Experimental Question 1: Levitation of Conductors in an Oscillating Magnetic Field

In an oscillating magnetic field of sufficient strength, levitation of a metal conductor becomes possible. The levitation occurs due to a non-zero mean magnetic force exerted on currents in the conductor. The currents are induced by the alternating field itself. The Lorentz force doesn't average to zero because of a phase shift between the current oscillations and the magnetic field oscillations. This phase shift is a result of the self-inductance of the current loops within the conductor.

In this experiment, we study this phenomenon and deduce the self-inductance of an aluminum ring from measuring the vertical force applied to it by a solenoid with an oscillating current. For measurement convenience, the mean force on the ring will be directed downwards, so levitation will not be observed.

On your desk, you have the following items (Figure 1):

(1) An AC power supply operating on 50Hz. The power supply has two pairs of terminals. The two smaller sockets (1a) supply a voltage of about 24V; use these only for running current through the solenoid. The two larger sockets (1b) supply a voltage of about 0.7V. The power supply is turned on only when the green button (1c) is pressed – this is in order to prevent overheating the system by accidentally leaving the current on, as further explained below. The red light bulb indicates when the power supply is on.

(2) A cylindrically symmetric solenoid filled with iron rods. The solenoid is connected to the 24V terminals of the power supply. The solenoid can be raised and lowered using a long screw. The screw's vertical step is $h = 1.41mm$.

(3-5) Three metal rings made of exactly the same material (an alloy of aluminum). One ring is closed. The second ring is identical to the first except for a short segment which was removed, making the ring open. The third ring is also open, and is much thinner than the first two.

(6-7) Two multimeters. They will be used as a voltmeter and an ammeter. The AC voltmeter's sensitivity is 0.1mV. The ammeter can measure currents of up to 20A (also in AC mode). **Note:** in AC mode, the multimeters display the RMS (root-mean-square) of the measured quantity, i.e. the amplitude divided by $\sqrt{2}$. See Figures 3 and 4 for detailed instructions.

(8) A battery-powered digital scale with sensitivity 0.01g. When the scale experiences a rapidly oscillating force, it displays the time-averaged force. **Note:** the scale has a "tare" option, which calibrates its reading under a given weight to zero. See Figure 2.

(9) An 8cm×7cm×7cm polystyrene block which can be used as a stand for the rings.

(10) Electric wires with various connectors.

(11) A ruler.

(12) Millimeter graph paper.

(13) A desktop lamp which can be turned on or off for your convenience.

Caution: when the closed ring is exposed to the solenoid's magnetic field, a large current flows through it, heating it up. As a consequence, the ring's electrical properties may change slightly. To avoid this, don't run a current through the solenoid for long periods of time.

The earth's gravity field in Tel Aviv is $g = 9.80 \pm 0.01\text{N/kg}$.

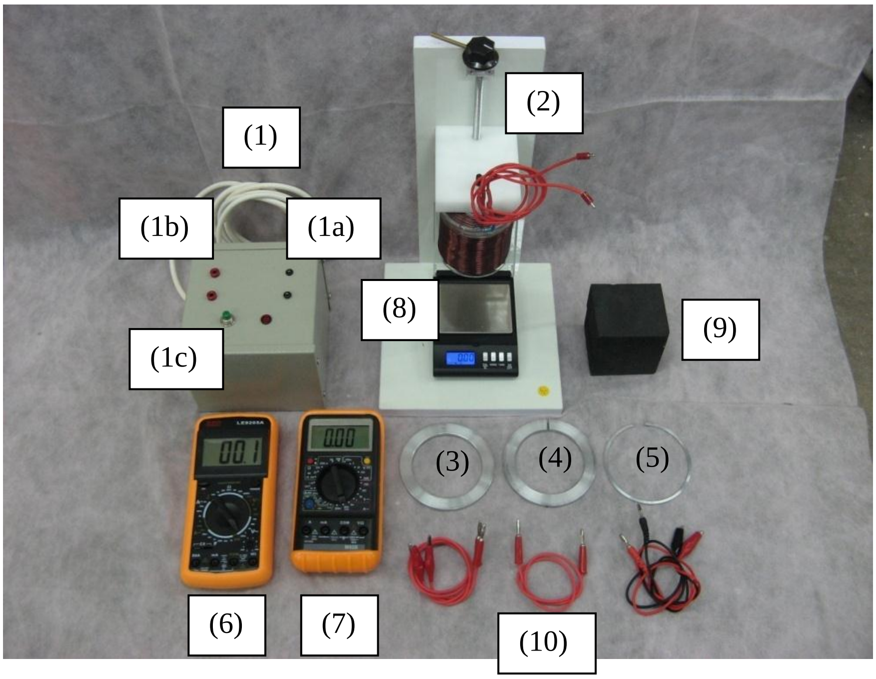

Figure 1 - Summary of the equipment.

(1) 50Hz power supply.
  (1a) 24V terminals.
  (1b) 0.7V terminals.
  (1c) Power button.
(2) Solenoid on a vertical screw.
(3) Broad closed ring.
(4) Broad open ring.
(5) Thin open ring.
(6) Voltmeter.
(7) Ammeter.
(8) Scale.
(9) Polystyrene block.
(10) Wires.

Figure 2: The digital scale

(1) On/Off button
(2) “Tare” button – sets the current weight as 0.

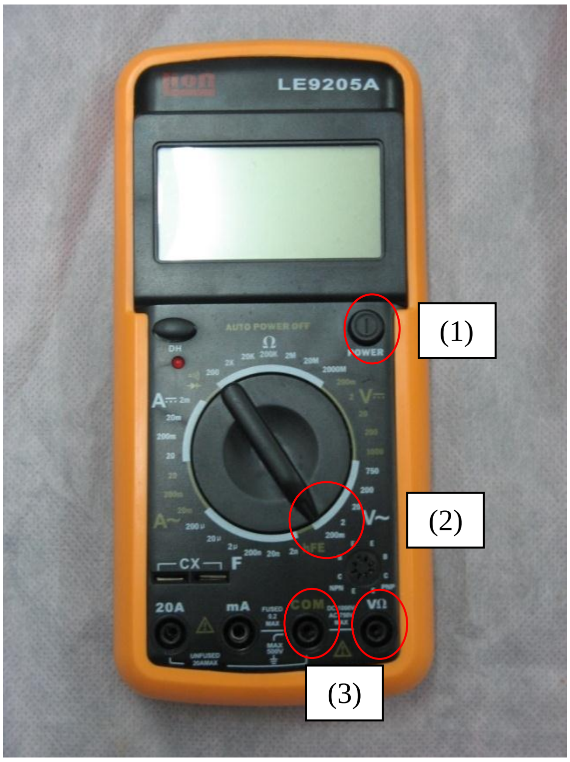

Figure 3: The voltmeter.

(1) On/Off button.
(2) The dial is set to 200mV AC.
(3) Connect your wires to the “COM” and “V/Ω” terminals.

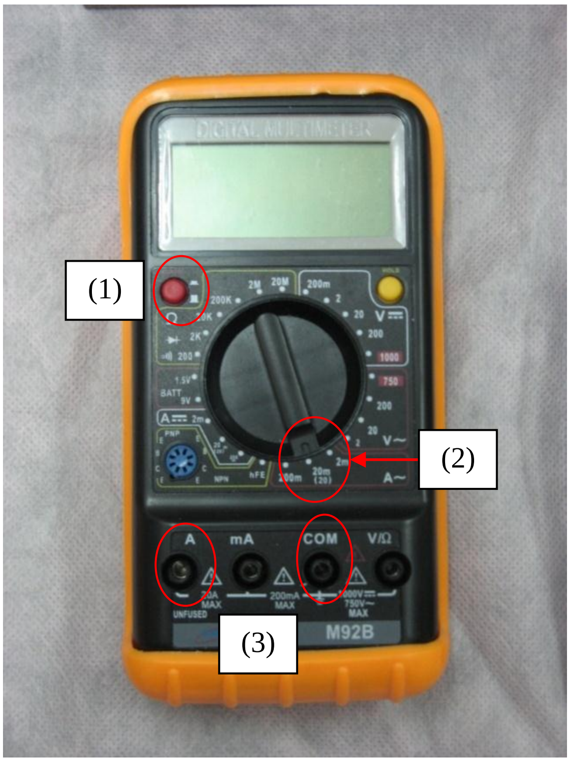

Figure 4: The ammeter.

(1) On/Off button.
(2) The dial is set to 20mA/20A AC.
(3) Connect your wires to the “COM” and “A” terminals.

Theory (1.3 points)

Consider a conducting ring of radius r placed in the solenoid's magnetic field. The symmetry axes of the solenoid and the ring coincide. Denote the ring's inductance by L, its resistance by R, and the angular frequency of the solenoid's current by ω. Define z as the coordinate along the common symmetry axis of the solenoid and the ring.

In this part only, you may neglect the small effect of the ring's magnetic field on the solenoid and the iron. Also, neglect the thickness of the ring.

In this part, we'll use Faraday's law, and the magnetic version of Gauss's law:

- Faraday's law: The induced EMF (electromotive force) on a loop generated by a changing magnetic flux is $\epsilon = -d\Phi_{\mathrm{B}}/dt$.
- The magnetic Gauss law: the total magnetic flux through a closed surface is zero.

A current loop placed in a cylindrically symmetric magnetic field $\vec{B}$ experiences a total force

$$F(t) = -2\pi r I(t) B_r(t)$$

where I is the current in the loop, and B_r is the radial component (in the direction of the loop's radius) of the external magnetic field in the loop's vicinity. The positive direction of the force F is downwards – in the z direction. The positive direction of the current I is shown in Figure 5.

a. (0.2 pts.) Consider an oscillating external magnetic flux $\Phi_{\mathrm{B}}(t) = \sqrt{2}\Phi_{\mathrm{B}}^{\mathrm{rms}} \sin(\omega t)$ through the ring.
Find $\epsilon(t)$ - the EMF induced by the given flux only, and $I(t)$ - the current induced in the ring, as functions of $\Phi_{\mathrm{B}}^{\mathrm{rms}}$, L, R, ω and t.
Hint: The EMF amplitude ϵ_0 and the current amplitude I_0on an AC circuit element with both resistance and inductance are related by $\epsilon_0 = I_0\sqrt{\omega^2 L^2 + R^2}$, and the current is delayed by a phase $\delta = \tan^{-1}\frac{\omega L}{R}$ with respect to the EMF.

b. (0.6 pts.) Find B_r in terms of r and $\frac{\mathrm{d}}{\mathrm{dz}}\Phi_{\mathrm{B}}$, where z is the coordinate along the axis perpendicular to the ring's plane.

c. (0.5 pts.) Show that $\langle F\rangle = \alpha \frac{L}{(R^2+\omega^2L^2)} \cdot \frac{d(\epsilon_{rms})^2}{dz}$, where $\langle F\rangle$ is the time averaged value of F , $\epsilon^{rms}(z)$ is the RMS (root-mean-square, i.e., amplitude divided by $\sqrt{2}$) of the EMF on a loop at height z. Find the constant α (if you do not find α, in later parts, take the magnitude of α to be 1).
Hint: you may find the following identities useful:

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\langle(\sin(\omega t))^2\rangle = \langle(\cos(\omega t))^2\rangle = \frac{1}{2}$$

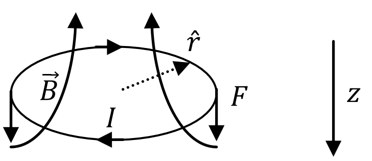

Figure 5: The metal ring in the solenoid's magnetic field.

Measurements (5.1 points)

In all of the following measurements and analysis, take into account that results with higher precision will receive higher grades. In all of your measurements and results, specify error estimations.

Resistance measurements (2.6 points)

In this part, you should use the 0.7V terminals of the AC power supply. Using the supplied wires to short-circuit the two 0.7V terminals should result in a current of $5\text{A} - 15\text{A}$, depending on the contacts. Note that the three shorter wires achieve a better contact with the ammeter than the two longer ones. CAUTION: Don't use the 24V terminals, to avoid overheating the components.

d. (1.3 pts.) Find the resistance R_{thin} of the thin ring. Draw your circuit on the answer form.
Hint: the resistance of each of the rings is much smaller than 0.1Ω. *For the thin ring, you can neglect the inductive impedance with respect to the resistance.*

e. (1.3 pts.) Find the resistance R of the closed ring. Make additional measurements as necessary.

Measurements of the induced EMF (1.5 points)

f. (1.5 pts.) Connect the solenoid to the 24V terminals of the power supply. Place the broad open ring so that its axis coincides with the axis of the solenoid. Measure the induced EMF ϵ_{rms} on the ring at different heights z, i.e., at different distances from the solenoid. Record your measurements in the provided table on the answer form. Plot a graph of ϵ_{rms} as a function of z (with a trend line).

Measurements of the force (1 point)

g. (1 pt.) Place the broad closed ring so that its axis coincides with the axis of the solenoid. Measure the time-averaged magnetic force $\langle F \rangle$ on the ring at different heights z, i.e. at different distances from the solenoid. Record your measurements in the provided table on the answer form.

Analysis (3.6 points)

h. (1.4 pts.) Find the absolute value of the derivative $|d\epsilon_{rms}^2/dz|$ of ϵ_{rms}^2 with respect to z, for values of z where you measured the force in part (g). Record your values in the provided table on the answer form. Error estimations are not required in this part.

i. (2.2 pts.) Analyze your results using a linear graph to find L - the inductance of the closed ring. You may use the fact that $\omega L < R$.
Note: Despite the noticeable thickness of the closed ring, the formula you derived in part (c) still applies with a high accuracy. Use it as an operational *definition* for the inductance of a broad ring.
Hint: When the ring is too close to the iron, the measurements will be distorted. Try to avoid this complication in your analysis.