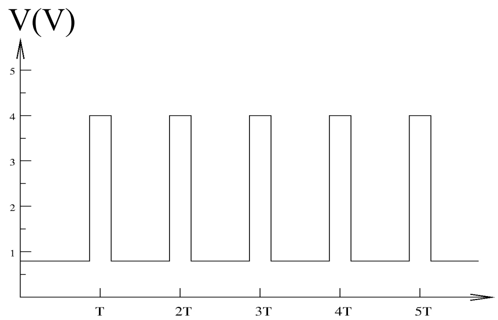
WIND POWER AND ITS METROLOGIES (20 points)

A. Theoretical Background (1.0 points)

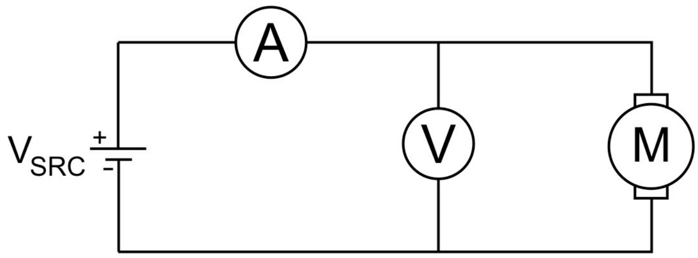
A.1 (0.4 pts)	$P_w = \frac{1}{2} v_0^2 \frac{dm}{dt}$ $P_w = \frac{1}{2} \rho A_0 v_0^3$ $n = 3$
A.2 (0.4 pts)	$P_R = \frac{1}{2} \rho A \frac{v_0^3}{2} (1+\lambda)\left(1-\lambda^2\right) = \frac{\rho A v_0^3}{4}(1+\lambda-\lambda^2-\lambda^3)$ $\frac{dP_R}{d\lambda} = 0 \rightarrow 1-2\lambda-3\lambda^2 = 0$ $\lambda = \frac{1}{3}$
A.3 (0.2 pts)	Betz efficiency: $C_P = \left.\frac{P_R}{P_W}\right\|_{\lambda} = \frac{16}{27} \sim 59.26\%$

B. The Wind Tunnel (3.2 points)

B.1 (0.8 pts)	We move the motor generator blade manually, and the voltage at the opto-sensor signal will increase every time the sensor hits the reflective sticker in the blade. This signal provides frequency signal to the meter. 
B.2 (2.4 pts)	$\eta_M = \frac{P_W}{P_M} = \frac{\rho_A A_0 v_0^n}{2P_M} \rightarrow P_M = \frac{\rho_A A_0 v_0^n}{2\eta_M}$ Wind tunnel diameter: $D_T = 13.5$ cm. $A_0 = \pi R_T^2 = 0.0143\,\text{m}^2$. $\ln P_M = \ln \frac{\rho_A A_0}{2\eta_M} + n \ln v \rightarrow y = a + bx$ From the plot and linear regression below we obtain the power factor: $n = 3.0$, *in good agreement with the theory* thus showing that the wind

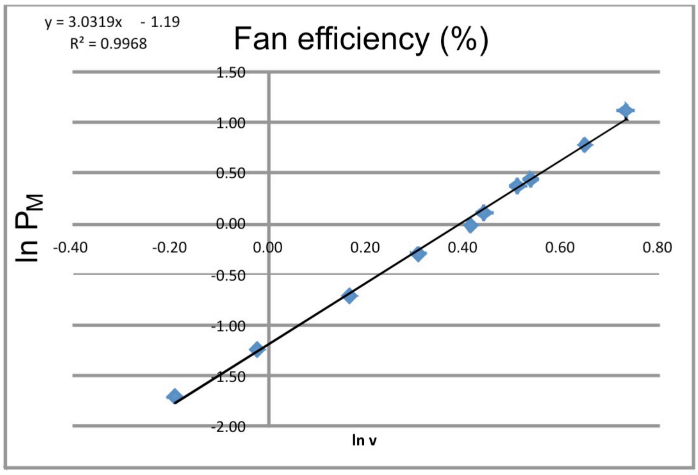
	power $P_W \sim v^3$. $\eta_M = \dfrac{\rho_A A_0}{2e^a} = 2.8\%$ Results for full score / grading scheme (sampling from several setups): $\eta_M = (3 \pm 2)\%$, to anticipate wide variability in motor quality.

Connection diagram:

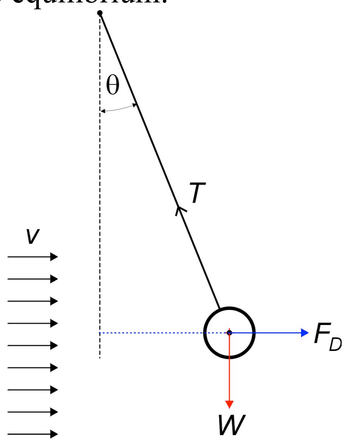


Note that for best results the voltmeter has to be placed right across the motor to avoid extra voltage drop across the amperemeter.

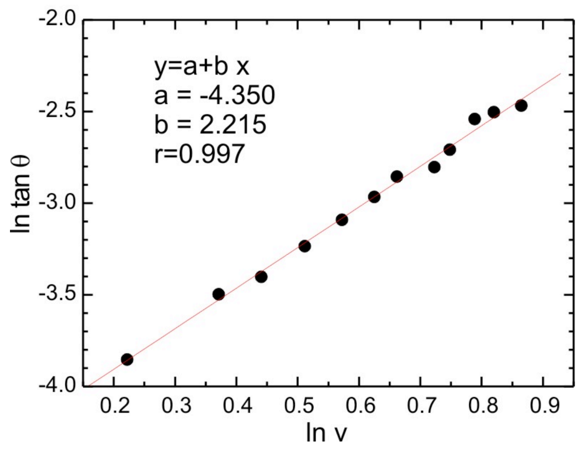
f (Hz)	v (m/s)	V (V)	I (A)	P_M (W)	ln v	ln P_M
27.2	2.38	15.920	0.350	5.57	0.87	1.72
26.0	2.27	14.910	0.330	4.92	0.82	1.59
25.0	2.18	13.410	0.300	4.02	0.78	1.39
23.8	2.08	11.830	0.260	3.08	0.73	1.12
21.9	1.91	9.880	0.220	2.17	0.65	0.78
19.6	1.71	8.100	0.190	1.54	0.54	0.43
17.3	1.51	6.540	0.150	0.98	0.41	-0.02
19.1	1.66	7.370	0.196	1.44	0.51	0.37
17.8	1.55	6.490	0.172	1.12	0.44	0.11
15.6	1.36	5.240	0.142	0.74	0.31	-0.30
13.5	1.18	4.210	0.116	0.49	0.16	-0.72
11.2	0.98	3.240	0.089	0.29	-0.02	-1.25
9.5	0.82	2.630	0.068	0.18	-0.19	-1.72
6.8	0.59	2.390	0.044	0.10	-0.52	-2.26



C. Ping Pong Ball Anemometer (3.5 points)

C.1 (0.7 pts)	Force diagram at static equilibrium:  $\tan\theta = \frac{F_D}{W_B} = \frac{C_D\,\rho_A\,A_B\,v^m}{2\,m_B\,g}$ $v = \sqrt[m]{\frac{2m_B g}{C_D \rho_A A}\tan\theta}$
C.2 (2.8 pts)	$\ln\tan\theta = \ln\frac{C_D\rho_A A_B}{2m_B g} + m\ln v \rightarrow y = a + bx$ The deflection can be calculated from: $\tan\theta = \Delta x / h$, where Δx is the displacement and h is the height from the ruler. The cross section of the ball is: $A_B = \pi\, d_B^{\;2}/4$ where $d_B = 0.0395\,\mathrm{m}$. In the tunnel the wind velocity is given as (Eq. 4) in the tunnel $v = c_1 f_M$ where $c_1 = 0.0873\,\mathrm{m}$. In this example we have: $m = b = 2.2$

	$C_D = \dfrac{2m_B g}{\rho_A A_B} e^a = 0.40$ Results for full score / grading scheme (sampling from several setups): $m = 2.0 \pm 0.3$ $C_D = 0.42 \pm 0.05$



f (Hz)	tan θ	V (m/s)	ln v	ln tan θ
27.2	0.0848	2.3746	0.8648	-2.47
26.0	0.0818	2.2698	0.8197	-2.50
25.2	0.0788	2.2	0.7884	-2.54
24.2	0.0667	2.1127	0.7479	-2.71
23.6	0.0606	2.0603	0.7228	-2.80
22.2	0.0576	1.9381	0.6617	-2.85
21.4	0.0515	1.8682	0.625	-2.97
20.3	0.0455	1.7722	0.5722	-3.09
19.1	0.0394	1.6674	0.5113	-3.23
17.8	0.0333	1.5539	0.4408	-3.40
16.6	0.0303	1.4492	0.371	-3.50
14.3	0.0212	1.2484	0.2219	-3.85

NOTE:

These values are in very good agreement with the theoretical and established value of $n = 2$, i.e. the drag force is proportional to the square of the velocity.

The drag coefficient C_D is in close to the ideal known value: $C_D = 0.47$ for smooth ball with particle Reynold number Re ~ $10^3 - 10^5$ as shown below. In this experiment the maximum Reynold number is:

$$\mathrm{Re}_B = \frac{\rho_A v D_B}{\mu_A} = \frac{1.2\times 2.5\times 0.038}{18.3\times 10^{-6}} = 6240 \qquad (1)$$

Discrepancy in our C_D values could be due to finite boundary of our wind tunnel.

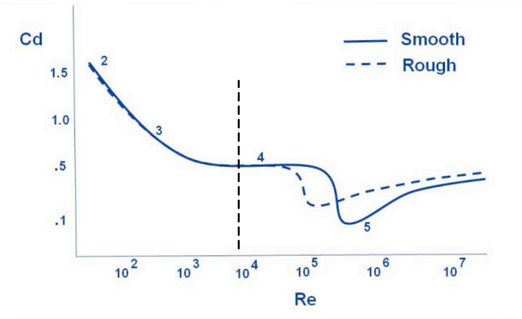


Figure 1. Drag coefficient as a function of the particle Reynold number

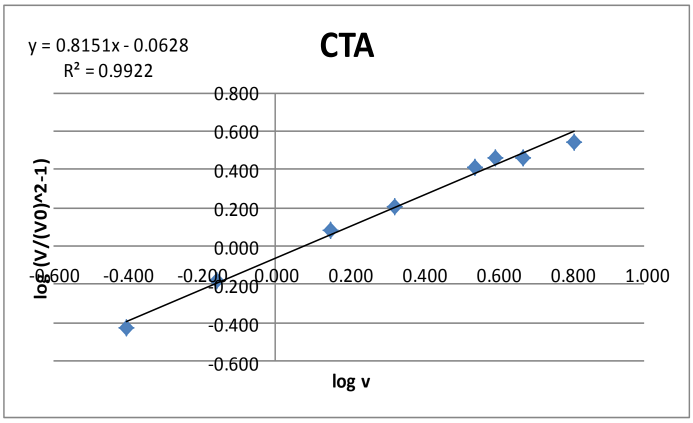
D. Hotwire Anemometer (HWA) (6.7 points)

[D.1] Constant Temperature (3.2 points)

D.1.1 (0.4 pts)	In the Wheatstone bridge we have: $V_W = V_{INP}\dfrac{R_W}{R_W+R_B} = c_1 V_{INP}$ $\dfrac{V_W^{\,2}}{R_W} = (a+bv^c)A_w(T_w - T_0) \rightarrow V_{INP}^{\,2} = \dfrac{R_W}{c_1^{\,2}}A_W(T_W-T_0)(a+bv^c)$ $V_{INP}^{\,2} = \dfrac{(R_W+R_B)^2}{R_W}A_W(T_W-T_0)(a+bv^c) = c_2(a+bv^c)$ Where $c_2 = \dfrac{(R_W+R_B)^2}{R_W}A_W(T_W-T_0)$ is a constant. $A = c_2 a$ $B = c_2 b$
D.1.2 (0.3 pts)	$y = \left(\dfrac{V_{INP}}{V_0}\right)^2 - 1$ with $V_0 = \sqrt{A}$ is the input potential when there is no wind.
D.1.3 (2.5 pts)	$c = 0.7 \pm 0.2$

	$\dfrac{b}{a} = 1.5 \pm 0.6$

f_M (Hz)	V_{INPUT} (V)	v (m/s)	ln v	$\ln((V/V_0)^2-1)$
25.70	1.785	2.244	0.808	0.546
22.40	1.740	1.956	0.671	0.464
20.80	1.737	1.816	0.597	0.459
19.70	1.710	1.720	0.542	0.407
15.83	1.615	1.382	0.324	0.209
13.30	1.560	1.161	0.149	0.079
9.76	1.464	0.852	-0.160	-0.181
7.65	1.390	0.668	-0.404	-0.426



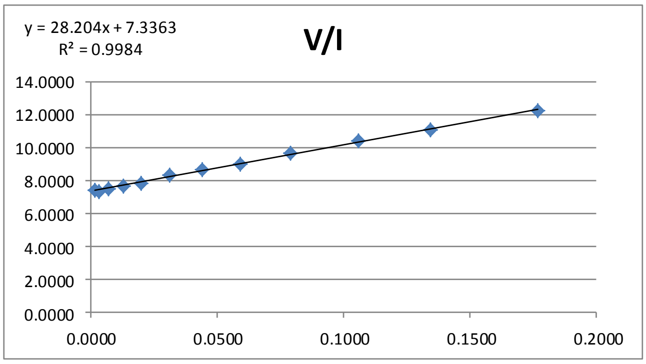
[D.2] Constant Current (3.5 points)

First we need to determine k and R_0.

D.2.1 (0.2 pts)	$k = \dfrac{aA_w}{\alpha}$
D.2.2 (1.2 pts)	$R_0 = (7.0 \pm 1.5)\,\Omega$

V_W (V)	I_W (mA)	P_W (W)	R_W (Ω)
0.109	14.7	0.0016	7.4150
0.153	20.8	0.0032	7.3558
0.225	30.1	0.0068	7.4751
0.308	40.3	0.0124	7.6427

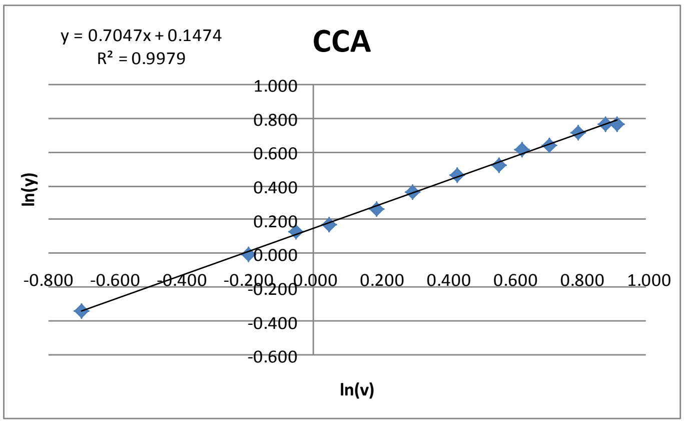
0.395	50.4	0.0199	7.8373
0.509	61.2	0.0312	8.3170
0.615	71.4	0.0439	8.6134
0.730	81.0	0.0591	9.0123
0.870	90.5	0.0787	9.6133
1.048	100.7	0.1055	10.4071
1.220	110.0	0.1342	11.0909
1.470	120.1	0.1765	12.2398



D.2.3 (0.2 pts)	$y = \dfrac{V_W I_W R_0}{k\left(\dfrac{V_W}{I_W} - R_0\right)} - 1$
D.2.4 (1.9 pts)	$\dfrac{b}{a} = 1.5 \pm 0.4$ $c = 0.77 \pm 0.07$

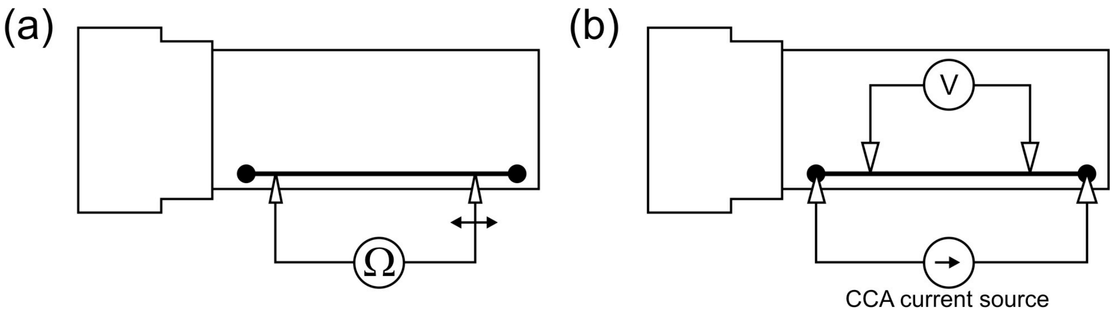
V_W (V)	f (Hz)	v (m/s)	y	$\ln(v)$	$\ln(y)$
1.41		0.498	0.711	-0.698	-0.342
1.33		0.821	0.991	-0.198	-0.009
1.30		0.945	1.134	-0.056	0.125
1.29		1.048	1.187	0.047	0.172
1.27		1.206	1.306	0.188	0.267
1.25		1.344	1.443	0.295	0.367
1.23		1.542	1.586	0.433	0.461
1.22		1.746	1.684	0.557	0.521
1.20		1.873	1.855	0.627	0.618
1.20		2.032	1.899	0.709	0.642
1.19		2.211	2.056	0.794	0.721

1.18	2.406	2.148	0.878	0.765
1.18	2.492	2.162	0.913	0.771



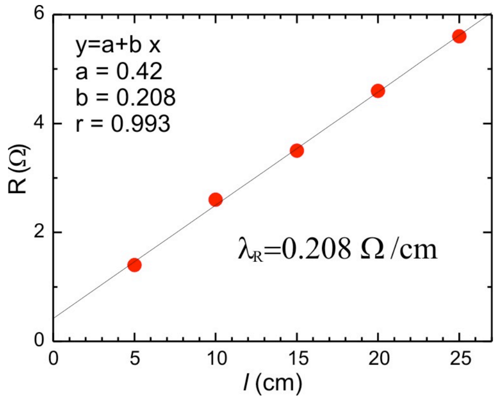
E. Wind Turbine (5.6 points)

E.1 (0.4 pts)	We use DMM in ohmmeter mode, we first measure the lead resistance of the ohmmeter cables by shorting them to get the lead resistance. $R_{DMM,0} \sim 0.2\ \Omega$ Then we measure the resistance of the motor directly with DMM several times, looking for its minimum values and subtract it with the lead resistance $R_{DMM,0}$. We get : $R_M = (0.8 \pm 0.2)\Omega$
E.2 (1.2 pts)	To eliminate the contact or lead resistance we can perform: (1) Two point resistance measurement at several lengths. (2) Four point resistance measurement with current source (using CCA box) ampmeter and voltmeter as shown below. This yields: $\lambda_R = (0.21 \pm 0.02)\Omega / \text{cm}$



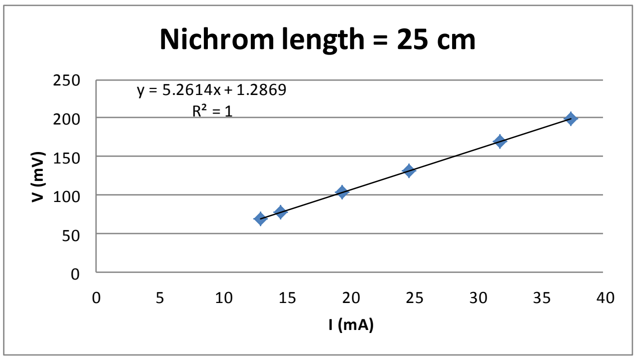
(a) Two-wire method (b) Four-wire method

Method #1: Using two-wire measurement



Method #2: Using four-wire method at $l = 250$ mm.

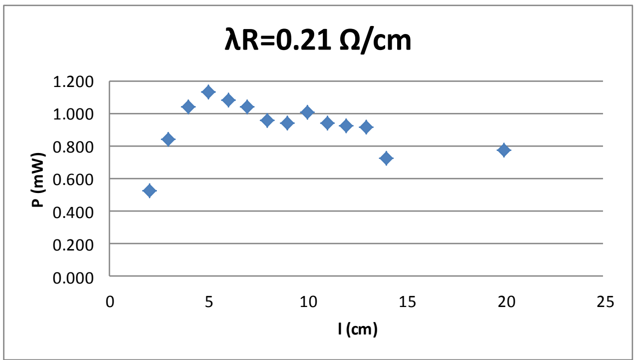
V (mV)	I (mA)
198.5	37.5
169.2	31.9
131.2	24.7
103.5	19.4
78.3	14.6
69.4	13



E.3 (2.4 pts)	We vary the nichrom wire length and calculate the power output, we obtain peak at: $R_L = 1.0\Omega$

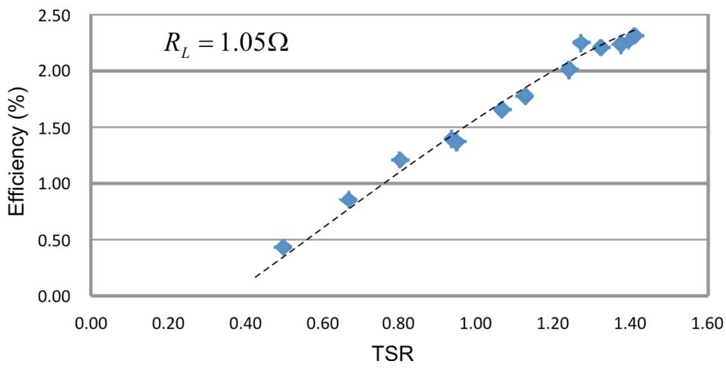
	Results for full score / grading scheme (sampling from several setups): $R_L = (1.0 \pm 0.4)\Omega$ $R_{L,theor} = R_M$

V (mV)	l (cm)	R (ohm)	P (mW)
14.8	2	0.42	0.522
23.0	3	0.63	0.840
29.5	4	0.84	1.036
34.5	5	1.05	1.134
36.9	6	1.26	1.081
39.0	7	1.47	1.035
40.1	8	1.68	0.957
42.1	9	1.89	0.938
45.9	10	2.10	1.003
46.5	11	2.31	0.936
48.2	12	2.52	0.922
49.9	13	2.73	0.912
46.0	14	2.94	0.720
57.0	20	4.20	0.774



E.4 (1.6 pts)	We use the optimum load by setting R_L = 1.05 Ω or setting the length of nichrom wire at l = 5 cm. The wind turbine efficiency : $\eta_{WT} = \frac{P_T}{P_W} = \frac{V_T^2 / R_L}{\frac{1}{2}\rho_A A_{WT} v^3}$ where $A_{WT} = \pi R^2$ is the wind turbine cross section area with R = 55 mm, and $v = c_1 f_M$ (Eq. 4) with c_1 = 0.0873 m and f_M is the frequency of motor generator. $TSR = \frac{2\pi f_T R}{c_1 f_M}$

f_M (Hz)	f_T (Hz)	V_T (mV)	TSR	P_T (mW)	P_W (mW)	η_{WT} (%)
27.8	9.8	44.0	1.40	1.84	81.48	2.26
27.9	9.7	44.0	1.38	1.84	82.36	2.24
27.2	9.7	43.0	1.41	1.76	76.32	2.31
26.3	8.8	40.0	1.32	1.52	68.99	2.21
25.5	8.2	38.5	1.27	1.41	62.88	2.24
24.6	7.7	34.5	1.24	1.13	56.46	2.01
23.6	6.7	30.5	1.12	0.89	49.85	1.78
22.3	6.0	27.0	1.07	0.69	42.06	1.65
21.2	5.0	23.0	0.93	0.50	36.13	1.39
21.3	5.1	22.9	0.95	0.50	36.65	1.36
19.8	4.0	19.3	0.80	0.35	29.44	1.20
18.4	3.1	14.5	0.67	0.20	23.62	0.85
16.8	2.1	9.0	0.49	0.08	17.98	0.43



Our turbine has range of efficiency <2.5% and tends to increase with higher TSR as the turbine spin faster. Note that at some point this efficiency will drop, unfortunately this is beyond the capability of our wind generator fan.