Introduction to Complex manifolds and Kähler geometry

Monday 14:50–18:15
Room 1418

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grading:
I will give some exercises which will be the basis for your grade. I will post problems and announcements on the above web page.

This course will be an introduction to complex manifolds. The main emphasis will be on Kähler manifolds. We will cover topics essential to differential geometry and complex algebraic geometry. We will spend some time on foundational material such as sheaf theory, cohomology and vector bundles. Our first goal will be the Hodge theorem, and the Hodge identities implied by the Kähler condition. Applications will be the Lefschetz decomposition, Kodaira vanishing, Lefschetz hyperplane theorem, Lefschetz theorem on $(1,1)$-classes, and the Kodaira embedding theorem. Interesting examples will come from complex algebraic geometry. We will cover concepts from complex algebraic geometry such as divisors, line bundles and algebraic varieties as needed. Time permitting, we will cover some deformation theory of complex manifolds.

Recommended reading:

- Philip Griffiths and Joe Harris. Principles of Algebraic Geometry. *Wiley-Interscience*, 1978, 813 pp.
- Raymond O. Wells. Differential Analysis on Complex Manifolds. *Springer*, 2008, 299 pp.
- Kunihiko Kodaira. Complex Manifolds and Deformations of Complex Structures. *Springer-Verlag*, 1986, 459 pp.

We will cover material roughly equivalent to the first 2 chapters of Griffiths and Harris "Principles of Algebraic Geometry". The second reference is easier to read. It mainly covers the Hodge theorem and some applications and contains more foundational material on manifolds. Kodaira's book covers the deformation theory of complex manifolds using harmonic theory as the fundamental technique. We may cover some deformation theory time permitting if there is enough interest.