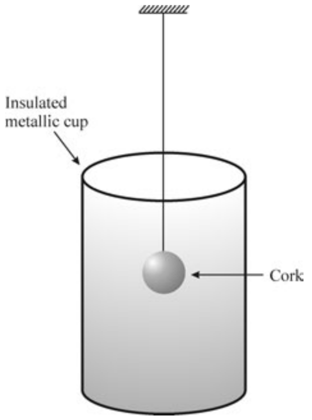

Figure 1. Franklin placed a small sphere made of cork inside a metallic cup, and when this was charged he observed that the sphere did not move, suggesting that there was no interaction between it and the cup. Priestley explained the phenomenon in 1767, providing a line of reasoning analogous to that used by Newton for the law of universal gravitation, implying that the field has an inverse square dependence on distance. In the subsequent tests, instead of the cork sphere inside a metallic cup, experimentalists considered a metallic shell enclosed within an outer charged shell or several concentric shells.

analogous to the gravitational case and he thus concluded that the electric force, like the gravitational force, must depend on distance as r^{-2}. The lack of an observed ponderable force on the cork sphere inside the charged spherical shell was thus evidence of an inverse square law behaviour in the electric force.

Franklin's work also served as inspiration for the efforts of Aepinus (1724–1802) [4], a German physicist[6]. He made experimental and theoretical contributions to the study of electricity and in 1759 proposed in a theoretical essay, written in Latin, the existence of two types of electric charges (positive and negative) and a $1/r^2$ behaviour of the electric force. His conjectures were made in analogy to what Newton had proposed in order to explain Kepler's Law, the free fall of bodies near the Earth's surface, and the outcome of Cavendish's laboratory experiment that investigated the gravitational attraction between lead spheres [5].

All of these early qualitative phenomenological and theoretical studies paved the way for the eventual quantitative verification of the basic law describing the electrical force. In fact, the essay of Aepinus was read by Robison (1739–1805) [6], an English physician who in 1769 carried out experimental tests of the inverse square law and used the results to surmise that it was indeed correct. His determination was made somewhat before that of Coulomb, but history has given the name of this interaction to the latter.

2. Early experimental verifications of Coulomb's Law

Robison's experiment was very straightforward. He measured the repulsive force between two charged masses, equilibrated by the force of gravity acting on them. By knowing their weight, and by repeating the measurements at different distances, it was possible to calculate the size of the electrical force, evaluate its dependence on distance, and thus verify the exactness of the hypothesized $1/r^2$ law.

From his observations, Robison deduced that the law must have the functional form

$$F \propto \frac{1}{r^{2\pm\varepsilon}}, \tag{1}$$

where ε represents a measure of the precision to which the $1/r^2$ behaviour is verified. He found an upper limit of 0.06 for ε and thus could state that for masses in electrical repulsion to each other, the force went as $r^{-2.06}$. However, for electrical attraction his limit was weaker, stated as r^{-c} where $c < 2$, but still essentially confirming the expected r^{-2} dependence. Unfortunately, Robison did not publish his results until 1801, and by then Coulomb [7] had already presented his. The parameter ε appearing in equation (1) and related to the precision of the $1/r^2$ behaviour, is not very useful from a theoretical point of view but is retained here because of its common historical use. Subsequent theoretical developments and improved understanding of the foundations for high precision tests of Coulomb's Law have led to the use of the quantity $\mu = m_\gamma c/\hbar = \lambda_C^{-1}$ which, as considered below, is well motivated theoretically and represents the inverse Compton wavelength of a photon of mass m_γ. Coulomb's Law is violated if $\mu \neq 0$, i.e. if the photon mass is not zero.

Before discussing Coulomb's experiment, we note that Cavendish, in addition to his celebrated measurement of the mean density of the Earth, also carried out an early experiment on the physics of the electrical force. Inspired by the same idea that motivated his predecessors, he too considered a metallic spherical shell enclosed within an outer shell consisting of two hemispheres that could be opened or closed. In the closed position, the two hemispheres were connected electrically to an electrostatic machine and charged while in ohmic contact with the inner sphere. The hemispheres were then disconnected from the inner sphere and opened, and it was verified that they remained charged. At this point, an electrometer was used to check that the inner sphere was still uncharged, thus confirming the $1/r^2$ law but with an uncertainty that was smaller than that of Robison (less than 1/60 of the charge moved to the inner shell over the thin wire interconnecting the two spheres). With reference to equation (1), Cavendish obtained $\varepsilon \leqslant 0.03$. An improved version of the experiment was later performed by Maxwell [8], who increased the precision of the test and found that the exponent of r in Coulomb's Law could differ from 2 by no more than $\varepsilon \simeq 5 \times 10^{-5}$.

We now turn to the famous experiment of Coulomb of 1788. Charles Augustin de Coulomb (1736–1806) was a French physicist and a pioneer in electrical theory. He was born in Angoulême. He served as a military engineer for France in the West Indies, but retired to Blois at the time of the French Revolution to continue his research on magnetism, friction, and electricity. In 1777, he invented the torsion balance for the purpose of measuring the force of magnetic and electrical attractions. With this device, Coulomb was able to formulate the principle, now known as Coulomb's Law, governing the interaction between electric charges. In

[6] He studied at Jena and Rostock and taught mathematics at Rostock from 1747 to 1755. After a brief stay in Berlin he went to St Petersburg as professor of physics and academician, remaining there until 1798 and rising to a high position as courtier to Catherine the Great.

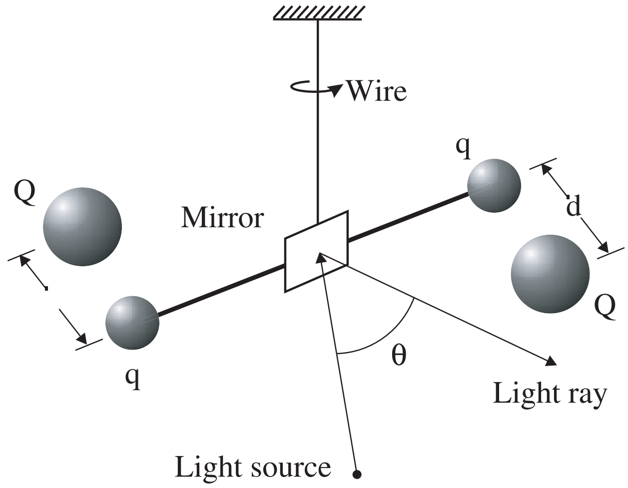

Figure 2. The Coulomb torsion balance was similar in principle to the torsion balance used by Cavendish for measurement of the gravitational attraction between masses. The interaction between the charged spheres produces a measurable twist in the torsion fibre, which lets the apparatus rotate until equilibrium is reached. By accurately measuring the torsion angle, Coulomb confirmed the $1/r^2$ law with a precision surpassing that of the previous experiments of Robison and Cavendish.

1779, Coulomb published the treatise *Théorie des machines simples* (*Theory of Simple Machines*), an analysis of friction in machinery. After the war Coulomb came out of retirement and assisted the new government in devising a metric system of weights and measures. The unit of quantity used to measure electrical charges, the *coulomb*, was named for him.

His torsion balance, shown in figure 2, was similar in principle to that used by Cavendish for the measurement of the mean density of the Earth (now interpreted as a seminal laboratory test of Newtonian gravitation). The interaction between the charged spheres produces a torque that acts on the torsion fibre, with the apparatus then rotating until equilibrium is reached. By accurately measuring the torsion angle, Coulomb found a limiting value for ε in equation (1) of $\varepsilon \leqslant 0.01$, thus surpassing the precision of the previous experiments of Robison and Cavendish. The scalar expression for what has come to be known as Coulomb's Law says that the force F between two charges q_1 and q_2 separated by a distance r may be written in the simple form

$$F = k\frac{q_1 q_2}{r^2}, \tag{2}$$

where the value of the constant of proportionality, k, will be considered in section 7.

The reasons why Coulomb achieved greater success and recognition than did his predecessors are essentially two. First, he performed his tests with combinations of both negative and positive charges. Cavendish used only charges of the same sign, but Coulomb sought to measure both attractive and repulsive forces. Second, he published his results immediately, while Robison did not make his findings available until 1801, thirteen years after Coulomb. Cavendish too delayed the dissemination of his work, and it thus garnered no attention until nearly a century later when a citation to it was given by Maxwell in his famous essay [8]. Coulomb's prompt publication of his results may signal that he was more aware than his colleagues of how fundamental and important this work was.

3. Null tests of Coulomb's Law: theory

The technique employed by Cavendish has been used in most of the experimental work done since then, as it turned out to be potentially the most sensitive. It is intrinsically a *null experiment*, in the sense that the experimentalist seeks to verify with great precision the absence of charge from the inner sphere, rather than having to measure with less precision a non-null physical quantity, such as the twist in the fibre, as in the torsion balance approach.

Following Robison and Maxwell and supposing that the exponent in Coulomb's Law is not -2 but $-(2+\varepsilon)$, to first order in ε the electric potential at a point $\boldsymbol{r}$ due to the charge density distribution $\rho(\boldsymbol{r}')$ is given by

$$V(\boldsymbol{r}) = \int \mathrm{d}^3\boldsymbol{r}' \frac{\rho(\boldsymbol{r}')}{|\,\boldsymbol{r}-\boldsymbol{r}'\,|^{1+\varepsilon}}. \tag{3}$$

If the charge is uniformly distributed on a spherical shell of radius $a > r$, then $\rho(r', \theta', \varphi') = \sigma\delta(r-a)$ and we may arbitrarily choose r to coincide with the z axis and expression (3) becomes

$$\begin{aligned} V(r) &= \int \frac{\sigma \sin\theta' \,\mathrm{d}\theta' \,\mathrm{d}\varphi'}{(r^2+a^2-2ar\cos\theta')^{(1+\varepsilon)/2}} \\ &= \int_{-1}^{+1} \frac{\mathrm{d}\cos\theta'}{(r^2+a^2-2ar\cos\theta')^{(1+\varepsilon)/2}} \\ &= \frac{f(r+a)-f(r-a)}{2ar}. \end{aligned} \tag{4}$$

In the case where Coulomb's Law is perfectly valid, $\varepsilon = 0$ and

$$V(r) = 2\pi\rho \int_{-1}^{+1} \frac{\mathrm{d}(\cos\theta')}{(r^2+a^2-2ar\cos\theta')^{1/2}} = \frac{1}{a} = \text{const}, \tag{5}$$

so that $V(r) - V(a) = 0$ and the electric field inside the charged spherical shell vanishes. Thus, in tests of Coulomb's Law, we are interested in the potential induced on a sphere of radius r by a charge distributed uniformly on a concentric sphere of radius $a > r$, i.e.

$$\frac{V(r)-V(a)}{V(a)} = \frac{a}{r}\left[\frac{f(a+r)-f(a-r)}{f(2a)}\right] - 1. \tag{6}$$

To first order in ε, equation (6) yields

$$\frac{V(r)-V(a)}{V(a)} = \varepsilon M(a, r), \tag{7}$$

where

$$M(a, r) = \frac{1}{2}\left[\frac{a}{r}\ln\left(\frac{a+r}{a-r}\right) - \ln\left(\frac{4a^2}{a^2-r^2}\right)\right]. \tag{8}$$

Since $M(a, r)$ turns out to be of order unity, ε is essentially the quotient $[V(r) - V(a)]/V(a)$ of the measured potential difference, $V(r) - V(a)$, and the applied voltage, $V(a)$.

As an alternative to equations (7) and (8), de Broglie [9] considered a simple generalization of Maxwell's equations involving a small non-zero rest mass of the photon. In this case, two charges will repel each other by a Yukawa force derived from the potential

$$U(r) = \frac{e^{-\mu r}}{r} = \frac{e^{-r/\lambda_C}}{r}, \tag{9}$$

where $\mu = m_\gamma c/\hbar = \lambda_C^{-1}$ is the inverse Compton wavelength of the photon. In the limit $\mu a \ll 1$, $U(r) = 1/r - \mu + \frac{1}{2}\mu^2 r$ and equation (6) yields

$$\frac{V(r) - V(a)}{V(a)} = -\frac{1}{6}\mu^2(a^2 - r^2). \tag{10}$$

Although $U(r)$ contains the term μ, what is tested experimentally is the result of equation (10). Since the other tests of Coulomb's Law are explicitly sensitive to μ^2 and not to μ, the quadratic dependence of $V(r) - V(a)$ on μ makes a test based on this approach comparable to other tests. Thus, the potential difference $V(r) - V(a)$ is not zero if Coulomb's Law is invalid or, equivalently, if the photon rest mass is non-zero. For direct tests of Coulomb's Law that consist of measuring the static potential difference of charged concentric shells, one may use either equation (7) or equation (10). However, one can also test Coulomb's Law by determining μ with independent, indirect methods. In general, these would rely on either finding possible variations due to the presence of the Yukawa potential (9) or on the standard fields of massless electrodynamics, such as, e.g., measurements at either large distances or long times, where the percentage effect would be much higher. Typical of these approaches are those that involve the magnetic field of the Earth. For example, one might consider (a) satellite verification that the magnetic field of the Earth falls off as $1/r^3$ out to distances at which the solar wind is appreciable [10], (b) observation of the propagation of hydromagnetic waves through the magnetosphere [11], (c) application of the Schrödinger external field method [12], or other methods such as those described below. The three approaches outlined above should all give roughly the same limit, $\mu \leqslant 10^{-11}\,\mathrm{cm}^{-1}$.

In the high-frequency (direct) null test of Coulomb's Law described below, it is convenient to start from a relativistically invariant linear generalization of Maxwell's equations, namely the Proca equations [13], which allow for a finite rest mass of the photon. Proca's equations for a particle of spin 1 and mass m_γ are [14]

$$(\Box + \mu^2)A_\nu = \frac{4\pi}{c}J_\nu \tag{11}$$

and Gauss' Law becomes

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = 4\pi\rho - \mu^2\varphi. \tag{12}$$

Equation (12) may be applied to two concentric, conducting, spherical shells of radii r and a ($a > r$) with an inductor across (i.e. in parallel with) this spherical capacitor. If a potential $V_0 e^{i\omega t}$ is applied to the outer shell, the resulting electric field is [15]

$$\boldsymbol{E}(r) = \left(qr^{-2} - \tfrac{1}{3}\mu^2 V_0 e^{i\omega t} r\right)\hat{r}, \tag{13}$$

where q is the total charge on the inner shell.

The voltage across the inductor of capacity C is then given [15] by

$$V(r) - V(a) = \int_r^a \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{l} = \frac{q}{C} - \mu^2\,\frac{V_0 e^{i\omega t}}{6}(a^2 - r^2). \tag{14}$$

Save for the standard term q/C (which is zero when there is no charge on the inner shell), the term dependent on μ in equation (14) coincides with that of equation (10).

4. Direct tests of Coulomb's Law

After the development of the phase-sensitive detectors such as lock-in amplifiers, new and more sensitive attempts to test Coulomb's Law were made such as the ones by Plimpton and Lawton [16], Cochran and Franken [17], and Bartlett and Phillips [18]. In this section, we consider Maxwell's derivation (equations (6) and (7)) applied to the simple case of a conducting sphere containing a smaller concentric sphere. The potential of the outer sphere is raised to a value V and the potential difference between them is measured. The actual shape of these conductors should not be relevant because the electric field inside a cavity of any shape vanishes unless Coulomb's Law is violated. Thus, Cochran and Franken [17] could use conducting rectangular boxes in their experiment and set a limit of $\varepsilon \leqslant 0.9 \times 10^{-11}$.

The experiments of both Cavendish and Maxwell required connecting the inner sphere to an electrometer. The accuracy of the experiment was thus limited by fluctuations in the contact potentials while measuring the inner sphere's voltage. Another problem was that of spontaneous ionization between the spheres. These problems were overcome by Plimpton and Lawton [16] by using alternating potentials. They developed a quasi-static method and charged the outer sphere with a slowly alternating current. The potential difference between the inner and outer spheres was detected with a resonant frequency electrometer. It consisted of an undamped galvanometer with amplifier, placed within the globes, and with the input resistor of the amplifier forming a permanent link connecting them, so as to measure any variable potential difference. No effect was observed when a harmonically alternating high potential V (>3000 V), from a condenser generator operating at the low resonance frequency of the galvanometer, was applied to the outer globe. The sensitivity was such that a voltage of 10^{-6} V was easily observable above the small level of background noise. With this technique they succeeded in reducing Maxwell's limit to $\varepsilon \simeq 2 \times 10^{-9}$.

Another of the classic 'null experiments' that tests the exactness of the electrostatic inverse square law was performed by Bartlett *et al* [19]. In this experiment, the outer shell of a spherical capacitor was raised to a potential V with respect to a distant ground and the potential difference $V(r) - V(a)$ of equations (7) and (10) induced between the inner and outer shells was measured. Five concentric spheres were used and a potential difference of 40 kV at 2500 Hz was imposed between the two outer spheres. A lock-in detector with a sensitivity of about 0.2 nV measured the potential difference between the inner two spheres. Any deviation in Coulomb's law should lead to a non-null result for $V(r) - V(a)$ proportional to ε as shown by equation (7). The result obtained by these authors was $\varepsilon \leqslant 1 \times 10^{-13}$. A comparable result was found even when the frequency was reduced to 250 Hz and the detector was synchronized with the charging current rather than with the charge itself.

The best result obtained so far through developments of the original Cavendish technique is still that from 1971 by Williams *et al* [15], who improved an earlier experiment [20]. They used five concentric metallic shells in the form of icosahedra rather than spheres in order to reduce the errors due to charge dispersion. A high voltage and frequency signal was applied to the external shells and a very sensitive detector checked for any trace of a signal related to variable charging of the internal shell. The detector worked by amplifying the signal of the internal shell and comparing it with an identical reference signal, progressively out of phase at a rhythm of 360° per half hour. Any signal from the detector would indicate a violation of Coulomb's Law. In order to avoid introducing unrelated fields, the reference signal and the detector output signal were transmitted by means of optical fibres. The outer shell, of about 1.5 m diameter, was charged 10 kV peak-to-peak with a 4 MHz sinusoidal voltage. Centred inside this charged conducting shell is a smaller conducting shell. Any deviation from Coulomb's Law is detected by measuring the line integral of the electric field between these two shells with a detection sensitivity of about 10^{-12} V peak-to-peak.

The null result of this experiment expressed in the form of the photon rest mass squared (equation (14) or equation (10)) is $\mu^2 = 2.3 \times 10^{-19}\,\mathrm{cm}^{-2}$. Expressed as a deviation from Coulomb's Law in the form of equation (7), their result is $\varepsilon = 6 \times 10^{-16}$, extending the validity of Coulomb's Law by two orders of magnitude beyond the findings of Bartlett *et al*.

5. Limits due to the effects of gravity

We have mentioned above that null experiments that test the validity of Coulomb's Law are typically more precise than those that attempt to directly measure the interaction force between charges. One of the problems arising when making direct measurements of the force between two macroscopic charged bodies, as done when using a torsion balance, is that the charges are distributed over conducting surfaces of finite size. In the ideal case, Coulomb's Law describes the interaction between two point charges separated by a precisely known distance. In any practical arrangement, even the charge on a microscopically small conducting ball cannot be considered to be truly point-like—as if placed at the centre—but rather distributed over the ball's surface. If the charged ball is interacting with another charged ball, the distribution on the surface is no longer uniform and has to be determined using the method of images. Saranin [21] has studied in detail the departures from Coulomb's Law that can occur when two conducting spheres interact electrostatically with each other. By computing forces on them as a function of their separation, he found that at small distances a switch from repulsion to attraction occurs in the general case of arbitrarily but similarly charged spheres. The only exception—and in it they always repel each other—is the case in which the charges on the spheres are related as the squares of their radii. The results of Saranin help corroborate the idea that, even in principle, null experiments can be more precise than tests based on the direct measurement of interaction forces between two macroscopic charged bodies.

In view of the high levels of precision achieved in several of these tests, it is interesting to consider what possible competing effects gravity might introduce into them. The result $f(a, r)$ in equation (5) was derived strictly from classical electrodynamics. In it, a uniform charge distribution is assumed and the effect of gravity is neglected. As noted by Plimpton and Lawton [16], if electrons have weight $m_e g$ the electron density on the conducting sphere must be asymmetrical, being greater at the bottom where the electrons are pulled by the force of gravity. For the experiment of Plimpton and Lawton this effect is insignificant as it leads to a maximum potential difference over the globe of 10^{-10} V, which is far less than the minimum detectable voltage of 10^{-6} V.

Thus, while such a gravitational effect should be negligible in the relatively low sensitivity experiment of Plimpton and Lawton, it could conceivably be important in experimental tests of higher sensitivity. According to this model, the overall effect of gravity is to produce a distortion in what should otherwise be a uniform charge distribution. Of course, the more general problem is to account for effects of a non-uniform charge distribution regardless of the origin of the non-uniformity. In equation (5), the null result comes from the assumption that Coulomb's Law is valid and that the charge is distributed uniformly on the sphere. However, Shaw [22] objected to the assumption that the charge will distribute itself uniformly over a conducting spherical shell, even in the absence of any gravitational effect. In conventional electrostatics, the uniform charge distribution for Coulomb and Yukawa potentials follows from the symmetry of the problem and the uniqueness of the solution. If these potentials are not valid there is no guarantee of a uniform charge distribution and thus irregularities in the spherical surface would bias the concept that the inner potential does not depend on the shape of the outer sphere. However, considering that any violation of Coulomb's Law is very small, departures from the expected uniformity should give [19] only second-order corrections to equations (7) and (8).

6. Indirect tests of Coulomb's Law

In addition to the tests discussed in the previous sections, there have also been a number of indirect experimental verifications of Coulomb's Law, and these will be discussed briefly in what follows.

6.1. Geomagnetic and astronomical tests

A consequence of Coulomb's Law is that the magnetic field produced by a dipole goes as $1/r^3$ at distances from its centre for which the dipole approximation is valid. For the magnetic field of a planet, this distance is equivalent to about two planetary radii (at least). If the photon rest mass is not zero—which is equivalent to a violation of Coulomb's Law—a Yukawa factor $\mathrm{e}^{-r/\lambda_C}$ is introduced in the $1/r$ terms for the electrostatic and magnetostatic potentials. In this case, the magnetic field produced by a dipole no longer goes as $1/r^3$ but contains corrections related to the Compton wavelength $\lambda_C = \mu^{-1} = \hbar/m_\gamma c$ where m_γ is the photon mass.

In terms of the Compton wavelength and photon mass, the sensitive explorations of the validity of Coulomb's Law reported by Williams *et al* [20] yielded an upper limit of 1.05×10^{10} cm for λ_C and 2×10^{-47} g for m_γ, while their subsequent direct test with concentric icosahedra [15] yielded the slightly better findings of 1.95×10^{10} cm for λ_C and 1.6×10^{-47} g for m_γ.

The first results obtained from orbital data were those of Goldhaber and Nieto [14], who used satellite measurements of the Earth's magnetic field to set limits of 5.5×10^{10} cm for λ_C and 4×10^{-48} g for m_γ. This corresponded to an equivalent value of $\varepsilon \leqslant 1.7 \times 10^{-16}$. Davis *et al* [23] verified the $1/r^3$ behaviour of the magnetic field of Jupiter (much more intense than that of the Earth) from observations of the Pioneer 10 spacecraft, and were able to improve the precision of the limit such that $\lambda_C \geqslant 3.14 \times 10^{11}$, i.e. $m_\gamma \leqslant 2.8 \times 10^{-11}\ \mathrm{cm}^{-1} = 8 \times 10^{-49}$ g.

Further studies on the photon mass and planetary magnetic fields were carried out by Bicknell [24]. He extended Schrödinger's [12] expression for a dipole field to a complete spherical harmonic analysis of a static planetary field. Additional components of the magnetic field were identified which would arise from a non-zero photon mass. However, no new limits were set on λ_C because the most important terms in the expressions for the geomagnetic field were of the same order as Schrödinger's apparent external field. An analysis of Earth's magnetic field, performed by Fischbach *et al* [25], led to an upper limit value of $m_\gamma = 1 \times 10^{-48}$ g.

It is important to note that there is still another way of characterizing deviations from Coulomb's Law, based on an electromagnetic analogue of a fifth force, and that this line of thought leads to additional suggestions for experimental tests. The relevant works in this area are those of Bartlett and Logl [26], Fischbach *et al* [25], and also Kloor *et al* [27].

An upper limit for the photon mass was also found by Lowenthal [28], who used astronomical observations of the gravitational deflection of electromagnetic radiation. Although this method does not lead to a better upper limit than those mentioned above, the method is nevertheless interesting and, because it is not related to the other techniques, it constitutes an independent approach to restricting the magnitude of the photon mass. The question posed by Lowenthal was the following: if the general theory of relativity predicts a deflection of starlight by the Sun of 1.75 s of arc, how is this deflection altered if the photon has a small rest mass μ (in units of $c = \hbar = 1$)? Lowenthal showed that the deflection varies proportionally to μ^2 and he set the correction term equal to the difference between the measured deflection angle and the calculated deflection angle for photons of zero rest mass. After taking into account the accuracy of the deflection measurements, he obtained an upper limit of $\mu < 7 \times 10^{-40}$ g, which is not as stringent as the other limits, but still a useful result.

6.2. Lumped-circuit tests and the photon mass

Another indirect method to verify the $1/r^2$ form of Coulomb's Law involves observations of electromagnetic waves of increasing wavelength, λ. Underlying it is the concept that if ε is small but $\neq 0$, there is a $\lambda_{\max}(\varepsilon)$ beyond which there cannot be electromagnetic wave propagation. With $\varepsilon \neq 0$ the equation for the scalar electromagnetic potential φ and vector potential $\boldsymbol{A}$ takes the form of a Klein–Gordon equation and the velocity of propagation *in vacuo* of sinusoidal electromagnetic waves decreases as the wavelength λ increases, and it vanishes for a value $\lambda_{\max}(\varepsilon)$ that depends on ε. The greater the observed $\lambda_{\max}(\varepsilon)$, the smaller is ε.

A very sensitive test of this concept using an LCR circuit was carried out by Franken and Ampulski [29] in 1970. These authors point out that the free-space phase velocity of light may be expressed as

$$\left(\frac{v_\varphi}{c}\right)^2 = \frac{\omega^2}{\omega^2 - \omega_c^2}, \qquad (15)$$

where ω is the angular frequency of the electromagnetic waves in a resonant cavity and $\omega_c = 2\pi c/\lambda_C$. The latter quantity can be rewritten as $\hbar\,\omega_c = mc^2$, which introduces a mass into the argument. In fact, the lowest resonant frequency of a cavity is ω_c, irrespective of the size of the cavity. The phase velocity becomes infinite at the critical frequency ω_c while the group velocity approaches zero at this frequency, corresponding to a massive photon with zero momentum and hence an infinite uncertainty in position, which, in turn, requires an infinitely large cavity for confinement. The results of the test [29] yield the values of 2×10^{12} cm for λ_C and 1×10^{-49} g for m_γ, corresponding to the equivalent value $\varepsilon \leqslant 4.3 \times 10^{-8}$. A difficulty with this test is that so-called 'warped' conducting cavities have been used in order to make up for the requirement of a very large cavity and there is no rigorous proof that the theoretical considerations valid for a rectangular box apply also to warped cavities.

Several authors have discussed possible limitations of the test by Franken and Ampulski. In one analysis of lumped-circuit tests of photon mass, Boulware [30] showed that the only effect of a photon mass, if there is one, is to produce small changes in the inductance and capacitance of the circuit as well as changes in its radiative half-life. The behaviour of a low-frequency, lumped LC circuit is essentially independent of the dynamics of the electromagnetic fields. Also Park and Williams [31], and Kroll [32] have noted that in reducing the size of their apparatus to table-top dimensions the authors have lowered the sensitivity of their experiment by the same ratio, thus making it difficult to set significant limits on the photon rest mass.

In examining the physical significance of the experiment by Franken and Ampulski, Goldhaber and Nieto [33] show that in the massive-photon case the fields and currents of the system are changed only by order $(\mu D)^2$ from those of the massless-photon case (D is the dimension of the system). This implies that the above-mentioned 'table-top' experiment is only weakly sensitive to small Yukawa-like deviations from Coulomb's Law and they argue that it is therefore unlikely that this test can improve present limits on the photon mass.

6.3. Cryogenic experiments

Modern theories that appeal to the concept of spontaneous symmetry breaking assume that particles, which are massless above a certain critical temperature T_c, acquire mass below this temperature. Within this framework it is natural to

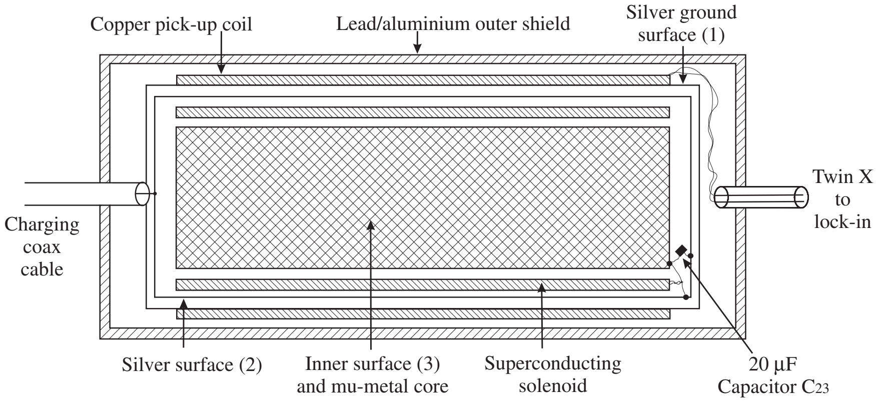

Figure 3. Schematic diagram of the cryogenic photon-mass experiment of Ryan *et al*. The system measures the current that flows between two closed surfaces in response to an impressed voltage difference. The apparatus is kept at very low temperature (1.36 K). (Figure 1 from Ryan *et al* [34]. Copyright (1985) by the American Physical Society (with permission of Prof. Austin).)

speculate that the photon could also be massless above a critical temperature and acquire a rest mass below it.

A cryogenic photon-mass experiment was performed by Ryan *et al* [34]. As shown diagrammatically in figure 3, it consisted essentially of a null experiment with concentric containers (closed surfaces), similar in a way to the previous direct tests of Coulomb's Law such as the one by Williams *et al* [15]. Unlike a standard Coulomb's Law experiment, this method measured the current that flows between two closed surfaces in response to an impressed voltage difference, not the voltage difference itself. The apparatus was immersed in liquid helium. The result of the experiment set a limit on the mass of the photon of $m_\gamma \leqslant (1.5 \pm 1.38) \times 10^{-42}$ g at 1.36 K. The sensitivity is lower than that of other previous tests. However, the result is still important because the thermodynamic validity of Coulomb's Law is now extended from the standard terrestrial ('room') temperatures to those typical of a planet far away from the Sun, or to those of the galactic environment.

6.4. The Aharonov–Bohm effects and the mass of the photon

As discussed in the literature, several conjectures related to the Aharonov–Bohm effect [35] have been developed assuming electromagnetic interaction of fields of infinite range, i.e. zero photon mass. The possibility that any associated effects, specifically the Aharonov–Bohm effect itself, might be manifest within the context of finite-range electrodynamics has been discussed by Boulware and Deser [36].

These authors consider the coupling of the photon mass m_γ, as predicted by the Proca equation in the form

$$\partial_\nu F^{\mu\nu} + m_\gamma^2 A^\mu = J^\mu \tag{16}$$

and show that the component of the magnetic field for a solenoid of the type that might be used in a test of the Aharonov–Bohm effect is

$$B = B_0 + m_\gamma^2 \Pi(\rho), \tag{17}$$

where the first term, B_0, is the standard magnetic field for zero photon mass—the field confined inside a long solenoid of radius a and carrying the current j—and the second term represents a correction due to the photon non-vanishing rest mass m_γ. The quantity $\Pi(\rho)$, which can be expressed in terms of the Bessel functions I_0 and K_0 that are regular at the origin and infinity respectively, reads

$$\begin{aligned}\Pi(\rho) = j\theta(a-\rho)\bigg[& K_0(m_\gamma \rho)\int_0^\rho \rho'\,\mathrm{d}\rho'\, I_0(m_\gamma \rho') \\ & + I_0(m_\gamma \rho)\int_\rho^a \rho'\,\mathrm{d}\rho'\, K_0(m_\gamma \rho')\bigg] \\ & - j\theta(\rho - a) K_0(m_\gamma \rho)\int_0^a \rho'\,\mathrm{d}\rho'\, I_0(m_\gamma \rho'). \end{aligned} \tag{18}$$

Thus, for the standard solenoid configuration, finite range would lead to a small $\boldsymbol{B}$ field leakage outside the solenoid represented by $m_\gamma^2 \Pi(\rho)$. In principle, one could arrange for the leakage flux addition to be an integer, in which case only the interior field would contribute, making the scenario just as non-local as in the standard massless case.

Because of the extra mass-dependent term, some non-trivial limits on the range of the transverse photon might thus be obtained from a table-top experiment. By comparing the theoretical corrections to the flux through a small circular region of radius ρ with the flux for the massless case, Boulware and Deser were able to estimate the size of a possible experimental limit on the range m_γ^{-1}. For scale sizes of $\rho \sim 10$ cm, the observable ranges are $m_\gamma^{-1} \leqslant 10^2$ km.

It would also be interesting to consider the other non-local effects of the Aharonov–Bohm type, such as those associated with neutral particles that have an intrinsic magnetic or electric dipole moment [37]. The goal would be to see if they provide similar correction terms that might be suitable for setting more precise limits on the range of m_γ^{-1}. In the case of the standard Aharonov–Casher effect, this analysis has been performed by Fuchs [38] who points out that, unlike the case for the Aharonov–Bohm effect, a neutral particle with a magnetic dipole moment that couples to non-gauge fields has no classical acceleration regardless of the photon mass, so that no observable corrections would be expected.

However, this may not be the case for a neutral particle with an electric dipole moment. In fact, in the Aharonov–Casher effect a particle with magnetic dipole $\boldsymbol{\mu}_\mathrm{m}$ acquires an electromagnetic momentum $c^{-1}\boldsymbol{E} \times \boldsymbol{\mu}_{\mathrm{m}}$ because $\boldsymbol{\mu}_{\mathrm{m}}$ couples with the electric field $\boldsymbol{E}$, while a particle with an electric dipole $\boldsymbol{d}_{\mathrm{e}}$ couples with the vector potential $\boldsymbol{A}$ [37]—as happens in the case of the Aharonov–Bohm effect—thus acquiring an electromagnetic momentum $c^{-1}(\boldsymbol{d}_{\mathrm{e}} \cdot \nabla)\boldsymbol{A}$. For field-free effects [37] on electric dipoles, a magnetic sheet is the source of $\boldsymbol{A}$. Small leakages of the field—confined within the sheet in the massless photon case—may lead to observable corrections of the type predicted by Boulware and Deser, thus presenting an interesting possibility for future study.

6.5. Recent limits on the photon mass and the cosmic magnetic vector potential

A novel experimental approach for measuring the photon mass m has been based on a toroidal Cavendish balance used to evaluate the product $\mu^2 A$ of the photon mass squared ($\mu^{-1} = \hbar/m_\gamma c$) and the ambient cosmic vector potential A. This approach, developed by Lakes [39], points out that the Maxwell–Proca equations modify the standard equations for the curl of $\boldsymbol{B}$ to

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{j} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} - \mu^2\boldsymbol{A}. \qquad (19)$$

Gauge invariance is lost if $\mu \neq 0$, since in these equations the potentials themselves have physical significance, in addition to that of the usual fields. If a toroid carries an electric current, or if it is permanently magnetized with a field $\boldsymbol{B}$ confined within it, the corresponding magnetic vector potential may be represented by a dipole field $\boldsymbol{a}_{\mathrm{d}}$. If $\mu \neq 0$, this dipole field interacts with the ambient vector potential $\boldsymbol{A}$ to produce a torque $\boldsymbol{\tau} = \boldsymbol{a}_{\mathrm{d}} \times \boldsymbol{A}\mu^2$ via the energy density of the vector potential. Thus, the method is based on the energy density of $\boldsymbol{A}$ in the presence of m_γ, not on measurement of the magnetic field.

The experiment does not yield a direct limit for μ, but rather on $\boldsymbol{A}\mu^2$. The modified Cavendish balance used to determine the product $\boldsymbol{A}\mu^2$ consisted of a toroid of electrical steel wound with many turns of wire that carried a current (see figure 4). The apparatus was supported by water flotation [40]. The experiment yielded $\boldsymbol{A}\mu^2 < 2 \times 10^{-9}\,\mathrm{T\,m\,m^{-2}}$ and, if the ambient magnetic vector potential is $A \approx 10^{12}\,\mathrm{T\,m}$ due to cluster level fields, $\mu^{-1} > 2 \times 10^{10}\,\mathrm{m}$. Even with more conservative values for A, the limit set by this experiment improves the precision attained with the Jovian magnetic field.

In the preceding section on lumped circuits we quoted the result of Goldhaber and Nieto [33], which suggests that for table-top experiments the sensitivity of any limit on μ^2 goes as $1/D^2$ where D is the dimension of the system. This line of reasoning has become an often-used 'rule of thumb' in the field, and the inference of it is that table-top experiments will be less sensitive than others. If so, it is then interesting to consider why the experiment of Lakes [39] sets such a good limit. We believe that the difference is due mainly to the fact that in table-top tests such as those using lumped circuits, the modification of the vector potential $\boldsymbol{A}$ due to a massive photon possesses a range that is logically limited to the dimensions of the circuit or cavity where the electromagnetic waves are produced and confined. However, in the Lakes experiment, the mass μ couples with a vector potential $\boldsymbol{A}$ that is of cosmic

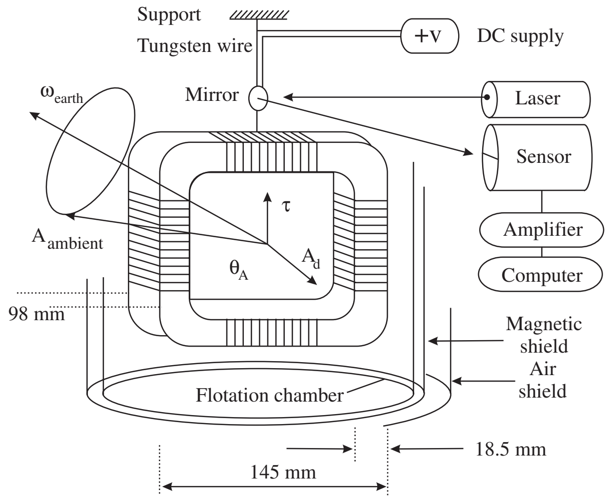

Figure 4. Block diagram of the sophisticated Cavendish balance used by Lakes to evaluate the product of the photon mass squared and the ambient cosmic vector potential. The toroid carries an electric current giving rise to a dipole field $\boldsymbol{a}_{\mathrm{d}}$ that, if $m_\gamma \neq 0$, interacts with the ambient vector potential $\boldsymbol{A}_{\mathrm{ambient}}$ to produce a torque τ on the toroid. (Figure 1 from Lakes [39]. Copyright (1998) by the American Physical Society (with permission of Prof. Lakes).)

origin so that the dimensions of the sources of $\boldsymbol{A}$ correspond to astronomical or galactic distances as opposed to those of laboratory size. Thus, experiments of that type are able to sidestep the implications of the argument of Goldhaber and Nieto [33].

The experimental arrangement used by Lakes is such that the torque on the torsion balance will vary with the rotation of the Earth, making it possible to experimentally detect the variations. However, if the cosmic ambient vector potential were to be fortuitously aligned with the Earth's rotation axis, then this approach would fail. In order to avoid this possibility, Luo *et al* [41] performed an improved experiment by rotating the torsion balance to ensure the effectiveness of detection for all possible orientations of the vector potential. They were also able to remove the influences of sidereal disturbances in the environment by virtue of this method of modulation. The experimental result was $\boldsymbol{A}\mu^2 < 1.1 \times 10^{-11}\,\mathrm{T\,m\,m^{-2}}$. If the ambient cosmic magnetic vector potential is $A \approx 10^{12}\,\mathrm{T\,m}$, it yields the new upper limit $\mu^{-1} > 1.66 \times 10^{11}\,\mathrm{m}$ and the photon mass is $m_\gamma < 1.2 \times 10^{-51}\,\mathrm{g}$.

Those latest results are certainly remarkable if one considers that, according to the uncertainty principle, a purely theoretical estimate of the photon mass is given by $m_\gamma = \hbar/(\Delta t)c^2$, which yields an order of magnitude number of $10^{-66}\,\mathrm{g}$, where the age of the universe is taken to be roughly 10^{10} years. Table 1 summarizes the results of the various experiments conducted since the time of Cavendish, and figure 5 presents a graphical comparison of the related precisions.

In table 1, direct (d) and indirect (i) tests of Coulomb's Law are listed. The experiments performed up to 1971 directly measured the parameter ε that indicates the degree of precision of Coulomb's Law written in the form $1/r^{2+\varepsilon}$; most of the successive experiments are instead tests of the photon rest mass m_γ ($\mu^{-1} = \hbar/m_\gamma c$) and determine its upper limit. For these latter experiments, the corresponding 'precision'

Table 1. Coulomb's Law and photon mass experiments.

Author	Date		ε	μ/cm^{-1}	m_γ/g $\leqslant$
Cavendish	1773	d	3×10^{-2}	1×10^{-2}	2.8×10^{-40}
Coulomb	1779	d	1×10^{-2}	1×10^{-1}	2.8×10^{-39}
Robison	1801	d	6×10^{-2}	6×10^{-2}	2.8×10^{-40}
Maxwell	1892	d	5×10^{-5}	1×10^{-3}	2.8×10^{-41}
Plimpton and Lawton	1936	d	2×10^{-9}	1×10^{-6}	2.8×10^{-44}
Cochran and Franken	1968	d	9×10^{-12}	9×10^{-8}	2.6×10^{-45}
Bartlett *et al*	1970	d	1×10^{-13}	1×10^{-8}	2.8×10^{-46}
Williams *et al*	1971	d	6×10^{-16}	5×10^{-10}	1.4×10^{-47}
Goldhaber and Nieto	1968	i	1.7×10^{-16}	1.4×10^{-10}	4.0×10^{-48}
Franken and Ampulski	1971	i	4.3×10^{-18}	3.6×10^{-12}	1.0×10^{-49}
Lowenthal	1973	i	3.0×10^{-08}	2.5×10^{-2}	7.0×10^{-40}
Davis *et al*	1975	i	3.4×10^{-17}	2.8×10^{-11}	8.0×10^{-49}
Crandall	1983	d	3.4×10^{-16}	2.8×10^{-10}	8.0×10^{-48}
Ryan *et al*	1985	i	6.4×10^{-11}	5.3×10^{-5}	1.5×10^{-42}
Boulware and Deser	1989	i	1.2×10^{-13}	1×10^{-7}	2.8×10^{-45}
Chernikov *et al*	1992	i	3.6×10^{-14}	3×10^{-8}	8.4×10^{-46}
Fischbach *et al*	1994	i	4.3×10^{-17}	3.5×10^{-11}	1.0×10^{-48}
Lakes	1998	i	6.8×10^{-19}	5.7×10^{-13}	1.6×10^{-50}
Schaefer	1999	i	1.8×10^{-12}	1.5×10^{-6}	4.2×10^{-44}
Lou *et al*	2002	i	5.1×10^{-20}	4.3×10^{-14}	1.2×10^{-45}

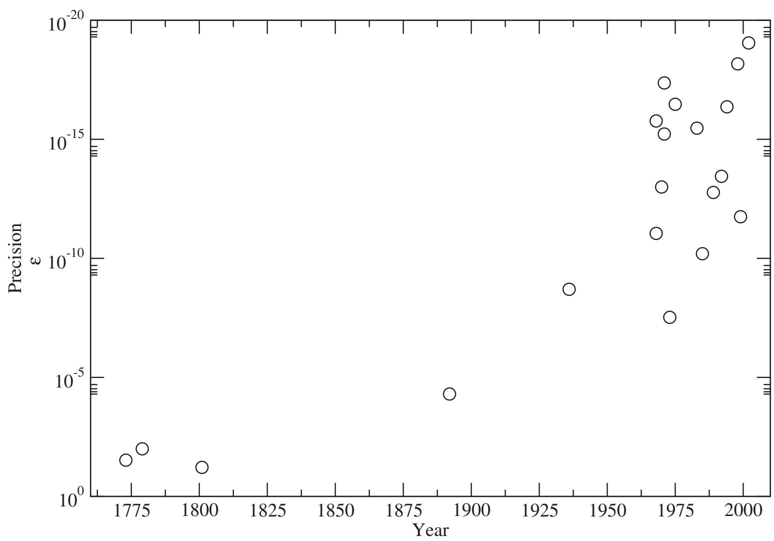

Figure 5. Graph showing the increasing 'precision' ε by which Coulomb's Law in the form $1/r^{2+\varepsilon}$ has been verified since the 1700s. The law would be verified exactly when $\varepsilon = 0$. Although some of the recent experiments are relatively imprecise, they are nevertheless important because they extend the validity of Coulomb's Law as a function of the physical conditions of the measurement.

ε is here evaluated indirectly as if, instead of a photon test mass, equivalent experiments of the type by Williams *et al*, maintaining the same proportion between ε and μ, were performed.

7. Coulomb's Law and units of measurement

The various tests discussed above have confirmed the validity of Coulomb's Law to a high degree of precision. We may write it vectorially as

$$\boldsymbol{F} = kq_1q_2\frac{\hat{\boldsymbol{r}}_{21}}{|\,\boldsymbol{r}_2 - \boldsymbol{r}_1\,|^2}, \tag{20}$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the position vectors of the charges q_1 and q_2, respectively. We are left with the task of determining the proportionality constant k, the value of which is strictly related to the system of units chosen. Although arbitrary, in general, the choice has usually been made in such a way as to obtain unitary and dimensionless constants.

Three systems of units have historically been used the most. The first two have been superseded by the third:

(1) The *non-rationalized Gaussian system*, which coincides with the old *cgs* electrostatic system.
(2) The *rationalized Gaussian system*.
(3) The presently accepted *International System* (*SI*).

In case (1), the constant k is dimensionless and unitary, i.e. $k = 1$. From the expression of the force $\boldsymbol{F}$ above, we can immediately establish the physical dimensions of the electric charge $[q]$ in terms of mechanical dimension such that

$$[q] = [r]\sqrt{[F]}, \tag{21}$$

that is, the charge has the dimensions of length times the square root of a force. The corresponding unit was called the *Franklin* (or simply $u.e.s$) and was such that a force of 1 dyne would act between two unit charges separated by a distance of 1 cm (1 Franklin = 1 u.e.s = 1 $\mathrm{cm}^{3/2}\,\mathrm{g}^{1/2}\,\mathrm{s}^{-1}$).

In the rationalized Gaussian system, k was also dimensionless and had a numerical value of $k = 1/4\pi$. The added complexity of incorporating a numerical constant into the statement of Coulomb's Law is amply compensated by the disappearance of the 4π in the integral relations (as in Gauss' Law). In this system, the unit of the electric charge was derived using arguments similar to those of the first case, but the unit was seldom used.

The accepted standard, of course, is the International System or SI, within which the unit of electric charge is the coulomb [C]. The constant k now takes the value $(4\pi\epsilon_0)^{-1}$ where ϵ_0, which has the defined value $\epsilon_0 = 8.854.... \times 10^{-12}\,\mathrm{F\,m}^{-1}$, is the 'electric constant', i.e. the permittivity of free space or the dielectric constant *in vacuo*. In this case, k is neither a pure number nor a dimensionless ratio, instead having the dimensions of $[\epsilon_0]^{-1}$.

In spite of the ubiquity of the SI, use of the Gaussian system persists among many theoretical physicists because it offers a convenient form of k and velocities appear in dimensionless form (instead of v the quotient v/c is used, where c is the speed of light *in vacuo*). Of course, the SI is used uniquely for any measurements and all modern electrical instruments and devices are calibrated accordingly.

8. Modern interpretations and conclusions

In considering the various tests of Coulomb's Law, or in applications of it to diverse physical phenomena, it is important to remember that this fundamental tenet of electrostatics is an idealization. Therefore, care must be taken when applying it and when exploring its limitations, as pointed out by Saranin [21]. Even so, it finds use in all fields of science. For example, it is employed in meteorology to test models of thunderstorm charge distributions, as done by Stolzenburg and Marshall [42]. They considered the charge distribution in a stratiform region of a mesoscale convective system and determined that the general vertical charge structure of that region is horizontally extensive over at least 100 km. Charge distributions were modelled in a three-dimensional domain using an approach that correctly calculates the electric field due to all the point charges and their corresponding images on the basis of Coulomb's Law. Atmospheric physics, planetary physics, astrophysics, and plasma physics are vast domains of application for efforts of this type, and this hints at the range of scales over which it is routinely applied.

Coulomb's torsion balance was in essence the first precision mechanical detector of charge, i.e. the first high-precision electrometer. The modern analogues of such instruments are semiconductor-based field-effect devices, the most sensitive of which are cryogenically cooled transistors that function at the single-electrons level [43, 44]. Also, recently, a working nanometre-scale mechanical electrometer was constructed by Cleland and Roukes [45], who state that, in principle, devices such as theirs should ultimately reach sensitivities comparable with charge detection capabilities of the cryogenic single-electron transistors.

Coulomb's Law is also at the heart of pedagogical physics. It is measured and tested by students in laboratory classes virtually every day. There are several types of precision apparatus that are commercially available for student experiments on Coulomb's Law. In fact, a Coulomb null experiment that enables physics students to obtain rigorous upper bounds on the photon mass using an apparatus that operates with subnanovolt signals has been devised by Crandall [46]. It is inexpensive to assemble and use, and the experimental arrangement can be adapted for use at several college levels. Of course, interest in the pedagogic aspects of Coulomb's Law goes well beyond the obvious applications of it to electrostatics. By introducing the superposition principle and the theory of special relativity, students are taught how generalizations of Coulomb's Law lead to Maxwell's equations [47].

In this review, we have presented a description of the ongoing development of empirical studies of the forces between charges, beginning with the very early work by people like Cavendish and others that established the nature of the inverse square law, and then proceeding to the modern experiments that search for deviations from exactness of the inverse square law. Both Newton's Law of gravity and Coulomb's Law possess the same spatial dependence, $1/r^2$, and it is fair to say that these statements of the inverse square law have served as critical markers in guiding much of the development of theoretical physics, in parallel with the long history of improvement of the experimental situation. With the development of modern physics, Newton's Law is now understood to be the weak-field limit of Einstein's general relativity, while Coulomb's Law, as one statement of the electromagnetic interaction, has been absorbed into the unified theory of electro-weak interactions.

However, according to general relativity, matter introduces curvature into space–time and this is what modifies the law of gravitation as originally formulated by Newton. The result is that the theory is nonlinear in the sense that the principle of superposition of effects is not valid. On the contrary, Coulomb's Law is not altered in the description of physical phenomena provided by modern unified theories. In the unified theory, the interaction (attraction or repulsion) between charges is no longer described in terms of forces but rather in terms of an exchange of virtual particles (photons) that yields the same effect as the classical electrostatic force. If the photon mass is zero, Coulomb's Law remains a fundamental law of electrodynamics, where linearity and the principle of superposition of effects are valid. That is, generally speaking, even unified field theories would predict, for the observable interaction between stationary charges, the same result predicted by Coulomb's Law, always supposing that the photon mass is zero. All this can introduce the thought that Coulomb's Law is thus more fundamental than Newton's Law and that electromagnetic interactions are a more primary kind of fundamental interaction at the basis of any attempt of unification of the forces and interactions of nature.

Finally, in discussing the physical implications of Coulomb's Law, it is tempting to consider the remarkably high degree of precision with which the inverse square law holds, and thus speculate that its mathematical simplicity may be at the basis of the successful incorporation of the electromagnetic interactions into unified theories. The precisions achieved by the experimental tests are increasingly exact. In recent times they go from the value of $\varepsilon = (2.7 \pm 3.1) \times 10^{-16}$ from the direct test of Williams *et al* [15] to the more recent limit on the photon mass of $m_\gamma < 1.2 \times 10^{-51}$ g from the indirect test of Luo *et al* [41].

This glimpse at several implications of Coulomb's Law reconfirms its importance and centrality in several areas of physics. Some far-reaching implications of the non-validity of Coulomb's Law or, equivalently, the existence of a non-zero photon mass, include the wavelength dependence of the speed of light in free space [48], possible variability in the speed of light [49], deviations from exactness in Ampère Law [50], the existence of longitudinal electromagnetic waves [51], and an additional Yukawa potential [25, 23] in magnetic dipole fields, as discussed earlier. The mass of the photon has been under scrutiny since the early days of quantum mechanics. However, the stringent limits imposed by the experiments discussed earlier seem to exclude the existence of a non-zero mass at least in the normal conditions of the universe as we know it. Nevertheless, in cosmological inflation scenarios where electrodynamics and gravitation are present in a unified way, the dynamical generation of a photon mass could be more likely. In this context, the cosmologists Prokopec *et al* [52] point out some far-reaching implications of the non-validity of Coulomb's Law. In describing the early stage of the universe, they find that quantum electrodynamics and curved space may do the trick: in a locally de Sitter inflationary space–time and in the presence of a light, minimally coupled, charged scalar field, the polarization of the vacuum induces a photon mass that may result in the generation of intragalactic magnetic fields responsible for seeding the galactic dynamo mechanism [53]. Apart from indirect tests of potentially observable effects of a dynamically generated photon mass in inflation, a direct test would have been possible only in the extreme conditions of the early universe described by cosmological inflation scenarios, hence it is unfortunately difficult to see how this far-reaching implication of the non-validity of Coulomb's Law might be examined. Finally, stepping away from electrodynamics, quantum mechanics, and cosmology, and entering the arena of thermodynamics, still other studies suggest that a non-zero photon mass would impact our understanding of blackbody radiation and other fundamental aspects of thermodynamics [54].

Regarding the limits of validity of Coulomb's Law, particularly as a function of distance r, the Jovian measurements show that it holds at least up to distances of the order of $r \approx 10^6$ km. From the microscopic point of view, the inferior limits found from scattering experiments show that it holds down at the atomic ($r \simeq 10^{-8}$ cm) and nuclear scales ($r \simeq 10^{-13}$ cm), and of course the cryogenic test extends its validity to very low temperatures. Coulomb's Law provides a useful and exciting window on fundamental physics and it opens experimental doors to a number of diverse fields of study. With continued imagination and through careful laboratory technique, it promises to be a stimulating tool for still more far-reaching tests of the limits of our knowledge of the physical world.

Acknowledgments

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