

## Solution of Experimental Problem No.2

### Black box

#### 1. The type of the elements Z

Adjust the oscilloscope to obtain the same gain for the two channels. Use the sine wave from the generator.

Use the circuit in Figure 1 to determine the type of the elements.

We find finally:

$Z_3$ ,  $Z'_1$  and  $Z'_2$  are resistors and

$$Z'_1 = Z'_2 = 2 Z_3 = (10.0 \pm 0.5) \text{ k}\Omega$$

$Z_1$ ,  $Z_2$  and  $Z'_3$  are capacitors, and

$$C_1 = C_2 = \frac{1}{2} C'_3 = (47 \pm 2) \text{ nF}$$

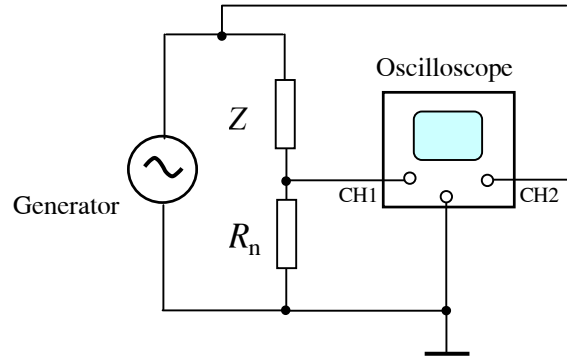


Figure 1

#### 2.

a. The electric circuit of the black box DD'A' is shown in Figure 2.

b. Apply a sine wave signal to connectors D and A'. Connect D and A' to channel 1 (CH1) and D' and A' to channel 2 (CH2).

The ratio  $K = \frac{U_{D'A'}}{U_{DA'}}$  is determined from the amplitudes of

the signals  $U_{D'A'}$  và  $U_{DA'}$ . The phase shift  $\varphi$  can be determined directly from the traces of the signals (or from the Lissajous patterns).

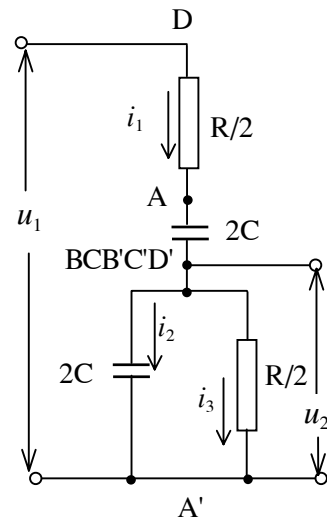


Figure 2

Tabulating  $K$  and  $\varphi$  versus  $f$ , we get for example:

$f$	100	150	200	300	330	400	600	800
$K$		0.29	0.30	0.32	0.33	0.32	0.3	0.28
$\varphi$	44	28	16	2	0	-8	-24	-38

The experimental error of  $f$  is  $\pm 1\text{Hz}$ , of  $\varphi$  is  $\pm 5^\circ$  and of  $K$  is  $\pm 0.02$ .

Here are the plots of  $K$  and  $\varphi$  as functions of  $f$ .

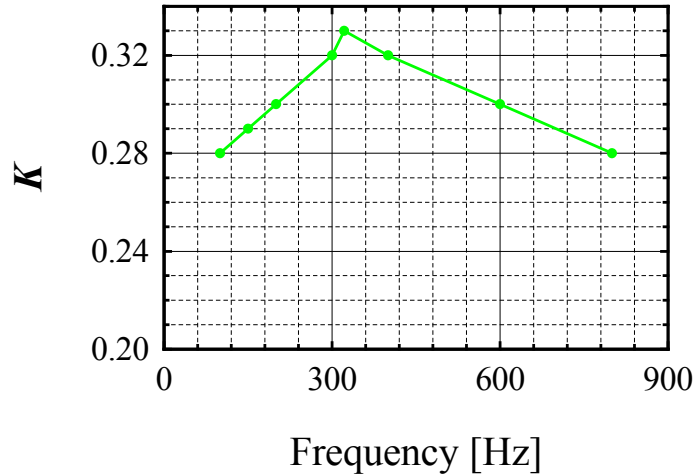


Figure 3

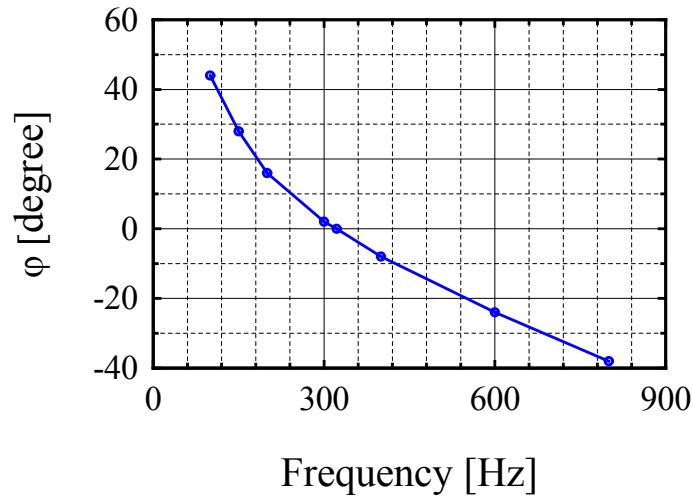


Figure 4

c. The graphs possess a particular point at  $f_0 = 330 \pm 1$  Hz, at which  $\varphi$  equals zero and  $K$  has a maximum of  $K = 0.33 \pm 0.01$ .  $\varphi$  changes its sign from positive to negative with increasing of the frequency  $f$  across  $f_0$ .

The value of  $f_0$  may vary from 325 Hz to 335 Hz depending on the set of experiment, due to the deviation in the value of the resistance and capacitance in the set.

d. The phasor diagram for the circuit is shown in Figure 5, where  $u_1$  is the instantaneous voltage between D and D', and  $U_1$  - its amplitude.

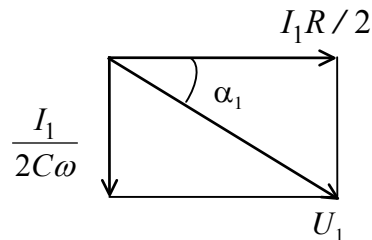


Figure 5

We have  $\tan \alpha_1 = \frac{1}{\omega CR}$  and  $U_1 = I_1 \sqrt{\left(\frac{R}{2}\right)^2 + \frac{1}{4C^2 \omega^2}}$

For the D'A' parallel circuit,  $i_1 = i_2 + i_3$ , and the phasor diagram is shown in Figure 6.  $u_2$  is in phase with  $i_3$ .

$\tan \alpha_2 = \frac{I_2}{I_3} = \omega \cdot 2C \cdot R / 2 = \omega CR$ . Let  $u_2$  the voltage between D'

and A', we have  $I_3 = \frac{U_2}{R/2}$ ;  $I_2 = U_2 \cdot 2C \cdot \omega$ . Hence:

$$U_2 = I_1 \cdot \frac{1}{\sqrt{\left(\frac{R}{2}\right)^2 + 4C^2 \omega^2}}$$

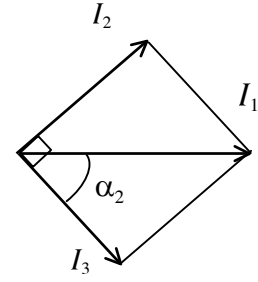


Figure 6

By combining Figure 5 and Figure 6, we obtain Figure 7, with  $u = u_1 + u_2$  being the instantaneous voltage between D và A'.

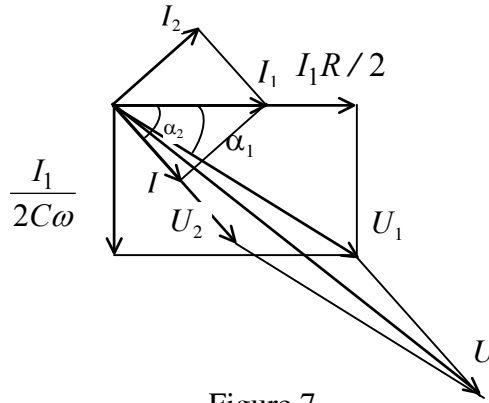


Figure 7

For  $\omega = 2\pi f = \frac{1}{CR}$ ,  $\tan \alpha_1 = \frac{1}{\omega CR} = \tan \alpha_2 = \omega CR$ . In this condition,  $u_1$ ,  $u_2$  and  $u$  are in phase, so that  $\varphi = \alpha_2 - \alpha_1 = 0$ . Hence  $K = \frac{U_{D'A'}}{U_{DA'}} = \frac{U_2}{U_1 + U_2}$ . Substituting  $\omega = \frac{1}{CR}$

and  $U_2$ , we obtain  $K = \frac{1}{3}$ .

That is, for  $f_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi RC} = \frac{1}{2\pi \cdot 10^4 \cdot 48 \cdot 10^{-9}} = 331 \text{ Hz}$ ,  $K = \frac{1}{3} = 0.33$ ,  $\varphi = 0$ , which is observed in the experiment.

For  $\omega \neq \frac{1}{CR}$ ,  $u_1$  and  $u_2$  are out of phase, and  $K$  has values smaller than  $1/3$ .