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BOSE-EINSTEIN CONDENSATION

Green's function method to the ground state properties of a two-component Bose–Einstein condensate

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The elementary excitation spectrum of a two-component Bose–Einstein condensate is obtained by Green's function method. It is found to have two branches. In the long-wave limit, the two branches of the excitation spectrum are reduced to one phonon excitation and one single-particle excitation. With the obtained excitation spectrum and the Green's functions, the depletion of the condensate and the ground state energy have also been calculated in this paper. © *2011 American Institute of Physics*. [doi: 10.1063/1.3633685]

I. INTRODUCTION

The realization of Bose–Einstein condensation (BEC) (Refs. 1 and 2) has attracted much interest in the past years, because it provides the unique opportunities for exploring quantum phenomena on a macroscopic scale. The Bose–Einstein condensation for noninteracting particles is characterized as the macroscopic occupation number for one of the single-particle energy levels. For interacting systems, the criterion for BEC is generalized by Penrose and Onsager and by Yang as off-diagonal long range order.[3,4] The experimentally realization of BEC is in dilute atomic gases, in which mean field theory is well applied in the nearly zero temperature.[5,6] The condition for diluteness is $na_s^3 \ll 1$, where n is the density of the gas and a_s is the s-wave scattering length. The interaction between atoms is characterized by the s-wave scattering length a_s, which can be manipulated by the use of lasers and magnetic fields. The transition between repulsive and attractive interaction can be controlled by a Feshbach resonance. The properties of a gas in a trap are usually studied by Thomas–Fermi approximation. When the length scale of the trap is much greater than the coherence length ξ, the gas is assumed to be homogeneous.

The subject of two-component Bose–Einstein condensate has attracted many experimental and theoretical studies.[7–15] Much work has been devoted to the case of double-well trapping,[9–13] which is similar to the Josephson junctions of superconductors. The recent experiments have directly observed the phenomena of plasmon oscillation and macroscopic quantum self-trapping (MQST),[12,13] which have been theoretically predicted earlier.[9–11] Another possible case is the mixtures of the same isotope, but in different internal spin states, such as ^{87}Rb.[7,8] In this case, atoms can undergo transitions between hyperfine states by an external field, which corresponds to the tunneling effects in the double-well case. The two-body interaction between the atoms of the same state may be different from the interaction of the different hyperfine states. The dynamics of this case is similar to the double-well condensate.[11] The thermal effects can act as the damping term and the system under damping will evolve into a stationary state of two equivalent components.[10] The π phase difference of the two components corresponds to the energy minimum in the mean field theory in the zero temperature.[14]

In this paper we will study the ground state properties of a two-component BEC. The concept of elementary excitations is important for the ground state BEC, and it can be studied by several ways. The excitation spectrum can be achieved by linearizing the hydrodynamic equations derived from the Gross–Pitaevskii equation.[6,15] However for a quantum Bose gas, the excitation spectrum was first obtained by Bogoliubov by a special transformation,[16] which has been well extended for many quantum theories. It is well known that the method of Green's function can be applied to find the elementary excitations in many fields of condensed matter physics.[17] In this paper, we will extend the method developed by Beliaev[18] to the Bose–Einstein condensate of two equivalent components. In the mean field approximation, the elementary excitation spectrum is found to have two branches. In the long-wave limit, the two branches of the excitation spectrum are reduced to one phonon excitation and one single-particle excitation. By use of the obtained excitation spectrum and the Green's functions, we have also calculated the depletion and the ground state energy of the condensate.

In Sec. II, we introduce the Green's method for a homogeneous Bose–Einstein condensate in the mean field approximation. In Sec. III, we apply the method to find the elementary excitation spectrum for a two-component BEC. The depletion of the condensate and the ground state energy have also been calculated in this section. In Sec. IV, we make a conclusion of the paper.

II. GREEN'S FUNCTION METHOD TO BOSE–EINSTEIN CONDENSATE

The ground-canonical Hamiltonian for a homogeneous Bose gas in a volume V is often written as[17]

$$K = \sum_p \left(\frac{p^2}{2m} - \mu\right) a_p^+ a_p + \frac{U_0}{2V} \sum_{k_1 k_2 q} a_{k_1+q}^+ a_{k_2-q}^+ a_{k_1} a_{k_2}, \quad (1)$$

where U_0 characterize the two-body interaction, $\mu = U_0 n_0$ is the chemical potential used to keep the conservation of the total number of particles, and a_p^+ and a_p are the Bose creation and annihilation operators in the momentum representation. In zero temperature the condensate is well described by a field. For simplicity, the phase of the condensate can be assumed to be zero,[17] and we can get the useful average $\langle a_0 \rangle = \langle a_0^+ \rangle = \sqrt{N_0}$.

In order to get the excitation spectrum, we introduce two single-particle Green's functions in zero temperature:[17]

$$G(p, t-t') = -i\langle T a_p(t) a_p^+(t') \rangle, \tag{2}$$

$$F(p, t-t') = -i\langle T a_{-p}^+(t) a_p^+(t') \rangle. \tag{3}$$

where the T denotes the chronological product and the operators are in the Heisenberg picture. The first one is the normal Green's function and the second is abnormal. We will try to derive their dynamical equations. For example:

$$\begin{aligned} i\frac{\partial}{\partial t} G(p, t-t') &= \delta(t-t') - i\langle T[a_p, K] a_p^+(t') \rangle \\ &= \delta(t-t') - i\left\langle T\left(\frac{p^2}{2m} - \mu\right) a_p(t) a_p^+(t') \right\rangle \\ &\quad - i\frac{U_0}{V}\sum_{kq}\left\langle T a_{k+q}^+(t) a_k(t) a_{p+q}(t) a_p^+(t') \right\rangle. \end{aligned} \tag{4}$$

The brackets of four operators in the interaction term must be reduced to products of pair operators by Wick's theorem:[17]

$$\begin{aligned} & - i\frac{U_0}{\Omega}\sum_{kq}\langle T a_{k+q}^+(t) a_k(t) a_{p+q}(t) a_p^+(t') \rangle \\ &= -i\frac{U_0}{V}\sum_{k} 2\langle a_k^+(t) a_k(t) \rangle \langle T a_p(t) a_p^+(t') \rangle \\ &\quad - i\frac{U_0}{V}\sum_{q}\langle a_{-p-q}^+(t) a_{p+q}(t) \rangle \langle T a_{-p}^+(t) a_p^+(t') \rangle \\ &\quad \approx 2nU_0 G(p, t-t') + \frac{U_0}{V}\langle a_{10}^+(t) a_{10}(t) \rangle F(p, t-t') \\ &= 2nU_0 G(p, t-t') + nU_0 F(p, t-t'). \end{aligned} \tag{5}$$

To deduce (5), we have made mean field approximation and the condensate density n_0 is replaced by n. If we make Fourier transformation of (5) into energy representation, we can get

$$\omega G(p, \omega) = 1 + \left[\left(\frac{p^2}{2m} - \mu\right) + 2nU_0\right] G(p, \omega) + nU_0 F(p, \omega). \tag{6}$$

Similarly, we can get another equation for the abnormal Green's function as

$$\omega F(p, \omega) = -\left[\left(\frac{p^2}{2m} - \mu\right) + 2nU_0\right] F(p, \omega) - nU_0 G(p, \omega). \tag{7}$$

From (6) and (7), we can get the two Green's functions as

$$G(p, \omega) = \frac{\omega + \dfrac{p^2}{2m} + nU_0}{\omega^2 - \varepsilon^2 + i0}, \tag{8}$$

$$F(p, \omega) = -\frac{nU_0}{\omega^2 - \varepsilon^2 + i0}, \tag{9}$$

where

$$\varepsilon(p) = \sqrt{\frac{p^2}{2m}\left(\frac{p^2}{2m} + 2nU_0\right)}. \tag{10}$$

The result (10) is the well-known elementary excitation spectrum. In the long wave-length limit $p \to 0$, the excitation (10) can be reduced to the phonon form $\varepsilon = sp$, where $s = \sqrt{nU_0/m}$ is the sound speed.

From the obtained Green's function (8), we can get the density of the noncondensate atoms as[17,19]

$$n_{\text{ex}} = \lim_{t \to -0} \int \frac{d^3p}{(2\pi\hbar)^3} \int d\omega \frac{i}{2\pi} e^{-i\omega t} G(p, \omega) = \frac{1}{3\pi^2\hbar^3}(mnU_0)^{3/2}, \tag{11}$$

and the energy density of the condensate can be calculated as

$$\begin{aligned} \frac{E}{V} &= \frac{1}{2}U_0 n^2\left(1 + mU_0\frac{1}{V}\sum_{p\neq 0}\frac{1}{p^2}\right) \\ &\quad + \lim_{t \to -0} \int \frac{d^3p}{(2\pi\hbar)^3} \int d\omega \frac{i}{2\pi} \mathrm{e}^{-i\omega t} \varepsilon(p) G(p, \omega) \\ &= \frac{U_0 n^2}{2} + \frac{8}{15}\frac{m^{3/2}}{\pi^2\hbar^3}(U_0 n)^{5/2}. \end{aligned} \tag{12}$$

To deduce the result (12), we have replaced the bare coupling constant

$$U_0 = \frac{4\pi\hbar^2 a}{m} \quad \text{by} \quad U_0 = \frac{4\pi\hbar^2 a}{m}\left(1 + \frac{4\pi\hbar^2 a}{V}\sum_{p\neq 0}\frac{1}{p^2}\right)$$

Refs. 17 and 20, where a is the scattering length. The first part of (12) is the interaction energy of the condensate, and the second part comes from the contributions of the noncondensed atoms. The result (12) was first obtained by Lee and Yang.[20]

The Green's function method used here was first introduced by Beliaev,[18] which is base on the mean-field approximation. The excitation spectrum can also be obtained by the method of Bogoliubov transformation. For a homogeneous BEC, the Hamiltonian should be simplified before the transformation:[16,17]

$$\begin{aligned} H &= \frac{N^2 U_0}{2V} + \sum_p \frac{p^2}{2m} a_p^+ a_p \\ &\quad + \frac{NU_0}{2V}\sum_{p\neq 0}(a_p a_{-p} + a_p^+ a_{-p}^+ + 2a_p^+ a_p). \end{aligned} \tag{13}$$

The above Hamiltonian (13) is deduced by mean field approximation and the fact that the fluctuation in the particle number is small. This was first done by Bogoliubov,[16] and the method of Bogoliubov transformation is to construct new Bose operators to make the simplified Hamiltonian (13) into diagonal form. However, if we begin from the above simplified Hamiltonian, we will be easier to get the dynamical equations of the Green's functions. The excitation spectrums obtained by the two methods are completely same in the form.[17] The Bogoliubov transformation method and the Green's function method in this paper are all based on mean field approximation, which is the first order approximation to the many-body theory.[17,19]

III. EXCITATION SPECTRUM AND THE GROUND STATE OF A TWO-COMPONENT BEC

The Hamiltonian for a homogeneous two-component Bose gas in a volume V can be written in the form[14]

$$\begin{aligned} K = &\sum_{i,p}\left(\frac{p^2}{2m}-\mu\right)a^+_{ip}a_{ip} + \sum_p \eta\left(a^+_{1p}a_{2p}+a^+_{2p}a_{1p}\right) \\ &+\frac{1}{2V}\sum_{i,k_1k_2q} U_s a^+_{ik_1+q}a^+_{ik_2-q}a_{ik_1}a_{ik_2} \\ &+\frac{1}{V}\sum_{k_1k_2q} U_x a^+_{1k_1+q}a^+_{2k_2-q}a_{1k_1}a_{2k_2}, \end{aligned} \tag{14}$$

where η is a coupling parameter which shows the transition between the two kinds of particles, U_s and U_x characterize the two-body interaction of the same kinds of particles and two different kinds of particles, μ is the chemical potential used to keep the conservation of the total number of particles, and a^+_{ip} and a_{ip} are the Bose creation and annihilation operators in the momentum representation.

Bose–Einstein condensation occurs in a state of zero momentum and the condensate wave function is

$$\Psi_i = \frac{1}{\sqrt{V}}\langle a_{i0}\rangle = \sqrt{n_{i0}}\mathrm{e}^{i\theta_i},$$

where θ_i is the phase of the condensate and the angle brackets denote averaging with respect to the ground state. When the depletion is small in nearly zero temperature, the condensate density n_{i0} can be replaced by the density n_i. As in Ref. 14, the phase difference of π corresponds to the minimum of the ground state energy of the system. In this paper, we will consider the case of the phase difference being π. In zero temperature, the ground state energy density for the homogeneous two-component condensate with volume V is given as

$$\frac{E_0}{V} = \frac{1}{4}(U_s+U_x)n^2 - \frac{1}{2}\eta n. \tag{15}$$

In accordance with,[14] the chemical potential can be found as

$$\mu = -\frac{\eta}{2}+\frac{n}{2}(U_s+U_x). \tag{16}$$

For simplicity, we assume that the phase of the first kind of condensate is zero and the second kind is π. On this assumption, we can get

$$\langle a_{10}a_{20}\rangle = -\frac{n}{2}, \quad \langle a^+_{10}\,a_{20}\rangle = -\frac{n}{2}. \tag{17}$$

The Hamiltonian (14) is more complicated than the condensate of only one component. In order to get the excitation spectrum, we introduce four single-particle Green's functions in zero temperature:

$$G(p,t-t') = -i\langle Ta_{1p}(t)a^+_{1p}(t')\rangle, \tag{18}$$

$$F(p,t-t') = -i\langle Ta^+_{1-p}(t)a^+_{1p}(t')\rangle, \tag{19}$$

$$H(p,t-t') = -i\langle Ta_{2p}(t)a^+_{1p}(t')\rangle, \tag{20}$$

$$L(p,t-t') = -i\langle Ta^+_{2-p}(t)a^+_{1p}(t')\rangle, \tag{21}$$

where the T denotes the chronological product and the operators are in the Heisenberg representation. As the procedure in last section, we will try to derive their dynamical equations. For the normal Green's function, we can get

$$\begin{aligned} i\frac{\partial}{\partial t}G(p,t-t') &= \delta(t-t') - i\langle T[a_{1p},K]a^+_{1p}(t')\rangle \\ &= \delta(t-t') - i\Big\langle T\Big\{\left(\frac{p^2}{2m}-\mu\right)a_{1p}(t)+\frac{\eta}{2}a_{2p}(t) \\ &\quad +\frac{1}{V}\sum_{kq}V_s a^+_{1k+q}(t)a_{1k}(t)a_{1p+q}(t) \\ &\quad +\frac{1}{V}\sum_{kq}V_x a^+_{2k+q}(t)a_{2k}(t)a_{1p+q}(t)\Big\}a^+_{1p}(t')\Big\rangle. \end{aligned} \tag{22}$$

The brackets of four operators in the interaction terms must be reduced to products of pair operators by Wick's theorem.[17,18] Similar to the result (5), we will try to reduce the following interaction term:

$$\begin{aligned} &-i\frac{1}{V}\sum_{kq}U_x\langle Ta^+_{2k+q}(t)a_{2k}(t)a_{1p+q}(t)a^+_{1p}(t')\rangle \\ &= -i\frac{1}{V}\sum_k U_x\langle a^+_{2k}(t)a_{2k}(t)\rangle\langle Ta_{1p}(t)a^+_{1p}(t')\rangle \\ &\quad -i\frac{1}{V}\sum_q U_x\langle a^+_{2p+q}(t)a_{1p+q}(t)\rangle\langle Ta_{2p}(t)a^+_{1p}(t')\rangle \\ &\quad -i\frac{1}{V}\sum_q U_x\langle a_{2-p-q}(t)a_{1p+q}(t)\rangle\langle Ta^+_{2-p}(t)a^+_{1p}(t')\rangle \\ &\approx n_2U_xG(p,t-t')+\frac{1}{V}U_x\langle a^+_{20}a_{10}\rangle H(p,t-t') \\ &\quad +\frac{1}{V}U_x\langle a_{20}a_{10}\rangle L(p,t-t') \\ &= \frac{n}{2}U_xG(p,t-t')-\frac{n}{2}U_x[H(p,t-t')+L(p,t-t')]. \end{aligned} \tag{23}$$

To deduce (23), we have made mean field approximation and the condensate density n_{i0} is replaced by n_i. We have also made use of the assumption (17) and $n_1 = n_2 = n/2$. All the interaction terms in (22) can be dealt with similarly. If we make Fourier transformation of (22) into energy representation, we can get

$$\omega G(p,\omega)=1+\left[\left(\frac{p^2}{2m}-\mu\right)+nU_s+\frac{n}{2}U_x\right]G(p,\omega)+\frac{n}{2}U_sF(p,\omega)+\left(\frac{\eta}{2}-\frac{n}{2}U_x\right)H(p,\omega)-\frac{n}{2}U_xL(p,\omega). \tag{24}$$

As the same procedure, we can get three other equations:

$$\omega F(p,\omega)=-\left[\left(\frac{p^2}{2m}-\mu\right)+nU_s+\frac{n}{2}U_x\right]F(p,\omega)-\frac{n}{2}U_sG(p,\omega)+\frac{n}{2}U_xH(p,\omega)-\left(\frac{\eta}{2}-\frac{n}{2}U_x\right)L(p,\omega), \tag{25}$$

$$\omega H(p,\omega)=\left[\left(\frac{p^2}{2m}-\mu\right)+nU_s+\frac{n}{2}U_x\right]H(p,\omega)+\frac{n}{2}U_sL(p,\omega)-\frac{n}{2}U_xF(p,\omega)+\left(\frac{\eta}{2}-\frac{n}{2}U_x\right)G(p,\omega), \tag{26}$$

$$\omega L(p,\omega)=-\left[\left(\frac{p^2}{2m}-\mu\right)+nU_s+\frac{n}{2}U_x\right]L(p,\omega)-\frac{n}{2}U_sH(p,\omega)+\frac{n}{2}U_xG(p,\omega)-\left(\frac{\eta}{2}-\frac{n}{2}U_x\right)F(p,\omega). \tag{27}$$

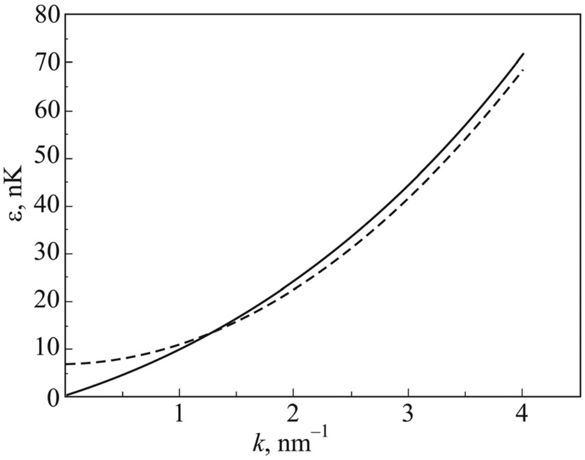

FIG. 1. Excitation energy as a function of the wave number k. The solid line shows the excitation energy ε_1, and the dash line resembles the branch ε_2 with the parameters: $n=4\cdot 10^{13}$ atoms/cm^3, $a_s=120a_0$, $a_x=80a_0$ and $\eta=n(U_s-U_x)/2$.

By the use of the chemical potential (16), we can solve the four algebra equations (24), (25), (26) and (27) with the aid of *Mathematica*. And the solutions are given as

$$G(p,\omega)=\frac{\left(\omega+\frac{p^2}{2m}+\frac{n}{2}U_s+\frac{\eta}{2}\right)\left(\omega^2-\frac{\varepsilon_1^2+\varepsilon_2^2}{2}\right)-\left(\frac{\eta}{2}-\frac{n}{2}U_x\right)\frac{\varepsilon_1^2-\varepsilon_2^2}{2}}{(\omega^2-\varepsilon_1^2)(\omega^2-\varepsilon_2^2)}, \tag{28}$$

$$F(p,\omega)=\frac{-\frac{n}{2}U_s\left(\omega^2-\frac{\varepsilon_1^2+\varepsilon_2^2}{2}\right)-\frac{n}{2}U_x\frac{\varepsilon_1^2-\varepsilon_2^2}{2}}{(\omega^2-\varepsilon_1^2)(\omega^2-\varepsilon_2^2)}, \tag{29}$$

$$H(p,\omega)=\frac{\left(\frac{\eta}{2}-\frac{n}{2}U_x\right)\left(\omega^2-\frac{\varepsilon_1^2+\varepsilon_2^2}{2}\right)-\left(\omega+\frac{p^2}{2m}+\frac{n}{2}U_s+\frac{\eta}{2}\right)\frac{\varepsilon_1^2-\varepsilon_2^2}{2}}{(\omega^2-\varepsilon_1^2)(\omega^2-\varepsilon_2^2)}, \tag{30}$$

$$L(p,\omega)=\frac{\frac{n}{2}U_x\left(\omega^2-\frac{\varepsilon_1^2+\varepsilon_2^2}{2}\right)+\frac{n}{2}U_s\frac{\varepsilon_1^2-\varepsilon_2^2}{2}}{(\omega^2-\varepsilon_1^2)(\omega^2-\varepsilon_2^2)}, \tag{31}$$

where

$$\varepsilon_1(p)=\sqrt{\frac{p^2}{2m}\left[\frac{p^2}{2m}+n(U_s+U_x)\right]}, \tag{32}$$

$$\varepsilon_2(p)=\sqrt{\left(\frac{p^2}{2m}+\eta\right)\left[\frac{p^2}{2m}+\eta+n(U_s-U_x)\right]}. \tag{33}$$

From the form of the denominator of the Green's functions, it is clear that the energy spectrum of the elementary excitations has two branches $\varepsilon_1(p)$ and $\varepsilon_2(p)$. It is possible for $\varepsilon_2(p)$ to be imaginary, and this situation corresponds to the instabilities as indicated in Refs. 14 and 15.

In the experiments of the cold atomic gas ^{87}Rb,[5] the scattering length is often in the range $85a_0<a_{\mathrm{Rb}}<140a_0$, where $a_0=0.5292$ Å is the Bohr radius. The density is $n\approx 10^{12}$–10^{14} atoms/cm^3, which can meet the condition of diluteness $na^3\ll 0$. And the sound speed $s=\sqrt{nU_0/m}$ is always in the order of 1 mm/s. In Fig. 1 we display the two branches of the excitation spectrum (32) and (33) with the parameters: the density of the ^{87}Rb gas $n=4\cdot 10^{13}$ atoms/cm^3, the two scattering length $a_s=120a_0$ and $a_x=80a_0$ (a_0 is the Bohr radius), and $\eta=n(U_s-U_x)/2$. From the selected parameters, we can obtain the sound speed $s=\sqrt{n(U_s+U_x)/2m}$ of the first branch $\varepsilon_1(p)$ to be 1.5 mm/s. It is clear that the two branches of excitation spectrum are of a single-particle excitation and a phonon one.

The normal Green's function (28) can be rewritten as

$$G(p,\omega)=\frac{\omega+\frac{p^2}{2m}+\frac{n}{2}(U_s+U_x)}{2(\omega^2-\varepsilon_1^2)+i0}+\frac{\omega+\frac{p^2}{2m}+\eta+\frac{n}{2}(U_s-U_x)}{2(\omega^2-\varepsilon_2^2)+i0}. \tag{34}$$

Based on the results of excitation spectrum (32) and (33), we can make some discussions about the ground state properties of the system. In the long-wavelength limit and $0 \ll \eta$, $n(U_s - U_x)$, (32) and (33) can be reduced as

$$\varepsilon_1(p) = \sqrt{\frac{n(U_s + U_x)}{2m}}p, \tag{35}$$

$$\varepsilon_2(p) = \sqrt{\eta[\eta + n(U_s - U_x)]} + \frac{p^2}{2m^*}, \tag{36}$$

where the effective mass m^* is given by

$$m^* = \frac{2\sqrt{\eta[\eta + n(U_s - U_x)]}}{2\eta + n(U_s - U_x)}m. \tag{37}$$

It is clear that $\varepsilon_1(p)$ is the phonon excitations with the sound speed $\sqrt{n(U_s + U_x)/2m}$ and $\varepsilon_2(p)$ corresponds to the single-particle excitations with an effective mass and a shift in the energy.

In the following, we will make some discussions for two special cases:

1. If $U_s = U_x$, the excitation spectrum will be simplified as

$$\varepsilon_1(p) = \sqrt{\frac{p^2}{2m}\left(\frac{p^2}{2m} + 2nU_s\right)}, \tag{38}$$

$$\varepsilon_2(p) = \frac{p^2}{2m} + \eta, \tag{39}$$

and the Green's function (34) can be reduced to

$$G(p,\omega) = \frac{\omega + \frac{p^2}{2m} + nU_s}{2(\omega^2 - \varepsilon_1^2) + i0} + \frac{1}{2(\omega - \varepsilon_2) + i0}. \tag{40}$$

From the form of (39) and (40), we can see that the second spectrum $\varepsilon_2(p)$ acts only as the transition state such as a_{1p} to a_{2p}^+, and there is no particles really occupying on this spectrum. From the Green's function (40), we can get the depletion from the condensate of the first kind of particles:

$$\begin{aligned} n_{1\mathrm{ex}} &= \lim_{t \to -0} \int \frac{d^3p}{(2\pi\hbar)^3} \int d\omega \frac{i}{2\pi} \mathrm{e}^{-i\omega t} G(p,\omega) \\ &= \frac{1}{2}\frac{1}{3\pi^2\hbar^3}(mnU_s)^{3/2}. \end{aligned} \tag{41}$$

The two components are equivalent and the results can be applied to the second kind of particles. Similar to the result (12), we can get the energy of the gases

$$\frac{E}{V} = \frac{U_s n^2}{2} - \frac{1}{2}\eta n + \frac{8}{15}\frac{m^{3/2}}{\pi^2\hbar^3}(U_s n)^{5/2}. \tag{42}$$

The results (41) and (42) are completely same to (11) and (12) as the case of one-component condensate. The phase difference makes no effect in this case. It is well known that the mean field theory is well applied for the cold dilute atomic gases.[5,6] In most experiments, the depletion of the ground state is of the order of one percent. And the energy from the contribution of the noncondensate atoms is only a small part of the whole energy. However, the mean field theory is not suitable to liquid ^{4}He for its strong interaction and high density.[6]

2. If $\eta = 0$, the excitation spectrum will be changed as

$$\varepsilon_1 = \sqrt{\frac{p^2}{2m}\left[\frac{p^2}{2m} + n(U_s + U_x)\right]}, \tag{43}$$

$$\varepsilon_2 = \sqrt{\frac{p^2}{2m}\left[\frac{p^2}{2m} + n(U_s - U_x)\right]}. \tag{44}$$

In this case the spectrum can be reduced to two phonon branches as $\varepsilon_1 = s_1 p$ and $\varepsilon_2 = s_2 p$, where the two sound speeds are $s_1 = \sqrt{n(U_s + U_x)/2m}$ and $s_2 = \sqrt{n(U_s - U_x)/2m}$, respectively. The two phonon speeds s_1 and s_2 can be different much from each other.

The normal Green's function (34) can be reduced to

$$G(p,\omega) = \frac{\omega + \frac{p^2}{2m} + \frac{n}{2}(U_s + U_x)}{2(\omega^2 - \varepsilon_1^2) + i0} + \frac{\omega + \frac{p^2}{2m} + \frac{n}{2}(U_s - U_x)}{2(\omega^2 - \varepsilon_2^2) + i0}. \tag{45}$$

Similarly we can get the depletion from the condensate of the first kind of particles:

$$n_{1\mathrm{ex}} = \frac{1}{2}\frac{1}{3\pi^2\hbar^3}\left(mn\frac{U_s + U_x}{2}\right)^{\frac{3}{2}} + \frac{1}{2}\frac{1}{3\pi^2\hbar^3}\left(mn\frac{U_s - U_x}{2}\right)^{\frac{3}{2}}. \tag{46}$$

The energy of the condensate can be calculated as

$$\begin{aligned} \frac{E}{V} &= \frac{1}{4}(U_s + U_x)n^2 + \frac{8}{15}\frac{m^{3/2}}{\pi^2\hbar^3} \\ &\times \left[\left(\frac{U_s + U_x}{2}n\right)^{5/2} + \left(\frac{U_s - U_x}{2}n\right)^{5/2}\right]. \end{aligned} \tag{47}$$

Comparing the results (41) and (46), we can see that the density of excitation $n_{1\mathrm{ex}}$ is reduced for the different interactions. The condition $U_x = 0$ corresponds to the minimum of the excitation density (46), which is $\sqrt{2}/2$ times the one of the case $U_s = U_x$. Comparing (47) with (42), the part of the ground state energy from the contribution of the noncondensed atoms is reduced correspondingly.

IV. CONCLUSION

In this paper, we extend the Green's function method to the equivalent two-component Bose–Einstein condensate. The elementary excitation spectrum is found to have two branches. On the condition of strong coupling, the two branches of the excitation spectrum are reduced to one phonon excitation and one single-particle excitation in the long wave-length limit. When the two different kinds of interaction are equal, there is no particle really occupying the branch of the single-particle excitation spectrum, which acts only as a transition state between two different atoms. The depletion of the condensate is same to the one of one component case. When the transition between the two different particles is forbidden, the excitation spectrum is reduced to two phonon forms in the long wave-length limit. In this case the depletion of the condensate is reduced for the two different kinds of interaction. With the obtained excitation spectrum and the Green's function, we have also calculated the ground state energy in this paper.

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