第五章

可压缩流动的数值解法

可压缩Navier－Stokes方程组

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \frac{\partial \mathbf{G}}{\partial z} = 0$$

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E_t \end{bmatrix} \qquad \mathbf{E} = \begin{bmatrix} \rho u \\ \rho u^2 + p - \tau_{xx} \\ \rho u v - \tau_{xy} \\ \rho u w - \tau_{xz} \\ (E_t + p)u - u\tau_{xx} - v\tau_{xy} - w\tau_{xz} + q_x \end{bmatrix}$$

可压缩Navier－Stokes方程组

$$\mathbf{F} = \begin{bmatrix} \rho v \\ \rho u v - \tau_{xy} \\ \rho v^2 + p - \tau_{yy} \\ \rho v w - \tau_{yz} \\ (E_t + p)v - u\tau_{xy} - v\tau_{yy} - w\tau_{yz} + q_y \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} \rho w \\ \rho u w - \tau_{xz} \\ \rho v w - \tau_{yz} \\ \rho w^2 + p - \tau_{zz} \\ (E_t + p)w - u\tau_{xz} - v\tau_{yz} - w\tau_{zz} + q_z \end{bmatrix}$$

可压缩Navier－Stokes方程组

$$\tau_{xx} = \frac{2}{3}\mu\left(2\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} - \frac{\partial w}{\partial z}\right) \qquad \tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \tau_{yx}$$

$$\tau_{yy} = \frac{2}{3}\mu\left(2\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} - \frac{\partial w}{\partial z}\right) \qquad \tau_{xz} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = \tau_{zx}$$

$$\tau_{zz} = \frac{2}{3}\mu\left(2\frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) \qquad \tau_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \tau_{zy}$$

$$\mathbf{q} = -k\,\nabla T \qquad E_t = \rho\left(e + \frac{V^2}{2} + \text{potential energy} + \cdots\right)$$

$$p = (\gamma - 1)\rho e \qquad T = \frac{(\gamma - 1)e}{R}$$

Stokes流体，量热完全气体，未考虑内部热源

无量纲化N-S方程组

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L} \qquad z^* = \frac{z}{L} \qquad t^* = \frac{t}{L/V_\infty}$$

$$u^* = \frac{u}{V_\infty} \qquad v^* = \frac{v}{V_\infty} \qquad w^* = \frac{w}{V_\infty} \qquad \mu^* = \frac{\mu}{\mu_\infty}$$

$$\rho^* = \frac{\rho}{\rho_\infty} \qquad p^* = \frac{p}{\rho_\infty V_\infty^2} \qquad T^* = \frac{T}{T_\infty} \qquad e^* = \frac{e}{V_\infty^2}$$

$$\mathrm{Re}_L = \frac{\rho_\infty V_\infty L}{\mu_\infty}$$

无量纲化N-S方程组

$$\frac{\partial \mathbf{U}^*}{\partial t^*} + \frac{\partial \mathbf{E}^*}{\partial x^*} + \frac{\partial \mathbf{F}^*}{\partial y^*} + \frac{\partial \mathbf{G}^*}{\partial z^*} = 0$$

$$\mathbf{U}^* = \begin{bmatrix} \rho^* \\ \rho^* u^* \\ \rho^* v^* \\ \rho^* w^* \\ E_t^* \end{bmatrix} \qquad \mathbf{E}^* = \begin{bmatrix} \rho^* u^* \\ \rho^* u^{*2} + p^* - \tau_{xx}^* \\ \rho^* u^* v^* - \tau_{xy}^* \\ \rho^* u^* w^* - \tau_{xz}^* \\ (E_t^* + p^*)u^* - u^*\tau_{xx}^* - v^*\tau_{xy}^* - w^*\tau_{xz}^* + q_x^* \end{bmatrix}$$

无量纲化N-S方程组

$$\mathbf{F}^* = \begin{bmatrix} \rho^* v^* \\ \rho^* u^* v^* - \tau_{xy}^* \\ \rho^* v^{*2} + p^* - \tau_{yy}^* \\ \rho^* v^* w^* - \tau_{yz}^* \\ (E_t^* + p^*) v^* - u^* \tau_{xy}^* - v^* \tau_{yy}^* - w^* \tau_{yz}^* + q_y^* \end{bmatrix}$$

$$\mathbf{G}^* = \begin{bmatrix} \rho^* w^* \\ \rho^* u^* w^* - \tau_{xz}^* \\ \rho^* v^* w^* - \tau_{yz}^* \\ \rho^* w^{*2} + p^* - \tau_{zz}^* \\ (E_t^* + p^*) w^* - u^* \tau_{xz}^* - v^* \tau_{yz}^* - w^* \tau_{zz}^* + q_z^* \end{bmatrix}$$

无量纲化N-S方程组

$$E_t^* = \rho^* \left(e^* + \frac{u^{*2} + v^{*2} + w^{*2}}{2} \right)$$

$$q_x^* = -\frac{\mu^*}{(\gamma - 1) M_\infty^2 \operatorname{Re}_L \operatorname{Pr}} \frac{\partial T^*}{\partial x^*}$$

$$q_y^* = -\frac{\mu^*}{(\gamma - 1) M_\infty^2 \operatorname{Re}_L \operatorname{Pr}} \frac{\partial T^*}{\partial y^*}$$

$$q_z^* = -\frac{\mu^*}{(\gamma - 1) M_\infty^2 \operatorname{Re}_L \operatorname{Pr}} \frac{\partial T^*}{\partial z^*}$$

$$M_\infty = \frac{V_\infty}{\sqrt{\gamma R T_\infty}} \qquad \operatorname{Pr} = \frac{c_p \mu}{k}$$

无量纲化N-S方程组

$$\tau_{xy}^* = \frac{\mu^*}{\mathrm{Re}_L}\left(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right) \qquad \tau_{xx}^* = \frac{2\mu^*}{3\,\mathrm{Re}_L}\left(2\frac{\partial u^*}{\partial x^*} - \frac{\partial v^*}{\partial y^*} - \frac{\partial w^*}{\partial z^*}\right)$$

$$\tau_{xz}^* = \frac{\mu^*}{\mathrm{Re}_L}\left(\frac{\partial u^*}{\partial z^*} + \frac{\partial w^*}{\partial x^*}\right) \qquad \tau_{yy}^* = \frac{2\mu^*}{3\,\mathrm{Re}_L}\left(2\frac{\partial v^*}{\partial y^*} - \frac{\partial u^*}{\partial x^*} - \frac{\partial w^*}{\partial z^*}\right)$$

$$\tau_{yz}^* = \frac{\mu^*}{\mathrm{Re}_L}\left(\frac{\partial v^*}{\partial z^*} + \frac{\partial w^*}{\partial y^*}\right) \qquad \tau_{zz}^* = \frac{2\mu^*}{3\,\mathrm{Re}_L}\left(2\frac{\partial w^*}{\partial z^*} - \frac{\partial u^*}{\partial x^*} - \frac{\partial v^*}{\partial y^*}\right)$$

$$p^* = (\gamma - 1)\rho^* e^*$$

$$T^* = \frac{\gamma M_\infty^2 p^*}{\rho^*}$$

可压缩Euler方程组

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \frac{\partial \mathbf{G}}{\partial z} = 0$$

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E_t \end{bmatrix} \qquad \mathbf{E} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ \rho u w \\ (E_t + p)u \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ \rho v w \\ (E_t + p)v \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} \rho w \\ \rho u w \\ \rho v w \\ \rho w^2 + p \\ (E_t + p)w \end{bmatrix}$$