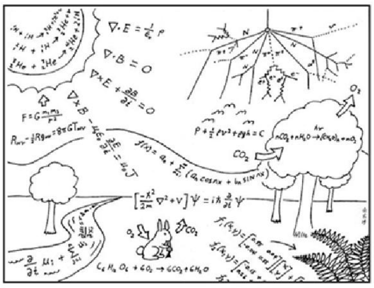
# 关于Ampère环路定理的讨论



This is how scientists see the world.

少年班学院
於得晟
PB14000284

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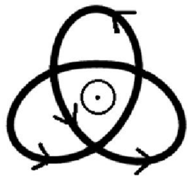
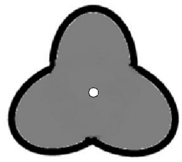
**A simple example**





平面“三叶草”回路

$$\oint_C \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 \iint_S \mathbf{J}\cdot d\mathbf{S} = \mu_0 I$$ ?

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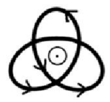

Parametric Equation:

$$\begin{cases} x=\sin t+2\sin 2t \\ y=-\cos t+2\cos 2t \end{cases} \qquad t: 2\pi \to 0$$

$\mathbf{B} = \frac{\mu_0 I \boldsymbol{\Phi}}{2\pi r}$, r=(x, y)$\boldsymbol{\Phi}$为环向单位矢量$\boldsymbol{\Phi} = \frac{(-y,\ x)}{r}$

$$\oint_c \mathbf{B}\cdot d\mathbf{l} = \frac{\mu_0 I}{2\pi}\oint_c \frac{(-y,x)\cdot d\mathbf{l}}{r^2}$$

$$= \frac{\mu_0 I}{2\pi}\int_c \frac{-y dx + x dy}{x^2+y^2}$$

$$= \frac{\mu_0 I}{2\pi}\int_{2\pi}^{0} \frac{(\cos t - 2\cos 2t)(\cos t + 4\cos 2t)dt + (\sin t + 2\sin 2t)(\sin t - 4\sin 2t)dt}{(\sin t + 2\sin 2t)^2 + (-\cos t + 2\cos 2t)^2}$$

$$= \frac{\mu_0 I}{2\pi}\int_{2\pi}^{0} \frac{2\cos(t)\cos(2t) - 2\sin(t)\sin(2t) - 7}{4\sin(t)\sin(2t) - 4\cos(t)\cos(2t) + 5} dt$$

$$= \frac{\mu_0 I}{2\pi}\cdot 4\pi$$

$$= 2\mu_0 I$$

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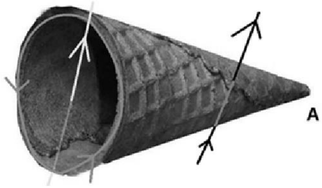
**Definition:** 任取空间中一点A，曲线上一动点绕曲线运动一周与A连线所扫过的面，定义为该曲线围成的曲面。

e.g.




**Remark:** 对静磁学A点确实是任意的，正如Stokes‘ theorem中的曲面的任意性，而任意的好处使得计算可以避开奇点。

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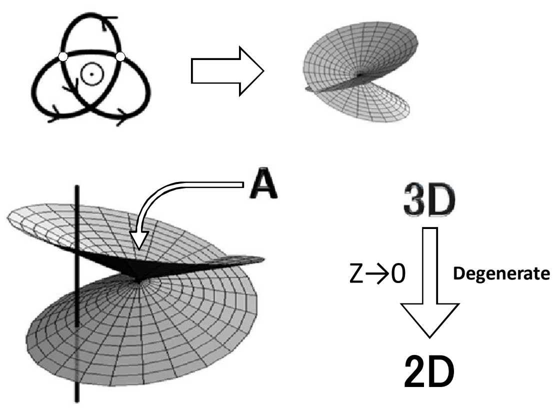


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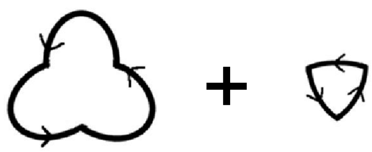
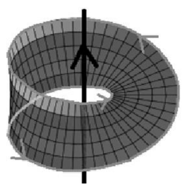
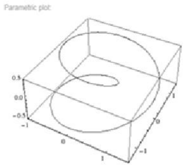
# 小技巧



# 并不具有一般性！

### II. The Boundary of Möbius Strip





①如图，电流I穿过Möbius带中间的空隙，求绕Möbius带边界的磁环量。

Parametric Equation (midcircle of radius 1):

$$x=(1+\frac{1}{2}\cos\frac{u}{2})\cos u$$

$$y=(1+\frac{1}{2}\cos\frac{u}{2})\sin u$$

$$z=\frac{1}{2}\sin\frac{u}{2}$$

$0 \le u < 4\pi$

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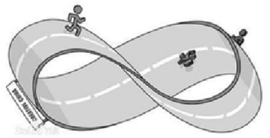
小技巧已经行不通了！

证1：Möbius曲面**不定向**，所以无论如何拆成一块块小环路，总会出现添加的相邻路径无法抵消的情况，否则由每一小块回路按右手法则确定的方向一致导出Möbius曲面定向，矛盾。

证2：如果拆分法可行，对每一小回路运用Stokes' Theorem，由Möbius曲面上各点旋度为0 $(\nabla\times\frac{\mu_0 I}{2\pi r}\mathbf{\Phi}=0)$可得磁环量为0



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**老老实实计算：**

$\mathbf{B}=\frac{\mu_0 I\mathbf{\Phi}}{2\pi r}$，r=(x,y,0) $\mathbf{\Phi}$为环向单位矢量$\mathbf{\Phi}=\frac{(-y,\ x,\ 0)}{r}$

$$\oint_c \mathbf{B}\cdot d\mathbf{l}=\frac{\mu_0 I}{2\pi}\oint_c\frac{(-y,x,0)\cdot d\mathbf{l}}{r^2}$$

$$=\frac{\mu_0 I}{2\pi}\int_c\frac{-ydx+xdy}{x^2+y^2}$$

$$=\frac{\mu_0 I}{2\pi}\int_0^{4\pi}1\cdot du$$

$$=\frac{\mu_0 I}{2\pi}\cdot 4\pi$$

$$=2\mu_0 I$$

# WHY?

**Möbius** 带边界围成的曲面 ≠ **Möbius** 曲面

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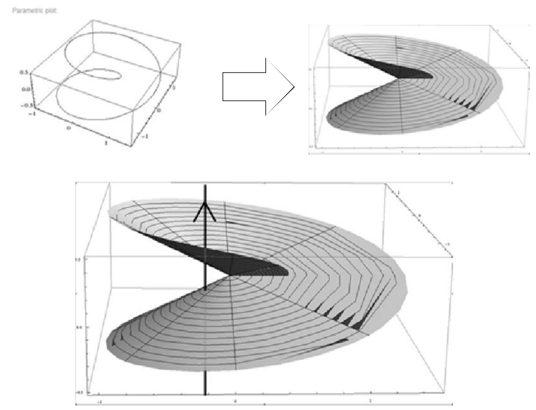

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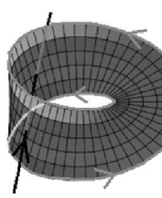
**Also**



②如图电流I穿过Möbius带，求绕Möbius带边界的磁环量。

导线方程：

$$y=-\frac{1}{4}, z=0$$

$\mathbf{B}=\frac{\mu_0 I\mathbf{\Phi}}{2\pi r}$, r=$(0, y+\frac{1}{4}, z)$ $\mathbf{\Phi}$为环向单位矢量$\mathbf{\Phi}=\frac{(0,+z,-y-\frac{1}{4})}{r}$

$$\oint_c \mathbf{B}\cdot d\mathbf{l}=\frac{\mu_0 I}{2\pi}\oint_c\frac{(0,+z,-y-\frac{1}{4})\cdot d\mathbf{l}}{r^2}$$

$$=\frac{\mu_0 I}{2\pi}\int_c\frac{+zdy-(y+\frac{1}{4})dz}{z^2+(y+\frac{1}{4})^2}$$

$$=\frac{\mu_0 I}{2\pi}\int_0^{4\pi}\frac{\frac{1}{2}\sin\frac{u}{2}\left(\cos(u)\left(\frac{\cos(\frac{u}{2})}{2}+1\right)-\frac{\sin(u)\sin(\frac{u}{2})}{4}\right)-((1+\frac{1}{2}\cos\frac{u}{2})\sin u+\frac{1}{4})\frac{\cos(\frac{u}{2})}{4}}{(\frac{1}{2}\sin\frac{u}{2})^2+((1+\frac{1}{2}\cos\frac{u}{2})\sin u+\frac{1}{4})^2}du$$

$$=0$$

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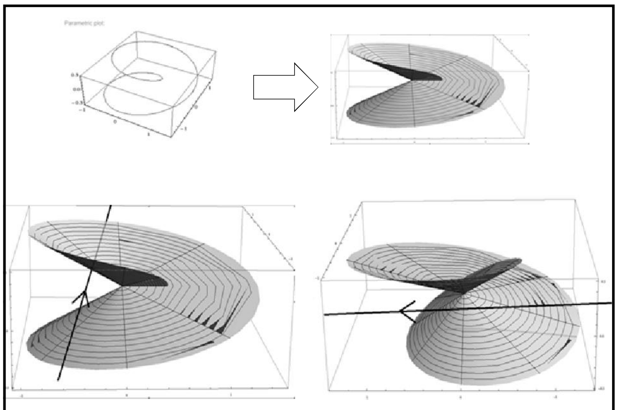


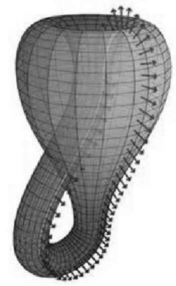
# 根本原因：不定向！

**Non-orientable!**

In the very definition of surface integral, integral can only be calculated on orientable surfaces.

To deal with such non-orientable or degenerated surfaces, our method is to redefine the exact surface of the circle, which is absolutely orientable.

**Orientation also matters in Gauss's law**



## Klein Bottle

$$\oint\!\!\oint_S \mathbf{E}\cdot d\mathbf{S} = \iiint \nabla\cdot\mathbf{E}dV = 0$$

**Trivial**

# 谢谢观看

## Xie Xie