

Fermion Masses and Mixing from Double Cover and Metaplectic Cover of A_5 Modular Symmetry Group

Supplementary Material

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We provide more details about the A_5' modular models in this supplementary file. We focus on the models with small number of free parameters. In the following, we will systematically classify the lepton and quark models based on A_5' modular symmetry, and quark-lepton unification would be also studied.

1 Lepton sector

1.1 Charged lepton sector

In the charged lepton sector, we consider six possible combinations of representation assignments of L and E^c , which are named C^i, \dots, C^{vi} respectively. As shown in table 1, lepton doublet L can be triplet representations $\mathbf{3}$ or $\mathbf{3}'$, while right-handed E^c can be $\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$, $\mathbf{2} \oplus \mathbf{1}$ or $\mathbf{2}' \oplus \mathbf{1}$. For a given representation, different weight assignments correspond to different models. The weights of the fields, the superpotential, and the mass matrix of the charged leptons will be shown one by one below.

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	C^i	C^{ii}	C^{iii}	C^{iv}	C^v	C^{vi}	C^{vii}	C^{viii}	C^{ix}	C^x
ρ_L	3	3'	3	3	3'	3'	3	3	3'	3'
$\rho_{E_i^c}$	1 \oplus 1 \oplus 1	1 \oplus 1 \oplus 1	2 \oplus 1	2' \oplus 1	2 \oplus 1	2' \oplus 1	3	3'	3	3'
# of models	1	1	6	8	8	6	2	3	3	2
(total number)	(1)	(1)	(6)	(9)	(9)	(6)	(3)	(3)	(3)	(3)

Table 1: The number (in bracket) of possible charged lepton models for different representation assignment of L and $E_{1,2,3}^c$ under the finite modular group A_5' up to weight 6 modular forms. We don't count the cases which give degenerate charged lepton masses. We also list the number (without bracket) of models which contain up to four independent terms in the charged lepton superpotential \mathcal{W}_e .

$$C_1^i: k_{E_{1,2,3}^c} + k_L = 2, 4, 6$$

$$\mathcal{W}_e = \alpha \left(Y_{\mathbf{3}}^{(2)} E_1^c L H_d \right)_1 + \beta \left(Y_{\mathbf{3}}^{(4)} E_2^c L H_d \right)_1 + \gamma \left(Y_{\mathbf{3}I}^{(6)} E_3^c L H_d \right)_1 + \delta \left(Y_{\mathbf{3}II}^{(6)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_{\mathbf{3},1}^{(2)} & \alpha Y_{\mathbf{3},3}^{(2)} & \alpha Y_{\mathbf{3},2}^{(2)} \\ \beta Y_{\mathbf{3},1}^{(4)} & \beta Y_{\mathbf{3},3}^{(4)} & \beta Y_{\mathbf{3},2}^{(4)} \\ \gamma Y_{\mathbf{3}I,1}^{(6)} + \delta Y_{\mathbf{3}II,1}^{(6)} & \gamma Y_{\mathbf{3}I,3}^{(6)} + \delta Y_{\mathbf{3}II,3}^{(6)} & \gamma Y_{\mathbf{3}I,2}^{(6)} + \delta Y_{\mathbf{3}II,2}^{(6)} \end{pmatrix} v_d$$

$$C_1^{ii}: k_{E_{1,2,3}^c} + k_L = 2, 4, 6$$

$$\mathcal{W}_e = \alpha \left(Y_{\mathbf{3}'}^{(2)} E_1^c L H_d \right)_1 + \beta \left(Y_{\mathbf{3}'}^{(4)} E_2^c L H_d \right)_1 + \gamma \left(Y_{\mathbf{3}'I}^{(6)} E_3^c L H_d \right)_1 + \delta \left(Y_{\mathbf{3}'II}^{(6)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_{\mathbf{3}',1}^{(2)} & \alpha Y_{\mathbf{3}',3}^{(2)} & \alpha Y_{\mathbf{3}',2}^{(2)} \\ \beta Y_{\mathbf{3}',1}^{(4)} & \beta Y_{\mathbf{3}',3}^{(4)} & \beta Y_{\mathbf{3}',2}^{(4)} \\ \gamma Y_{\mathbf{3}'I,1}^{(6)} + \delta Y_{\mathbf{3}'II,1}^{(6)} & \gamma Y_{\mathbf{3}'I,3}^{(6)} + \delta Y_{\mathbf{3}'II,3}^{(6)} & \gamma Y_{\mathbf{3}'I,2}^{(6)} + \delta Y_{\mathbf{3}'II,2}^{(6)} \end{pmatrix} v_d$$

$$C_1^{iii}: k_{E_{D,3}^c} + k_L = 2, 3$$

$$\mathcal{W}_e = \alpha \left(Y_{\mathbf{4}'}^{(3)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{3}}^{(2)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} -\sqrt{2}\alpha Y_{\mathbf{4}',3}^{(3)} & \alpha Y_{\mathbf{4}',2}^{(3)} & \sqrt{3}\alpha Y_{\mathbf{4}',4}^{(3)} \\ -\sqrt{2}\alpha Y_{\mathbf{4}',2}^{(3)} & \sqrt{3}\alpha Y_{\mathbf{4}',1}^{(3)} & -\alpha Y_{\mathbf{4}',3}^{(3)} \\ \beta Y_{\mathbf{3},1}^{(2)} & \beta Y_{\mathbf{3},3}^{(2)} & \beta Y_{\mathbf{3},2}^{(2)} \end{pmatrix} v_d$$

$$C_2^{iii}: k_{E_D^c,3} + k_L = 3, 4$$

$$W_e = \alpha \left(Y_{4'}^{(3)} E_D^c L H_d \right)_1 + \beta \left(Y_3^{(4)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} -\sqrt{2}\alpha Y_{4',3}^{(3)} & \alpha Y_{4',2}^{(3)} & \sqrt{3}\alpha Y_{4',4}^{(3)} \\ -\sqrt{2}\alpha Y_{4',2}^{(3)} & \sqrt{3}\alpha Y_{4',1}^{(3)} & -\alpha Y_{4',3}^{(3)} \\ \beta Y_{3,1}^{(4)} & \beta Y_{3,3}^{(4)} & \beta Y_{3,2}^{(4)} \end{pmatrix} v_d$$

$$C_3^{iii}: k_{E_D^c,3} + k_L = 5, 2$$

$$W_e = \alpha \left(Y_2^{(5)} E_D^c L H_d \right)_1 + \beta \left(Y_{4'}^{(5)} E_D^c L H_d \right)_1 + \gamma \left(Y_3^{(2)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_{2,2}^{(5)} - \sqrt{2}\beta Y_{4',3}^{(5)} & \sqrt{2}\alpha Y_{2,1}^{(5)} + \beta Y_{4',2}^{(5)} & \sqrt{3}\beta Y_{4',4}^{(5)} \\ \alpha Y_{2,1}^{(5)} - \sqrt{2}\beta Y_{4',2}^{(5)} & \sqrt{3}\beta Y_{4',1}^{(5)} & -\sqrt{2}\alpha Y_{2,2}^{(5)} - \beta Y_{4',3}^{(5)} \\ \gamma Y_{3,1}^{(2)} & \gamma Y_{3,3}^{(2)} & \gamma Y_{3,2}^{(2)} \end{pmatrix} v_d$$

$$C_4^{iii}: k_{E_D^c,3} + k_L = 5, 4$$

$$W_e = \alpha \left(Y_2^{(5)} E_D^c L H_d \right)_1 + \beta \left(Y_{4'}^{(5)} E_D^c L H_d \right)_1 + \gamma \left(Y_3^{(4)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_{2,2}^{(5)} - \sqrt{2}\beta Y_{4',3}^{(5)} & \sqrt{2}\alpha Y_{2,1}^{(5)} + \beta Y_{4',2}^{(5)} & \sqrt{3}\beta Y_{4',4}^{(5)} \\ \alpha Y_{2,1}^{(5)} - \sqrt{2}\beta Y_{4',2}^{(5)} & \sqrt{3}\beta Y_{4',1}^{(5)} & -\sqrt{2}\alpha Y_{2,2}^{(5)} - \beta Y_{4',3}^{(5)} \\ \gamma Y_{3,1}^{(4)} & \gamma Y_{3,3}^{(4)} & \gamma Y_{3,2}^{(4)} \end{pmatrix} v_d$$

$$C_5^{iii}: k_{E_D^c,3} + k_L = 3, 6$$

$$W_e = \alpha \left(Y_{4'}^{(3)} E_D^c L H_d \right)_1 + \beta \left(Y_{3I}^{(6)} E_3^c L H_d \right)_1 + \gamma \left(Y_{3II}^{(6)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} -\sqrt{2}\alpha Y_{4',3}^{(3)} & \alpha Y_{4',2}^{(3)} & \sqrt{3}\alpha Y_{4',4}^{(3)} \\ -\sqrt{2}\alpha Y_{4',2}^{(3)} & \sqrt{3}\alpha Y_{4',1}^{(3)} & -\alpha Y_{4',3}^{(3)} \\ \beta Y_{3I,1}^{(6)} + \gamma Y_{3II,1}^{(6)} & \beta Y_{3I,3}^{(6)} + \gamma Y_{3II,3}^{(6)} & \beta Y_{3I,2}^{(6)} + \gamma Y_{3II,2}^{(6)} \end{pmatrix} v_d$$

$$C_6^{iii}: k_{E_D^c,3} + k_L = 5, 6$$

$$W_e = \alpha \left(Y_2^{(5)} E_D^c L H_d \right)_1 + \beta \left(Y_{4'}^{(5)} E_D^c L H_d \right)_1 + \gamma \left(Y_{3I}^{(6)} E_3^c L H_d \right)_1 + \delta \left(Y_{3II}^{(6)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_{2,2}^{(5)} - \sqrt{2}\beta Y_{4',3}^{(5)} & \sqrt{2}\alpha Y_{2,1}^{(5)} + \beta Y_{4',2}^{(5)} & \sqrt{3}\beta Y_{4',4}^{(5)} \\ \alpha Y_{2,1}^{(5)} - \sqrt{2}\beta Y_{4',2}^{(5)} & \sqrt{3}\beta Y_{4',1}^{(5)} & -\sqrt{2}\alpha Y_{2,2}^{(5)} - \beta Y_{4',3}^{(5)} \\ \gamma Y_{3I,1}^{(6)} + \delta Y_{3II,1}^{(6)} & \gamma Y_{3I,3}^{(6)} + \delta Y_{3II,3}^{(6)} & \gamma Y_{3I,2}^{(6)} + \delta Y_{3II,2}^{(6)} \end{pmatrix} v_d$$

$$C_1^{iv}: k_{E_{D,3}^c} + k_L = 1, 2$$

$$W_e = \alpha \left(Y_{\mathbf{6}}^{(1)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{3}}^{(2)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{2}\alpha Y_6 & \sqrt{2}\alpha Y_5 & \alpha(Y_1 + Y_2) \\ \sqrt{2}\alpha Y_3 & \alpha(Y_1 - Y_2) & \sqrt{2}\alpha Y_4 \\ \beta Y_{\mathbf{3},1}^{(2)} & \beta Y_{\mathbf{3},3}^{(2)} & \beta Y_{\mathbf{3},2}^{(2)} \end{pmatrix} v_d$$

$$C_2^{iv}: k_{E_{D,3}^c} + k_L = 1, 4$$

$$W_e = \alpha \left(Y_{\mathbf{6}}^{(1)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{3}}^{(4)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{2}\alpha Y_6 & \sqrt{2}\alpha Y_5 & \alpha(Y_1 + Y_2) \\ \sqrt{2}\alpha Y_3 & \alpha(Y_1 - Y_2) & \sqrt{2}\alpha Y_4 \\ \beta Y_{\mathbf{3},1}^{(4)} & \beta Y_{\mathbf{3},3}^{(4)} & \beta Y_{\mathbf{3},2}^{(4)} \end{pmatrix} v_d$$

$$C_3^{iv}: k_{E_{D,3}^c} + k_L = 3, 2$$

$$\mathcal{W}_e = \alpha \left(Y_{\mathbf{6}I}^{(3)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{6}II}^{(3)} E_D^c L H_d \right)_1 + \gamma \left(Y_{\mathbf{3}}^{(2)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{2} \left(\alpha Y_{\mathbf{6}I,6}^{(3)} + \beta Y_{\mathbf{6}II,6}^{(3)} \right) & \sqrt{2} \left(\alpha Y_{\mathbf{6}I,5}^{(3)} + \beta Y_{\mathbf{6}II,5}^{(3)} \right) & \alpha \left(Y_{\mathbf{6}I,1}^{(3)} + Y_{\mathbf{6}I,2}^{(3)} \right) \\ & & + \beta \left(Y_{\mathbf{6}II,1}^{(3)} + Y_{\mathbf{6}II,2}^{(3)} \right) \\ \sqrt{2} \left(\alpha Y_{\mathbf{6}I,3}^{(3)} + \beta Y_{\mathbf{6}II,3}^{(3)} \right) & \alpha Y_{\mathbf{6}I,1}^{(3)} - \alpha Y_{\mathbf{6}I,2}^{(3)} \\ & + \beta \left(Y_{\mathbf{6}II,1}^{(3)} - Y_{\mathbf{6}II,2}^{(3)} \right) & \sqrt{2} \left(\alpha Y_{\mathbf{6}I,4}^{(3)} + \beta Y_{\mathbf{6}II,4}^{(3)} \right) \\ \gamma Y_{\mathbf{3},1}^{(2)} & \gamma Y_{\mathbf{3},3}^{(2)} & \gamma Y_{\mathbf{3},2}^{(2)} \end{pmatrix} v_d$$

$$C_4^{iv}: k_{E_{D,3}^c} + k_L = 3, 4$$

$$\mathcal{W}_e = \alpha \left(Y_{\mathbf{6}I}^{(3)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{6}II}^{(3)} E_D^c L H_d \right)_1 + \gamma \left(Y_{\mathbf{3}}^{(4)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{2} \left(\alpha Y_{\mathbf{6}I,6}^{(3)} + \beta Y_{\mathbf{6}II,6}^{(3)} \right) & \sqrt{2} \left(\alpha Y_{\mathbf{6}I,5}^{(3)} + \beta Y_{\mathbf{6}II,5}^{(3)} \right) & \alpha \left(Y_{\mathbf{6}I,1}^{(3)} + Y_{\mathbf{6}I,2}^{(3)} \right) \\ & & + \beta \left(Y_{\mathbf{6}II,1}^{(3)} + Y_{\mathbf{6}II,2}^{(3)} \right) \\ \sqrt{2} \left(\alpha Y_{\mathbf{6}I,3}^{(3)} + \beta Y_{\mathbf{6}II,3}^{(3)} \right) & \alpha Y_{\mathbf{6}I,1}^{(3)} - \alpha Y_{\mathbf{6}I,2}^{(3)} \\ & + \beta \left(Y_{\mathbf{6}II,1}^{(3)} - Y_{\mathbf{6}II,2}^{(3)} \right) & \sqrt{2} \left(\alpha Y_{\mathbf{6}I,4}^{(3)} + \beta Y_{\mathbf{6}II,4}^{(3)} \right) \\ \gamma Y_{\mathbf{3},1}^{(4)} & \gamma Y_{\mathbf{3},3}^{(4)} & \gamma Y_{\mathbf{3},2}^{(4)} \end{pmatrix} v_d$$

$$C_5^{iv}: k_{E_D^c,3} + k_L = 1, 6$$

$$W_e = \alpha \left(Y_{\mathbf{6}}^{(1)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{3I}}^{(6)} E_3^c L H_d \right)_1 + \gamma \left(Y_{\mathbf{3II}}^{(6)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{2}\alpha Y_6 & \sqrt{2}\alpha Y_5 & \alpha(Y_1 + Y_2) \\ \sqrt{2}\alpha Y_3 & \alpha(Y_1 - Y_2) & \sqrt{2}\alpha Y_4 \\ \beta Y_{\mathbf{3I},1}^{(6)} + \gamma Y_{\mathbf{3II},1}^{(6)} & \beta Y_{\mathbf{3I},3}^{(6)} + \gamma Y_{\mathbf{3II},3}^{(6)} & \beta Y_{\mathbf{3I},2}^{(6)} + \gamma Y_{\mathbf{3II},2}^{(6)} \end{pmatrix} v_d$$

$$C_6^{iv}: k_{E_D^c,3} + k_L = 5, 2$$

$$W_e = \alpha \left(Y_{\mathbf{6I}}^{(5)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{6II}}^{(5)} E_D^c L H_d \right)_1 + \gamma \left(Y_{\mathbf{6III}}^{(5)} E_D^c L H_d \right)_1 + \delta \left(Y_{\mathbf{3}}^{(2)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{2}(\alpha Y_{\mathbf{6I},6}^{(5)} + \beta Y_{\mathbf{6II},6}^{(5)} + \gamma Y_{\mathbf{6III},6}^{(5)}) & \sqrt{2}(\alpha Y_{\mathbf{6I},5}^{(5)} + \beta Y_{\mathbf{6II},5}^{(5)} + \gamma Y_{\mathbf{6III},5}^{(5)}) & \alpha(Y_{\mathbf{6I},1}^{(5)} + Y_{\mathbf{6I},2}^{(5)}) + \beta(Y_{\mathbf{6II},1}^{(5)} + Y_{\mathbf{6II},2}^{(5)}) + \gamma(Y_{\mathbf{6III},1}^{(5)} + Y_{\mathbf{6III},2}^{(5)}) \\ \sqrt{2}(\alpha Y_{\mathbf{6I},3}^{(5)} + \beta Y_{\mathbf{6II},3}^{(5)} + \gamma Y_{\mathbf{6III},3}^{(5)}) & \alpha(Y_{\mathbf{6I},1}^{(5)} - Y_{\mathbf{6I},2}^{(5)}) + \beta(Y_{\mathbf{6II},1}^{(5)} - Y_{\mathbf{6II},2}^{(5)}) + \gamma(Y_{\mathbf{6III},1}^{(5)} - Y_{\mathbf{6III},2}^{(5)}) & \sqrt{2}(\alpha Y_{\mathbf{6I},4}^{(5)} + \beta Y_{\mathbf{6II},4}^{(5)} + \gamma Y_{\mathbf{6III},4}^{(5)}) \\ \delta Y_{\mathbf{3},1}^{(2)} & \delta Y_{\mathbf{3},3}^{(2)} & \delta Y_{\mathbf{3},2}^{(2)} \end{pmatrix} v_d$$

$$C_7^{iv}: k_{E_D^c,3} + k_L = 3, 6$$

$$W_e = \alpha \left(Y_{\mathbf{6I}}^{(3)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{6II}}^{(3)} E_D^c L H_d \right)_1 + \gamma \left(Y_{\mathbf{3II}}^{(6)} E_3^c L H_d \right)_1 + \delta \left(Y_{\mathbf{3I}}^{(6)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{2}(\alpha Y_{\mathbf{6I},6}^{(3)} + \beta Y_{\mathbf{6II},6}^{(3)}) & \sqrt{2}(\alpha Y_{\mathbf{6I},5}^{(3)} + \beta Y_{\mathbf{6II},5}^{(3)}) & \alpha(Y_{\mathbf{6I},1}^{(3)} + Y_{\mathbf{6I},2}^{(3)}) + \beta(Y_{\mathbf{6II},1}^{(3)} + Y_{\mathbf{6II},2}^{(3)}) \\ \sqrt{2}(\alpha Y_{\mathbf{6I},3}^{(3)} + \beta Y_{\mathbf{6II},3}^{(3)}) & \alpha(Y_{\mathbf{6I},1}^{(3)} - Y_{\mathbf{6I},2}^{(3)}) + \beta(Y_{\mathbf{6II},1}^{(3)} - Y_{\mathbf{6II},2}^{(3)}) & \sqrt{2}(\alpha Y_{\mathbf{6I},4}^{(3)} + \beta Y_{\mathbf{6II},4}^{(3)}) \\ \delta Y_{\mathbf{3I},1}^{(6)} + \gamma Y_{\mathbf{3II},1}^{(6)} & \delta Y_{\mathbf{3I},3}^{(6)} + \gamma Y_{\mathbf{3II},3}^{(6)} & \delta Y_{\mathbf{3I},2}^{(6)} + \gamma Y_{\mathbf{3II},2}^{(6)} \end{pmatrix} v_d$$

$$C_8^{iv}: k_{E_D^c,3} + k_L = 5, 4$$

$$W_e = \alpha \left(Y_{\mathbf{6I}}^{(5)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{6II}}^{(5)} E_D^c L H_d \right)_1 + \gamma \left(Y_{\mathbf{6III}}^{(5)} E_D^c L H_d \right)_1 + \delta \left(Y_{\mathbf{3}}^{(4)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{2}(\alpha Y_{\mathbf{6I},6}^{(5)} + \beta Y_{\mathbf{6II},6}^{(5)} + \gamma Y_{\mathbf{6III},6}^{(5)}) & \sqrt{2}(\alpha Y_{\mathbf{6I},5}^{(5)} + \beta Y_{\mathbf{6II},5}^{(5)} + \gamma Y_{\mathbf{6III},5}^{(5)}) & \alpha(Y_{\mathbf{6I},1}^{(5)} + Y_{\mathbf{6I},2}^{(5)}) + \beta(Y_{\mathbf{6II},1}^{(5)} + Y_{\mathbf{6II},2}^{(5)}) + \gamma(Y_{\mathbf{6III},1}^{(5)} + Y_{\mathbf{6III},2}^{(5)}) \\ \sqrt{2}(\alpha Y_{\mathbf{6I},3}^{(5)} + \beta Y_{\mathbf{6II},3}^{(5)} + \gamma Y_{\mathbf{6III},3}^{(5)}) & \alpha(Y_{\mathbf{6I},1}^{(5)} - Y_{\mathbf{6I},2}^{(5)}) + \beta(Y_{\mathbf{6II},1}^{(5)} - Y_{\mathbf{6II},2}^{(5)}) + \gamma(Y_{\mathbf{6III},1}^{(5)} - Y_{\mathbf{6III},2}^{(5)}) & \sqrt{2}(\alpha Y_{\mathbf{6I},4}^{(5)} + \beta Y_{\mathbf{6II},4}^{(5)} + \gamma Y_{\mathbf{6III},4}^{(5)}) \\ \delta Y_{\mathbf{3},1}^{(4)} & \delta Y_{\mathbf{3},3}^{(4)} & \delta Y_{\mathbf{3},2}^{(4)} \end{pmatrix} v_d$$

$$C_1^v: k_{E_{D,3}^c} + k_L = 2, 1$$

$$W_e = \alpha \left(Y_{\mathbf{6}}^{(1)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{3}'}^{(2)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_5 & -\alpha Y_3 & \alpha Y_2 \\ -\alpha Y_4 & -\alpha Y_1 & -\alpha Y_6 \\ \beta Y_{\mathbf{3}',1}^{(2)} & \beta Y_{\mathbf{3}',3}^{(2)} & \beta Y_{\mathbf{3}',2}^{(2)} \end{pmatrix} v_d$$

$$C_2^v: k_{E_{D,3}^c} + k_L = 4, 1$$

$$W_e = \alpha \left(Y_{\mathbf{6}}^{(1)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{3}'}^{(4)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_5 & -\alpha Y_3 & \alpha Y_2 \\ -\alpha Y_4 & -\alpha Y_1 & -\alpha Y_6 \\ \beta Y_{\mathbf{3}',1}^{(4)} & \beta Y_{\mathbf{3}',3}^{(4)} & \beta Y_{\mathbf{3}',2}^{(4)} \end{pmatrix} v_d$$

$$C_3^v: k_{E_{D,3}^c} + k_L = 3, 2$$

$$W_e = \alpha \left(Y_{\mathbf{6}I}^{(3)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{6}II}^{(3)} E_D^c L H_d \right)_1 + \gamma \left(Y_{\mathbf{3}'}^{(2)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_{\mathbf{6}I,5}^{(3)} + \beta Y_{\mathbf{6}II,5}^{(3)} & -\alpha Y_{\mathbf{6}I,3}^{(3)} - \beta Y_{\mathbf{6}II,3}^{(3)} & \alpha Y_{\mathbf{6}I,2}^{(3)} + \beta Y_{\mathbf{6}II,2}^{(3)} \\ -\alpha Y_{\mathbf{6}I,4}^{(3)} - \beta Y_{\mathbf{6}II,4}^{(3)} & -\alpha Y_{\mathbf{6}I,1}^{(3)} - \beta Y_{\mathbf{6}II,1}^{(3)} & -\alpha Y_{\mathbf{6}I,6}^{(3)} - \beta Y_{\mathbf{6}II,6}^{(3)} \\ \gamma Y_{\mathbf{3}',1}^{(2)} & \gamma Y_{\mathbf{3}',3}^{(2)} & \gamma Y_{\mathbf{3}',2}^{(2)} \end{pmatrix} v_d$$

$$C_4^v: k_{E_{D,3}^c} + k_L = 3, 4$$

$$W_e = \alpha \left(Y_{\mathbf{6}I}^{(3)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{6}II}^{(3)} E_D^c L H_d \right)_1 + \gamma \left(Y_{\mathbf{3}'}^{(4)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_{\mathbf{6}I,5}^{(3)} + \beta Y_{\mathbf{6}II,5}^{(3)} & -\alpha Y_{\mathbf{6}I,3}^{(3)} - \beta Y_{\mathbf{6}II,3}^{(3)} & \alpha Y_{\mathbf{6}I,2}^{(3)} + \beta Y_{\mathbf{6}II,2}^{(3)} \\ -\alpha Y_{\mathbf{6}I,4}^{(3)} - \beta Y_{\mathbf{6}II,4}^{(3)} & -\alpha Y_{\mathbf{6}I,1}^{(3)} - \beta Y_{\mathbf{6}II,1}^{(3)} & -\alpha Y_{\mathbf{6}I,6}^{(3)} - \beta Y_{\mathbf{6}II,6}^{(3)} \\ \gamma Y_{\mathbf{3}',1}^{(4)} & \gamma Y_{\mathbf{3}',3}^{(4)} & \gamma Y_{\mathbf{3}',2}^{(4)} \end{pmatrix} v_d$$

$$C_5^v: k_{E_{D,3}^c} + k_L = 1, 6$$

$$W_e = \alpha \left(Y_{\mathbf{6}}^{(1)} E_D^c L H_d \right)_1 + \beta \left(Y_{\mathbf{3}'I}^{(6)} E_3^c L H_d \right)_1 + \gamma \left(Y_{\mathbf{3}'II}^{(6)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_5 & -\alpha Y_3 & \alpha Y_2 \\ -\alpha Y_4 & -\alpha Y_1 & -\alpha Y_6 \\ \beta Y_{\mathbf{3}'I,1}^{(6)} + \gamma Y_{\mathbf{3}'II,1}^{(6)} & \beta Y_{\mathbf{3}'I,3}^{(6)} + \gamma Y_{\mathbf{3}'II,3}^{(6)} & \beta Y_{\mathbf{3}'I,2}^{(6)} + \gamma Y_{\mathbf{3}'II,2}^{(6)} \end{pmatrix} v_d$$

$$C_6^v: k_{E_D^c,3} + k_L = 5, 2$$

$$W_e = \alpha \left(Y_{6I}^{(5)} E_D^c LH_d \right)_1 + \beta \left(Y_{6II}^{(5)} E_D^c LH_d \right)_1 + \gamma \left(Y_{6III}^{(5)} E_D^c LH_d \right)_1 + \delta \left(Y_{3'}^{(2)} E_3^c LH_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_{6I,5}^{(5)} + \beta Y_{6II,5}^{(5)} + \gamma Y_{6III,5}^{(5)} & -\alpha Y_{6I,3}^{(5)} - \beta Y_{6II,3}^{(5)} - \gamma Y_{6III,3}^{(5)} & \alpha Y_{6I,2}^{(5)} + \beta Y_{6II,2}^{(5)} + \gamma Y_{6III,2}^{(5)} \\ -\alpha Y_{6I,4}^{(5)} - \beta Y_{6II,4}^{(5)} - \gamma Y_{6III,4}^{(5)} & -\alpha Y_{6I,1}^{(5)} - \beta Y_{6II,1}^{(5)} - \gamma Y_{6III,1}^{(5)} & -\alpha Y_{6I,6}^{(5)} - \beta Y_{6II,6}^{(5)} - \gamma Y_{6III,6}^{(5)} \\ \delta Y_{3',1}^{(2)} & \delta Y_{3',3}^{(2)} & \delta Y_{3',2}^{(2)} \end{pmatrix} v_d$$

$$C_7^v: k_{E_D^c,3} + k_L = 3, 6$$

$$W_e = \alpha \left(Y_{6I}^{(3)} E_D^c LH_d \right)_1 + \beta \left(Y_{6II}^{(3)} E_D^c LH_d \right)_1 + \gamma \left(Y_{3',II}^{(6)} E_3^c LH_d \right)_1 + \delta \left(Y_{3',I}^{(6)} E_3^c LH_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_{6I,5}^{(3)} + \beta Y_{6II,5}^{(3)} & -\alpha Y_{6I,3}^{(3)} - \beta Y_{6II,3}^{(3)} & \alpha Y_{6I,2}^{(3)} + \beta Y_{6II,2}^{(3)} \\ -\alpha Y_{6I,4}^{(3)} - \beta Y_{6II,4}^{(3)} & -\alpha Y_{6I,1}^{(3)} - \beta Y_{6II,1}^{(3)} & -\alpha Y_{6I,6}^{(3)} - \beta Y_{6II,6}^{(3)} \\ \delta Y_{3',I,1}^{(6)} + \gamma Y_{3',II,1}^{(6)} & \delta Y_{3',I,3}^{(6)} + \gamma Y_{3',II,3}^{(6)} & \delta Y_{3',I,2}^{(6)} + \gamma Y_{3',II,2}^{(6)} \end{pmatrix} v_d$$

$$C_8^v: k_{E_D^c,3} + k_L = 5, 4$$

$$W_e = \alpha \left(Y_{6I}^{(5)} E_D^c LH_d \right)_1 + \beta \left(Y_{6II}^{(5)} E_D^c LH_d \right)_1 + \gamma \left(Y_{6III}^{(5)} E_D^c LH_d \right)_1 + \delta \left(Y_{3'}^{(4)} E_3^c LH_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_{6I,5}^{(5)} + \beta Y_{6II,5}^{(5)} + \gamma Y_{6III,5}^{(5)} & -\alpha Y_{6I,3}^{(5)} - \beta Y_{6II,3}^{(5)} - \gamma Y_{6III,3}^{(5)} & \alpha Y_{6I,2}^{(5)} + \beta Y_{6II,2}^{(5)} + \gamma Y_{6III,2}^{(5)} \\ -\alpha Y_{6I,4}^{(5)} - \beta Y_{6II,4}^{(5)} - \gamma Y_{6III,4}^{(5)} & -\alpha Y_{6I,1}^{(5)} - \beta Y_{6II,1}^{(5)} - \gamma Y_{6III,1}^{(5)} & -\alpha Y_{6I,6}^{(5)} - \beta Y_{6II,6}^{(5)} - \gamma Y_{6III,6}^{(5)} \\ \delta Y_{3',1}^{(4)} & \delta Y_{3',3}^{(4)} & \delta Y_{3',2}^{(4)} \end{pmatrix} v_d$$

$$C_1^{vi}: k_{E_D^c,3} + k_L = 3, 2$$

$$W_e = \alpha \left(Y_{4'}^{(3)} E_D^c LH_d \right)_1 + \beta \left(Y_{3'}^{(2)} E_3^c LH_d \right)_1$$

$$M_e = \begin{pmatrix} -\sqrt{2}\alpha Y_{4',4}^{(3)} & -\sqrt{3}\alpha Y_{4',2}^{(3)} & \alpha Y_{4',1}^{(3)} \\ \sqrt{2}\alpha Y_{4',1}^{(3)} & \alpha Y_{4',4}^{(3)} & -\sqrt{3}\alpha Y_{4',3}^{(3)} \\ \beta Y_{3',1}^{(2)} & \beta Y_{3',3}^{(2)} & \beta Y_{3',2}^{(2)} \end{pmatrix} v_d$$

$$C_2^{vi}: k_{E_D^c,3} + k_L = 3, 4$$

$$W_e = \alpha \left(Y_{4'}^{(3)} E_D^c LH_d \right)_1 + \beta \left(Y_{3'}^{(4)} E_3^c LH_d \right)_1$$

$$M_e = \begin{pmatrix} -\sqrt{2}\alpha Y_{4',4}^{(3)} & -\sqrt{3}\alpha Y_{4',2}^{(3)} & \alpha Y_{4',1}^{(3)} \\ \sqrt{2}\alpha Y_{4',1}^{(3)} & \alpha Y_{4',4}^{(3)} & -\sqrt{3}\alpha Y_{4',3}^{(3)} \\ \beta Y_{3',1}^{(4)} & \beta Y_{3',3}^{(4)} & \beta Y_{3',2}^{(4)} \end{pmatrix} v_d$$

$$C_3^{vi}: k_{E_{D,3}^c} + k_L = 5, 2$$

$$W_e = \alpha \left(Y_{2'}^{(5)} E_D^c L H_d \right)_1 + \beta \left(Y_{4'}^{(5)} E_D^c L H_d \right)_1 + \gamma \left(Y_{3'}^{(2)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} -\alpha Y_{2',2}^{(5)} - \sqrt{2}\beta Y_{4',4}^{(5)} & -\sqrt{3}\beta Y_{4',2}^{(5)} & \sqrt{2}\alpha Y_{2',1}^{(5)} + \beta Y_{4',1}^{(5)} \\ \sqrt{2}\beta Y_{4',1}^{(5)} - \alpha Y_{2',1}^{(5)} & \beta Y_{4',4}^{(5)} - \sqrt{2}\alpha Y_{2',2}^{(5)} & -\sqrt{3}\beta Y_{4',3}^{(5)} \\ \gamma Y_{3',1}^{(2)} & \gamma Y_{3',3}^{(2)} & \gamma Y_{3',2}^{(2)} \end{pmatrix} v_d$$

$$C_4^{vi}: k_{E_{D,3}^c} + k_L = 5, 4$$

$$W_e = \alpha \left(Y_{2'}^{(5)} E_D^c L H_d \right)_1 + \beta \left(Y_{4'}^{(5)} E_D^c L H_d \right)_1 + \gamma \left(Y_{3'}^{(4)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} -\alpha Y_{2',2}^{(5)} - \sqrt{2}\beta Y_{4',4}^{(5)} & -\sqrt{3}\beta Y_{4',2}^{(5)} & \sqrt{2}\alpha Y_{2',1}^{(5)} + \beta Y_{4',1}^{(5)} \\ \sqrt{2}\beta Y_{4',1}^{(5)} - \alpha Y_{2',1}^{(5)} & \beta Y_{4',4}^{(5)} - \sqrt{2}\alpha Y_{2',2}^{(5)} & -\sqrt{3}\beta Y_{4',3}^{(5)} \\ \gamma Y_{3',1}^{(4)} & \gamma Y_{3',3}^{(4)} & \gamma Y_{3',2}^{(4)} \end{pmatrix} v_d$$

$$C_5^{vi}: k_{E_{D,3}^c} + k_L = 3, 6$$

$$W_e = \alpha \left(Y_{4'}^{(3)} E_D^c L H_d \right)_1 + \beta \left(Y_{3'I}^{(6)} E_3^c L H_d \right)_1 + \gamma \left(Y_{3'II}^{(6)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} -\sqrt{2}\alpha Y_{4',4}^{(3)} & -\sqrt{3}\alpha Y_{4',2}^{(3)} & \alpha Y_{4',1}^{(3)} \\ \sqrt{2}\alpha Y_{4',1}^{(3)} & \alpha Y_{4',4}^{(3)} & -\sqrt{3}\alpha Y_{4',3}^{(3)} \\ \beta Y_{3'I,1}^{(6)} + \gamma Y_{3'II,1}^{(6)} & \beta Y_{3'I,3}^{(6)} + \gamma Y_{3'II,3}^{(6)} & \beta Y_{3'I,2}^{(6)} + \gamma Y_{3'II,2}^{(6)} \end{pmatrix} v_d$$

$$C_6^{vi}: k_{E_{D,3}^c} + k_L = 5, 6$$

$$W_e = \alpha \left(Y_{2'}^{(5)} E_D^c L H_d \right)_1 + \beta \left(Y_{4'}^{(5)} E_D^c L H_d \right)_1 + \gamma \left(Y_{3',I}^{(6)} E_3^c L H_d \right)_1 + \delta \left(Y_{3',II}^{(6)} E_3^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} -\alpha Y_{2',2}^{(5)} - \sqrt{2}\beta Y_{4',4}^{(5)} & -\sqrt{3}\beta Y_{4',2}^{(5)} & \sqrt{2}\alpha Y_{2',1}^{(5)} + \beta Y_{4',1}^{(5)} \\ \sqrt{2}\beta Y_{4',1}^{(5)} - \alpha Y_{2',1}^{(5)} & \beta Y_{4',4}^{(5)} - \sqrt{2}\alpha Y_{2',2}^{(5)} & -\sqrt{3}\beta Y_{4',3}^{(5)} \\ \gamma Y_{3',I,1}^{(6)} + \delta Y_{3',II,1}^{(6)} & \gamma Y_{3',I,3}^{(6)} + \delta Y_{3',II,3}^{(6)} & \gamma Y_{3',I,2}^{(6)} + \delta Y_{3',II,2}^{(6)} \end{pmatrix} v_d$$

$$C_1^{vii}: k_{E^c} + k_L = 2$$

$$W_e = \alpha \left(Y_3^{(2)} E^c L H_d \right)_1 + \beta \left(Y_5^{(2)} E^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} 2\beta Y_{5,1}^{(2)} & \alpha Y_{3,3}^{(2)} - \sqrt{3}\beta Y_{5,5}^{(2)} & -\alpha Y_{3,2}^{(2)} - \sqrt{3}\beta Y_{5,2}^{(2)} \\ -\alpha Y_{3,3}^{(2)} - \sqrt{3}\beta Y_{5,5}^{(2)} & \sqrt{6}\beta Y_{5,4}^{(2)} & \alpha Y_{3,1}^{(2)} - \beta Y_{5,1}^{(2)} \\ \alpha Y_{3,2}^{(2)} - \sqrt{3}\beta Y_{5,2}^{(2)} & -\alpha Y_{3,1}^{(2)} - \beta Y_{5,1}^{(2)} & \sqrt{6}\beta Y_{5,3}^{(2)} \end{pmatrix} v_d$$

$$C_2^{vii}: k_{E^c} + k_L = 4$$

$$W_e = \alpha \left(Y_1^{(4)} E^c L H_d \right)_1 + \beta \left(Y_3^{(4)} E^c L H_d \right)_1 + \gamma \left(Y_{5I}^{(4)} E^c L H_d \right)_1 + \delta \left(Y_{5II}^{(4)} E^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_{1,1}^{(4)} + 2\gamma Y_{5I,1}^{(4)} + 2\delta Y_{5II,1}^{(4)} & \beta Y_{3,3}^{(4)} - \sqrt{3} \left(\gamma Y_{5I,5}^{(4)} + \delta Y_{5II,5}^{(4)} \right) & -\beta Y_{3,2}^{(4)} - \sqrt{3} \left(\gamma Y_{5I,2}^{(4)} + \delta Y_{5II,2}^{(4)} \right) \\ -\beta Y_{3,3}^{(4)} - \sqrt{3} \left(\gamma Y_{5I,5}^{(4)} + \delta Y_{5II,5}^{(4)} \right) & \sqrt{6} \left(\gamma Y_{5I,4}^{(4)} + \delta Y_{5II,4}^{(4)} \right) & \alpha Y_{1,1}^{(4)} + \beta Y_{3,1}^{(4)} - \gamma Y_{5I,1}^{(4)} - \delta Y_{5II,1}^{(4)} \\ \beta Y_{3,2}^{(4)} - \sqrt{3} \left(\gamma Y_{5I,2}^{(4)} + \delta Y_{5II,2}^{(4)} \right) & \alpha Y_{1,1}^{(4)} - \beta Y_{3,1}^{(4)} - \gamma Y_{5I,1}^{(4)} - \delta Y_{5II,1}^{(4)} & \sqrt{6} \left(\gamma Y_{5I,3}^{(4)} + \delta Y_{5II,3}^{(4)} \right) \end{pmatrix} v_d$$

$$C_1^{viii}: k_{E^c} + k_L = 2$$

$$W_e = \alpha \left(Y_5^{(2)} E^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{3} Y_{5,1}^{(2)} & Y_{5,5}^{(2)} & Y_{5,2}^{(2)} \\ Y_{5,4}^{(2)} & -\sqrt{2} Y_{5,3}^{(2)} & -\sqrt{2} Y_{5,5}^{(2)} \\ Y_{5,3}^{(2)} & -\sqrt{2} Y_{5,2}^{(2)} & -\sqrt{2} Y_{5,4}^{(2)} \end{pmatrix} (\alpha v_d)$$

$$C_2^{viii}: k_{E^c} + k_L = 4$$

$$W_e = \alpha \left(Y_4^{(4)} E^c L H_d \right)_1 + \beta \left(Y_{5I}^{(4)} E^c L H_d \right)_1 + \gamma \left(Y_{5II}^{(4)} E^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{3} \left(\beta Y_{5I,1}^{(4)} + \gamma Y_{5II,1}^{(4)} \right) & \sqrt{2} \alpha Y_{4,4}^{(4)} + \beta Y_{5I,5}^{(4)} + \gamma Y_{5II,5}^{(4)} & \sqrt{2} \alpha Y_{4,1}^{(4)} + \beta Y_{5I,2}^{(4)} + \gamma Y_{5II,2}^{(4)} \\ -\sqrt{2} \alpha Y_{4,3}^{(4)} + \beta Y_{5I,4}^{(4)} + \gamma Y_{5II,4}^{(4)} & -\alpha Y_{4,2}^{(4)} - \sqrt{2} \left(\beta Y_{5I,3}^{(4)} + \gamma Y_{5II,3}^{(4)} \right) & \alpha Y_{4,4}^{(4)} - \sqrt{2} \left(\beta Y_{5I,5}^{(4)} + \gamma Y_{5II,5}^{(4)} \right) \\ -\sqrt{2} \alpha Y_{4,2}^{(4)} + \beta Y_{5I,3}^{(4)} + \gamma Y_{5II,3}^{(4)} & \alpha Y_{4,1}^{(4)} - \sqrt{2} \left(\beta Y_{5I,2}^{(4)} + \gamma Y_{5II,2}^{(4)} \right) & -\alpha Y_{4,3}^{(4)} - \sqrt{2} \left(\beta Y_{5I,4}^{(4)} + \gamma Y_{5II,4}^{(4)} \right) \end{pmatrix} v_d$$

$$C_3^{viii}: k_{E^c} + k_L = 6$$

$$W_e = \alpha \left(Y_{4I}^{(6)} E^c L H_d \right)_1 + \beta \left(Y_{4II}^{(6)} E^c L H_d \right)_1 + \gamma \left(Y_{5I}^{(6)} E^c L H_d \right)_1 + \delta \left(Y_{5II}^{(6)} E^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{3} \left(\gamma Y_{5I,1}^{(6)} + \delta Y_{5II,1}^{(6)} \right) & \sqrt{2} \alpha Y_{4I,4}^{(6)} + \sqrt{2} \beta Y_{4II,4}^{(6)} + \gamma Y_{5I,5}^{(6)} + \delta Y_{5II,5}^{(6)} & \sqrt{2} \alpha Y_{4I,1}^{(6)} + \sqrt{2} \beta Y_{4II,1}^{(6)} + \gamma Y_{5I,2}^{(6)} + \delta Y_{5II,2}^{(6)} \\ -\sqrt{2} \alpha Y_{4I,3}^{(6)} - \sqrt{2} \beta Y_{4II,3}^{(6)} + \gamma Y_{5I,4}^{(6)} + \delta Y_{5II,4}^{(6)} & -\alpha Y_{4I,2}^{(6)} - \beta Y_{4II,2}^{(6)} - \sqrt{2} \left(\gamma Y_{5I,3}^{(6)} + \delta Y_{5II,3}^{(6)} \right) & \alpha Y_{4I,4}^{(6)} + \beta Y_{4II,4}^{(6)} - \sqrt{2} \left(\gamma Y_{5I,5}^{(6)} + \delta Y_{5II,5}^{(6)} \right) \\ -\sqrt{2} \alpha Y_{4I,2}^{(6)} - \sqrt{2} \beta Y_{4II,2}^{(6)} + \gamma Y_{5I,3}^{(6)} + \delta Y_{5II,3}^{(6)} & \alpha Y_{4I,1}^{(6)} + \beta Y_{4II,1}^{(6)} - \sqrt{2} \left(\gamma Y_{5I,2}^{(6)} + \delta Y_{5II,2}^{(6)} \right) & -\alpha Y_{4I,3}^{(6)} - \beta Y_{4II,3}^{(6)} - \sqrt{2} \left(\gamma Y_{5I,4}^{(6)} + \delta Y_{5II,4}^{(6)} \right) \end{pmatrix} v_d$$

$$C_1^{ix}: k_{E^c} + k_L = 2$$

$$W_e = \alpha \left(Y_5^{(2)} E^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{3} Y_{5,1}^{(2)} & Y_{5,4}^{(2)} & Y_{5,3}^{(2)} \\ Y_{5,5}^{(2)} & -\sqrt{2} Y_{5,3}^{(2)} & -\sqrt{2} Y_{5,2}^{(2)} \\ Y_{5,2}^{(2)} & -\sqrt{2} Y_{5,5}^{(2)} & -\sqrt{2} Y_{5,4}^{(2)} \end{pmatrix} (\alpha v_d)$$

$$C_2^{ix}: k_{E^c} + k_L = 4$$

$$W_e = \alpha \left(Y_4^{(4)} E^c L H_d \right)_1 + \beta \left(Y_{5I}^{(4)} E^c L H_d \right)_1 + \gamma \left(Y_{5II}^{(4)} E^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{3} \left(\beta Y_{5I,1}^{(4)} + \gamma Y_{5II,1}^{(4)} \right) & -\sqrt{2} \alpha Y_{4,3}^{(4)} + \beta Y_{5I,4}^{(4)} + \gamma Y_{5II,4}^{(4)} & -\sqrt{2} \alpha Y_{4,2}^{(4)} + \beta Y_{5I,3}^{(4)} + \gamma Y_{5II,3}^{(4)} \\ \sqrt{2} \alpha Y_{4,4}^{(4)} + \beta Y_{5I,5}^{(4)} + \gamma Y_{5II,5}^{(4)} & -\alpha Y_{4,2}^{(4)} - \sqrt{2} \left(\beta Y_{5I,3}^{(4)} + \gamma Y_{5II,3}^{(4)} \right) & \alpha Y_{4,1}^{(4)} - \sqrt{2} \left(\beta Y_{5I,2}^{(4)} + \gamma Y_{5II,2}^{(4)} \right) \\ \sqrt{2} \alpha Y_{4,1}^{(4)} + \beta Y_{5I,2}^{(4)} + \gamma Y_{5II,2}^{(4)} & \alpha Y_{4,4}^{(4)} - \sqrt{2} \left(\beta Y_{5I,5}^{(4)} + \gamma Y_{5II,5}^{(4)} \right) & -\alpha Y_{4,3}^{(4)} - \sqrt{2} \left(\beta Y_{5I,4}^{(4)} + \gamma Y_{5II,4}^{(4)} \right) \end{pmatrix} v_d$$

$$C_3^{ix}: k_{E^c} + k_L = 6$$

$$W_e = \alpha \left(Y_{4I}^{(6)} E^c L H_d \right)_1 + \beta \left(Y_{4II}^{(6)} E^c L H_d \right)_1 + \gamma \left(Y_{5I}^{(6)} E^c L H_d \right)_1 + \delta \left(Y_{5II}^{(6)} E^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \sqrt{3} \left(\gamma Y_{5I,1}^{(6)} + \delta Y_{5II,1}^{(6)} \right) & -\sqrt{2} \alpha Y_{4I,3}^{(6)} - \sqrt{2} \beta Y_{4II,3}^{(6)} & -\sqrt{2} \alpha Y_{4I,2}^{(6)} - \sqrt{2} \beta Y_{4II,2}^{(6)} \\ & + \gamma Y_{5I,4}^{(6)} + \delta Y_{5II,4}^{(6)} & + \gamma Y_{5I,3}^{(6)} + \delta Y_{5II,3}^{(6)} \\ \sqrt{2} \alpha Y_{4I,4}^{(6)} + \sqrt{2} \beta Y_{4II,4}^{(6)} & -\alpha Y_{4I,2}^{(6)} - \beta Y_{4II,2}^{(6)} & \alpha Y_{4I,1}^{(6)} + \beta Y_{4II,1}^{(6)} \\ + \gamma Y_{5I,5}^{(6)} + \delta Y_{5II,5}^{(6)} & -\sqrt{2} \left(\gamma Y_{5I,3}^{(6)} + \delta Y_{5II,3}^{(6)} \right) & -\sqrt{2} \left(\gamma Y_{5I,2}^{(6)} + \delta Y_{5II,2}^{(6)} \right) \\ \sqrt{2} \alpha Y_{4I,1}^{(6)} + \sqrt{2} \beta Y_{4II,1}^{(6)} & \alpha Y_{4I,4}^{(6)} + \beta Y_{4II,4}^{(6)} & -\alpha Y_{4I,3}^{(6)} - \beta Y_{4II,3}^{(6)} \\ + \gamma Y_{5I,2}^{(6)} + \delta Y_{5II,2}^{(6)} & -\sqrt{2} \left(\gamma Y_{5I,5}^{(6)} + \delta Y_{5II,5}^{(6)} \right) & -\sqrt{2} \left(\gamma Y_{5I,4}^{(6)} + \delta Y_{5II,4}^{(6)} \right) \end{pmatrix} v_d$$

$$C_1^x: k_{E^c} + k_L = 2$$

$$W_e = \alpha \left(Y_{3'}^{(2)} E^c L H_d \right)_1 + \beta \left(Y_5^{(2)} E^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} 2\beta Y_{5,1}^{(2)} & \alpha Y_{3',3}^{(2)} - \sqrt{3} \beta Y_{5,4}^{(2)} & -\alpha Y_{3',2}^{(2)} - \sqrt{3} \beta Y_{5,3}^{(2)} \\ -\alpha Y_{3',3}^{(2)} - \sqrt{3} \beta Y_{5,4}^{(2)} & \sqrt{6} \beta Y_{5,2}^{(2)} & \alpha Y_{3',1}^{(2)} - \beta Y_{5,1}^{(2)} \\ \alpha Y_{3',2}^{(2)} - \sqrt{3} \beta Y_{5,3}^{(2)} & -\alpha Y_{3',1}^{(2)} - \beta Y_{5,1}^{(2)} & \sqrt{6} \beta Y_{5,5}^{(2)} \end{pmatrix} v_d$$

$$C_2^x: k_{E^c} + k_L = 4$$

$$W_e = \alpha \left(Y_1^{(4)} E^c L H_d \right)_1 + \beta \left(Y_{3'}^{(4)} E^c L H_d \right)_1 + \gamma \left(Y_{5I}^{(4)} E^c L H_d \right)_1 + \delta \left(Y_{5II}^{(4)} E^c L H_d \right)_1$$

$$M_e = \begin{pmatrix} \alpha Y_{1,1}^{(4)} + 2\gamma Y_{5I,1}^{(4)} + 2\delta Y_{5II,1}^{(4)} & \beta Y_{3',3}^{(4)} - \sqrt{3} (\gamma Y_{5I,4}^{(4)} + \delta Y_{5II,4}^{(4)}) & -\beta Y_{3',2}^{(4)} - \sqrt{3} (\gamma Y_{5I,3}^{(4)} + \delta Y_{5II,3}^{(4)}) \\ -\beta Y_{3',3}^{(4)} - \sqrt{3} (\gamma Y_{5I,4}^{(4)} + \delta Y_{5II,4}^{(4)}) & \sqrt{6} (\gamma Y_{5I,2}^{(4)} + \delta Y_{5II,2}^{(4)}) & \alpha Y_{1,1}^{(4)} + \beta Y_{3',1}^{(4)} - \gamma Y_{5I,1}^{(4)} - \delta Y_{5II,1}^{(4)} \\ \beta Y_{3',2}^{(4)} - \sqrt{3} (\gamma Y_{5I,3}^{(4)} + \delta Y_{5II,3}^{(4)}) & \alpha Y_{1,1}^{(4)} - \beta Y_{3',1}^{(4)} - \gamma Y_{5I,1}^{(4)} - \delta Y_{5II,1}^{(4)} & \sqrt{6} (\gamma Y_{5I,5}^{(4)} + \delta Y_{5II,5}^{(4)}) \end{pmatrix} v_d$$

1.2 Neutrino sector

Under the theoretical framework of Seesaw mechanism, the neutrino sector is rich in content. Heavy neutrinos can be three or two generations or be absent. The neutrino mass matrices corresponding to these three scenarios will be very different, and the corresponding models will be given in Sec. 1.2.1, Sec. 1.2.2 and Sec. 1.2.3.

In table 2, we give the representation assignments of L and N^c in these three scenarios. For heavy neutrino with three (two) generations or be absent, they are named $S^{i,\dots,iv}$ ($T^{i,\dots,iv}$) or $W^{i,ii}$ respectively. In the following subsections, we will give the corresponding models in turn, including weight assignments, superpotentials, mass matrices M_D , M_N and M_ν . The mass matrices of light neutrinos in the type-I seesaw can be obtained by the so-called seesaw formula $M_\nu = -M_D^T M_N^{-1} M_D$.

	S^i	S^{ii}	S^{iii}	S^{iv}	T^i	T^{ii}	T^{iii}	T^{iv}	W^i	W^{ii}
ρ_L	3	3	3'	3'	3	3	3'	3'	3	3'
ρ_{N^c}	3	3'	3	3'	2	2'	2	2'	—	—
# of models	3	2	2	3	5	5	5	5	3	3
(total number)	(15)	(12)	(12)	(15)	(6)	(9)	(9)	(6)	(3)	(3)

Table 2: The number (in bracket) of possible neutrino models for different representation assignments of L and N^c under the finite modular group A_5^t up to weight 6 modular forms, where the case without modular forms is not counted. We also list the number (without bracket) of models which contain up to three independent terms in the neutrino superpotential \mathcal{W}_ν .

1.2.1 Type-I seesaw with three right-handed neutrinos

$$S_1^i: k_{N^c} = 1, k_L = -1$$

$$\mathcal{W}_\nu = g (N^c L H_u)_1 + \Lambda \left(Y_5^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} g v_u \quad M_N = \begin{pmatrix} 2Y_{5,1}^{(2)} & -\sqrt{3}Y_{5,5}^{(2)} & -\sqrt{3}Y_{5,2}^{(2)} \\ -\sqrt{3}Y_{5,5}^{(2)} & \sqrt{6}Y_{5,4}^{(2)} & -Y_{5,1}^{(2)} \\ -\sqrt{3}Y_{5,2}^{(2)} & -Y_{5,1}^{(2)} & \sqrt{6}Y_{5,3}^{(2)} \end{pmatrix} \Lambda$$

$$S_2^i: k_{N^c} = 0, k_L = 2$$

$$\mathcal{W}_\nu = g_1 \left(Y_3^{(2)} N^c L H_u \right)_1 + g_2 \left(Y_5^{(2)} N^c L H_u \right)_1 + \Lambda (N^c N^c)_1$$

$$M_D = \begin{pmatrix} 2g_2 Y_{5,1}^{(2)} & g_1 Y_{3,3}^{(2)} - \sqrt{3} g_2 Y_{5,5}^{(2)} & -g_1 Y_{3,2}^{(2)} - \sqrt{3} g_2 Y_{5,2}^{(2)} \\ -g_1 Y_{3,3}^{(2)} - \sqrt{3} g_2 Y_{5,5}^{(2)} & \sqrt{6} g_2 Y_{5,4}^{(2)} & g_1 Y_{3,1}^{(2)} - g_2 Y_{5,1}^{(2)} \\ g_1 Y_{3,2}^{(2)} - \sqrt{3} g_2 Y_{5,2}^{(2)} & -g_1 Y_{3,1}^{(2)} - g_2 Y_{5,1}^{(2)} & \sqrt{6} g_2 Y_{5,3}^{(2)} \end{pmatrix} v_u$$

$$M_N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \Lambda$$

$$S_3^i: k_{N^c} = 1, k_L = 1$$

$$\mathcal{W}_\nu = g_1 \left(Y_3^{(2)} N^c L H_u \right)_1 + g_2 \left(Y_5^{(2)} N^c L H_u \right)_1 + \Lambda \left(Y_5^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} 2g_2 Y_{5,1}^{(2)} & g_1 Y_{3,3}^{(2)} - \sqrt{3} g_2 Y_{5,5}^{(2)} & -g_1 Y_{3,2}^{(2)} - \sqrt{3} g_2 Y_{5,2}^{(2)} \\ -g_1 Y_{3,3}^{(2)} - \sqrt{3} g_2 Y_{5,5}^{(2)} & \sqrt{6} g_2 Y_{5,4}^{(2)} & g_1 Y_{3,1}^{(2)} - g_2 Y_{5,1}^{(2)} \\ g_1 Y_{3,2}^{(2)} - \sqrt{3} g_2 Y_{5,2}^{(2)} & -g_1 Y_{3,1}^{(2)} - g_2 Y_{5,1}^{(2)} & \sqrt{6} g_2 Y_{5,3}^{(2)} \end{pmatrix} v_u$$

$$M_N = \begin{pmatrix} 2Y_{5,1}^{(2)} & -\sqrt{3}Y_{5,5}^{(2)} & -\sqrt{3}Y_{5,2}^{(2)} \\ -\sqrt{3}Y_{5,5}^{(2)} & \sqrt{6}Y_{5,4}^{(2)} & -Y_{5,1}^{(2)} \\ -\sqrt{3}Y_{5,2}^{(2)} & -Y_{5,1}^{(2)} & \sqrt{6}Y_{5,3}^{(2)} \end{pmatrix} \Lambda$$

$$S_1^{ii}: k_{N^c} = 0, k_L = 2$$

$$\mathcal{W}_\nu = g \left(Y_5^{(2)} N^c L H_u \right)_1 + \Lambda (N^c N^c)_1$$

$$M_D = \begin{pmatrix} \sqrt{3}Y_{5,1}^{(2)} & Y_{5,5}^{(2)} & Y_{5,2}^{(2)} \\ Y_{5,4}^{(2)} & -\sqrt{2}Y_{5,3}^{(2)} & -\sqrt{2}Y_{5,5}^{(2)} \\ Y_{5,3}^{(2)} & -\sqrt{2}Y_{5,2}^{(2)} & -\sqrt{2}Y_{5,4}^{(2)} \end{pmatrix} g v_u \quad M_N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \Lambda$$

$$S_2^{ii}: k_{N^c} = 1, k_L = 1$$

$$\mathcal{W}_\nu = g \left(Y_5^{(2)} N^c L H_u \right)_1 + \Lambda \left(Y_5^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} \sqrt{3} Y_{5,1}^{(2)} & Y_{5,5}^{(2)} & Y_{5,2}^{(2)} \\ Y_{5,4}^{(2)} & -\sqrt{2} Y_{5,3}^{(2)} & -\sqrt{2} Y_{5,5}^{(2)} \\ Y_{5,3}^{(2)} & -\sqrt{2} Y_{5,2}^{(2)} & -\sqrt{2} Y_{5,4}^{(2)} \end{pmatrix} g v_u$$

$$M_N = \begin{pmatrix} 2Y_{5,1}^{(2)} & -\sqrt{3} Y_{5,4}^{(2)} & -\sqrt{3} Y_{5,3}^{(2)} \\ -\sqrt{3} Y_{5,4}^{(2)} & \sqrt{6} Y_{5,2}^{(2)} & -Y_{5,1}^{(2)} \\ -\sqrt{3} Y_{5,3}^{(2)} & -Y_{5,1}^{(2)} & \sqrt{6} Y_{5,5}^{(2)} \end{pmatrix} \Lambda$$

$$S_1^{iii}: k_{N^c} = 0, k_L = 2$$

$$\mathcal{W}_\nu = g \left(Y_5^{(2)} N^c L H_u \right)_1 + \Lambda \left(N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} \sqrt{3} Y_{5,1}^{(2)} & Y_{5,4}^{(2)} & Y_{5,3}^{(2)} \\ Y_{5,5}^{(2)} & -\sqrt{2} Y_{5,3}^{(2)} & -\sqrt{2} Y_{5,2}^{(2)} \\ Y_{5,2}^{(2)} & -\sqrt{2} Y_{5,5}^{(2)} & -\sqrt{2} Y_{5,4}^{(2)} \end{pmatrix} g v_u \quad M_N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \Lambda$$

$$S_2^{iii}: k_{N^c} = 1, k_L = 1$$

$$\mathcal{W}_\nu = g \left(Y_5^{(2)} N^c L H_u \right)_1 + \Lambda \left(Y_5^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} \sqrt{3} Y_{5,1}^{(2)} & Y_{5,4}^{(2)} & Y_{5,3}^{(2)} \\ Y_{5,5}^{(2)} & -\sqrt{2} Y_{5,3}^{(2)} & -\sqrt{2} Y_{5,2}^{(2)} \\ Y_{5,2}^{(2)} & -\sqrt{2} Y_{5,5}^{(2)} & -\sqrt{2} Y_{5,4}^{(2)} \end{pmatrix} g v_u$$

$$M_N = \begin{pmatrix} 2Y_{5,1}^{(2)} & -\sqrt{3} Y_{5,5}^{(2)} & -\sqrt{3} Y_{5,2}^{(2)} \\ -\sqrt{3} Y_{5,5}^{(2)} & \sqrt{6} Y_{5,4}^{(2)} & -Y_{5,1}^{(2)} \\ -\sqrt{3} Y_{5,2}^{(2)} & -Y_{5,1}^{(2)} & \sqrt{6} Y_{5,3}^{(2)} \end{pmatrix} \Lambda$$

$$S_1^{iv}: k_{N^c} = 1, k_L = -1$$

$$\mathcal{W}_\nu = g \left(N^c L H_u \right)_1 + \Lambda \left(Y_5^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} g v_u \quad M_N = \begin{pmatrix} 2Y_{5,1}^{(2)} & -\sqrt{3} Y_{5,4}^{(2)} & -\sqrt{3} Y_{5,3}^{(2)} \\ -\sqrt{3} Y_{5,4}^{(2)} & \sqrt{6} Y_{5,2}^{(2)} & -Y_{5,1}^{(2)} \\ -\sqrt{3} Y_{5,3}^{(2)} & -Y_{5,1}^{(2)} & \sqrt{6} Y_{5,5}^{(2)} \end{pmatrix} \Lambda$$

$$S_2^{iv}: k_{N^c} = 0, k_L = 2$$

$$\mathcal{W}_\nu = g_1 \left(Y_{\mathbf{3}'}^{(2)} N^c L H_u \right)_1 + g_2 \left(Y_{\mathbf{5}}^{(2)} N^c L H_u \right)_1 + \Lambda \left(N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} 2g_2 Y_{\mathbf{5},1}^{(2)} & g_1 Y_{\mathbf{3}',3}^{(2)} - \sqrt{3}g_2 Y_{\mathbf{5},4}^{(2)} & -g_1 Y_{\mathbf{3}',2}^{(2)} - \sqrt{3}g_2 Y_{\mathbf{5},3}^{(2)} \\ -g_1 Y_{\mathbf{3}',3}^{(2)} - \sqrt{3}g_2 Y_{\mathbf{5},4}^{(2)} & \sqrt{6}g_2 Y_{\mathbf{5},2}^{(2)} & g_1 Y_{\mathbf{3}',1}^{(2)} - g_2 Y_{\mathbf{5},1}^{(2)} \\ g_1 Y_{\mathbf{3}',2}^{(2)} - \sqrt{3}g_2 Y_{\mathbf{5},3}^{(2)} & -g_1 Y_{\mathbf{3}',1}^{(2)} - g_2 Y_{\mathbf{5},1}^{(2)} & \sqrt{6}g_2 Y_{\mathbf{5},5}^{(2)} \end{pmatrix} v_u$$

$$M_N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \Lambda$$

$$S_3^{iv}: k_{N^c} = 1, k_L = 1$$

$$\mathcal{W}_\nu = g_1 \left(Y_{\mathbf{3}'}^{(2)} N^c L H_u \right)_1 + g_2 \left(Y_{\mathbf{5}}^{(2)} N^c L H_u \right)_1 + \Lambda \left(Y_{\mathbf{5}}^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} 2g_2 Y_{\mathbf{5},1}^{(2)} & g_1 Y_{\mathbf{3}',3}^{(2)} - \sqrt{3}g_2 Y_{\mathbf{5},4}^{(2)} & -g_1 Y_{\mathbf{3}',2}^{(2)} - \sqrt{3}g_2 Y_{\mathbf{5},3}^{(2)} \\ -g_1 Y_{\mathbf{3}',3}^{(2)} - \sqrt{3}g_2 Y_{\mathbf{5},4}^{(2)} & \sqrt{6}g_2 Y_{\mathbf{5},2}^{(2)} & g_1 Y_{\mathbf{3}',1}^{(2)} - g_2 Y_{\mathbf{5},1}^{(2)} \\ g_1 Y_{\mathbf{3}',2}^{(2)} - \sqrt{3}g_2 Y_{\mathbf{5},3}^{(2)} & -g_1 Y_{\mathbf{3}',1}^{(2)} - g_2 Y_{\mathbf{5},1}^{(2)} & \sqrt{6}g_2 Y_{\mathbf{5},5}^{(2)} \end{pmatrix} v_u$$

$$M_N = \begin{pmatrix} 2Y_{\mathbf{5},1}^{(2)} & -\sqrt{3}Y_{\mathbf{5},4}^{(2)} & -\sqrt{3}Y_{\mathbf{5},3}^{(2)} \\ -\sqrt{3}Y_{\mathbf{5},4}^{(2)} & \sqrt{6}Y_{\mathbf{5},2}^{(2)} & -Y_{\mathbf{5},1}^{(2)} \\ -\sqrt{3}Y_{\mathbf{5},3}^{(2)} & -Y_{\mathbf{5},1}^{(2)} & \sqrt{6}Y_{\mathbf{5},5}^{(2)} \end{pmatrix} \Lambda$$

1.2.2 Type-I seesaw with two right-handed neutrinos

$$T_1^i: k_{N^c} = 1, k_L = 2$$

$$\mathcal{W}_\nu = g \left(Y_{\mathbf{4}'}^{(3)} N^c L H_u \right)_1 + \Lambda \left(Y_{\mathbf{3}}^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} -\sqrt{2}Y_{\mathbf{4}',3}^{(3)} & Y_{\mathbf{4}',2}^{(3)} & \sqrt{3}Y_{\mathbf{4}',4}^{(3)} \\ -\sqrt{2}Y_{\mathbf{4}',2}^{(3)} & \sqrt{3}Y_{\mathbf{4}',1}^{(3)} & -Y_{\mathbf{4}',3}^{(3)} \end{pmatrix} g v_u \quad M_N = \begin{pmatrix} -\sqrt{2}Y_{\mathbf{3},2}^{(2)} & -Y_{\mathbf{3},1}^{(2)} \\ -Y_{\mathbf{3},1}^{(2)} & \sqrt{2}Y_{\mathbf{3},3}^{(2)} \end{pmatrix} \Lambda$$

$$T_2^i: k_{N^c} = 2, k_L = 1$$

$$\mathcal{W}_\nu = g \left(Y_{\mathbf{4}'}^{(3)} N^c L H_u \right)_1 + \Lambda \left(Y_{\mathbf{3}}^{(4)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} -\sqrt{2}Y_{\mathbf{4}',3}^{(3)} & Y_{\mathbf{4}',2}^{(3)} & \sqrt{3}Y_{\mathbf{4}',4}^{(3)} \\ -\sqrt{2}Y_{\mathbf{4}',2}^{(3)} & \sqrt{3}Y_{\mathbf{4}',1}^{(3)} & -Y_{\mathbf{4}',3}^{(3)} \end{pmatrix} g v_u \quad M_N = \begin{pmatrix} -\sqrt{2}Y_{\mathbf{3},2}^{(4)} & -Y_{\mathbf{3},1}^{(4)} \\ -Y_{\mathbf{3},1}^{(4)} & \sqrt{2}Y_{\mathbf{3},3}^{(4)} \end{pmatrix} \Lambda$$

$$T_3^i: k_{N^c} = 1, k_L = 4$$

$$\mathcal{W}_\nu = g_1 \left(Y_2^{(5)} N^c L H_u \right)_1 + g_2 \left(Y_{4'}^{(5)} N^c L H_u \right)_1 + \Lambda \left(Y_3^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} g_1 Y_{2,2}^{(5)} - \sqrt{2} g_2 Y_{4',3}^{(5)} & \sqrt{2} g_1 Y_{2,1}^{(5)} + g_2 Y_{4',2}^{(5)} & \sqrt{3} g_2 Y_{4',4}^{(5)} \\ g_1 Y_{2,1}^{(5)} - \sqrt{2} g_2 Y_{4',2}^{(5)} & \sqrt{3} g_2 Y_{4',1}^{(5)} & -\sqrt{2} g_1 Y_{2,2}^{(5)} - g_2 Y_{4',3}^{(5)} \end{pmatrix} v_u$$

$$M_N = \begin{pmatrix} -\sqrt{2} Y_{3,2}^{(2)} & -Y_{3,1}^{(2)} \\ -Y_{3,1}^{(2)} & \sqrt{2} Y_{3,3}^{(2)} \end{pmatrix} \Lambda$$

$$T_4^i: k_{N^c} = 2, k_L = 3$$

$$\mathcal{W}_\nu = g_1 \left(Y_2^{(5)} N^c L H_u \right)_1 + g_2 \left(Y_{4'}^{(5)} N^c L H_u \right)_1 + \Lambda \left(Y_3^{(4)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} g_1 Y_{2,2}^{(5)} - \sqrt{2} g_2 Y_{4',3}^{(5)} & \sqrt{2} g_1 Y_{2,1}^{(5)} + g_2 Y_{4',2}^{(5)} & \sqrt{3} g_2 Y_{4',4}^{(5)} \\ g_1 Y_{2,1}^{(5)} - \sqrt{2} g_2 Y_{4',2}^{(5)} & \sqrt{3} g_2 Y_{4',1}^{(5)} & -\sqrt{2} g_1 Y_{2,2}^{(5)} - g_2 Y_{4',3}^{(5)} \end{pmatrix} v_u$$

$$M_N = \begin{pmatrix} -\sqrt{2} Y_{3,2}^{(4)} & -Y_{3,1}^{(4)} \\ -Y_{3,1}^{(4)} & \sqrt{2} Y_{3,3}^{(4)} \end{pmatrix} \Lambda$$

$$T_5^i: k_{N^c} = 3, k_L = 0$$

$$\mathcal{W}_\nu = g \left(Y_{4'}^{(3)} N^c L H_u \right)_1 + \Lambda_1 \left(Y_{3I}^{(6)} N^c N^c \right)_1 + \Lambda_2 \left(Y_{3II}^{(6)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} -\sqrt{2} Y_{4',3}^{(3)} & Y_{4',2}^{(3)} & \sqrt{3} Y_{4',4}^{(3)} \\ -\sqrt{2} Y_{4',2}^{(3)} & \sqrt{3} Y_{4',1}^{(3)} & -Y_{4',3}^{(3)} \end{pmatrix} g v_u$$

$$M_N = \begin{pmatrix} -\sqrt{2} \left(\Lambda_1 Y_{3I,2}^{(6)} + \Lambda_2 Y_{3II,2}^{(6)} \right) & -\Lambda_1 Y_{3I,1}^{(6)} - \Lambda_2 Y_{3II,1}^{(6)} \\ -\Lambda_1 Y_{3I,1}^{(6)} - \Lambda_2 Y_{3II,1}^{(6)} & \sqrt{2} \left(\Lambda_1 Y_{3I,3}^{(6)} + \Lambda_2 Y_{3II,3}^{(6)} \right) \end{pmatrix}$$

$$T_1^{ii}: k_{N^c} = 1, k_L = 0$$

$$\mathcal{W}_\nu = g \left(Y_6^{(1)} N^c L H_u \right)_1 + \Lambda \left(Y_{3'}^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} \sqrt{2} Y_6 & \sqrt{2} Y_5 & Y_1 + Y_2 \\ \sqrt{2} Y_3 & Y_1 - Y_2 & \sqrt{2} Y_4 \end{pmatrix} g v_u \quad M_N = \begin{pmatrix} -\sqrt{2} Y_{3',3}^{(2)} & Y_{3',1}^{(2)} \\ Y_{3',1}^{(2)} & \sqrt{2} Y_{3',2}^{(2)} \end{pmatrix} \Lambda$$

$$T_2^{ii}: k_{N^c} = 2, k_L = -1$$

$$\mathcal{W}_\nu = g \left(Y_{\mathbf{6}}^{(1)} N^c L H_u \right)_1 + \Lambda \left(Y_{\mathbf{3}'}^{(4)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} \sqrt{2}Y_6 & \sqrt{2}Y_5 & Y_1 + Y_2 \\ \sqrt{2}Y_3 & Y_1 - Y_2 & \sqrt{2}Y_4 \end{pmatrix} g v_u \quad M_N = \begin{pmatrix} -\sqrt{2}Y_{\mathbf{3}',3}^{(4)} & Y_{\mathbf{3}',1}^{(4)} \\ Y_{\mathbf{3}',1}^{(4)} & \sqrt{2}Y_{\mathbf{3}',2}^{(4)} \end{pmatrix} \Lambda$$

$$T_3^{ii}: k_{N^c} = 1, k_L = 2$$

$$\mathcal{W}_\nu = g_1 \left(Y_{\mathbf{6}I}^{(3)} N^c L H_u \right)_1 + g_2 \left(Y_{\mathbf{6}II}^{(3)} N^c L H_u \right)_1 + \Lambda \left(Y_{\mathbf{3}'}^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} \sqrt{2} \left(g_1 Y_{\mathbf{6}I,6}^{(3)} + g_2 Y_{\mathbf{6}II,6}^{(3)} \right) & \sqrt{2} \left(g_1 Y_{\mathbf{6}I,5}^{(3)} + g_2 Y_{\mathbf{6}II,5}^{(3)} \right) & g_1 \left(Y_{\mathbf{6}I,1}^{(3)} + Y_{\mathbf{6}I,2}^{(3)} \right) \\ & & + g_2 \left(Y_{\mathbf{6}II,1}^{(3)} + Y_{\mathbf{6}II,2}^{(3)} \right) \\ \sqrt{2} \left(g_1 Y_{\mathbf{6}I,3}^{(3)} + g_2 Y_{\mathbf{6}II,3}^{(3)} \right) & g_1 \left(Y_{\mathbf{6}I,1}^{(3)} - Y_{\mathbf{6}I,2}^{(3)} \right) & \\ & + g_2 \left(Y_{\mathbf{6}II,1}^{(3)} - Y_{\mathbf{6}II,2}^{(3)} \right) & \sqrt{2} \left(g_1 Y_{\mathbf{6}I,4}^{(3)} + g_2 Y_{\mathbf{6}II,4}^{(3)} \right) \end{pmatrix} v_u$$

$$M_N = \begin{pmatrix} -\sqrt{2}Y_{\mathbf{3}',3}^{(2)} & Y_{\mathbf{3}',1}^{(2)} \\ Y_{\mathbf{3}',1}^{(2)} & \sqrt{2}Y_{\mathbf{3}',2}^{(2)} \end{pmatrix} \Lambda$$

$$T_4^{ii}: k_{N^c} = 2, k_L = 1$$

$$\mathcal{W}_\nu = g_1 \left(Y_{\mathbf{6}I}^{(3)} N^c L H_u \right)_1 + g_2 \left(Y_{\mathbf{6}II}^{(3)} N^c L H_u \right)_1 + \Lambda \left(Y_{\mathbf{3}'}^{(4)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} \sqrt{2} \left(g_1 Y_{\mathbf{6}I,6}^{(3)} + g_2 Y_{\mathbf{6}II,6}^{(3)} \right) & \sqrt{2} \left(g_1 Y_{\mathbf{6}I,5}^{(3)} + g_2 Y_{\mathbf{6}II,5}^{(3)} \right) & g_1 \left(Y_{\mathbf{6}I,1}^{(3)} + Y_{\mathbf{6}I,2}^{(3)} \right) \\ & & + g_2 \left(Y_{\mathbf{6}II,1}^{(3)} + Y_{\mathbf{6}II,2}^{(3)} \right) \\ \sqrt{2} \left(g_1 Y_{\mathbf{6}I,3}^{(3)} + g_2 Y_{\mathbf{6}II,3}^{(3)} \right) & g_1 \left(Y_{\mathbf{6}I,1}^{(3)} - Y_{\mathbf{6}I,2}^{(3)} \right) & \\ & + g_2 \left(Y_{\mathbf{6}II,1}^{(3)} - Y_{\mathbf{6}II,2}^{(3)} \right) & \sqrt{2} \left(g_1 Y_{\mathbf{6}I,4}^{(3)} + g_2 Y_{\mathbf{6}II,4}^{(3)} \right) \end{pmatrix} v_u$$

$$M_N = \begin{pmatrix} -\sqrt{2}Y_{\mathbf{3}',3}^{(4)} & Y_{\mathbf{3}',1}^{(4)} \\ Y_{\mathbf{3}',1}^{(4)} & \sqrt{2}Y_{\mathbf{3}',2}^{(4)} \end{pmatrix} \Lambda$$

$$T_5^{ii}: k_{N^c} = 3, k_L = -2$$

$$\mathcal{W}_\nu = g \left(Y_6^{(1)} N^c L H_u \right)_1 + \Lambda_1 \left(Y_{3'I}^{(6)} N^c N^c \right)_1 + \Lambda_2 \left(Y_{3''I}^{(6)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} \sqrt{2}Y_6 & \sqrt{2}Y_5 & Y_1 + Y_2 \\ \sqrt{2}Y_3 & Y_1 - Y_2 & \sqrt{2}Y_4 \end{pmatrix} g v_u$$

$$M_N = \begin{pmatrix} -\sqrt{2} \left(\Lambda_1 Y_{3'I,3}^{(6)} + \Lambda_2 Y_{3''I,3}^{(6)} \right) & \Lambda_1 Y_{3'I,1}^{(6)} + \Lambda_2 Y_{3''I,1}^{(6)} \\ \Lambda_1 Y_{3'I,1}^{(6)} + \Lambda_2 Y_{3''I,1}^{(6)} & \sqrt{2} \left(\Lambda_1 Y_{3'I,2}^{(6)} + \Lambda_2 Y_{3''I,2}^{(6)} \right) \end{pmatrix}$$

$$T_1^{iii}: k_{N^c} = 1, k_L = 0$$

$$\mathcal{W}_\nu = g \left(Y_6^{(1)} N^c L H_u \right)_1 + \Lambda \left(Y_3^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} Y_5 & -Y_3 & Y_2 \\ -Y_4 & -Y_1 & -Y_6 \end{pmatrix} g v_u \quad M_N = \begin{pmatrix} -\sqrt{2}Y_{3,2}^{(2)} & -Y_{3,1}^{(2)} \\ -Y_{3,1}^{(2)} & \sqrt{2}Y_{3,3}^{(2)} \end{pmatrix} \Lambda$$

$$T_2^{iii}: k_{N^c} = 2, k_L = -1$$

$$\mathcal{W}_\nu = g \left(Y_6^{(1)} N^c L H_u \right)_1 + \Lambda \left(Y_3^{(4)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} Y_5 & -Y_3 & Y_2 \\ -Y_4 & -Y_1 & -Y_6 \end{pmatrix} g v_u \quad M_N = \begin{pmatrix} -\sqrt{2}Y_{3,2}^{(4)} & -Y_{3,1}^{(4)} \\ -Y_{3,1}^{(4)} & \sqrt{2}Y_{3,3}^{(4)} \end{pmatrix} \Lambda$$

$$T_3^{iii}: k_{N^c} = 1, k_L = 2$$

$$\mathcal{W}_\nu = g_1 \left(Y_{6I}^{(3)} N^c L H_u \right)_1 + g_2 \left(Y_{6II}^{(3)} N^c L H_u \right)_1 + \Lambda \left(Y_3^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} g_1 Y_{6I,5}^{(3)} + g_2 Y_{6II,5}^{(3)} & -g_1 Y_{6I,3}^{(3)} - g_2 Y_{6II,3}^{(3)} & g_1 Y_{6I,2}^{(3)} + g_2 Y_{6II,2}^{(3)} \\ -g_1 Y_{6I,4}^{(3)} - g_2 Y_{6II,4}^{(3)} & -g_1 Y_{6I,1}^{(3)} - g_2 Y_{6II,1}^{(3)} & -g_1 Y_{6I,6}^{(3)} - g_2 Y_{6II,6}^{(3)} \end{pmatrix} v_u$$

$$M_N = \begin{pmatrix} -\sqrt{2}Y_{3,2}^{(2)} & -Y_{3,1}^{(2)} \\ -Y_{3,1}^{(2)} & \sqrt{2}Y_{3,3}^{(2)} \end{pmatrix} \Lambda$$

$$T_4^{iii}: k_{N^c} = 2, k_L = 1$$

$$\mathcal{W}_\nu = g_1 \left(Y_{6I}^{(3)} N^c L H_u \right)_1 + g_2 \left(Y_{6II}^{(3)} N^c L H_u \right)_1 + \Lambda \left(Y_{\mathbf{3}}^{(4)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} g_1 Y_{6I,5}^{(3)} + g_2 Y_{6II,5}^{(3)} & -g_1 Y_{6I,3}^{(3)} - g_2 Y_{6II,3}^{(3)} & g_1 Y_{6I,2}^{(3)} + g_2 Y_{6II,2}^{(3)} \\ -g_1 Y_{6I,4}^{(3)} - g_2 Y_{6II,4}^{(3)} & -g_1 Y_{6I,1}^{(3)} - g_2 Y_{6II,1}^{(3)} & -g_1 Y_{6I,6}^{(3)} - g_2 Y_{6II,6}^{(3)} \end{pmatrix} v_u$$

$$M_N = \begin{pmatrix} -\sqrt{2} Y_{\mathbf{3},2}^{(4)} & -Y_{\mathbf{3},1}^{(4)} \\ -Y_{\mathbf{3},1}^{(4)} & \sqrt{2} Y_{\mathbf{3},3}^{(4)} \end{pmatrix} \Lambda$$

$$T_5^{iii}: k_{N^c} = 3, k_L = -2$$

$$\mathcal{W}_\nu = g \left(Y_{\mathbf{6}}^{(1)} N^c L H_u \right)_1 + \Lambda_1 \left(Y_{\mathbf{3I}}^{(6)} N^c N^c \right)_1 + \Lambda_2 \left(Y_{\mathbf{3II}}^{(6)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} Y_5 & -Y_3 & Y_2 \\ -Y_4 & -Y_1 & -Y_6 \end{pmatrix} g v_u$$

$$M_N = \begin{pmatrix} -\sqrt{2} \left(\Lambda_1 Y_{\mathbf{3I},2}^{(6)} + \Lambda_2 Y_{\mathbf{3II},2}^{(6)} \right) & -\Lambda_1 Y_{\mathbf{3I},1}^{(6)} - \Lambda_2 Y_{\mathbf{3II},1}^{(6)} \\ -\Lambda_1 Y_{\mathbf{3I},1}^{(6)} - \Lambda_2 Y_{\mathbf{3II},1}^{(6)} & \sqrt{2} \left(\Lambda_1 Y_{\mathbf{3I},3}^{(6)} + \Lambda_2 Y_{\mathbf{3II},3}^{(6)} \right) \end{pmatrix}$$

$$T_1^{iv}: k_{N^c} = 1, k_L = 2$$

$$\mathcal{W}_\nu = g \left(Y_{\mathbf{4}'}^{(3)} N^c L H_u \right)_1 + \Lambda \left(Y_{\mathbf{3}'}^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} -\sqrt{2} Y_{\mathbf{4}',4}^{(3)} & -\sqrt{3} Y_{\mathbf{4}',2}^{(3)} & Y_{\mathbf{4}',1}^{(3)} \\ \sqrt{2} Y_{\mathbf{4}',1}^{(3)} & Y_{\mathbf{4}',4}^{(3)} & -\sqrt{3} Y_{\mathbf{4}',3}^{(3)} \end{pmatrix} g v_u \quad M_N = \begin{pmatrix} -\sqrt{2} Y_{\mathbf{3}',3}^{(2)} & Y_{\mathbf{3}',1}^{(2)} \\ Y_{\mathbf{3}',1}^{(2)} & \sqrt{2} Y_{\mathbf{3}',2}^{(2)} \end{pmatrix} \Lambda$$

$$T_2^{iv}: k_{N^c} = 2, k_L = 1$$

$$\mathcal{W}_\nu = g \left(Y_{\mathbf{4}'}^{(3)} N^c L H_u \right)_1 + \Lambda \left(Y_{\mathbf{3}'}^{(4)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} -\sqrt{2} Y_{\mathbf{4}',4}^{(3)} & -\sqrt{3} Y_{\mathbf{4}',2}^{(3)} & Y_{\mathbf{4}',1}^{(3)} \\ \sqrt{2} Y_{\mathbf{4}',1}^{(3)} & Y_{\mathbf{4}',4}^{(3)} & -\sqrt{3} Y_{\mathbf{4}',3}^{(3)} \end{pmatrix} g v_u \quad M_N = \begin{pmatrix} -\sqrt{2} Y_{\mathbf{3}',3}^{(4)} & Y_{\mathbf{3}',1}^{(4)} \\ Y_{\mathbf{3}',1}^{(4)} & \sqrt{2} Y_{\mathbf{3}',2}^{(4)} \end{pmatrix} \Lambda$$

$$T_3^{iv}: k_{N^c} = 1, k_L = 4$$

$$\mathcal{W}_\nu = g_1 \left(Y_{2'}^{(5)} N^c L H_u \right)_1 + g_2 \left(Y_{4'}^{(5)} N^c L H_u \right)_1 + \Lambda \left(Y_{3'}^{(2)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} -g_1 Y_{2',2}^{(5)} - \sqrt{2} g_2 Y_{4',4}^{(5)} & -\sqrt{3} g_2 Y_{4',2}^{(5)} & \sqrt{2} g_1 Y_{2',1}^{(5)} + g_2 Y_{4',1}^{(5)} \\ \sqrt{2} g_2 Y_{4',1}^{(5)} - g_1 Y_{2',1}^{(5)} & g_2 Y_{4',4}^{(5)} - \sqrt{2} g_1 Y_{2',2}^{(5)} & -\sqrt{3} g_2 Y_{4',3}^{(5)} \end{pmatrix} v_u$$

$$M_N = \begin{pmatrix} -\sqrt{2} Y_{3',3}^{(2)} & Y_{3',1}^{(2)} \\ Y_{3',1}^{(2)} & \sqrt{2} Y_{3',2}^{(2)} \end{pmatrix} \Lambda$$

$$T_4^{iv}: k_{N^c} = 2, k_L = 3$$

$$\mathcal{W}_\nu = g_1 \left(Y_{2'}^{(5)} N^c L H_u \right)_1 + g_2 \left(Y_{4'}^{(5)} N^c L H_u \right)_1 + \Lambda \left(Y_{3'}^{(4)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} -g_1 Y_{2',2}^{(5)} - \sqrt{2} g_2 Y_{4',4}^{(5)} & -\sqrt{3} g_2 Y_{4',2}^{(5)} & \sqrt{2} g_1 Y_{2',1}^{(5)} + g_2 Y_{4',1}^{(5)} \\ \sqrt{2} g_2 Y_{4',1}^{(5)} - g_1 Y_{2',1}^{(5)} & g_2 Y_{4',4}^{(5)} - \sqrt{2} g_1 Y_{2',2}^{(5)} & -\sqrt{3} g_2 Y_{4',3}^{(5)} \end{pmatrix} v_u$$

$$M_N = \begin{pmatrix} -\sqrt{2} Y_{3',3}^{(4)} & Y_{3',1}^{(4)} \\ Y_{3',1}^{(4)} & \sqrt{2} Y_{3',2}^{(4)} \end{pmatrix} \Lambda$$

$$T_5^{iv}: k_{N^c} = 3, k_L = 0$$

$$\mathcal{W}_\nu = g \left(Y_{4'}^{(3)} N^c L H_u \right)_1 + \Lambda_1 \left(Y_{3'I}^{(6)} N^c N^c \right)_1 + \Lambda_2 \left(Y_{3'II}^{(6)} N^c N^c \right)_1$$

$$M_D = \begin{pmatrix} -\sqrt{2} Y_{4',4}^{(3)} & -\sqrt{3} Y_{4',2}^{(3)} & Y_{4',1}^{(3)} \\ \sqrt{2} Y_{4',1}^{(3)} & Y_{4',4}^{(3)} & -\sqrt{3} Y_{4',3}^{(3)} \end{pmatrix} g v_u$$

$$M_N = \begin{pmatrix} -\sqrt{2} \left(\Lambda_1 Y_{3'I,3}^{(6)} + \Lambda_2 Y_{3'II,3}^{(6)} \right) & \Lambda_1 Y_{3'I,1}^{(6)} + \Lambda_2 Y_{3'II,1}^{(6)} \\ \Lambda_1 Y_{3'I,1}^{(6)} + \Lambda_2 Y_{3'II,1}^{(6)} & \sqrt{2} \left(\Lambda_1 Y_{3'I,2}^{(6)} + \Lambda_2 Y_{3'II,2}^{(6)} \right) \end{pmatrix}$$

1.2.3 Models without right-handed neutrino

$$W_1^i: k_L = 1$$

$$W_\nu = \frac{1}{\Lambda_1} \left(Y_5^{(2)} L^2 H_u^2 \right)_1$$

$$M_\nu = \begin{pmatrix} 2Y_{5,1}^{(2)} & -\sqrt{3}Y_{5,5}^{(2)} & -\sqrt{3}Y_{5,2}^{(2)} \\ -\sqrt{3}Y_{5,5}^{(2)} & \sqrt{6}Y_{5,4}^{(2)} & -Y_{5,1}^{(2)} \\ -\sqrt{3}Y_{5,2}^{(2)} & -Y_{5,1}^{(2)} & \sqrt{6}Y_{5,3}^{(2)} \end{pmatrix} \frac{v_u^2}{\Lambda_1}$$

$$W_2^i: k_L = 2$$

$$W_\nu = \frac{1}{\Lambda_1} \left(Y_1^{(4)} L^2 H_u^2 \right)_1 + \frac{1}{\Lambda_2} \left(Y_{5I}^{(4)} L^2 H_u^2 \right)_1 + \frac{1}{\Lambda_3} \left(Y_{5II}^{(4)} L^2 H_u^2 \right)_1$$

$$M_\nu = \begin{pmatrix} \frac{Y_{1,1}^{(4)}}{\Lambda_1} + \frac{2Y_{5I,1}^{(4)}}{\Lambda_2} + \frac{2Y_{5II,1}^{(4)}}{\Lambda_3} & -\frac{\sqrt{3}(\Lambda_3 Y_{5I,5}^{(4)} + \Lambda_2 Y_{5II,5}^{(4)})}{\Lambda_2 \Lambda_3} & -\frac{\sqrt{3}(\Lambda_3 Y_{5I,2}^{(4)} + \Lambda_2 Y_{5II,2}^{(4)})}{\Lambda_2 \Lambda_3} \\ -\frac{\sqrt{3}(\Lambda_3 Y_{5I,5}^{(4)} + \Lambda_2 Y_{5II,5}^{(4)})}{\Lambda_2 \Lambda_3} & \sqrt{6} \left(\frac{Y_{5I,4}^{(4)}}{\Lambda_2} + \frac{Y_{5II,4}^{(4)}}{\Lambda_3} \right) & \frac{Y_{1,1}^{(4)}}{\Lambda_1} - \frac{Y_{5I,1}^{(4)}}{\Lambda_2} - \frac{Y_{5II,1}^{(4)}}{\Lambda_3} \\ -\frac{\sqrt{3}(\Lambda_3 Y_{5I,2}^{(4)} + \Lambda_2 Y_{5II,2}^{(4)})}{\Lambda_2 \Lambda_3} & \frac{Y_{1,1}^{(4)}}{\Lambda_1} - \frac{Y_{5I,1}^{(4)}}{\Lambda_2} - \frac{Y_{5II,1}^{(4)}}{\Lambda_3} & \sqrt{6} \left(\frac{Y_{5I,3}^{(4)}}{\Lambda_2} + \frac{Y_{5II,3}^{(4)}}{\Lambda_3} \right) \end{pmatrix} v_u^2$$

$$W_3^i: k_L = 3$$

$$W_\nu = \frac{1}{\Lambda_1} \left(Y_1^{(6)} L^2 H_u^2 \right)_1 + \frac{1}{\Lambda_2} \left(Y_{5I}^{(6)} L^2 H_u^2 \right)_1 + \frac{1}{\Lambda_3} \left(Y_{5II}^{(6)} L^2 H_u^2 \right)_1$$

$$M_\nu = \begin{pmatrix} \frac{Y_{1,1}^{(6)}}{\Lambda_1} + \frac{2Y_{5I,1}^{(6)}}{\Lambda_2} + \frac{2Y_{5II,1}^{(6)}}{\Lambda_3} & -\frac{\sqrt{3}(\Lambda_3 Y_{5I,5}^{(6)} + \Lambda_2 Y_{5II,5}^{(6)})}{\Lambda_2 \Lambda_3} & -\frac{\sqrt{3}(\Lambda_3 Y_{5I,2}^{(6)} + \Lambda_2 Y_{5II,2}^{(6)})}{\Lambda_2 \Lambda_3} \\ -\frac{\sqrt{3}(\Lambda_3 Y_{5I,5}^{(6)} + \Lambda_2 Y_{5II,5}^{(6)})}{\Lambda_2 \Lambda_3} & \sqrt{6} \left(\frac{Y_{5I,4}^{(6)}}{\Lambda_2} + \frac{Y_{5II,4}^{(6)}}{\Lambda_3} \right) & \frac{Y_{1,1}^{(6)}}{\Lambda_1} - \frac{Y_{5I,1}^{(6)}}{\Lambda_2} - \frac{Y_{5II,1}^{(6)}}{\Lambda_3} \\ -\frac{\sqrt{3}(\Lambda_3 Y_{5I,2}^{(6)} + \Lambda_2 Y_{5II,2}^{(6)})}{\Lambda_2 \Lambda_3} & \frac{Y_{1,1}^{(6)}}{\Lambda_1} - \frac{Y_{5I,1}^{(6)}}{\Lambda_2} - \frac{Y_{5II,1}^{(6)}}{\Lambda_3} & \sqrt{6} \left(\frac{Y_{5I,3}^{(6)}}{\Lambda_2} + \frac{Y_{5II,3}^{(6)}}{\Lambda_3} \right) \end{pmatrix} v_u^2$$

$$W_1^{ii}: k_L = 1$$

$$W_\nu = \frac{1}{\Lambda_0} \left(Y_5^{(2)} L^2 H_u^2 \right)_1$$

$$M_\nu = \begin{pmatrix} 2Y_{5,1}^{(2)} & -\sqrt{3}Y_{5,4}^{(2)} & -\sqrt{3}Y_{5,3}^{(2)} \\ -\sqrt{3}Y_{5,4}^{(2)} & \sqrt{6}Y_{5,2}^{(2)} & -Y_{5,1}^{(2)} \\ -\sqrt{3}Y_{5,3}^{(2)} & -Y_{5,1}^{(2)} & \sqrt{6}Y_{5,5}^{(2)} \end{pmatrix} \frac{v_u^2}{\Lambda_0}$$

$$W_2^{ii}: k_L = 2$$

$$W_\nu = \frac{1}{\Lambda_1} \left(Y_1^{(4)} L^2 H_u^2 \right)_1 + \frac{1}{\Lambda_2} \left(Y_{5I}^{(4)} L^2 H_u^2 \right)_1 + \frac{1}{\Lambda_3} \left(Y_{5II}^{(4)} L^2 H_u^2 \right)_1$$

$$M_\nu = \begin{pmatrix} \frac{Y_{1,1}^{(4)}}{\Lambda_1} + \frac{2Y_{5I,1}^{(4)}}{\Lambda_2} + \frac{2Y_{5II,1}^{(4)}}{\Lambda_3} & -\frac{\sqrt{3}(\Lambda_3 Y_{5I,4}^{(4)} + \Lambda_2 Y_{5II,4}^{(4)})}{\Lambda_2 \Lambda_3} & -\frac{\sqrt{3}(\Lambda_3 Y_{5I,3}^{(4)} + \Lambda_2 Y_{5II,3}^{(4)})}{\Lambda_2 \Lambda_3} \\ -\frac{\sqrt{3}(\Lambda_3 Y_{5I,4}^{(4)} + \Lambda_2 Y_{5II,4}^{(4)})}{\Lambda_2 \Lambda_3} & \sqrt{6} \left(\frac{Y_{5I,2}^{(4)}}{\Lambda_2} + \frac{Y_{5II,2}^{(4)}}{\Lambda_3} \right) & \frac{Y_{1,1}^{(4)}}{\Lambda_1} - \frac{Y_{5I,1}^{(4)}}{\Lambda_2} - \frac{Y_{5II,1}^{(4)}}{\Lambda_3} \\ -\frac{\sqrt{3}(\Lambda_3 Y_{5I,3}^{(4)} + \Lambda_2 Y_{5II,3}^{(4)})}{\Lambda_2 \Lambda_3} & \frac{Y_{1,1}^{(4)}}{\Lambda_1} - \frac{Y_{5I,1}^{(4)}}{\Lambda_2} - \frac{Y_{5II,1}^{(4)}}{\Lambda_3} & \sqrt{6} \left(\frac{Y_{5I,5}^{(4)}}{\Lambda_2} + \frac{Y_{5II,5}^{(4)}}{\Lambda_3} \right) \end{pmatrix} v_u^2$$

$$W_3^{ii}: k_L = 3$$

$$W_\nu = \frac{1}{\Lambda_1} \left(Y_1^{(6)} L^2 H_u^2 \right)_1 + \frac{1}{\Lambda_2} \left(Y_{5I}^{(6)} L^2 H_u^2 \right)_1 + \frac{1}{\Lambda_3} \left(Y_{5II}^{(6)} L^2 H_u^2 \right)_1$$

$$M_\nu = \begin{pmatrix} \frac{Y_{1,1}^{(6)}}{\Lambda_1} + \frac{2Y_{5I,1}^{(6)}}{\Lambda_2} + \frac{2Y_{5II,1}^{(6)}}{\Lambda_3} & -\frac{\sqrt{3}(\Lambda_3 Y_{5I,4}^{(6)} + \Lambda_2 Y_{5II,4}^{(6)})}{\Lambda_2 \Lambda_3} & -\frac{\sqrt{3}(\Lambda_3 Y_{5I,3}^{(6)} + \Lambda_2 Y_{5II,3}^{(6)})}{\Lambda_2 \Lambda_3} \\ -\frac{\sqrt{3}(\Lambda_3 Y_{5I,4}^{(6)} + \Lambda_2 Y_{5II,4}^{(6)})}{\Lambda_2 \Lambda_3} & \sqrt{6} \left(\frac{Y_{5I,2}^{(6)}}{\Lambda_2} + \frac{Y_{5II,2}^{(6)}}{\Lambda_3} \right) & \frac{Y_{1,1}^{(6)}}{\Lambda_1} - \frac{Y_{5I,1}^{(6)}}{\Lambda_2} - \frac{Y_{5II,1}^{(6)}}{\Lambda_3} \\ -\frac{\sqrt{3}(\Lambda_3 Y_{5I,3}^{(6)} + \Lambda_2 Y_{5II,3}^{(6)})}{\Lambda_2 \Lambda_3} & \frac{Y_{1,1}^{(6)}}{\Lambda_1} - \frac{Y_{5I,1}^{(6)}}{\Lambda_2} - \frac{Y_{5II,1}^{(6)}}{\Lambda_3} & \sqrt{6} \left(\frac{Y_{5I,5}^{(6)}}{\Lambda_2} + \frac{Y_{5II,5}^{(6)}}{\Lambda_3} \right) \end{pmatrix} v_u^2$$

1.3 Lepton Models

In this chapter, we will give all possible models through combinations based on the results of charged lepton sector and neutrino sector. In the combination, we only need to require the lepton doublet L in both sectors to have the same representation $\mathbf{3}$ or $\mathbf{3}'$ under A_5 group.

In table 3, We present 720 possible models, they are named $\mathcal{L}_1 \cdots \mathcal{L}_{720}$, their representation assignments and weights, as well as the χ_{\min}^2 of normal ordering(NO) and inverted ordering(IO) neutrino models are also shown. We find that 25(49) of the 720 models with 9 or 10 parameters can give the best fit results within the experimental 3σ limit for NO(IO) models. There are also some models with $\chi_{\min}^2 < 50$, but the best fit values of the observables slightly deviate from the 3σ limit. We have marked them in the table with \star and \star respectively. We have selected some models that can accommodate the experimental data for detailed discussion in the article.

Models	#P	Combinations	$(\rho_{Ec}, \rho_L, \rho_{Nc})$	k_{Ec}	k_L	k_{Nc}	χ^2 (NO)	χ^2 (IO)
\mathcal{L}_1	8	C_1^i, S_1^i	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	3, 5, 7	-1	1	1571.8	18886.
\mathcal{L}_2	10	C_1^i, S_2^i	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	0, 2, 4	2	0	2.6459×10^{-8} \star	2.8006×10^{-6} \star
\mathcal{L}_3	10	C_1^i, S_3^i	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	1, 3, 5	1	1	3.9239×10^{-8} \star	1.1577×10^{-6} \star
\mathcal{L}_4	8	C_1^i, S_1^{ii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3}')$	0, 2, 4	2	0	1610.	554.1
\mathcal{L}_5	8	C_1^i, S_2^{ii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3}')$	1, 3, 5	1	1	2379.1	4580.
\mathcal{L}_6	8	C_1^{ii}, S_1^{iii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{3})$	0, 2, 4	2	0	1138.5	775460.
\mathcal{L}_7	8	C_1^{ii}, S_2^{iii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{3})$	1, 3, 5	1	1	1318.6	6957.4
\mathcal{L}_8	8	C_1^{ii}, S_1^{iv}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{3}')$	3, 5, 7	-1	1	251.77	560.64
\mathcal{L}_9	10	C_1^{ii}, S_2^{iv}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{3}')$	0, 2, 4	2	0	2.1424×10^{-8} \star	2.7143×10^{-6} \star
\mathcal{L}_{10}	10	C_1^{ii}, S_3^{iv}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{3}')$	1, 3, 5	1	1	4.5697×10^{-8} \star	8.5578×10^{-8} \star
\mathcal{L}_{11}	5	C_1^{iii}, S_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	4, 3	-1	1	14413.	279190.
\mathcal{L}_{12}	7	C_1^{iii}, S_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	1, 0	2	0	14515.	14636.
\mathcal{L}_{13}	7	C_1^{iii}, S_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	2, 1	1	1	14801.	14807.
\mathcal{L}_{14}	5	C_1^{iii}, S_1^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3}')$	1, 0	2	0	15793.	593320.
\mathcal{L}_{15}	5	C_1^{iii}, S_2^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3}')$	2, 1	1	1	16415.	72912.
\mathcal{L}_{16}	5	C_2^{iii}, S_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	4, 5	-1	1	1.0806×10^6	594180.
\mathcal{L}_{17}	7	C_2^{iii}, S_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	1, 2	2	0	14569.	14629.
\mathcal{L}_{18}	7	C_2^{iii}, S_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	2, 3	1	1	14235.	266840.
\mathcal{L}_{19}	5	C_2^{iii}, S_1^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3}')$	1, 2	2	0	16175.	17733.
\mathcal{L}_{20}	5	C_2^{iii}, S_2^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3}')$	2, 3	1	1	16431.	1.2156×10^6
\mathcal{L}_{21}	7	C_3^{iii}, S_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	6, 3	-1	1	516.84	260830.
\mathcal{L}_{22}	9	C_3^{iii}, S_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	3, 0	2	0	8.7252 \star	379.05
\mathcal{L}_{23}	9	C_3^{iii}, S_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	4, 1	1	1	479.19	390.79
\mathcal{L}_{24}	7	C_3^{iii}, S_1^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3}')$	3, 0	2	0	1910.2	1958.
\mathcal{L}_{25}	7	C_3^{iii}, S_2^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3}')$	4, 1	1	1	1826.3	8857.9
\mathcal{L}_{26}	7	C_4^{iii}, S_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	6, 5	-1	1	1.0804×10^6	594180.
\mathcal{L}_{27}	9	C_4^{iii}, S_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	3, 2	2	0	14579.	14298.
\mathcal{L}_{28}	9	C_4^{iii}, S_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	4, 3	1	1	14502.	254740.
\mathcal{L}_{29}	7	C_4^{iii}, S_1^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3}')$	3, 2	2	0	16583.	15851.
\mathcal{L}_{30}	7	C_4^{iii}, S_2^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3}')$	4, 3	1	1	16441.	1.2155×10^6
\mathcal{L}_{31}	7	C_5^{iii}, S_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	4, 7	-1	1	14175.	14343.
\mathcal{L}_{32}	9	C_5^{iii}, S_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	1, 4	2	0	14977.	13932.
\mathcal{L}_{33}	9	C_5^{iii}, S_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	2, 5	1	1	15259.	15924.
\mathcal{L}_{34}	7	C_5^{iii}, S_1^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3}')$	1, 4	2	0	16456.	14769.
\mathcal{L}_{35}	7	C_5^{iii}, S_2^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3}')$	2, 5	1	1	15252.	32085.
\mathcal{L}_{36}	9	C_6^{iii}, S_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{3})$	6, 7	-1	1	113.07	336.49

\mathcal{L}_{37}	11	C_6^{iii}, S_2^i	$(2 \oplus 1, 3, 3)$	3, 4	2	0	4.22×10^{-8} ★	9.0628×10^{-6} ★
\mathcal{L}_{38}	11	C_6^{iii}, S_3^i	$(2 \oplus 1, 3, 3)$	4, 5	1	1	4.3012×10^{-7} ★	6.4597×10^{-6} ★
\mathcal{L}_{39}	9	C_6^{iii}, S_1^{ii}	$(2 \oplus 1, 3, 3')$	3, 4	2	0	700.57	561.51
\mathcal{L}_{40}	9	C_6^{iii}, S_2^{ii}	$(2 \oplus 1, 3, 3')$	4, 5	1	1	1149.2	28215.
\mathcal{L}_{41}	5	C_1^{iv}, S_1^i	$(2' \oplus 1, 3, 3)$	2, 3	-1	1	3386.8	352230.
\mathcal{L}_{42}	7	C_1^{iv}, S_2^i	$(2' \oplus 1, 3, 3)$	-1, 0	2	0	2253.8	1795.8
\mathcal{L}_{43}	7	C_1^{iv}, S_3^i	$(2' \oplus 1, 3, 3)$	0, 1	1	1	240020.	448.29
\mathcal{L}_{44}	5	C_1^{iv}, S_1^{ii}	$(2' \oplus 1, 3, 3')$	-1, 0	2	0	600640.	565490.
\mathcal{L}_{45}	5	C_1^{iv}, S_2^{ii}	$(2' \oplus 1, 3, 3')$	0, 1	1	1	161970.	1.4956×10^6
\mathcal{L}_{46}	5	C_2^{iv}, S_1^i	$(2' \oplus 1, 3, 3)$	2, 5	-1	1	8707.	352230.
\mathcal{L}_{47}	7	C_2^{iv}, S_2^i	$(2' \oplus 1, 3, 3)$	-1, 2	2	0	1403.1	767.31
\mathcal{L}_{48}	7	C_2^{iv}, S_3^i	$(2' \oplus 1, 3, 3)$	0, 3	1	1	1088.	35251.
\mathcal{L}_{49}	5	C_2^{iv}, S_1^{ii}	$(2' \oplus 1, 3, 3')$	-1, 2	2	0	9837.9	947960.
\mathcal{L}_{50}	5	C_2^{iv}, S_2^{ii}	$(2' \oplus 1, 3, 3')$	0, 3	1	1	3508.8	2800.7
\mathcal{L}_{51}	7	C_3^{iv}, S_1^i	$(2' \oplus 1, 3, 3)$	4, 3	-1	1	522.18	1253.7
\mathcal{L}_{52}	9	C_3^{iv}, S_2^i	$(2' \oplus 1, 3, 3)$	1, 0	2	0	161.84	0.00036262 ★
\mathcal{L}_{53}	9	C_3^{iv}, S_3^i	$(2' \oplus 1, 3, 3)$	2, 1	1	1	277.13	23.797 ☆
\mathcal{L}_{54}	7	C_3^{iv}, S_1^{ii}	$(2' \oplus 1, 3, 3')$	1, 0	2	0	2057.2	13592.
\mathcal{L}_{55}	7	C_3^{iv}, S_2^{ii}	$(2' \oplus 1, 3, 3')$	2, 1	1	1	1851.3	62776.
\mathcal{L}_{56}	7	C_4^{iv}, S_1^i	$(2' \oplus 1, 3, 3)$	4, 5	-1	1	630.29	1253.7
\mathcal{L}_{57}	9	C_4^{iv}, S_2^i	$(2' \oplus 1, 3, 3)$	1, 2	2	0	78.769	0.022598 ★
\mathcal{L}_{58}	9	C_4^{iv}, S_3^i	$(2' \oplus 1, 3, 3)$	2, 3	1	1	402.4	23.795 ☆
\mathcal{L}_{59}	7	C_4^{iv}, S_1^{ii}	$(2' \oplus 1, 3, 3')$	1, 2	2	0	2693.	9940.5
\mathcal{L}_{60}	7	C_4^{iv}, S_2^{ii}	$(2' \oplus 1, 3, 3')$	2, 3	1	1	1849.4	2339.9
\mathcal{L}_{61}	7	C_5^{iv}, S_1^i	$(2' \oplus 1, 3, 3)$	2, 7	-1	1	161.86	7777.4
\mathcal{L}_{62}	9	C_5^{iv}, S_2^i	$(2' \oplus 1, 3, 3)$	-1, 4	2	0	44.091 ☆	1157.5
\mathcal{L}_{63}	9	C_5^{iv}, S_3^i	$(2' \oplus 1, 3, 3)$	0, 5	1	1	434.31	7054.7
\mathcal{L}_{64}	7	C_5^{iv}, S_1^{ii}	$(2' \oplus 1, 3, 3')$	-1, 4	2	0	18201.	529890.
\mathcal{L}_{65}	7	C_5^{iv}, S_2^{ii}	$(2' \oplus 1, 3, 3')$	0, 5	1	1	11135.	306800.
\mathcal{L}_{66}	9	C_6^{iv}, S_1^i	$(2' \oplus 1, 3, 3)$	6, 3	-1	1	0.027304 ★	517.49
\mathcal{L}_{67}	11	C_6^{iv}, S_2^i	$(2' \oplus 1, 3, 3)$	3, 0	2	0	5.7442×10^{-7} ★	4.6436×10^{-6} ★
\mathcal{L}_{68}	11	C_6^{iv}, S_3^i	$(2' \oplus 1, 3, 3)$	4, 1	1	1	2.3688×10^{-6} ★	4.8531×10^{-6} ★
\mathcal{L}_{69}	9	C_6^{iv}, S_1^{ii}	$(2' \oplus 1, 3, 3')$	3, 0	2	0	854.45	149.03
\mathcal{L}_{70}	9	C_6^{iv}, S_2^{ii}	$(2' \oplus 1, 3, 3')$	4, 1	1	1	1138.5	20566.
\mathcal{L}_{71}	9	C_7^{iv}, S_1^i	$(2' \oplus 1, 3, 3)$	4, 7	-1	1	0.16787 ★	387.22
\mathcal{L}_{72}	11	C_7^{iv}, S_2^i	$(2' \oplus 1, 3, 3)$	1, 4	2	0	9.3298×10^{-6} ★	0.031896 ★
\mathcal{L}_{73}	11	C_7^{iv}, S_3^i	$(2' \oplus 1, 3, 3)$	2, 5	1	1	2.4193×10^{-6} ★	0.000011215 ★
\mathcal{L}_{74}	9	C_7^{iv}, S_1^{ii}	$(2' \oplus 1, 3, 3')$	1, 4	2	0	1179.1	13590.
\mathcal{L}_{75}	9	C_7^{iv}, S_2^{ii}	$(2' \oplus 1, 3, 3')$	2, 5	1	1	1514.8	11485.
\mathcal{L}_{76}	9	C_8^{iv}, S_1^i	$(2' \oplus 1, 3, 3)$	6, 5	-1	1	0.027269 ★	496.91
\mathcal{L}_{77}	11	C_8^{iv}, S_2^i	$(2' \oplus 1, 3, 3)$	3, 2	2	0	2.9021×10^{-7} ★	6.2417×10^{-6} ★
\mathcal{L}_{78}	11	C_8^{iv}, S_3^i	$(2' \oplus 1, 3, 3)$	4, 3	1	1	50.42	1.8376×10^{-6} ★
\mathcal{L}_{79}	9	C_8^{iv}, S_1^{ii}	$(2' \oplus 1, 3, 3')$	3, 2	2	0	584.94	566.62
\mathcal{L}_{80}	9	C_8^{iv}, S_2^{ii}	$(2' \oplus 1, 3, 3')$	4, 3	1	1	1138.5	1829.8
\mathcal{L}_{81}	5	C_1^v, S_1^{iii}	$(2 \oplus 1, 3', 3)$	-1, 0	2	0	4264.	776200.
\mathcal{L}_{82}	5	C_1^v, S_2^{iii}	$(2 \oplus 1, 3', 3)$	0, 1	1	1	472710.	442330.
\mathcal{L}_{83}	5	C_1^v, S_1^{iv}	$(2 \oplus 1, 3', 3')$	2, 3	-1	1	550520.	568190.
\mathcal{L}_{84}	7	C_1^v, S_2^{iv}	$(2 \oplus 1, 3', 3')$	-1, 0	2	0	992.93	1078.7
\mathcal{L}_{85}	7	C_1^v, S_3^{iv}	$(2 \oplus 1, 3', 3')$	0, 1	1	1	148.97	1947.7
\mathcal{L}_{86}	5	C_2^v, S_1^{iii}	$(2 \oplus 1, 3', 3)$	-1, 2	2	0	13096.	966560.

\mathcal{L}_{87}	5	C_2^v, S_2^{iii}	$(2 \oplus 1, 3', 3)$	0,3	1	1	4134.9	6434.3
\mathcal{L}_{88}	5	C_2^v, S_1^{iv}	$(2 \oplus 1, 3', 3')$	2,5	-1	1	64433.	144240.
\mathcal{L}_{89}	7	C_2^v, S_2^{iv}	$(2 \oplus 1, 3', 3')$	-1,2	2	0	19419.	932.59
\mathcal{L}_{90}	7	C_2^v, S_3^{iv}	$(2 \oplus 1, 3', 3')$	0,3	1	1	3173.	58583.
\mathcal{L}_{91}	7	C_3^v, S_1^{iii}	$(2 \oplus 1, 3', 3)$	1,0	2	0	2106.3	753930.
\mathcal{L}_{92}	7	C_3^v, S_2^{iii}	$(2 \oplus 1, 3', 3)$	2,1	1	1	1739.2	98721.
\mathcal{L}_{93}	7	C_3^v, S_1^{iv}	$(2 \oplus 1, 3', 3')$	4,3	-1	1	1325.4	8188.4
\mathcal{L}_{94}	9	C_3^v, S_2^{iv}	$(2 \oplus 1, 3', 3')$	1,0	2	0	4.1601×10^{-6} ★	0.0489 ★
\mathcal{L}_{95}	9	C_3^v, S_3^{iv}	$(2 \oplus 1, 3', 3')$	2,1	1	1	3.8817 ★	0.49665 ★
\mathcal{L}_{96}	7	C_4^v, S_1^{iii}	$(2 \oplus 1, 3', 3)$	1,2	2	0	1994.7	790420.
\mathcal{L}_{97}	7	C_4^v, S_2^{iii}	$(2 \oplus 1, 3', 3)$	2,3	1	1	1885.	35634.
\mathcal{L}_{98}	7	C_4^v, S_1^{iv}	$(2 \oplus 1, 3', 3')$	4,5	-1	1	1325.5	8188.4
\mathcal{L}_{99}	9	C_4^v, S_2^{iv}	$(2 \oplus 1, 3', 3')$	1,2	2	0	3.6746×10^{-6} ★	0.0016914 ★
\mathcal{L}_{100}	9	C_4^v, S_3^{iv}	$(2 \oplus 1, 3', 3')$	2,3	1	1	3.8879 ★	0.000010449 ★
\mathcal{L}_{101}	7	C_5^v, S_1^{iii}	$(2 \oplus 1, 3', 3)$	-1,4	2	0	4063.8	776710.
\mathcal{L}_{102}	7	C_5^v, S_2^{iii}	$(2 \oplus 1, 3', 3)$	0,5	1	1	18711.	510830.
\mathcal{L}_{103}	7	C_5^v, S_1^{iv}	$(2 \oplus 1, 3', 3')$	2,7	-1	1	18246.	248560.
\mathcal{L}_{104}	9	C_5^v, S_2^{iv}	$(2 \oplus 1, 3', 3')$	-1,4	2	0	230.85	750.93
\mathcal{L}_{105}	9	C_5^v, S_3^{iv}	$(2 \oplus 1, 3', 3')$	0,5	1	1	1.6783 ★	1931.9
\mathcal{L}_{106}	9	C_6^v, S_1^{iii}	$(2 \oplus 1, 3', 3)$	3,0	2	0	1243.9	753490.
\mathcal{L}_{107}	9	C_6^v, S_2^{iii}	$(2 \oplus 1, 3', 3)$	4,1	1	1	1285.4	8190.8
\mathcal{L}_{108}	9	C_6^v, S_1^{iv}	$(2 \oplus 1, 3', 3')$	6,3	-1	1	4.4337 ★	565.99
\mathcal{L}_{109}	11	C_6^v, S_2^{iv}	$(2 \oplus 1, 3', 3')$	3,0	2	0	5.7955×10^{-6} ★	3.011×10^{-7} ★
\mathcal{L}_{110}	11	C_6^v, S_3^{iv}	$(2 \oplus 1, 3', 3')$	4,1	1	1	1.4515×10^{-7} ★	6.2481×10^{-7} ★
\mathcal{L}_{111}	9	C_7^v, S_1^{iii}	$(2 \oplus 1, 3', 3)$	1,4	2	0	1900.2	857850.
\mathcal{L}_{112}	9	C_7^v, S_2^{iii}	$(2 \oplus 1, 3', 3)$	2,5	1	1	1666.3	29604.
\mathcal{L}_{113}	9	C_7^v, S_1^{iv}	$(2 \oplus 1, 3', 3')$	4,7	-1	1	3.2411 ★	909.64
\mathcal{L}_{114}	11	C_7^v, S_2^{iv}	$(2 \oplus 1, 3', 3')$	1,4	2	0	8.1926×10^{-6} ★	6.2147×10^{-6} ★
\mathcal{L}_{115}	11	C_7^v, S_3^{iv}	$(2 \oplus 1, 3', 3')$	2,5	1	1	2.5482×10^{-6} ★	0.000010614 ★
\mathcal{L}_{116}	9	C_8^v, S_1^{iii}	$(2 \oplus 1, 3', 3)$	3,2	2	0	1158.5	763300.
\mathcal{L}_{117}	9	C_8^v, S_2^{iii}	$(2 \oplus 1, 3', 3)$	4,3	1	1	1551.1	9481.8
\mathcal{L}_{118}	9	C_8^v, S_1^{iv}	$(2 \oplus 1, 3', 3')$	6,5	-1	1	4.4337 ★	423.84
\mathcal{L}_{119}	11	C_8^v, S_2^{iv}	$(2 \oplus 1, 3', 3')$	3,2	2	0	2.9937×10^{-6} ★	7.6585×10^{-6} ★
\mathcal{L}_{120}	11	C_8^v, S_3^{iv}	$(2 \oplus 1, 3', 3')$	4,3	1	1	1.1411×10^{-6} ★	3.5164×10^{-6} ★
\mathcal{L}_{121}	5	C_1^{vi}, S_1^{iii}	$(2' \oplus 1, 3', 3)$	1,0	2	0	17089.	1.2165×10^6
\mathcal{L}_{122}	5	C_1^{vi}, S_2^{iii}	$(2' \oplus 1, 3', 3)$	2,1	1	1	16588.	314100.
\mathcal{L}_{123}	5	C_1^{vi}, S_1^{iv}	$(2' \oplus 1, 3', 3')$	4,3	-1	1	1.2445×10^6	593190.
\mathcal{L}_{124}	7	C_1^{vi}, S_2^{iv}	$(2' \oplus 1, 3', 3')$	1,0	2	0	14194.	13939.
\mathcal{L}_{125}	7	C_1^{vi}, S_3^{iv}	$(2' \oplus 1, 3', 3')$	2,1	1	1	13986.	13890.
\mathcal{L}_{126}	5	C_2^{vi}, S_1^{iii}	$(2' \oplus 1, 3', 3)$	1,2	2	0	16672.	1.2159×10^6
\mathcal{L}_{127}	5	C_2^{vi}, S_2^{iii}	$(2' \oplus 1, 3', 3)$	2,3	1	1	17214.	316900.
\mathcal{L}_{128}	5	C_2^{vi}, S_1^{iv}	$(2' \oplus 1, 3', 3')$	4,5	-1	1	1.2447×10^6	593550.
\mathcal{L}_{129}	7	C_2^{vi}, S_2^{iv}	$(2' \oplus 1, 3', 3')$	1,2	2	0	14099.	14274.
\mathcal{L}_{130}	7	C_2^{vi}, S_3^{iv}	$(2' \oplus 1, 3', 3')$	2,3	1	1	14582.	14622.
\mathcal{L}_{131}	7	C_3^{vi}, S_1^{iii}	$(2' \oplus 1, 3', 3)$	3,0	2	0	1823.2	1.1307×10^6
\mathcal{L}_{132}	7	C_3^{vi}, S_2^{iii}	$(2' \oplus 1, 3', 3)$	4,1	1	1	1939.5	11163.
\mathcal{L}_{133}	7	C_3^{vi}, S_1^{iv}	$(2' \oplus 1, 3', 3')$	6,3	-1	1	2153.7	1961.1
\mathcal{L}_{134}	9	C_3^{vi}, S_2^{iv}	$(2' \oplus 1, 3', 3')$	3,0	2	0	0.013825 ★	2.7009×10^{-6} ★
\mathcal{L}_{135}	9	C_3^{vi}, S_3^{iv}	$(2' \oplus 1, 3', 3')$	4,1	1	1	82.433	0.48312 ★
\mathcal{L}_{136}	7	C_4^{vi}, S_1^{iii}	$(2' \oplus 1, 3', 3)$	3,2	2	0	15362.	916580.

\mathcal{L}_{137}	7	C_4^{vi}, S_2^{iii}	$(2' \oplus 1, 3', 3)$	4, 3	1	1	17000.	18184.
\mathcal{L}_{138}	7	C_4^{vi}, S_1^{iv}	$(2' \oplus 1, 3', 3')$	6, 5	-1	1	14418.	15581.
\mathcal{L}_{139}	9	C_4^{vi}, S_2^{iv}	$(2' \oplus 1, 3', 3')$	3, 2	2	0	13898.	13955.
\mathcal{L}_{140}	9	C_4^{vi}, S_3^{iv}	$(2' \oplus 1, 3', 3')$	4, 3	1	1	13970.	14366.
\mathcal{L}_{141}	7	C_5^{vi}, S_1^{iii}	$(2' \oplus 1, 3', 3)$	1, 4	2	0	3612.2	1.2027×10^6
\mathcal{L}_{142}	7	C_5^{vi}, S_2^{iii}	$(2' \oplus 1, 3', 3)$	2, 5	1	1	2723.	312050.
\mathcal{L}_{143}	7	C_5^{vi}, S_1^{iv}	$(2' \oplus 1, 3', 3')$	4, 7	-1	1	943440.	220370.
\mathcal{L}_{144}	9	C_5^{vi}, S_2^{iv}	$(2' \oplus 1, 3', 3')$	1, 4	2	0	292.34	7.3372 ★
\mathcal{L}_{145}	9	C_5^{vi}, S_3^{iv}	$(2' \oplus 1, 3', 3')$	2, 5	1	1	90.884	0.26011 ★
\mathcal{L}_{146}	9	C_6^{vi}, S_1^{iii}	$(2' \oplus 1, 3', 3)$	3, 4	2	0	1223.9	915870.
\mathcal{L}_{147}	9	C_6^{vi}, S_2^{iii}	$(2' \oplus 1, 3', 3)$	4, 5	1	1	1256.9	16814.
\mathcal{L}_{148}	9	C_6^{vi}, S_1^{iv}	$(2' \oplus 1, 3', 3')$	6, 7	-1	1	253.63	808.7
\mathcal{L}_{149}	11	C_6^{vi}, S_2^{iv}	$(2' \oplus 1, 3', 3')$	3, 4	2	0	9.25×10^{-7} ★	0.00012515 ★
\mathcal{L}_{150}	11	C_6^{vi}, S_3^{iv}	$(2' \oplus 1, 3', 3')$	4, 5	1	1	17.788 ☆	0.0066197 ★
\mathcal{L}_{151}	6	C_1^{vii}, S_1^i	$(3, 3, 3)$	3	-1	1	49099.	1.1913×10^6
\mathcal{L}_{152}	8	C_1^{vii}, S_2^i	$(3, 3, 3)$	0	2	0	47898.	46776.
\mathcal{L}_{153}	8	C_1^{vii}, S_3^i	$(3, 3, 3)$	1	1	1	46514.	113290.
\mathcal{L}_{154}	6	C_1^{vii}, S_1^{ii}	$(3, 3, 3')$	0	2	0	142290.	47617.
\mathcal{L}_{155}	6	C_1^{vii}, S_2^{ii}	$(3, 3, 3')$	1	1	1	48747.	160700.
\mathcal{L}_{156}	10	C_2^{vii}, S_1^i	$(3, 3, 3)$	5	-1	1	697.01	502.07
\mathcal{L}_{157}	12	C_2^{vii}, S_2^i	$(3, 3, 3)$	2	2	0	0.082778 ★	0.22999 ★
\mathcal{L}_{158}	12	C_2^{vii}, S_3^i	$(3, 3, 3)$	3	1	1	328.81	0.0038639 ★
\mathcal{L}_{159}	10	C_2^{vii}, S_1^{ii}	$(3, 3, 3')$	2	2	0	1362.3	926.28
\mathcal{L}_{160}	10	C_2^{vii}, S_2^{ii}	$(3, 3, 3')$	3	1	1	1791.1	73520.
\mathcal{L}_{161}	4	C_1^{viii}, S_1^i	$(3', 3, 3)$	3	-1	1	1.2001×10^6	601670.
\mathcal{L}_{162}	6	C_1^{viii}, S_2^i	$(3', 3, 3)$	0	2	0	15013.	98289.
\mathcal{L}_{163}	6	C_1^{viii}, S_3^i	$(3', 3, 3)$	1	1	1	15002.	392860.
\mathcal{L}_{164}	4	C_1^{viii}, S_1^{ii}	$(3', 3, 3')$	0	2	0	17474.	1.2165×10^6
\mathcal{L}_{165}	4	C_1^{viii}, S_2^{ii}	$(3', 3, 3')$	1	1	1	20453.	1.2193×10^6
\mathcal{L}_{166}	8	C_2^{viii}, S_1^i	$(3', 3, 3)$	5	-1	1	491.1	9783.5
\mathcal{L}_{167}	10	C_2^{viii}, S_2^i	$(3', 3, 3)$	2	2	0	785.1	545.85
\mathcal{L}_{168}	10	C_2^{viii}, S_3^i	$(3', 3, 3)$	3	1	1	814.27	98.177
\mathcal{L}_{169}	8	C_2^{viii}, S_1^{ii}	$(3', 3, 3')$	2	2	0	2052.5	1.2019×10^6
\mathcal{L}_{170}	8	C_2^{viii}, S_2^{ii}	$(3', 3, 3')$	3	1	1	5262.4	1.1792×10^6
\mathcal{L}_{171}	10	C_3^{viii}, S_1^i	$(3', 3, 3)$	7	-1	1	114.8	10.001 ★
\mathcal{L}_{172}	12	C_3^{viii}, S_2^i	$(3', 3, 3)$	4	2	0	1.6244×10^{-7} ★	6.7774×10^{-7} ★
\mathcal{L}_{173}	12	C_3^{viii}, S_3^i	$(3', 3, 3)$	5	1	1	3.236 ★	9.7519×10^{-6} ★
\mathcal{L}_{174}	10	C_3^{viii}, S_1^{ii}	$(3', 3, 3')$	4	2	0	1728.6	99249.
\mathcal{L}_{175}	10	C_3^{viii}, S_2^{ii}	$(3', 3, 3')$	5	1	1	2094.8	123890.
\mathcal{L}_{176}	4	C_1^{ix}, S_1^{iii}	$(3, 3', 3)$	0	2	0	17727.	1.2171×10^6
\mathcal{L}_{177}	4	C_1^{ix}, S_2^{iii}	$(3, 3', 3)$	1	1	1	17728.	1.2171×10^6
\mathcal{L}_{178}	4	C_1^{ix}, S_1^{iv}	$(3, 3', 3')$	3	-1	1	1.4398×10^6	681190.
\mathcal{L}_{179}	6	C_1^{ix}, S_2^{iv}	$(3, 3', 3')$	0	2	0	16091.	14847.
\mathcal{L}_{180}	6	C_1^{ix}, S_3^{iv}	$(3, 3', 3')$	1	1	1	16109.	14829.
\mathcal{L}_{181}	8	C_2^{ix}, S_1^{iii}	$(3, 3', 3)$	2	2	0	3048.3	1.2024×10^6
\mathcal{L}_{182}	8	C_2^{ix}, S_2^{iii}	$(3, 3', 3)$	3	1	1	3199.8	1.1378×10^6
\mathcal{L}_{183}	8	C_2^{ix}, S_1^{iv}	$(3, 3', 3')$	5	-1	1	2325.7	25253.
\mathcal{L}_{184}	10	C_2^{ix}, S_2^{iv}	$(3, 3', 3')$	2	2	0	27.772 ☆	4.6365 ★
\mathcal{L}_{185}	10	C_2^{ix}, S_3^{iv}	$(3, 3', 3')$	3	1	1	591.84	1.3997 ★
\mathcal{L}_{186}	10	C_3^{ix}, S_1^{iii}	$(3, 3', 3)$	4	2	0	2293.1	1.0043×10^6

\mathcal{L}_{187}	10	C_3^{ix}, S_2^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{3})$	5	1	1	1853.4	10629.
\mathcal{L}_{188}	10	C_3^{ix}, S_1^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{3}')$	7	-1	1	371.57	20311.
\mathcal{L}_{189}	12	C_3^{ix}, S_2^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{3}')$	4	2	0	9.9122	★ 0.000083367
\mathcal{L}_{190}	12	C_3^{ix}, S_3^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{3}')$	5	1	1	156.67	187.17
\mathcal{L}_{191}	6	C_1^x, S_1^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{3})$	0	2	0	2.4405×10^6	3.5204×10^6
\mathcal{L}_{192}	6	C_1^x, S_2^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{3})$	1	1	1	2.4405×10^6	2.5876×10^6
\mathcal{L}_{193}	6	C_1^x, S_1^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{3}')$	3	-1	1	2.5471×10^6	2.4393×10^6
\mathcal{L}_{194}	8	C_1^x, S_2^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{3}')$	0	2	0	2.4379×10^6	2.4374×10^6
\mathcal{L}_{195}	8	C_1^x, S_3^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{3}')$	1	1	1	2.4396×10^6	2.4384×10^6
\mathcal{L}_{196}	10	C_2^x, S_1^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{3})$	2	2	0	3066.3	760350.
\mathcal{L}_{197}	10	C_2^x, S_2^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{3})$	3	1	1	1888.9	65468.
\mathcal{L}_{198}	10	C_2^x, S_1^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{3}')$	5	-1	1	1203.8	1767.8
\mathcal{L}_{199}	12	C_2^x, S_2^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{3}')$	2	2	0	0.0014246	★ 6.7467×10^{-7} ★
\mathcal{L}_{200}	12	C_2^x, S_3^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{3}')$	3	1	1	0.0016262	★ 64.922
\mathcal{L}_{201}	8	C_1^i, T_1^i	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	0, 2, 4	2	1	1140.7	1.2004×10^6
\mathcal{L}_{202}	8	C_1^i, T_2^i	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	1, 3, 5	1	2	3091.2	665500.
\mathcal{L}_{203}	10	C_1^i, T_3^i	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	-2, 0, 2	4	1	3.4216×10^{-7}	★ 7.3653×10^{-7} ★
\mathcal{L}_{204}	10	C_1^i, T_4^i	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	-1, 1, 3	3	2	340.6	★ 1.0964×10^{-7} ★
\mathcal{L}_{205}	10	C_1^i, T_5^i	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	2, 4, 6	0	3	1145.8	1.2004×10^6
\mathcal{L}_{206}	8	C_1^i, T_1^{ii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2}')$	2, 4, 6	0	1	566.08	18795.
\mathcal{L}_{207}	8	C_1^i, T_2^{ii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2}')$	3, 5, 7	-1	2	1159.4	506.21
\mathcal{L}_{208}	10	C_1^i, T_3^{ii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2}')$	0, 2, 4	2	1	10.205	★ 1.9213×10^{-6} ★
\mathcal{L}_{209}	10	C_1^i, T_4^{ii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2}')$	1, 3, 5	1	2	1.4197×10^{-8}	★ 1.8291×10^{-7} ★
\mathcal{L}_{210}	10	C_1^i, T_5^{ii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2}')$	4, 6, 8	-2	3	3.0531×10^{-7}	★ 2.5318×10^{-6} ★
\mathcal{L}_{211}	8	C_1^{ii}, T_1^{iii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{2})$	2, 4, 6	0	1	687.91	1112.1
\mathcal{L}_{212}	8	C_1^{ii}, T_2^{iii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{2})$	3, 5, 7	-1	2	736.98	536.37
\mathcal{L}_{213}	10	C_1^{ii}, T_3^{iii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{2})$	0, 2, 4	2	1	1.7068×10^{-7}	★ 8.8418×10^{-9} ★
\mathcal{L}_{214}	10	C_1^{ii}, T_4^{iii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{2})$	1, 3, 5	1	2	8.3469×10^{-8}	★ 6.2284×10^{-8} ★
\mathcal{L}_{215}	10	C_1^{ii}, T_5^{iii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{2})$	4, 6, 8	-2	3	195.93	★ 1.2967×10^{-7} ★
\mathcal{L}_{216}	8	C_1^{ii}, T_1^{iv}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{2}')$	0, 2, 4	2	1	1852.2	529.17
\mathcal{L}_{217}	8	C_1^{ii}, T_2^{iv}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{2}')$	1, 3, 5	1	2	1036.3	546.04
\mathcal{L}_{218}	10	C_1^{ii}, T_3^{iv}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{2}')$	-2, 0, 2	4	1	612.89	36.251
\mathcal{L}_{219}	10	C_1^{ii}, T_4^{iv}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{2}')$	-1, 1, 3	3	2	190.36	539.61
\mathcal{L}_{220}	10	C_1^{ii}, T_5^{iv}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', \mathbf{2}')$	2, 4, 6	0	3	254.45	★ 4.1772×10^{-6} ★
\mathcal{L}_{221}	5	C_1^{iii}, T_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	1, 0	2	1	16943.	1.2162×10^6
\mathcal{L}_{222}	5	C_1^{iii}, T_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	2, 1	1	2	17957.	713460.
\mathcal{L}_{223}	7	C_1^{iii}, T_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	-1, -2	4	1	14437.	14376.
\mathcal{L}_{224}	7	C_1^{iii}, T_4^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	0, -1	3	2	14327.	14436.
\mathcal{L}_{225}	7	C_1^{iii}, T_5^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	3, 2	0	3	16943.	1.2162×10^6
\mathcal{L}_{226}	5	C_1^{iii}, T_1^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2}')$	3, 2	0	1	18874.	287460.
\mathcal{L}_{227}	5	C_1^{iii}, T_2^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2}')$	4, 3	-1	2	1.188×10^6	593290.
\mathcal{L}_{228}	7	C_1^{iii}, T_3^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2}')$	1, 0	2	1	13929.	14346.
\mathcal{L}_{229}	7	C_1^{iii}, T_4^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2}')$	2, 1	1	2	14278.	14434.
\mathcal{L}_{230}	7	C_1^{iii}, T_5^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2}')$	5, 4	-2	3	17293.	15665.
\mathcal{L}_{231}	5	C_2^i, T_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	1, 2	2	1	16369.	1.2156×10^6
\mathcal{L}_{232}	5	C_2^i, T_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	2, 3	1	2	18999.	1.2184×10^6
\mathcal{L}_{233}	7	C_2^i, T_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	-1, 0	4	1	15771.	15833.
\mathcal{L}_{234}	7	C_2^i, T_4^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	0, 1	3	2	15771.	15833.
\mathcal{L}_{235}	7	C_2^i, T_5^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2})$	3, 4	0	3	16342.	1.2156×10^6
\mathcal{L}_{236}	5	C_2^{ii}, T_1^{ii}	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, \mathbf{2}')$	3, 4	0	1	1.3373×10^6	595370.

\mathcal{L}_{237}	5	C_2^{iii}, T_2^i	$(2 \oplus 1, 3, 2')$	4, 5	-1	2	18842.	538760.
\mathcal{L}_{238}	7	C_2^{iii}, T_3^i	$(2 \oplus 1, 3, 2')$	1, 2	2	1	15618.	16756.
\mathcal{L}_{239}	7	C_2^{iii}, T_4^i	$(2 \oplus 1, 3, 2')$	2, 3	1	2	375140.	14339.
\mathcal{L}_{240}	7	C_2^{iii}, T_5^i	$(2 \oplus 1, 3, 2')$	5, 6	-2	3	17755.	17278.
\mathcal{L}_{241}	7	C_3^{iii}, T_1^i	$(2 \oplus 1, 3, 2)$	3, 0	2	1	1938.2	1.2012×10^6
\mathcal{L}_{242}	7	C_3^{iii}, T_2^i	$(2 \oplus 1, 3, 2)$	4, 1	1	2	3093.4	666810.
\mathcal{L}_{243}	9	C_3^{iii}, T_3^i	$(2 \oplus 1, 3, 2)$	1, -2	4	1	229.19	457.03
\mathcal{L}_{244}	9	C_3^{iii}, T_4^i	$(2 \oplus 1, 3, 2)$	2, -1	3	2	409.77	55.131
\mathcal{L}_{245}	9	C_3^{iii}, T_5^i	$(2 \oplus 1, 3, 2)$	5, 2	0	3	2125.7	1.2013×10^6
\mathcal{L}_{246}	7	C_3^{iii}, T_1^i	$(2 \oplus 1, 3, 2')$	5, 2	0	1	1340.6	268940.
\mathcal{L}_{247}	7	C_3^{iii}, T_2^i	$(2 \oplus 1, 3, 2')$	6, 3	-1	2	1867.7	1462.9
\mathcal{L}_{248}	9	C_3^{iii}, T_3^i	$(2 \oplus 1, 3, 2')$	3, 0	2	1	39.286	☆ 514.84
\mathcal{L}_{249}	9	C_3^{iii}, T_4^i	$(2 \oplus 1, 3, 2')$	4, 1	1	2	301.67	380.77
\mathcal{L}_{250}	9	C_3^{iii}, T_5^i	$(2 \oplus 1, 3, 2')$	7, 4	-2	3	479.	437.49
\mathcal{L}_{251}	7	C_4^{iii}, T_1^i	$(2 \oplus 1, 3, 2)$	3, 2	2	1	16324.	1.2156×10^6
\mathcal{L}_{252}	7	C_4^{iii}, T_2^i	$(2 \oplus 1, 3, 2)$	4, 3	1	2	16981.	1.2163×10^6
\mathcal{L}_{253}	9	C_4^{iii}, T_3^i	$(2 \oplus 1, 3, 2)$	1, 0	4	1	15771.	15833.
\mathcal{L}_{254}	9	C_4^{iii}, T_4^i	$(2 \oplus 1, 3, 2)$	2, 1	3	2	15771.	15833.
\mathcal{L}_{255}	9	C_4^{iii}, T_5^i	$(2 \oplus 1, 3, 2)$	5, 4	0	3	16359.	1.2156×10^6
\mathcal{L}_{256}	7	C_4^{iii}, T_1^i	$(2 \oplus 1, 3, 2')$	5, 4	0	1	1.3372×10^6	595370.
\mathcal{L}_{257}	7	C_4^{iii}, T_2^i	$(2 \oplus 1, 3, 2')$	6, 5	-1	2	17247.	538920.
\mathcal{L}_{258}	9	C_4^{iii}, T_3^i	$(2 \oplus 1, 3, 2')$	3, 2	2	1	15618.	16755.
\mathcal{L}_{259}	9	C_4^{iii}, T_4^i	$(2 \oplus 1, 3, 2')$	4, 3	1	2	375080.	14275.
\mathcal{L}_{260}	9	C_4^{iii}, T_5^i	$(2 \oplus 1, 3, 2')$	7, 6	-2	3	17752.	17431.
\mathcal{L}_{261}	7	C_5^{iii}, T_1^i	$(2 \oplus 1, 3, 2)$	1, 4	2	1	16271.	1.2155×10^6
\mathcal{L}_{262}	7	C_5^{iii}, T_2^i	$(2 \oplus 1, 3, 2)$	2, 5	1	2	17005.	679450.
\mathcal{L}_{263}	9	C_5^{iii}, T_3^i	$(2 \oplus 1, 3, 2)$	-1, 2	4	1	14499.	15978.
\mathcal{L}_{264}	9	C_5^{iii}, T_4^i	$(2 \oplus 1, 3, 2)$	0, 3	3	2	14676.	16319.
\mathcal{L}_{265}	9	C_5^{iii}, T_5^i	$(2 \oplus 1, 3, 2)$	3, 6	0	3	16282.	1.2155×10^6
\mathcal{L}_{266}	7	C_5^{iii}, T_1^i	$(2 \oplus 1, 3, 2')$	3, 6	0	1	17730.	14832.
\mathcal{L}_{267}	7	C_5^{iii}, T_2^i	$(2 \oplus 1, 3, 2')$	4, 7	-1	2	17359.	16266.
\mathcal{L}_{268}	9	C_5^{iii}, T_3^i	$(2 \oplus 1, 3, 2')$	1, 4	2	1	14434.	14462.
\mathcal{L}_{269}	9	C_5^{iii}, T_4^i	$(2 \oplus 1, 3, 2')$	2, 5	1	2	14153.	14393.
\mathcal{L}_{270}	9	C_5^{iii}, T_5^i	$(2 \oplus 1, 3, 2')$	5, 8	-2	3	16416.	15450.
\mathcal{L}_{271}	9	C_6^{iii}, T_1^i	$(2 \oplus 1, 3, 2)$	3, 4	2	1	1374.3	1.2005×10^6
\mathcal{L}_{272}	9	C_6^{iii}, T_2^i	$(2 \oplus 1, 3, 2)$	4, 5	1	2	3114.4	665120.
\mathcal{L}_{273}	11	C_6^{iii}, T_3^i	$(2 \oplus 1, 3, 2)$	1, 2	4	1	205.15	1.1012×10^{-7} ★
\mathcal{L}_{274}	11	C_6^{iii}, T_4^i	$(2 \oplus 1, 3, 2)$	2, 3	3	2	174.83	36.217 ☆
\mathcal{L}_{275}	11	C_6^{iii}, T_5^i	$(2 \oplus 1, 3, 2)$	5, 6	0	3	1377.	1.2008×10^6
\mathcal{L}_{276}	9	C_6^{iii}, T_1^i	$(2 \oplus 1, 3, 2')$	5, 6	0	1	639.65	194.22
\mathcal{L}_{277}	9	C_6^{iii}, T_2^i	$(2 \oplus 1, 3, 2')$	6, 7	-1	2	195.29	73.069
\mathcal{L}_{278}	11	C_6^{iii}, T_3^i	$(2 \oplus 1, 3, 2')$	3, 4	2	1	1.0984×10^{-6} ★	3.8508×10^{-7} ★
\mathcal{L}_{279}	11	C_6^{iii}, T_4^i	$(2 \oplus 1, 3, 2')$	4, 5	1	2	3.5421×10^{-7} ★	5.4817×10^{-6} ★
\mathcal{L}_{280}	11	C_6^{iii}, T_5^i	$(2 \oplus 1, 3, 2')$	7, 8	-2	3	81.444	0.97818 ★
\mathcal{L}_{281}	5	C_1^{iv}, T_1^i	$(2' \oplus 1, 3, 2)$	-1, 0	2	1	355550.	1.5543×10^6
\mathcal{L}_{282}	5	C_1^{iv}, T_2^i	$(2' \oplus 1, 3, 2)$	0, 1	1	2	65130.	667250.
\mathcal{L}_{283}	7	C_1^{iv}, T_3^i	$(2' \oplus 1, 3, 2)$	-3, -2	4	1	5316.8	3575.
\mathcal{L}_{284}	7	C_1^{iv}, T_4^i	$(2' \oplus 1, 3, 2)$	-2, -1	3	2	354480.	433120.
\mathcal{L}_{285}	7	C_1^{iv}, T_5^i	$(2' \oplus 1, 3, 2)$	1, 2	0	3	355550.	1.5543×10^6
\mathcal{L}_{286}	5	C_1^{iv}, T_1^i	$(2' \oplus 1, 3, 2')$	1, 2	0	1	3839.5	352200.

\mathcal{L}_{287}	5	C_1^{iv}, T_2^{ii}	$(2' \oplus 1, 3, 2')$	2, 3	-1	2	3732.8	352360.
\mathcal{L}_{288}	7	C_1^{iv}, T_3^{ii}	$(2' \oplus 1, 3, 2')$	-1, 0	2	1	3196.8	3762.1
\mathcal{L}_{289}	7	C_1^{iv}, T_4^{ii}	$(2' \oplus 1, 3, 2')$	0, 1	1	2	3200.	349980.
\mathcal{L}_{290}	7	C_1^{iv}, T_5^{ii}	$(2' \oplus 1, 3, 2')$	3, 4	-2	3	2327.2	1961.2
\mathcal{L}_{291}	5	C_2^{iv}, T_1^i	$(2' \oplus 1, 3, 2)$	-1, 2	2	1	19897.	1.219×10^6
\mathcal{L}_{292}	5	C_2^{iv}, T_2^i	$(2' \oplus 1, 3, 2)$	0, 3	1	2	65644.	667730.
\mathcal{L}_{293}	7	C_2^{iv}, T_3^i	$(2' \oplus 1, 3, 2)$	-3, 0	4	1	5210.5	3600.4
\mathcal{L}_{294}	7	C_2^{iv}, T_4^i	$(2' \oplus 1, 3, 2)$	-2, 1	3	2	18360.	19464.
\mathcal{L}_{295}	7	C_2^{iv}, T_5^i	$(2' \oplus 1, 3, 2)$	1, 4	0	3	18440.	1.2185×10^6
\mathcal{L}_{296}	5	C_2^{iv}, T_1^{ii}	$(2' \oplus 1, 3, 2')$	1, 4	0	1	6640.5	352200.
\mathcal{L}_{297}	5	C_2^{iv}, T_2^{ii}	$(2' \oplus 1, 3, 2')$	2, 5	-1	2	190910.	1971.9
\mathcal{L}_{298}	7	C_2^{iv}, T_3^{ii}	$(2' \oplus 1, 3, 2')$	-1, 2	2	1	3869.	137340.
\mathcal{L}_{299}	7	C_2^{iv}, T_4^{ii}	$(2' \oplus 1, 3, 2')$	0, 3	1	2	31555.	2296.7
\mathcal{L}_{300}	7	C_2^{iv}, T_5^{ii}	$(2' \oplus 1, 3, 2')$	3, 6	-2	3	2594.3	1941.8
\mathcal{L}_{301}	7	C_3^{iv}, T_1^i	$(2' \oplus 1, 3, 2)$	1, 0	2	1	2081.4	1.2014×10^6
\mathcal{L}_{302}	7	C_3^{iv}, T_2^i	$(2' \oplus 1, 3, 2)$	2, 1	1	2	18503.	665980.
\mathcal{L}_{303}	9	C_3^{iv}, T_3^i	$(2' \oplus 1, 3, 2)$	-1, -2	4	1	547.46	299.61
\mathcal{L}_{304}	9	C_3^{iv}, T_4^i	$(2' \oplus 1, 3, 2)$	0, -1	3	2	437.77	309.13
\mathcal{L}_{305}	9	C_3^{iv}, T_5^i	$(2' \oplus 1, 3, 2)$	3, 2	0	3	2080.7	1.2014×10^6
\mathcal{L}_{306}	7	C_3^{iv}, T_1^{ii}	$(2' \oplus 1, 3, 2')$	3, 2	0	1	859.71	851.81
\mathcal{L}_{307}	7	C_3^{iv}, T_2^{ii}	$(2' \oplus 1, 3, 2')$	4, 3	-1	2	2434.5	554.97
\mathcal{L}_{308}	9	C_3^{iv}, T_3^{ii}	$(2' \oplus 1, 3, 2')$	1, 0	2	1	37.258	☆ 384.58
\mathcal{L}_{309}	9	C_3^{iv}, T_4^{ii}	$(2' \oplus 1, 3, 2')$	2, 1	1	2	346.85	537.37
\mathcal{L}_{310}	9	C_3^{iv}, T_5^{ii}	$(2' \oplus 1, 3, 2')$	5, 4	-2	3	4.2439×10^{-6}	★ 468.26
\mathcal{L}_{311}	7	C_4^{iv}, T_1^i	$(2' \oplus 1, 3, 2)$	1, 2	2	1	1962.5	1.2012×10^6
\mathcal{L}_{312}	7	C_4^{iv}, T_2^i	$(2' \oplus 1, 3, 2)$	2, 3	1	2	4631.4	665980.
\mathcal{L}_{313}	9	C_4^{iv}, T_3^i	$(2' \oplus 1, 3, 2)$	-1, 0	4	1	1080.9	299.64
\mathcal{L}_{314}	9	C_4^{iv}, T_4^i	$(2' \oplus 1, 3, 2)$	0, 1	3	2	481.76	309.17
\mathcal{L}_{315}	9	C_4^{iv}, T_5^i	$(2' \oplus 1, 3, 2)$	3, 4	0	3	1999.6	1.2012×10^6
\mathcal{L}_{316}	7	C_4^{iv}, T_1^{ii}	$(2' \oplus 1, 3, 2')$	3, 4	0	1	751.94	851.79
\mathcal{L}_{317}	7	C_4^{iv}, T_2^{ii}	$(2' \oplus 1, 3, 2')$	4, 5	-1	2	2434.5	526.76
\mathcal{L}_{318}	9	C_4^{iv}, T_3^{ii}	$(2' \oplus 1, 3, 2')$	1, 2	2	1	729.86	422.69
\mathcal{L}_{319}	9	C_4^{iv}, T_4^{ii}	$(2' \oplus 1, 3, 2')$	2, 3	1	2	347.07	336.91
\mathcal{L}_{320}	9	C_4^{iv}, T_5^{ii}	$(2' \oplus 1, 3, 2')$	5, 6	-2	3	16.094	☆ 539.89
\mathcal{L}_{321}	7	C_5^{iv}, T_1^i	$(2' \oplus 1, 3, 2)$	-1, 4	2	1	5454.9	1.204×10^6
\mathcal{L}_{322}	7	C_5^{iv}, T_2^i	$(2' \oplus 1, 3, 2)$	0, 5	1	2	64731.	666820.
\mathcal{L}_{323}	9	C_5^{iv}, T_3^i	$(2' \oplus 1, 3, 2)$	-3, 2	4	1	3817.2	2821.8
\mathcal{L}_{324}	9	C_5^{iv}, T_4^i	$(2' \oplus 1, 3, 2)$	-2, 3	3	2	4006.2	17832.
\mathcal{L}_{325}	9	C_5^{iv}, T_5^i	$(2' \oplus 1, 3, 2)$	1, 6	0	3	5690.7	1.2041×10^6
\mathcal{L}_{326}	7	C_5^{iv}, T_1^{ii}	$(2' \oplus 1, 3, 2')$	1, 6	0	1	1882.1	5601.1
\mathcal{L}_{327}	7	C_5^{iv}, T_2^{ii}	$(2' \oplus 1, 3, 2')$	2, 7	-1	2	2292.5	2098.8
\mathcal{L}_{328}	9	C_5^{iv}, T_3^{ii}	$(2' \oplus 1, 3, 2')$	-1, 4	2	1	561.2	1084.3
\mathcal{L}_{329}	9	C_5^{iv}, T_4^{ii}	$(2' \oplus 1, 3, 2')$	0, 5	1	2	273.4	606.72
\mathcal{L}_{330}	9	C_5^{iv}, T_5^{ii}	$(2' \oplus 1, 3, 2')$	3, 8	-2	3	1881.2	1942.1
\mathcal{L}_{331}	9	C_6^{iv}, T_1^i	$(2' \oplus 1, 3, 2)$	3, 0	2	1	1827.8	1.2011×10^6
\mathcal{L}_{332}	9	C_6^{iv}, T_2^i	$(2' \oplus 1, 3, 2)$	4, 1	1	2	3133.3	664950.
\mathcal{L}_{333}	11	C_6^{iv}, T_3^i	$(2' \oplus 1, 3, 2)$	1, -2	4	1	527.05	9.1714×10^{-6} ★
\mathcal{L}_{334}	11	C_6^{iv}, T_4^i	$(2' \oplus 1, 3, 2)$	2, -1	3	2	212.75	4.6061×10^{-6} ★
\mathcal{L}_{335}	11	C_6^{iv}, T_5^i	$(2' \oplus 1, 3, 2)$	5, 2	0	3	1821.5	1.2011×10^6
\mathcal{L}_{336}	9	C_6^{iv}, T_1^{ii}	$(2' \oplus 1, 3, 2')$	5, 2	0	1	560.47	551.63

\mathcal{L}_{337}	9	C_6^{iv}, T_2^{ii}	$(2' \oplus 1, 3, 2')$	6,3	-1	2	634.92	0.33874	★
\mathcal{L}_{338}	11	C_6^{iv}, T_3^{ii}	$(2' \oplus 1, 3, 2')$	3,0	2	1	2.3314×10^{-6}	8.6728×10^{-6}	★
\mathcal{L}_{339}	11	C_6^{iv}, T_4^{ii}	$(2' \oplus 1, 3, 2')$	4,1	1	2	6.9389	8.3984×10^{-6}	★
\mathcal{L}_{340}	11	C_6^{iv}, T_5^{ii}	$(2' \oplus 1, 3, 2')$	7,4	-2	3	0.000018297	5.0812×10^{-8}	★
\mathcal{L}_{341}	9	C_7^{iv}, T_1^i	$(2' \oplus 1, 3, 2)$	1,4	2	1	2790.1	1.2012×10^6	
\mathcal{L}_{342}	9	C_7^{iv}, T_2^i	$(2' \oplus 1, 3, 2)$	2,5	1	2	62812.	665160.	
\mathcal{L}_{343}	11	C_7^{iv}, T_3^i	$(2' \oplus 1, 3, 2)$	-1,2	4	1	35.602	8.5021×10^{-6}	★
\mathcal{L}_{344}	11	C_7^{iv}, T_4^i	$(2' \oplus 1, 3, 2)$	0,3	3	2	7.3244×10^{-7}	0.0070242	★
\mathcal{L}_{345}	11	C_7^{iv}, T_5^i	$(2' \oplus 1, 3, 2)$	3,6	0	3	3041.1	1.2016×10^6	
\mathcal{L}_{346}	9	C_7^{iv}, T_1^{ii}	$(2' \oplus 1, 3, 2')$	3,6	0	1	503.88	238.41	
\mathcal{L}_{347}	9	C_7^{iv}, T_2^{ii}	$(2' \oplus 1, 3, 2')$	4,7	-1	2	643.15	92.843	
\mathcal{L}_{348}	11	C_7^{iv}, T_3^{ii}	$(2' \oplus 1, 3, 2')$	1,4	2	1	1.4136×10^{-7}	2.522×10^{-6}	★
\mathcal{L}_{349}	11	C_7^{iv}, T_4^{ii}	$(2' \oplus 1, 3, 2')$	2,5	1	2	0.90088	191.99	
\mathcal{L}_{350}	11	C_7^{iv}, T_5^{ii}	$(2' \oplus 1, 3, 2')$	5,8	-2	3	2.1671×10^{-7}	22.786	★
\mathcal{L}_{351}	9	C_8^{iv}, T_1^i	$(2' \oplus 1, 3, 2)$	3,2	2	1	1423.	1.2004×10^6	
\mathcal{L}_{352}	9	C_8^{iv}, T_2^i	$(2' \oplus 1, 3, 2)$	4,3	1	2	16971.	664950.	
\mathcal{L}_{353}	11	C_8^{iv}, T_3^i	$(2' \oplus 1, 3, 2)$	1,0	4	1	565.78	7.1908×10^{-6}	★
\mathcal{L}_{354}	11	C_8^{iv}, T_4^i	$(2' \oplus 1, 3, 2)$	2,1	3	2	212.76	1.5524×10^{-6}	★
\mathcal{L}_{355}	11	C_8^{iv}, T_5^i	$(2' \oplus 1, 3, 2)$	5,4	0	3	1368.8	1.2004×10^6	
\mathcal{L}_{356}	9	C_8^{iv}, T_1^{ii}	$(2' \oplus 1, 3, 2')$	5,4	0	1	560.33	546.3	
\mathcal{L}_{357}	9	C_8^{iv}, T_2^{ii}	$(2' \oplus 1, 3, 2')$	6,5	-1	2	646.21	0.33932	★
\mathcal{L}_{358}	11	C_8^{iv}, T_3^{ii}	$(2' \oplus 1, 3, 2')$	3,2	2	1	7.343×10^{-7}	2.9507×10^{-6}	★
\mathcal{L}_{359}	11	C_8^{iv}, T_4^{ii}	$(2' \oplus 1, 3, 2')$	4,3	1	2	6.9387	5.7437×10^{-6}	★
\mathcal{L}_{360}	11	C_8^{iv}, T_5^{ii}	$(2' \oplus 1, 3, 2')$	7,6	-2	3	178.45	7.6032×10^{-6}	★
\mathcal{L}_{361}	5	C_1^v, T_1^{iii}	$(2 \oplus 1, 3', 2)$	1,2	0	1	3807.8	477220.	
\mathcal{L}_{362}	5	C_1^v, T_2^{iii}	$(2 \oplus 1, 3', 2)$	2,3	-1	2	3728.2	88280.	
\mathcal{L}_{363}	7	C_1^v, T_3^{iii}	$(2 \oplus 1, 3', 2)$	-1,0	2	1	3150.2	3660.1	
\mathcal{L}_{364}	7	C_1^v, T_4^{iii}	$(2 \oplus 1, 3', 2)$	0,1	1	2	3189.3	429580.	
\mathcal{L}_{365}	7	C_1^v, T_5^{iii}	$(2 \oplus 1, 3', 2)$	3,4	-2	3	2296.9	1951.	
\mathcal{L}_{366}	5	C_1^v, T_1^{iv}	$(2 \oplus 1, 3', 2')$	-1,0	2	1	1.9627×10^6	904630.	
\mathcal{L}_{367}	5	C_1^v, T_2^{iv}	$(2 \oplus 1, 3', 2')$	0,1	1	2	6558.7	801450.	
\mathcal{L}_{368}	7	C_1^v, T_3^{iv}	$(2 \oplus 1, 3', 2')$	-3,-2	4	1	3339.8	3543.1	
\mathcal{L}_{369}	7	C_1^v, T_4^{iv}	$(2 \oplus 1, 3', 2')$	-2,-1	3	2	712200.	464840.	
\mathcal{L}_{370}	7	C_1^v, T_5^{iv}	$(2 \oplus 1, 3', 2')$	1,2	0	3	467680.	461540.	
\mathcal{L}_{371}	5	C_2^v, T_1^{iii}	$(2 \oplus 1, 3', 2)$	1,4	0	1	405420.	6086.8	
\mathcal{L}_{372}	5	C_2^v, T_2^{iii}	$(2 \oplus 1, 3', 2)$	2,5	-1	2	19938.	1948.8	
\mathcal{L}_{373}	7	C_2^v, T_3^{iii}	$(2 \oplus 1, 3', 2)$	-1,2	2	1	41516.	798.73	
\mathcal{L}_{374}	7	C_2^v, T_4^{iii}	$(2 \oplus 1, 3', 2)$	0,3	1	2	128350.	547.52	
\mathcal{L}_{375}	7	C_2^v, T_5^{iii}	$(2 \oplus 1, 3', 2)$	3,6	-2	3	2593.9	1955.3	
\mathcal{L}_{376}	5	C_2^v, T_1^{iv}	$(2 \oplus 1, 3', 2')$	-1,2	2	1	241290.	164240.	
\mathcal{L}_{377}	5	C_2^v, T_2^{iv}	$(2 \oplus 1, 3', 2')$	0,3	1	2	407520.	247900.	
\mathcal{L}_{378}	7	C_2^v, T_3^{iv}	$(2 \oplus 1, 3', 2')$	-3,0	4	1	2450.	3547.1	
\mathcal{L}_{379}	7	C_2^v, T_4^{iv}	$(2 \oplus 1, 3', 2')$	-2,1	3	2	18440.	618.93	
\mathcal{L}_{380}	7	C_2^v, T_5^{iv}	$(2 \oplus 1, 3', 2')$	1,4	0	3	4677.5	3129.1	
\mathcal{L}_{381}	7	C_3^v, T_1^{iii}	$(2 \oplus 1, 3', 2)$	3,2	0	1	2130.3	407.97	
\mathcal{L}_{382}	7	C_3^v, T_2^{iii}	$(2 \oplus 1, 3', 2)$	4,3	-1	2	2487.8	4646.	
\mathcal{L}_{383}	9	C_3^v, T_3^{iii}	$(2 \oplus 1, 3', 2)$	1,0	2	1	755.23	5.9632×10^{-7}	★
\mathcal{L}_{384}	9	C_3^v, T_4^{iii}	$(2 \oplus 1, 3', 2)$	2,1	1	2	168.63	540.2	
\mathcal{L}_{385}	9	C_3^v, T_5^{iii}	$(2 \oplus 1, 3', 2)$	5,4	-2	3	666.	10.461	☆
\mathcal{L}_{386}	7	C_3^v, T_1^{iv}	$(2 \oplus 1, 3', 2')$	1,0	2	1	1505.3	568.34	

\mathcal{L}_{387}	7	C_3^v, T_2^{iv}	$(2 \oplus 1, 3', 2')$	2, 1	1	2	1024.4	1961.8
\mathcal{L}_{388}	9	C_3^v, T_3^{iv}	$(2 \oplus 1, 3', 2')$	-1, -2	4	1	187.1	96.913
\mathcal{L}_{389}	9	C_3^v, T_4^{iv}	$(2 \oplus 1, 3', 2')$	0, -1	3	2	271.23	1.6819×10^{-7} ★
\mathcal{L}_{390}	9	C_3^v, T_5^{iv}	$(2 \oplus 1, 3', 2')$	3, 2	0	3	593.6	460.62
\mathcal{L}_{391}	7	C_4^v, T_1^{iii}	$(2 \oplus 1, 3', 2)$	3, 4	0	1	2130.3	407.98
\mathcal{L}_{392}	7	C_4^v, T_2^{iii}	$(2 \oplus 1, 3', 2)$	4, 5	-1	2	2487.8	1145.5
\mathcal{L}_{393}	9	C_4^v, T_3^{iii}	$(2 \oplus 1, 3', 2)$	1, 2	2	1	755.23	9.2856×10^{-7} ★
\mathcal{L}_{394}	9	C_4^v, T_4^{iii}	$(2 \oplus 1, 3', 2)$	2, 3	1	2	180.35	124.03
\mathcal{L}_{395}	9	C_4^v, T_5^{iii}	$(2 \oplus 1, 3', 2)$	5, 6	-2	3	666.	10.554 ☆
\mathcal{L}_{396}	7	C_4^v, T_1^{iv}	$(2 \oplus 1, 3', 2')$	1, 2	2	1	2467.5	568.34
\mathcal{L}_{397}	7	C_4^v, T_2^{iv}	$(2 \oplus 1, 3', 2')$	2, 3	1	2	1068.4	567.26
\mathcal{L}_{398}	9	C_4^v, T_3^{iv}	$(2 \oplus 1, 3', 2')$	-1, 0	4	1	187.1	238.24
\mathcal{L}_{399}	9	C_4^v, T_4^{iv}	$(2 \oplus 1, 3', 2')$	0, 1	3	2	683.6	1.6396×10^{-6} ★
\mathcal{L}_{400}	9	C_4^v, T_5^{iv}	$(2 \oplus 1, 3', 2')$	3, 4	0	3	593.6	465.88
\mathcal{L}_{401}	7	C_5^v, T_1^{iii}	$(2 \oplus 1, 3', 2)$	1, 6	0	1	3004.	4389.5
\mathcal{L}_{402}	7	C_5^v, T_2^{iii}	$(2 \oplus 1, 3', 2)$	2, 7	-1	2	2623.8	52862.
\mathcal{L}_{403}	9	C_5^v, T_3^{iii}	$(2 \oplus 1, 3', 2)$	-1, 4	2	1	2353.1	2154.8
\mathcal{L}_{404}	9	C_5^v, T_4^{iii}	$(2 \oplus 1, 3', 2)$	0, 5	1	2	2990.7	2350.9
\mathcal{L}_{405}	9	C_5^v, T_5^{iii}	$(2 \oplus 1, 3', 2)$	3, 8	-2	3	1869.3	1934.1
\mathcal{L}_{406}	7	C_5^v, T_1^{iv}	$(2 \oplus 1, 3', 2')$	-1, 4	2	1	1.1072×10^6	39729.
\mathcal{L}_{407}	7	C_5^v, T_2^{iv}	$(2 \oplus 1, 3', 2')$	0, 5	1	2	5099.2	405720.
\mathcal{L}_{408}	9	C_5^v, T_3^{iv}	$(2 \oplus 1, 3', 2')$	-3, 2	4	1	4604.1	3333.3
\mathcal{L}_{409}	9	C_5^v, T_4^{iv}	$(2 \oplus 1, 3', 2')$	-2, 3	3	2	262570.	6698.3
\mathcal{L}_{410}	9	C_5^v, T_5^{iv}	$(2 \oplus 1, 3', 2')$	1, 6	0	3	24083.	13954.
\mathcal{L}_{411}	9	C_6^v, T_1^{iii}	$(2 \oplus 1, 3', 2)$	5, 2	0	1	797.18	6.2842×10^{-6} ★
\mathcal{L}_{412}	9	C_6^v, T_2^{iii}	$(2 \oplus 1, 3', 2)$	6, 3	-1	2	490.2	502.65
\mathcal{L}_{413}	11	C_6^v, T_3^{iii}	$(2 \oplus 1, 3', 2)$	3, 0	2	1	9.6773×10^{-8} ★	1.13×10^{-6} ★
\mathcal{L}_{414}	11	C_6^v, T_4^{iii}	$(2 \oplus 1, 3', 2)$	4, 1	1	2	6.2373×10^{-8} ★	6.7794×10^{-6} ★
\mathcal{L}_{415}	11	C_6^v, T_5^{iii}	$(2 \oplus 1, 3', 2)$	7, 4	-2	3	8.4667×10^{-7} ★	4.3284×10^{-7} ★
\mathcal{L}_{416}	9	C_6^v, T_1^{iv}	$(2 \oplus 1, 3', 2')$	3, 0	2	1	781.9	8.1908×10^{-6} ★
\mathcal{L}_{417}	9	C_6^v, T_2^{iv}	$(2 \oplus 1, 3', 2')$	4, 1	1	2	661.16	4.2934×10^{-7} ★
\mathcal{L}_{418}	11	C_6^v, T_3^{iv}	$(2 \oplus 1, 3', 2')$	1, -2	4	1	4.0709×10^{-7} ★	1.3395×10^{-6} ★
\mathcal{L}_{419}	11	C_6^v, T_4^{iv}	$(2 \oplus 1, 3', 2')$	2, -1	3	2	9.4405×10^{-8} ★	6.1299×10^{-7} ★
\mathcal{L}_{420}	11	C_6^v, T_5^{iv}	$(2 \oplus 1, 3', 2')$	5, 2	0	3	7.0651×10^{-6} ★	6.2426×10^{-6} ★
\mathcal{L}_{421}	9	C_7^v, T_1^{iii}	$(2 \oplus 1, 3', 2)$	3, 6	0	1	1756.9	5.2284×10^{-6} ★
\mathcal{L}_{422}	9	C_7^v, T_2^{iii}	$(2 \oplus 1, 3', 2)$	4, 7	-1	2	527.42	560.86
\mathcal{L}_{423}	11	C_7^v, T_3^{iii}	$(2 \oplus 1, 3', 2)$	1, 4	2	1	0.000012628 ★	7.2321×10^{-7} ★
\mathcal{L}_{424}	11	C_7^v, T_4^{iii}	$(2 \oplus 1, 3', 2)$	2, 5	1	2	9.3807×10^{-8} ★	6.6696×10^{-6} ★
\mathcal{L}_{425}	11	C_7^v, T_5^{iii}	$(2 \oplus 1, 3', 2)$	5, 8	-2	3	9.997×10^{-7} ★	3.7355×10^{-6} ★
\mathcal{L}_{426}	9	C_7^v, T_1^{iv}	$(2 \oplus 1, 3', 2')$	1, 4	2	1	698.06	544.5
\mathcal{L}_{427}	9	C_7^v, T_2^{iv}	$(2 \oplus 1, 3', 2')$	2, 5	1	2	282.74	545.49
\mathcal{L}_{428}	11	C_7^v, T_3^{iv}	$(2 \oplus 1, 3', 2')$	-1, 2	4	1	187.1	4.6762×10^{-6} ★
\mathcal{L}_{429}	11	C_7^v, T_4^{iv}	$(2 \oplus 1, 3', 2')$	0, 3	3	2	19.909 ☆	1.9177×10^{-6} ★
\mathcal{L}_{430}	11	C_7^v, T_5^{iv}	$(2 \oplus 1, 3', 2')$	3, 6	0	3	54.162	0.000025247 ★
\mathcal{L}_{431}	9	C_8^v, T_1^{iii}	$(2 \oplus 1, 3', 2)$	5, 4	0	1	1249.8	5.3274×10^{-7} ★
\mathcal{L}_{432}	9	C_8^v, T_2^{iii}	$(2 \oplus 1, 3', 2)$	6, 5	-1	2	490.23	503.52
\mathcal{L}_{433}	11	C_8^v, T_3^{iii}	$(2 \oplus 1, 3', 2)$	3, 2	2	1	1.9113×10^{-6} ★	7.0267×10^{-6} ★
\mathcal{L}_{434}	11	C_8^v, T_4^{iii}	$(2 \oplus 1, 3', 2)$	4, 3	1	2	5.8142×10^{-7} ★	1.5269×10^{-7} ★
\mathcal{L}_{435}	11	C_8^v, T_5^{iii}	$(2 \oplus 1, 3', 2)$	7, 6	-2	3	1.312×10^{-6} ★	6.3935×10^{-7} ★
\mathcal{L}_{436}	9	C_8^v, T_1^{iv}	$(2 \oplus 1, 3', 2')$	3, 2	2	1	781.95	8.3729×10^{-6} ★

\mathcal{L}_{437}	9	C_8^v, T_2^{iv}	$(2 \oplus 1, 3', 2')$	4, 3	1	2	680.47	7.0434×10^{-6} ★
\mathcal{L}_{438}	11	C_8^v, T_3^{iv}	$(2 \oplus 1, 3', 2')$	1, 0	4	1	8.486×10^{-8} ★	5.1971×10^{-6} ★
\mathcal{L}_{439}	11	C_8^v, T_4^{iv}	$(2 \oplus 1, 3', 2')$	2, 1	3	2	2.017×10^{-7} ★	1.1158×10^{-6} ★
\mathcal{L}_{440}	11	C_8^v, T_5^{iv}	$(2 \oplus 1, 3', 2')$	5, 4	0	3	1.5132×10^{-6} ★	3.98×10^{-6} ★
\mathcal{L}_{441}	5	C_1^{vi}, T_1^{iii}	$(2' \oplus 1, 3', 2)$	3, 2	0	1	19206.	40149.
\mathcal{L}_{442}	5	C_1^{vi}, T_2^{iii}	$(2' \oplus 1, 3', 2)$	4, 3	-1	2	1.0275×10^6	497970.
\mathcal{L}_{443}	7	C_1^{vi}, T_3^{iii}	$(2' \oplus 1, 3', 2)$	1, 0	2	1	14888.	13936.
\mathcal{L}_{444}	7	C_1^{vi}, T_4^{iii}	$(2' \oplus 1, 3', 2)$	2, 1	1	2	14093.	14434.
\mathcal{L}_{445}	7	C_1^{vi}, T_5^{iii}	$(2' \oplus 1, 3', 2)$	5, 4	-2	3	17135.	16428.
\mathcal{L}_{446}	5	C_1^{vi}, T_1^{iv}	$(2' \oplus 1, 3', 2')$	1, 0	2	1	16419.	15838.
\mathcal{L}_{447}	5	C_1^{vi}, T_2^{iv}	$(2' \oplus 1, 3', 2')$	2, 1	1	2	16222.	15851.
\mathcal{L}_{448}	7	C_1^{vi}, T_3^{iv}	$(2' \oplus 1, 3', 2')$	-1, -2	4	1	14804.	13988.
\mathcal{L}_{449}	7	C_1^{vi}, T_4^{iv}	$(2' \oplus 1, 3', 2')$	0, -1	3	2	14163.	14436.
\mathcal{L}_{450}	7	C_1^{vi}, T_5^{iv}	$(2' \oplus 1, 3', 2')$	3, 2	0	3	15771.	15833.
\mathcal{L}_{451}	5	C_2^{vi}, T_1^{iii}	$(2' \oplus 1, 3', 2)$	3, 4	0	1	17603.	40458.
\mathcal{L}_{452}	5	C_2^{vi}, T_2^{iii}	$(2' \oplus 1, 3', 2)$	4, 5	-1	2	1.0277×10^6	498180.
\mathcal{L}_{453}	7	C_2^{vi}, T_3^{iii}	$(2' \oplus 1, 3', 2)$	1, 2	2	1	15272.	13914.
\mathcal{L}_{454}	7	C_2^{vi}, T_4^{iii}	$(2' \oplus 1, 3', 2)$	2, 3	1	2	14131.	14435.
\mathcal{L}_{455}	7	C_2^{vi}, T_5^{iii}	$(2' \oplus 1, 3', 2)$	5, 6	-2	3	17108.	16318.
\mathcal{L}_{456}	5	C_2^{vi}, T_1^{iv}	$(2' \oplus 1, 3', 2')$	1, 2	2	1	16590.	15833.
\mathcal{L}_{457}	5	C_2^{vi}, T_2^{iv}	$(2' \oplus 1, 3', 2')$	2, 3	1	2	16366.	15835.
\mathcal{L}_{458}	7	C_2^{vi}, T_3^{iv}	$(2' \oplus 1, 3', 2')$	-1, 0	4	1	14798.	14455.
\mathcal{L}_{459}	7	C_2^{vi}, T_4^{iv}	$(2' \oplus 1, 3', 2')$	0, 1	3	2	14576.	14457.
\mathcal{L}_{460}	7	C_2^{vi}, T_5^{iv}	$(2' \oplus 1, 3', 2')$	3, 4	0	3	15771.	15833.
\mathcal{L}_{461}	7	C_3^{vi}, T_1^{iii}	$(2' \oplus 1, 3', 2)$	5, 2	0	1	1508.7	1750.4
\mathcal{L}_{462}	7	C_3^{vi}, T_2^{iii}	$(2' \oplus 1, 3', 2)$	6, 3	-1	2	1472.4	1728.6
\mathcal{L}_{463}	9	C_3^{vi}, T_3^{iii}	$(2' \oplus 1, 3', 2)$	3, 0	2	1	786.57	7.278×10^{-6} ★
\mathcal{L}_{464}	9	C_3^{vi}, T_4^{iii}	$(2' \oplus 1, 3', 2)$	4, 1	1	2	201.4	3.6306 ★
\mathcal{L}_{465}	9	C_3^{vi}, T_5^{iii}	$(2' \oplus 1, 3', 2)$	7, 4	-2	3	572.41	474.46
\mathcal{L}_{466}	7	C_3^{vi}, T_1^{iv}	$(2' \oplus 1, 3', 2')$	3, 0	2	1	2530.2	585.82
\mathcal{L}_{467}	7	C_3^{vi}, T_2^{iv}	$(2' \oplus 1, 3', 2')$	4, 1	1	2	2331.2	568.5
\mathcal{L}_{468}	9	C_3^{vi}, T_3^{iv}	$(2' \oplus 1, 3', 2')$	1, -2	4	1	896.9	92.888
\mathcal{L}_{469}	9	C_3^{vi}, T_4^{iv}	$(2' \oplus 1, 3', 2')$	2, -1	3	2	260.76	8.0307×10^{-7} ★
\mathcal{L}_{470}	9	C_3^{vi}, T_5^{iv}	$(2' \oplus 1, 3', 2')$	5, 2	0	3	625.3	566.28
\mathcal{L}_{471}	7	C_4^{vi}, T_1^{iii}	$(2' \oplus 1, 3', 2)$	5, 4	0	1	17916.	15505.
\mathcal{L}_{472}	7	C_4^{vi}, T_2^{iii}	$(2' \oplus 1, 3', 2)$	6, 5	-1	2	14643.	14441.
\mathcal{L}_{473}	9	C_4^{vi}, T_3^{iii}	$(2' \oplus 1, 3', 2)$	3, 2	2	1	13909.	13905.
\mathcal{L}_{474}	9	C_4^{vi}, T_4^{iii}	$(2' \oplus 1, 3', 2)$	4, 3	1	2	13925.	13890.
\mathcal{L}_{475}	9	C_4^{vi}, T_5^{iii}	$(2' \oplus 1, 3', 2)$	7, 6	-2	3	14469.	13899.
\mathcal{L}_{476}	7	C_4^{vi}, T_1^{iv}	$(2' \oplus 1, 3', 2')$	3, 2	2	1	15931.	14466.
\mathcal{L}_{477}	7	C_4^{vi}, T_2^{iv}	$(2' \oplus 1, 3', 2')$	4, 3	1	2	15129.	14455.
\mathcal{L}_{478}	9	C_4^{vi}, T_3^{iv}	$(2' \oplus 1, 3', 2')$	1, 0	4	1	14621.	14216.
\mathcal{L}_{479}	9	C_4^{vi}, T_4^{iv}	$(2' \oplus 1, 3', 2')$	2, 1	3	2	14167.	14444.
\mathcal{L}_{480}	9	C_4^{vi}, T_5^{iv}	$(2' \oplus 1, 3', 2')$	5, 4	0	3	14454.	14455.
\mathcal{L}_{481}	7	C_5^{vi}, T_1^{iii}	$(2' \oplus 1, 3', 2)$	3, 6	0	1	5308.6	25927.
\mathcal{L}_{482}	7	C_5^{vi}, T_2^{iii}	$(2' \oplus 1, 3', 2)$	4, 7	-1	2	802860.	175390.
\mathcal{L}_{483}	9	C_5^{vi}, T_3^{iii}	$(2' \oplus 1, 3', 2)$	1, 4	2	1	996.51	0.00005391 ★
\mathcal{L}_{484}	9	C_5^{vi}, T_4^{iii}	$(2' \oplus 1, 3', 2)$	2, 5	1	2	184.76	538.57
\mathcal{L}_{485}	9	C_5^{vi}, T_5^{iii}	$(2' \oplus 1, 3', 2)$	5, 8	-2	3	3228.3	2518.
\mathcal{L}_{486}	7	C_5^{vi}, T_1^{iv}	$(2' \oplus 1, 3', 2')$	1, 4	2	1	2530.3	1947.1

\mathcal{L}_{487}	7	C_5^{vi}, T_2^{iv}	$(2' \oplus 1, 3', 2')$	2,5	1	2	2329.5	1954.2
\mathcal{L}_{488}	9	C_5^{vi}, T_3^{iv}	$(2' \oplus 1, 3', 2')$	-1,2	4	1	879.57	91.884
\mathcal{L}_{489}	9	C_5^{vi}, T_4^{iv}	$(2' \oplus 1, 3', 2')$	0,3	3	2	264.89	541.25
\mathcal{L}_{490}	9	C_5^{vi}, T_5^{iv}	$(2' \oplus 1, 3', 2')$	3,6	0	3	1881.6	1943.8
\mathcal{L}_{491}	9	C_6^{vi}, T_1^{iii}	$(2' \oplus 1, 3', 2)$	5,6	0	1	686.97	1106.3
\mathcal{L}_{492}	9	C_6^{vi}, T_2^{iii}	$(2' \oplus 1, 3', 2)$	6,7	-1	2	776.06	5847.2
\mathcal{L}_{493}	11	C_6^{vi}, T_3^{iii}	$(2' \oplus 1, 3', 2)$	3,4	2	1	0.000017897	0.000025566
\mathcal{L}_{494}	11	C_6^{vi}, T_4^{iii}	$(2' \oplus 1, 3', 2)$	4,5	1	2	0.000015641	1.2548×10^{-6}
\mathcal{L}_{495}	11	C_6^{vi}, T_5^{iii}	$(2' \oplus 1, 3', 2)$	7,8	-2	3	197.93	6.6962×10^{-6}
\mathcal{L}_{496}	9	C_6^{vi}, T_1^{iv}	$(2' \oplus 1, 3', 2')$	3,4	2	1	1854.1	518.18
\mathcal{L}_{497}	9	C_6^{vi}, T_2^{iv}	$(2' \oplus 1, 3', 2')$	4,5	1	2	1037.5	520.58
\mathcal{L}_{498}	11	C_6^{vi}, T_3^{iv}	$(2' \oplus 1, 3', 2')$	1,2	4	1	605.95	36.29
\mathcal{L}_{499}	11	C_6^{vi}, T_4^{iv}	$(2' \oplus 1, 3', 2')$	2,3	3	2	181.58	3.3821×10^{-7}
\mathcal{L}_{500}	11	C_6^{vi}, T_5^{iv}	$(2' \oplus 1, 3', 2')$	5,6	0	3	500.9	0.00010974
\mathcal{L}_{501}	6	C_1^{vii}, T_1^i	$(3, 3, 2)$	0	2	1	48764.	1.248×10^6
\mathcal{L}_{502}	6	C_1^{vii}, T_2^i	$(3, 3, 2)$	1	1	2	48769.	749510.
\mathcal{L}_{503}	8	C_1^{vii}, T_3^i	$(3, 3, 2)$	-2	4	1	48230.	47817.
\mathcal{L}_{504}	8	C_1^{vii}, T_4^i	$(3, 3, 2)$	-1	3	2	47821.	47594.
\mathcal{L}_{505}	8	C_1^{vii}, T_5^i	$(3, 3, 2)$	2	0	3	48760.	1.248×10^6
\mathcal{L}_{506}	6	C_1^{vii}, T_1^{ii}	$(3, 3, 2')$	2	0	1	50369.	1.2446×10^6
\mathcal{L}_{507}	6	C_1^{vii}, T_2^{ii}	$(3, 3, 2')$	3	-1	2	49101.	253200.
\mathcal{L}_{508}	8	C_1^{vii}, T_3^{ii}	$(3, 3, 2')$	0	2	1	47451.	46731.
\mathcal{L}_{509}	8	C_1^{vii}, T_4^{ii}	$(3, 3, 2')$	1	1	2	47970.	47592.
\mathcal{L}_{510}	8	C_1^{vii}, T_5^{ii}	$(3, 3, 2')$	4	-2	3	50916.	180840.
\mathcal{L}_{511}	10	C_2^{vii}, T_1^i	$(3, 3, 2)$	2	2	1	1889.7	1.2011×10^6
\mathcal{L}_{512}	10	C_2^{vii}, T_2^i	$(3, 3, 2)$	3	1	2	3311.9	665370.
\mathcal{L}_{513}	12	C_2^{vii}, T_3^i	$(3, 3, 2)$	0	4	1	1302.2	368.07
\mathcal{L}_{514}	12	C_2^{vii}, T_4^i	$(3, 3, 2)$	1	3	2	679.58	339.15
\mathcal{L}_{515}	12	C_2^{vii}, T_5^i	$(3, 3, 2)$	4	0	3	1921.1	1.2023×10^6
\mathcal{L}_{516}	10	C_2^{vii}, T_1^{ii}	$(3, 3, 2')$	4	0	1	1193.8	568.5
\mathcal{L}_{517}	10	C_2^{vii}, T_2^{ii}	$(3, 3, 2')$	5	-1	2	925.91	57.246
\mathcal{L}_{518}	12	C_2^{vii}, T_3^{ii}	$(3, 3, 2')$	2	2	1	822.06	39.114
\mathcal{L}_{519}	12	C_2^{vii}, T_4^{ii}	$(3, 3, 2')$	3	1	2	568.28	417.44
\mathcal{L}_{520}	12	C_2^{vii}, T_5^{ii}	$(3, 3, 2')$	6	-2	3	376.6	5.0468
\mathcal{L}_{521}	4	C_1^{viii}, T_1^i	$(3', 3, 2)$	0	2	1	17609.	1.2169×10^6
\mathcal{L}_{522}	4	C_1^{viii}, T_2^i	$(3', 3, 2)$	1	1	2	2.5472×10^6	3.0715×10^6
\mathcal{L}_{523}	6	C_1^{viii}, T_3^i	$(3', 3, 2)$	-2	4	1	15934.	15833.
\mathcal{L}_{524}	6	C_1^{viii}, T_4^i	$(3', 3, 2)$	-1	3	2	16012.	15836.
\mathcal{L}_{525}	6	C_1^{viii}, T_5^i	$(3', 3, 2)$	2	0	3	17609.	1.2169×10^6
\mathcal{L}_{526}	4	C_1^{viii}, T_1^{ii}	$(3', 3, 2')$	2	0	1	1.3943×10^6	604730.
\mathcal{L}_{527}	4	C_1^{viii}, T_2^{ii}	$(3', 3, 2')$	3	-1	2	853880.	569500.
\mathcal{L}_{528}	6	C_1^{viii}, T_3^{ii}	$(3', 3, 2')$	0	2	1	16975.	461770.
\mathcal{L}_{529}	6	C_1^{viii}, T_4^{ii}	$(3', 3, 2')$	1	1	2	1.2148×10^6	583690.
\mathcal{L}_{530}	6	C_1^{viii}, T_5^{ii}	$(3', 3, 2')$	4	-2	3	785680.	465790.
\mathcal{L}_{531}	8	C_2^{viii}, T_1^i	$(3', 3, 2)$	2	2	1	1902.	1.2011×10^6
\mathcal{L}_{532}	8	C_2^{viii}, T_2^i	$(3', 3, 2)$	3	1	2	4663.1	666160.
\mathcal{L}_{533}	10	C_2^{viii}, T_3^i	$(3', 3, 2)$	0	4	1	187.76	424.64
\mathcal{L}_{534}	10	C_2^{viii}, T_4^i	$(3', 3, 2)$	1	3	2	385.31	53.129
\mathcal{L}_{535}	10	C_2^{viii}, T_5^i	$(3', 3, 2)$	4	0	3	2267.3	1.2014×10^6
\mathcal{L}_{536}	8	C_2^{viii}, T_1^{ii}	$(3', 3, 2')$	4	0	1	1460.7	10080.

\mathcal{L}_{537}	8	C_2^{iiii}, T_2^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2}')$	5	-1	2	1804.6	827.15
\mathcal{L}_{538}	10	C_2^{iiii}, T_3^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2}')$	2	2	1	61.498	494.96
\mathcal{L}_{539}	10	C_2^{iiii}, T_4^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2}')$	3	1	2	388.24	543.55
\mathcal{L}_{540}	10	C_2^{iiii}, T_5^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2}')$	6	-2	3	460.72	162.48
\mathcal{L}_{541}	10	C_3^{iiii}, T_1^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2})$	4	2	1	1979.9	1.2012×10^6
\mathcal{L}_{542}	10	C_3^{iiii}, T_2^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2})$	5	1	2	63096.	665420.
\mathcal{L}_{543}	12	C_3^{iiii}, T_3^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2})$	2	4	1	1.2137	7.9894×10^{-6} ★
\mathcal{L}_{544}	12	C_3^{iiii}, T_4^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2})$	3	3	2	228.73	0.000021417 ★
\mathcal{L}_{545}	12	C_3^{iiii}, T_5^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2})$	6	0	3	2089.4	1.2012×10^6
\mathcal{L}_{546}	10	C_3^{iiii}, T_1^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2}')$	6	0	1	746.74	3.0906×10^{-6} ★
\mathcal{L}_{547}	10	C_3^{iiii}, T_2^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2}')$	7	-1	2	920.88	20.815 ☆
\mathcal{L}_{548}	12	C_3^{iiii}, T_3^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2}')$	4	2	1	73.527	1.3855×10^{-6} ★
\mathcal{L}_{549}	12	C_3^{iiii}, T_4^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2}')$	5	1	2	88.114	4.1593×10^{-6} ★
\mathcal{L}_{550}	12	C_3^{iiii}, T_5^{ii}	$(\mathbf{3}', \mathbf{3}, \mathbf{2}')$	8	-2	3	0.000042377	7.1351×10^{-6} ★
\mathcal{L}_{551}	4	C_1^{ix}, T_1^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	2	0	1	1.2803×10^6	217580.
\mathcal{L}_{552}	4	C_1^{ix}, T_2^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	3	-1	2	1.5723×10^6	677650.
\mathcal{L}_{553}	6	C_1^{ix}, T_3^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	0	2	1	319130.	15089.
\mathcal{L}_{554}	6	C_1^{ix}, T_4^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	1	1	2	261260.	14612.
\mathcal{L}_{555}	6	C_1^{ix}, T_5^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	4	-2	3	450660.	216880.
\mathcal{L}_{556}	4	C_1^{ix}, T_1^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	0	2	1	859540.	16526.
\mathcal{L}_{557}	4	C_1^{ix}, T_2^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	1	1	2	447210.	15855.
\mathcal{L}_{558}	6	C_1^{ix}, T_3^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	-2	4	1	14822.	14453.
\mathcal{L}_{559}	6	C_1^{ix}, T_4^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	-1	3	2	286800.	14452.
\mathcal{L}_{560}	6	C_1^{ix}, T_5^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	2	0	3	15886.	15846.
\mathcal{L}_{561}	8	C_2^{ix}, T_1^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	4	0	1	3499.5	4480.8
\mathcal{L}_{562}	8	C_2^{ix}, T_2^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	5	-1	2	1479.5	21711.
\mathcal{L}_{563}	10	C_2^{ix}, T_3^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	2	2	1	991.98	393.09
\mathcal{L}_{564}	10	C_2^{ix}, T_4^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	3	1	2	200.42	85.593
\mathcal{L}_{565}	10	C_2^{ix}, T_5^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	6	-2	3	918.36	2596.8
\mathcal{L}_{566}	8	C_2^{ix}, T_1^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	2	2	1	2528.3	603.43
\mathcal{L}_{567}	8	C_2^{ix}, T_2^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	3	1	2	2313.6	566.86
\mathcal{L}_{568}	10	C_2^{ix}, T_3^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	0	4	1	894.72	548.97
\mathcal{L}_{569}	10	C_2^{ix}, T_4^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	1	3	2	253.16	547.84
\mathcal{L}_{570}	10	C_2^{ix}, T_5^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	4	0	3	633.14	565.62
\mathcal{L}_{571}	10	C_3^{ix}, T_1^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	6	0	1	1107.9	1688.6
\mathcal{L}_{572}	10	C_3^{ix}, T_2^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	7	-1	2	823.66	544.89
\mathcal{L}_{573}	12	C_3^{ix}, T_3^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	4	2	1	3.1642	9.4269×10^{-6} ★
\mathcal{L}_{574}	12	C_3^{ix}, T_4^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	5	1	2	0.000071568	1.338×10^{-6} ★
\mathcal{L}_{575}	12	C_3^{ix}, T_5^{iii}	$(\mathbf{3}, \mathbf{3}', \mathbf{2})$	8	-2	3	631.23	566.27
\mathcal{L}_{576}	10	C_3^{ix}, T_1^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	4	2	1	1183.7	526.27
\mathcal{L}_{577}	10	C_3^{ix}, T_2^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	5	1	2	633.21	18.676 ☆
\mathcal{L}_{578}	12	C_3^{ix}, T_3^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	2	4	1	419.77	6.4264×10^{-6} ★
\mathcal{L}_{579}	12	C_3^{ix}, T_4^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	3	3	2	63.734	7.0782×10^{-6} ★
\mathcal{L}_{580}	12	C_3^{ix}, T_5^{iv}	$(\mathbf{3}, \mathbf{3}', \mathbf{2}')$	6	0	3	385.91	3.0992×10^{-7} ★
\mathcal{L}_{581}	6	C_1^x, T_1^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{2})$	2	0	1	2.4404×10^6	2.7688×10^6
\mathcal{L}_{582}	6	C_1^x, T_2^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{2})$	3	-1	2	2.4407×10^6	2.609×10^6
\mathcal{L}_{583}	8	C_1^x, T_3^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{2})$	0	2	1	2.4395×10^6	2.4382×10^6
\mathcal{L}_{584}	8	C_1^x, T_4^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{2})$	1	1	2	2.4395×10^6	2.4393×10^6
\mathcal{L}_{585}	8	C_1^x, T_5^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{2})$	4	-2	3	2.4425×10^6	2.5455×10^6
\mathcal{L}_{586}	6	C_1^x, T_1^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{2}')$	0	2	1	2.4409×10^6	2.5713×10^6

\mathcal{L}_{587}	6	C_1^x, T_2^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{2}')$	1	1	2	2.4408×10^6	2.5503×10^6
\mathcal{L}_{588}	8	C_1^x, T_3^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{2}')$	-2	4	1	2.4396×10^6	2.4383×10^6
\mathcal{L}_{589}	8	C_1^x, T_4^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{2}')$	-1	3	2	2.4396×10^6	2.4393×10^6
\mathcal{L}_{590}	8	C_1^x, T_5^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{2}')$	2	0	3	2.4396×10^6	2.4393×10^6
\mathcal{L}_{591}	10	C_2^x, T_1^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{2})$	4	0	1	1254.7	3480.4
\mathcal{L}_{592}	10	C_2^x, T_2^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{2})$	5	-1	2	2445.9	4652.5
\mathcal{L}_{593}	12	C_2^x, T_3^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{2})$	2	2	1	780.06	1.4494 ★
\mathcal{L}_{594}	12	C_2^x, T_4^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{2})$	3	1	2	657.74	1.8528 ★
\mathcal{L}_{595}	12	C_2^x, T_5^{iii}	$(\mathbf{3}', \mathbf{3}', \mathbf{2})$	6	-2	3	352.33	332.56
\mathcal{L}_{596}	10	C_2^x, T_1^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{2}')$	2	2	1	1394.7	811.01
\mathcal{L}_{597}	10	C_2^x, T_2^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{2}')$	3	1	2	2443.3	533.21
\mathcal{L}_{598}	12	C_2^x, T_3^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{2}')$	0	4	1	507.02	2.2044 ★
\mathcal{L}_{599}	12	C_2^x, T_4^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{2}')$	1	3	2	656.56	4.8755 ★
\mathcal{L}_{600}	12	C_2^x, T_5^{iv}	$(\mathbf{3}', \mathbf{3}', \mathbf{2}')$	4	0	3	365.3	337.42
\mathcal{L}_{601}	8	C_1^i, W_1^i	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, -)$	1, 3, 5	1	-	1793.6	497.52
\mathcal{L}_{602}	12	C_1^i, W_2^i	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, -)$	0, 2, 4	2	-	8.2969×10^{-8} ★	2.9786×10^{-7} ★
\mathcal{L}_{603}	12	C_1^i, W_3^i	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}, -)$	-1, 1, 3	3	-	4.5597×10^{-9} ★	1.2025×10^{-7} ★
\mathcal{L}_{604}	8	C_1^{ii}, W_1^{ii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', -)$	1, 3, 5	1	-	1926.1	540.66
\mathcal{L}_{605}	12	C_1^{ii}, W_2^{ii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', -)$	0, 2, 4	2	-	4.3831×10^{-8} ★	1.034×10^{-6} ★
\mathcal{L}_{606}	12	C_1^{ii}, W_3^{ii}	$(\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}, \mathbf{3}', -)$	-1, 1, 3	3	-	5.9865×10^{-8} ★	1.6773×10^{-7} ★
\mathcal{L}_{607}	5	C_1^{iii}, W_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	2, 1	1	-	14926.	553780.
\mathcal{L}_{608}	9	C_1^{iii}, W_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	1, 0	2	-	13891.	13889.
\mathcal{L}_{609}	9	C_1^{iii}, W_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	0, -1	3	-	13891.	13891.
\mathcal{L}_{610}	5	C_2^{iii}, W_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	2, 3	1	-	14923.	382030.
\mathcal{L}_{611}	9	C_2^{iii}, W_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	1, 2	2	-	14263.	14287.
\mathcal{L}_{612}	9	C_2^{iii}, W_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	0, 1	3	-	14323.	13892.
\mathcal{L}_{613}	7	C_3^{iii}, W_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	4, 1	1	-	501.26	766.07
\mathcal{L}_{614}	11	C_3^{iii}, W_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	3, 0	2	-	2.168×10^{-8} ★	7.0231×10^{-7} ★
\mathcal{L}_{615}	11	C_3^{iii}, W_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	2, -1	3	-	0.06423 ★	3.7879×10^{-6} ★
\mathcal{L}_{616}	7	C_4^{iii}, W_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	4, 3	1	-	14783.	382030.
\mathcal{L}_{617}	11	C_4^{iii}, W_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	3, 2	2	-	13926.	13899.
\mathcal{L}_{618}	11	C_4^{iii}, W_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	2, 1	3	-	13898.	13893.
\mathcal{L}_{619}	7	C_5^{iii}, W_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	2, 5	1	-	16904.	17671.
\mathcal{L}_{620}	11	C_5^{iii}, W_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	1, 4	2	-	14960.	14720.
\mathcal{L}_{621}	11	C_5^{iii}, W_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	0, 3	3	-	14922.	16430.
\mathcal{L}_{622}	9	C_6^{iii}, W_1^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	4, 5	1	-	603.11	540.77
\mathcal{L}_{623}	13	C_6^{iii}, W_2^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	3, 4	2	-	4.7758×10^{-6} ★	1.584×10^{-7} ★
\mathcal{L}_{624}	13	C_6^{iii}, W_3^i	$(\mathbf{2} \oplus \mathbf{1}, \mathbf{3}, -)$	2, 3	3	-	8.0053×10^{-7} ★	7.7523×10^{-7} ★
\mathcal{L}_{625}	5	C_1^{iv}, W_1^i	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}, -)$	0, 1	1	-	643700.	2477.7
\mathcal{L}_{626}	9	C_1^{iv}, W_2^i	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}, -)$	-1, 0	2	-	903.82	1708.1
\mathcal{L}_{627}	9	C_1^{iv}, W_3^i	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}, -)$	-2, -1	3	-	277.89	335.84
\mathcal{L}_{628}	5	C_2^{iv}, W_1^i	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}, -)$	0, 3	1	-	20077.	36162.
\mathcal{L}_{629}	9	C_2^{iv}, W_2^i	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}, -)$	-1, 2	2	-	1345.	737.69
\mathcal{L}_{630}	9	C_2^{iv}, W_3^i	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}, -)$	-2, 1	3	-	18002.	249.64
\mathcal{L}_{631}	7	C_3^{iv}, W_1^i	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}, -)$	2, 1	1	-	1137.9	683.35
\mathcal{L}_{632}	11	C_3^{iv}, W_2^i	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}, -)$	1, 0	2	-	6.7733×10^{-7} ★	2.9602×10^{-7} ★
\mathcal{L}_{633}	11	C_3^{iv}, W_3^i	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}, -)$	0, -1	3	-	2.539×10^{-6} ★	9.3844×10^{-7} ★
\mathcal{L}_{634}	7	C_4^{iv}, W_1^i	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}, -)$	2, 3	1	-	1131.7	538.58
\mathcal{L}_{635}	11	C_4^{iv}, W_2^i	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}, -)$	1, 2	2	-	9.203×10^{-6} ★	7.9597×10^{-6} ★
\mathcal{L}_{636}	11	C_4^{iv}, W_3^i	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}, -)$	0, 1	3	-	2.0268×10^{-6} ★	4.9713×10^{-6} ★

\mathcal{L}_{637}	7	C_5^{iv}, W_1^i	$(2' \oplus 1, 3, -)$	0,5	1	-	17691.	1908.6
\mathcal{L}_{638}	11	C_5^{iv}, W_2^i	$(2' \oplus 1, 3, -)$	-1,4	2	-	8.4048×10^{-7} ★	0.00014465 ★
\mathcal{L}_{639}	11	C_5^{iv}, W_3^i	$(2' \oplus 1, 3, -)$	-2,3	3	-	4.3825×10^{-7} ★	4.0431×10^{-6} ★
\mathcal{L}_{640}	9	C_6^{iv}, W_1^i	$(2' \oplus 1, 3, -)$	4,1	1	-	804.22	6.2024×10^{-6} ★
\mathcal{L}_{641}	13	C_6^{iv}, W_2^i	$(2' \oplus 1, 3, -)$	3,0	2	-	1.433×10^{-6} ★	4.3132×10^{-6} ★
\mathcal{L}_{642}	13	C_6^{iv}, W_3^i	$(2' \oplus 1, 3, -)$	2,-1	3	-	2.3067×10^{-8} ★	1.1938×10^{-7} ★
\mathcal{L}_{643}	9	C_7^{iv}, W_1^i	$(2' \oplus 1, 3, -)$	2,5	1	-	916.23	523.86
\mathcal{L}_{644}	13	C_7^{iv}, W_2^i	$(2' \oplus 1, 3, -)$	1,4	2	-	3.4225×10^{-7} ★	1.5807×10^{-6} ★
\mathcal{L}_{645}	13	C_7^{iv}, W_3^i	$(2' \oplus 1, 3, -)$	0,3	3	-	5.9604×10^{-6} ★	8.5215×10^{-7} ★
\mathcal{L}_{646}	9	C_8^{iv}, W_1^i	$(2' \oplus 1, 3, -)$	4,3	1	-	708.63	6.315×10^{-6} ★
\mathcal{L}_{647}	13	C_8^{iv}, W_2^i	$(2' \oplus 1, 3, -)$	3,2	2	-	6.7924×10^{-6} ★	5.3925×10^{-7} ★
\mathcal{L}_{648}	13	C_8^{iv}, W_3^i	$(2' \oplus 1, 3, -)$	2,1	3	-	7.5352×10^{-8} ★	2.0431×10^{-7} ★
\mathcal{L}_{649}	5	C_1^v, W_1^{ii}	$(2 \oplus 1, 3', -)$	0,1	1	-	2.0198×10^6	993620.
\mathcal{L}_{650}	9	C_1^v, W_2^{ii}	$(2 \oplus 1, 3', -)$	-1,0	2	-	904.74	1039.4
\mathcal{L}_{651}	9	C_1^v, W_3^{ii}	$(2 \oplus 1, 3', -)$	-2,-1	3	-	852.89	825.4
\mathcal{L}_{652}	5	C_2^v, W_1^{ii}	$(2 \oplus 1, 3', -)$	0,3	1	-	20947.	261710.
\mathcal{L}_{653}	9	C_2^v, W_2^{ii}	$(2 \oplus 1, 3', -)$	-1,2	2	-	688.88	221.94
\mathcal{L}_{654}	9	C_2^v, W_3^{ii}	$(2 \oplus 1, 3', -)$	-2,1	3	-	7.8802 ★	105.8
\mathcal{L}_{655}	7	C_3^v, W_1^{ii}	$(2 \oplus 1, 3', -)$	2,1	1	-	1374.	8795.8
\mathcal{L}_{656}	11	C_3^v, W_2^{ii}	$(2 \oplus 1, 3', -)$	1,0	2	-	1.3986×10^{-6} ★	2.6253×10^{-6} ★
\mathcal{L}_{657}	11	C_3^v, W_3^{ii}	$(2 \oplus 1, 3', -)$	0,-1	3	-	3.7538×10^{-8} ★	5.6406×10^{-7} ★
\mathcal{L}_{658}	7	C_4^v, W_1^{ii}	$(2 \oplus 1, 3', -)$	2,3	1	-	1375.3	1091.2
\mathcal{L}_{659}	11	C_4^v, W_2^{ii}	$(2 \oplus 1, 3', -)$	1,2	2	-	7.8693×10^{-7} ★	2.5969×10^{-6} ★
\mathcal{L}_{660}	11	C_4^v, W_3^{ii}	$(2 \oplus 1, 3', -)$	0,1	3	-	5.6216×10^{-7} ★	4.1554×10^{-7} ★
\mathcal{L}_{661}	7	C_5^v, W_1^{ii}	$(2 \oplus 1, 3', -)$	0,5	1	-	20809.	189640.
\mathcal{L}_{662}	11	C_5^v, W_2^{ii}	$(2 \oplus 1, 3', -)$	-1,4	2	-	79.493	784.6
\mathcal{L}_{663}	11	C_5^v, W_3^{ii}	$(2 \oplus 1, 3', -)$	-2,3	3	-	1501.4	1083.
\mathcal{L}_{664}	9	C_6^v, W_1^{ii}	$(2 \oplus 1, 3', -)$	4,1	1	-	793.2	0.40969 ★
\mathcal{L}_{665}	13	C_6^v, W_2^{ii}	$(2 \oplus 1, 3', -)$	3,0	2	-	1.0203×10^{-6} ★	4.9244×10^{-7} ★
\mathcal{L}_{666}	13	C_6^v, W_3^{ii}	$(2 \oplus 1, 3', -)$	2,-1	3	-	3.0222×10^{-8} ★	1.6573×10^{-7} ★
\mathcal{L}_{667}	9	C_7^v, W_1^{ii}	$(2 \oplus 1, 3', -)$	2,5	1	-	726.64	511.71
\mathcal{L}_{668}	13	C_7^v, W_2^{ii}	$(2 \oplus 1, 3', -)$	1,4	2	-	1.0471×10^{-6} ★	0.00054006 ★
\mathcal{L}_{669}	13	C_7^v, W_3^{ii}	$(2 \oplus 1, 3', -)$	0,3	3	-	3.0553×10^{-7} ★	3.1028×10^{-6} ★
\mathcal{L}_{670}	9	C_8^v, W_1^{ii}	$(2 \oplus 1, 3', -)$	4,3	1	-	790.94	0.41065 ★
\mathcal{L}_{671}	13	C_8^v, W_2^{ii}	$(2 \oplus 1, 3', -)$	3,2	2	-	1.3029×10^{-6} ★	1.487×10^{-6} ★
\mathcal{L}_{672}	13	C_8^v, W_3^{ii}	$(2 \oplus 1, 3', -)$	2,1	3	-	7.5059×10^{-7} ★	6.5796×10^{-8} ★
\mathcal{L}_{673}	5	C_1^{wi}, W_1^{ii}	$(2' \oplus 1, 3', -)$	2,1	1	-	16027.	29609.
\mathcal{L}_{674}	9	C_1^{wi}, W_2^{ii}	$(2' \oplus 1, 3', -)$	1,0	2	-	13992.	14653.
\mathcal{L}_{675}	9	C_1^{wi}, W_3^{ii}	$(2' \oplus 1, 3', -)$	0,-1	3	-	13889.	13906.
\mathcal{L}_{676}	5	C_2^{wi}, W_1^{ii}	$(2' \oplus 1, 3', -)$	2,3	1	-	16141.	29781.
\mathcal{L}_{677}	9	C_2^{wi}, W_2^{ii}	$(2' \oplus 1, 3', -)$	1,2	2	-	13889.	14579.
\mathcal{L}_{678}	9	C_2^{wi}, W_3^{ii}	$(2' \oplus 1, 3', -)$	0,1	3	-	13945.	14328.
\mathcal{L}_{679}	7	C_3^{wi}, W_1^{ii}	$(2' \oplus 1, 3', -)$	4,1	1	-	2048.9	15589.
\mathcal{L}_{680}	11	C_3^{wi}, W_2^{ii}	$(2' \oplus 1, 3', -)$	3,0	2	-	1.6387×10^{-6} ★	7.216×10^{-6} ★
\mathcal{L}_{681}	11	C_3^{wi}, W_3^{ii}	$(2' \oplus 1, 3', -)$	2,-1	3	-	1.186×10^{-7} ★	7.3538×10^{-8} ★
\mathcal{L}_{682}	7	C_4^{wi}, W_1^{ii}	$(2' \oplus 1, 3', -)$	4,3	1	-	15953.	14457.
\mathcal{L}_{683}	11	C_4^{wi}, W_2^{ii}	$(2' \oplus 1, 3', -)$	3,2	2	-	13893.	13889.
\mathcal{L}_{684}	11	C_4^{wi}, W_3^{ii}	$(2' \oplus 1, 3', -)$	2,1	3	-	13889.	13890.
\mathcal{L}_{685}	7	C_5^{wi}, W_1^{ii}	$(2' \oplus 1, 3', -)$	2,5	1	-	2127.3	15123.
\mathcal{L}_{686}	11	C_5^{wi}, W_2^{ii}	$(2' \oplus 1, 3', -)$	1,4	2	-	2.3472×10^{-6} ★	1168.7

\mathcal{L}_{687}	11	C_5^{wi}, W_3^{ii}	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}', -)$	0, 3	3	-	3.806×10^{-7} ★	1.5723×10^{-7} ★
\mathcal{L}_{688}	9	C_6^{wi}, W_1^{ii}	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}', -)$	4, 5	1	-	1571.1	547.18
\mathcal{L}_{689}	13	C_6^{wi}, W_2^{ii}	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}', -)$	3, 4	2	-	2.2998×10^{-7} ★	5.5402×10^{-6} ★
\mathcal{L}_{690}	13	C_6^{wi}, W_3^{ii}	$(\mathbf{2}' \oplus \mathbf{1}, \mathbf{3}', -)$	2, 3	3	-	3.5309×10^{-8} ★	1.2902×10^{-7} ★
\mathcal{L}_{691}	6	C_1^{wii}, W_1^i	$(\mathbf{3}, \mathbf{3}, -)$	1	1	-	446650.	252810.
\mathcal{L}_{692}	10	C_1^{wii}, W_2^i	$(\mathbf{3}, \mathbf{3}, -)$	0	2	-	45685.	45686.
\mathcal{L}_{693}	10	C_1^{wii}, W_3^i	$(\mathbf{3}, \mathbf{3}, -)$	-1	3	-	45714.	45972.
\mathcal{L}_{694}	10	C_2^{wii}, W_1^i	$(\mathbf{3}, \mathbf{3}, -)$	3	1	-	1333.3	88.846
\mathcal{L}_{695}	14	C_2^{wii}, W_2^i	$(\mathbf{3}, \mathbf{3}, -)$	2	2	-	9.5573×10^{-6} ★	0.000023496 ★
\mathcal{L}_{696}	14	C_2^{wii}, W_3^i	$(\mathbf{3}, \mathbf{3}, -)$	1	3	-	0.0022952 ★	0.00096709 ★
\mathcal{L}_{697}	4	C_1^{wiii}, W_1^i	$(\mathbf{3}', \mathbf{3}, -)$	1	1	-	15281.	964190.
\mathcal{L}_{698}	8	C_1^{wiii}, W_2^i	$(\mathbf{3}', \mathbf{3}, -)$	0	2	-	14869.	14906.
\mathcal{L}_{699}	8	C_1^{wiii}, W_3^i	$(\mathbf{3}', \mathbf{3}, -)$	-1	3	-	14855.	14906.
\mathcal{L}_{700}	8	C_2^{wiii}, W_1^i	$(\mathbf{3}', \mathbf{3}, -)$	3	1	-	959.87	549.46
\mathcal{L}_{701}	12	C_2^{wiii}, W_2^i	$(\mathbf{3}', \mathbf{3}, -)$	2	2	-	4.6402 ★	0.000023064 ★
\mathcal{L}_{702}	12	C_2^{wiii}, W_3^i	$(\mathbf{3}', \mathbf{3}, -)$	1	3	-	7.4548×10^{-6} ★	9.5782×10^{-7} ★
\mathcal{L}_{703}	10	C_3^{wiii}, W_1^i	$(\mathbf{3}', \mathbf{3}, -)$	5	1	-	858.55	492.5
\mathcal{L}_{704}	14	C_3^{wiii}, W_2^i	$(\mathbf{3}', \mathbf{3}, -)$	4	2	-	6.4208×10^{-8} ★	3.6295×10^{-6} ★
\mathcal{L}_{705}	14	C_3^{wiii}, W_3^i	$(\mathbf{3}', \mathbf{3}, -)$	3	3	-	2.1561×10^{-6} ★	3.2065×10^{-6} ★
\mathcal{L}_{706}	4	C_1^{ix}, W_1^{ii}	$(\mathbf{3}, \mathbf{3}', -)$	1	1	-	20607.	540020.
\mathcal{L}_{707}	8	C_1^{ix}, W_2^{ii}	$(\mathbf{3}, \mathbf{3}', -)$	0	2	-	14681.	14844.
\mathcal{L}_{708}	8	C_1^{ix}, W_3^{ii}	$(\mathbf{3}, \mathbf{3}', -)$	-1	3	-	14677.	14761.
\mathcal{L}_{709}	8	C_2^{ix}, W_1^{ii}	$(\mathbf{3}, \mathbf{3}', -)$	3	1	-	2169.1	15652.
\mathcal{L}_{710}	12	C_2^{ix}, W_2^{ii}	$(\mathbf{3}, \mathbf{3}', -)$	2	2	-	9.0072×10^{-7} ★	1.5691×10^{-6} ★
\mathcal{L}_{711}	12	C_2^{ix}, W_3^{ii}	$(\mathbf{3}, \mathbf{3}', -)$	1	3	-	3.2782×10^{-6} ★	0.011709 ★
\mathcal{L}_{712}	10	C_3^{ix}, W_1^{ii}	$(\mathbf{3}, \mathbf{3}', -)$	5	1	-	882.55	0.00473 ★
\mathcal{L}_{713}	14	C_3^{ix}, W_2^{ii}	$(\mathbf{3}, \mathbf{3}', -)$	4	2	-	3.7776×10^{-7} ★	1.0746×10^{-7} ★
\mathcal{L}_{714}	14	C_3^{ix}, W_3^{ii}	$(\mathbf{3}, \mathbf{3}', -)$	3	3	-	1.5696×10^{-7} ★	5.6074×10^{-6} ★
\mathcal{L}_{715}	6	C_1^x, W_1^{ii}	$(\mathbf{3}', \mathbf{3}', -)$	1	1	-	2.4403×10^6	2.609×10^6
\mathcal{L}_{716}	10	C_1^x, W_2^{ii}	$(\mathbf{3}', \mathbf{3}', -)$	0	2	-	2.4373×10^6	2.438×10^6
\mathcal{L}_{717}	10	C_1^x, W_3^{ii}	$(\mathbf{3}', \mathbf{3}', -)$	-1	3	-	2.4374×10^6	2.4382×10^6
\mathcal{L}_{718}	10	C_2^x, W_1^{ii}	$(\mathbf{3}', \mathbf{3}', -)$	3	1	-	921.86	155.46
\mathcal{L}_{719}	14	C_2^x, W_2^{ii}	$(\mathbf{3}', \mathbf{3}', -)$	2	2	-	0.000011234 ★	0.000048871 ★
\mathcal{L}_{720}	14	C_2^x, W_3^{ii}	$(\mathbf{3}', \mathbf{3}', -)$	1	3	-	7.8338×10^{-6} ★	0.000037615 ★

Table 3: All models generated by combinations of charged lepton sector and neutrino sector. We have list the number of parameters involved in each model in the second column. The combinations between charged lepton sector and neutrino sector are given in the third column. At the following columns, we have given the representation assignments, weights, and best fit values of χ^2 respectively. If the χ_{\min}^2 is marked with ★, then the model can accommodate the experimental data within 3σ limits, if it is marked with ☆, then the best fit values of observables slightly deviate the 3σ limits, for other cases, the experimental data can not be explained by the corresponding model.

2 Quark Sector

In the quark sector, we tried the same representation assignments in the lepton sector as given in table 4 for $\rho_Q, \rho_{U^c}/\rho_{D^c}$, but the experimentally favored data can not be produced by these models. Considering the Cabibbo mixing pattern as an approximation of CKM matrix, we further consider the assignment $\mathbf{2} \oplus \mathbf{1}$ or $\mathbf{2}' \oplus \mathbf{1}$ for both ρ_Q and ρ_{U^c}/ρ_{D^c} . In table 4, we list the possible representations assignments for quark fields, and the numbers of models. As

what we have done in the charged lepton sector, we can write out the superpotential and the corresponding quark mass matrix for each possible model. Due to the large number of models, we don't provide all the relevant details here, and we only show the models that are in good agreement with the experiments like $\mathcal{Q}_{1\dots 4}$ (see text below), all other models can be constructed in a similar fashion. Through the combination of the up quark and down quark sectors, we have obtained 664 models with 9 parameters, 892 models with 11 parameters and 358 models with 13 parameters. We cannot construct a model with 10 or 12 parameters since the real parameters increase in pair when we add more complex parameters. After the numerical fitting, we find that the models with 9 parameters cannot explain the experimental data of quarks, while four models with 11 parameters and 18 models with 13 parameters can explain the experimental results well.

	G^i	G^{ii}	G^{iii}	G^{iv}
ρ_Q	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$
ρ_{U^c}/ρ_{D^c}	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2}' \oplus \mathbf{1}$
# of models	30	21	21	30
(total number)	(46)	(26)	(26)	(46)

Table 4: The number (in bracket) of possible quark models for different representation assignments of Q and ρ_{U^c}/ρ_{D^c} under the finite modular group A_5^* up to weight 6 modular forms, where the case without modular forms is not counted. We also list the number (without bracket) of models which contain three or four independent terms in the quark superpotential $\mathcal{W}_{u,d}$.

The four models $\mathcal{Q}_{1\dots 4}$ with 11 parameters are given as follows

$\mathcal{Q}_1 \sim \langle G_1^i, G_1^{ii} \rangle$
$G_1^i: k_{Q_D} = k_{Q_3} + 5, k_{U_D} = -k_{Q_3} - 3, k_{U_3} = -k_{Q_3}$ $W_u = \alpha_u \left(Y_3^{(2)} U_D^c Q_D H_u \right)_1 + \beta_u \left(Y_2^{(5)} U_3^c Q_D H_u \right)_1 + \gamma_u \left(U_3^c Q_3 H_u \right)_1$ $M_u = \begin{pmatrix} -\sqrt{2}\alpha_u Y_{3,2}^{(2)} & -\alpha_u Y_{3,1}^{(2)} & 0 \\ -\alpha_u Y_{3,1}^{(2)} & \sqrt{2}\alpha_u Y_{3,3}^{(2)} & 0 \\ -\beta_u Y_{2,2}^{(5)} & \beta_u Y_{2,1}^{(5)} & \gamma_u \end{pmatrix} v_u$
$G_1^{ii}: k_{Q_D} = k_{Q_3} + 5, k_{D_3} = -k_{Q_3}, k_{D_D} = 1 - k_{Q_3}$ $W_d = \alpha_d \left(Y_{4I}^{(6)} D_D^c Q_D H_d \right)_1 + \beta_d \left(Y_{4II}^{(6)} D_D^c Q_D H_d \right)_1 + \gamma_d \left(Y_2^{(5)} D_3^c Q_D H_d \right)_1 + \delta_d \left(D_3^c Q_3 H_d \right)_1$ $M_d = \begin{pmatrix} -\alpha_d Y_{4I,2}^{(6)} - \beta_d Y_{4II,2}^{(6)} & \alpha_d Y_{4I,1}^{(6)} + \beta_d Y_{4II,1}^{(6)} & 0 \\ \alpha_d Y_{4I,4}^{(6)} + \beta_d Y_{4II,4}^{(6)} & \alpha_d Y_{4I,3}^{(6)} + \beta_d Y_{4II,3}^{(6)} & 0 \\ -\gamma_d Y_{2,2}^{(5)} & \gamma_d Y_{2,1}^{(5)} & \delta_d \end{pmatrix} v_d$

$\mathcal{Q}_2 \sim \langle G_2^i, G_2^{ii} \rangle$
$G_2^i: k_{Q_D} = k_{Q_3} + 1, k_{U_3} = 4 - k_{Q_3}, k_{U_D} = 1 - k_{Q_3}$ $W_u = \alpha_u \left(Y_{\mathbf{3}}^{(2)} U_D^c Q_D H_u \right)_1 + \beta_u \left(Y_{\mathbf{2}}^{(5)} U_3^c Q_D H_u \right)_1 + \gamma_u \left(Y_{\mathbf{1}}^{(4)} U_3^c Q_3 H_u \right)_1$ $M_u = \begin{pmatrix} -\sqrt{2}\alpha_u Y_{\mathbf{3},2}^{(2)} & -\alpha_u Y_{\mathbf{3},1}^{(2)} & 0 \\ -\alpha_u Y_{\mathbf{3},1}^{(2)} & \sqrt{2}\alpha_u Y_{\mathbf{3},3}^{(2)} & 0 \\ -\beta_u Y_{\mathbf{2},2}^{(5)} & \beta_u Y_{\mathbf{2},1}^{(5)} & \gamma_u Y_{\mathbf{1},1}^{(4)} \end{pmatrix} v_u$
$G_2^{ii}: k_{Q_D} = k_{Q_3} + 1, k_{D_3} = 4 - k_{Q_3}, k_{D_D} = 5 - k_{Q_3}$ $W_d = \alpha_d \left(Y_{\mathbf{4I}}^{(6)} D_D^c Q_D H_d \right)_1 + \beta_d \left(Y_{\mathbf{4II}}^{(6)} D_D^c Q_D H_d \right)_1 + \gamma_d \left(Y_{\mathbf{2}}^{(5)} D_3^c Q_D H_d \right)_1 + \delta_d \left(Y_{\mathbf{1}}^{(4)} D_3^c Q_3 H_d \right)_1$ $M_d = \begin{pmatrix} -\alpha_d Y_{\mathbf{4I},2}^{(6)} - \beta_d Y_{\mathbf{4II},2}^{(6)} & \alpha_d Y_{\mathbf{4I},1}^{(6)} + \beta_d Y_{\mathbf{4II},1}^{(6)} & 0 \\ \alpha_d Y_{\mathbf{4I},4}^{(6)} + \beta_d Y_{\mathbf{4II},4}^{(6)} & \alpha_d Y_{\mathbf{4I},3}^{(6)} + \beta_d Y_{\mathbf{4II},3}^{(6)} & 0 \\ -\gamma_d Y_{\mathbf{2},2}^{(5)} & \gamma_d Y_{\mathbf{2},1}^{(5)} & \delta_d Y_{\mathbf{1},1}^{(4)} \end{pmatrix} v_d$

$\mathcal{Q}_3 \sim \langle G_1^{iv}, G_2^{iv} \rangle$
$G_1^{iv}: k_{Q_D} = k_{Q_3} + 5, k_{U_D} = -k_{Q_3} - 3, k_{U_3} = -k_{Q_3}$ $W_u = \alpha_u \left(Y_{\mathbf{3}'}^{(2)} U_D^c Q_D H_u \right)_1 + \beta_u \left(Y_{\mathbf{2}'}^{(5)} U_3^c Q_D H_u \right)_1 + \gamma_u \left(U_3^c Q_3 H_u \right)_1$ $M_u = \begin{pmatrix} -\sqrt{2}\alpha_u Y_{\mathbf{3}',3}^{(2)} & \alpha_u Y_{\mathbf{3}',1}^{(2)} & 0 \\ \alpha_u Y_{\mathbf{3}',1}^{(2)} & \sqrt{2}\alpha_u Y_{\mathbf{3}',2}^{(2)} & 0 \\ -\beta_u Y_{\mathbf{2}',2}^{(5)} & \beta_u Y_{\mathbf{2}',1}^{(5)} & \gamma_u \end{pmatrix} v_u$
$G_2^{iv}: k_{Q_D} = k_{Q_3} + 5, k_{D_D} = -k_{Q_3} - 1, k_{D_3} = -k_{Q_3}$ $W_d = \alpha_d \left(Y_{\mathbf{1}}^{(4)} D_D^c Q_D H_d \right)_1 + \beta_d \left(Y_{\mathbf{3}'}^{(4)} D_D^c Q_D H_d \right)_1 + \gamma_d \left(Y_{\mathbf{2}'}^{(5)} D_3^c Q_D H_d \right)_1 + \delta_d \left(D_3^c Q_3 H_d \right)_1$ $M_d = \begin{pmatrix} -\sqrt{2}\beta_d Y_{\mathbf{3}',3}^{(4)} & \beta_d Y_{\mathbf{3}',1}^{(4)} - \alpha_d Y_{\mathbf{1},1}^{(4)} & 0 \\ \alpha_d Y_{\mathbf{1},1}^{(4)} + \beta_d Y_{\mathbf{3}',1}^{(4)} & \sqrt{2}\beta_d Y_{\mathbf{3}',2}^{(4)} & 0 \\ -\gamma_d Y_{\mathbf{2}',2}^{(5)} & \gamma_d Y_{\mathbf{2}',1}^{(5)} & \delta_d \end{pmatrix} v_d$

$\mathcal{Q}_4 \sim \langle G_3^{iv}, G_4^{iv} \rangle$
$G_3^{iv}: k_{Q_D} = 1 + k_{Q_3}, k_{U_D} = 1 - k_{Q_3}, k_{U_3} = 4 - k_{Q_3}$ $W_u = \alpha_u \left(Y_{\mathbf{3}'}^{(2)} U_D^c Q_D H_u \right)_1 + \beta_u \left(Y_{\mathbf{2}'}^{(5)} U_3^c Q_D H_u \right)_1 + \gamma_u \left(Y_{\mathbf{1}}^{(4)} U_3^c Q_3 H_u \right)_1$ $M_u = \begin{pmatrix} -\sqrt{2}\alpha_u Y_{\mathbf{3}',3}^{(2)} & \alpha_u Y_{\mathbf{3}',1}^{(2)} & 0 \\ \alpha_u Y_{\mathbf{3}',1}^{(2)} & \sqrt{2}\alpha_u Y_{\mathbf{3}',2}^{(2)} & 0 \\ -\beta_u Y_{\mathbf{2}',2}^{(5)} & \beta_u Y_{\mathbf{2}',1}^{(5)} & \gamma_u Y_{\mathbf{1},1}^{(4)} \end{pmatrix} v_u$
$G_4^{iv}: k_{Q_D} = k_{Q_3} + 1, k_{D_3} = 4 - k_{Q_3}, k_{D_D} = 3 - k_{Q_3}$ $W_d = \alpha_d \left(Y_{\mathbf{1}}^{(4)} D_D^c Q_D H_d \right)_1 + \beta_d \left(Y_{\mathbf{3}'}^{(4)} D_D^c Q_D H_d \right)_1 + \gamma_d \left(Y_{\mathbf{2}'}^{(5)} D_3^c Q_D H_d \right)_1 + \delta_d \left(Y_{\mathbf{1}}^{(4)} D_3^c Q_3 H_d \right)_1$ $M_d = \begin{pmatrix} -\sqrt{2}\beta_d Y_{\mathbf{3}',3}^{(4)} & \beta_d Y_{\mathbf{3}',1}^{(4)} - \alpha_d Y_{\mathbf{1},1}^{(4)} & 0 \\ \alpha_d Y_{\mathbf{1},1}^{(4)} + \beta_d Y_{\mathbf{3}',1}^{(4)} & \sqrt{2}\beta_d Y_{\mathbf{3}',2}^{(4)} & 0 \\ -\gamma_d Y_{\mathbf{2}',2}^{(5)} & \gamma_d Y_{\mathbf{2}',1}^{(5)} & \delta_d Y_{\mathbf{1},1}^{(4)} \end{pmatrix} v_d$

Due to limited space, we will not give the results of models with 13 parameters. We can find from the above four models that \mathcal{Q}_1 is similar to \mathcal{Q}_2 , and \mathcal{Q}_3 is similar to \mathcal{Q}_4 . Their

predictions of quark mass matrices M_u, M_d only differ in (3, 3) element, and this difference can be eliminated by redefining γ_u and δ_d . Therefore, each pair of the models will give the same predictions. So we only need to focus on the models \mathcal{Q}_1 and \mathcal{Q}_3 . In this section, we present the numerical results of model \mathcal{Q}_1 , the best fit results are given in table 5. We found that only $m_u/m_c = 0.003002$ slightly deviates from the experimental 1σ limit $[0.001327, 0.002530]$, but it is within 3σ limits.

The numerical results of \mathcal{Q}_3 is not shown here, because it can not only meet the experimental requirements of quark sector separately, but also interpret lepton and quark experimental data simultaneously under the unified framework of modular symmetry A'_5 . We will show the results of \mathcal{Q}_3 in the next section when we present the unified model in table 6.

quark model \mathcal{Q}_1			
Input		Observable	
$\text{Re}(\tau)$	-0.4973	θ_{13}^q	0.003464
$\text{Im}(\tau)$	2.221	θ_{12}^q	0.2273
β_u/α_u	384.6	θ_{23}^q	0.04015
γ_u/α_u	7.438	δ^q/π	0.3870
$ \beta_d/\alpha_d $	0.3804	m_u/GeV	0.0007407
$ \delta_d/\alpha_d $	0.06164	m_c/GeV	0.2467
γ_d/α_d	1.207	m_t/GeV	87.46
$\arg(\beta_d/\alpha_d)/\pi$	-0.01782	m_d/GeV	0.0009206
$\arg(\delta_d/\alpha_d)/\pi$	-0.2672	m_s/GeV	0.01753
$ \alpha_u v_u /\text{GeV}$	0.05121	m_b/GeV	0.9682
$ \alpha_d v_d /\text{GeV}$	0.07099	χ_{\min}^2	3.382

Table 5: Numerical results of the quark model \mathcal{Q}_1 . On the left, the best fit values of the input parameters are given, on the right, we list the quark mixing parameters and predictions of quark masses, and χ_{\min}^2 .

3 Unified model of lepton and quark

A model that can explain lepton and quark experimental data simultaneously should explain them separately. Therefore, in order to construct a unified model of lepton and quark, we have selected the 25 NO models in table 3 that marked with \star (currently, NO is more favored with the experimental data, for simplicity, we only consider this scenario). For quark sector, to find the minimal unified model, we only focus on two independent models \mathcal{Q}_1 and \mathcal{Q}_3 with 11 parameters as given in the previous section. By combining the lepton sector and the quark sector, we have obtained 50 unified models. We have performed a systematic numerical analysis of these models, and find that the unified models generated by lepton model with 9 parameters and quark model with 11 parameters cannot explain the experimental data. In the remaining models, we found that the model combination \mathcal{L}_2 and \mathcal{Q}_3 can well explain the experimental results. We show the best fit results of this unified model in table 6. We can find that both the lepton model and quark model can accommodate the experimental data within the 3σ limits.

best fit results of the unified model			
lepton model \mathcal{L}_2		quark model \mathcal{Q}_3	
input	value	input	value
β/α	24.5400	β_u/α_u	32.6296
γ/α	654.9320	γ_u/α_u	11.1981
$ \delta/\alpha $	100.8429	$ \beta_d/\alpha_d $	6.3229
$\arg(\delta/\alpha)/\pi$	1.3895	γ_d/α_d	1.7462
$ g_2/g_1 $	0.3818	$ \delta_d/\alpha_d $	1.9895
$\arg(g_2/g_1)/\pi$	1.7542	$\arg(\beta_d/\alpha_d)/\pi$	0.0002176
$ \alpha v_d /\text{MeV}$	0.04361	$\arg(\delta_d/\delta'_d)/\pi$	1.7336
$(g_1^2 v_u^2/\Lambda)/\text{meV}$	1.0895	$ \alpha_u v_u /\text{GeV}$	0.03432
lepton observable	value	$ \alpha_d v_d /\text{GeV}$	0.002429
$\sin^2 \theta_{13}$	0.02231	quark observable	value
$\sin^2 \theta_{12}$	0.3019	θ_{13}^q	0.003498
$\sin^2 \theta_{23}$	0.4570	θ_{12}^q	0.2276
δ_{CP}/π	1.1671	θ_{23}^q	0.04023
α_{21}/π	1.3593	δ^q/π	0.3843
α_{31}/π	0.4762	m_u/GeV	0.00005300
m_1/meV	77.2709	m_c/GeV	0.2467
m_2/meV	77.7495	m_t/GeV	87.4555
m_3/meV	92.1717	m_d/GeV	0.0005920
m_β/meV	77.7755	m_s/GeV	0.01712
$m_{\beta\beta}/\text{meV}$	48.9783	m_b/GeV	0.9682
$\chi_{\min,L}^2$	33.9453	$\chi_{\min,Q}^2$	15.2279
common input	value		
$\text{Re}(\tau)$	0.5000		
$\text{Im}(\tau)$	0.8944		

Table 6: The best fit results of the unified model of \mathcal{L}_2 and \mathcal{Q}_3 . The values of input parameters and predictions of observables are given in both lepton and quark sectors.