微分几何选讲(I): Special metrics in Kähler geometry

(preliminary) Monday 14:00–17:30
Room 1418

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This course is an introduction to canonical metrics in Kähler geometry. This has been a very active area of research since S.-T. Yau solved the Calabi conjecture in 1977. A consequence is that any Kähler manifold X with $c_1(X) = 0$ admits unique Ricci-flat Kähler metric in each Kähler class. Independently in 1976 S.-T. Yau and T. Aubin proved that any complex manifold X with $c_1(X) < 0$ admits a unique Kähler-Einstein metric with negative scalar curvature. The existence problem for Kähler-Einstein metrics when $c_1(X) > 0$ remained open, while some obstructions were found by Y. Matsushima, A. Lichnerowicz, and A. Futaki which also obstructed the more general case of constant scalar curvature Kähler (cscK) metrics.

This lead to the Yau-Tian-Donaldson conjecture which states that the existence of a cscK metric on a polarised Kähler manifold (X, L) should be equivalent to some notion of stability in geometric invariant theory, such as K-stability. In 2012 this conjecture was solved in the Kähler-Einstein case by X. X. Chen, S. Donaldson, and S. Song, and independently by G. Tian.

We will start with the Monge-Ampère equation and the proof Calabi conjecture and existence Kähler-Einstein metrics of negative scalar curvature. We will then consider more general cscK and extremal metrics, the structure of their automorphism groups, and the Futaki invariant. Then depending on time and interest we will consider more advanced topics related to the Yau-Tian-Donaldson conjecture, such as K-stability, balanced metrics, Chow and Hilbert stability.

The prerequisites are just a basic background on complex manifolds, such as Hodge theory on a Kähler manifold, basics on holomorphic line bundles and divisors, and the Kodaira embedding theorem.

Grading:

I will give some exercises which will be the basis for your grade. I will post problems and announcements on the above web page.

References:

- Thierry Aubin, Some Nonlinear Problems in Riemannian Geometry, Springer-Verlag, 1998, 395 pp.
- Dominic Joyce, Compact Manifold with Special Holonomy, Oxford University Press, 2000, 436 pp.
- Akito Futaki, Kähler-Einstein Metrics and Integral Invariants, Lecture Notes in Mathematics 1314, Springer-Verlag, 1988, 140 pp.
- D. H. Phong and Jacob Sturm, Lectures on Stability and Constant Scalar Curvature, Handbook of geometric analysis, No. 3, Adv. Lect. Math., 14, Int. Press, Somerville, MA, 2010.