Sphere-based Cut Construction for Planar Parameterizations

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Applications of Parameterization

- Texture mapping  
  (many papers)

- Remeshing  
  [Bommes et al., 2009]

- Inter-surface mapping  
  [Aigerman et al., 2014]

- ...

Previous Work

Geometry Images
[Gu et al., 2002]

Seamster
[Sheffer and Hart, 2002]

Autocuts
[Poranne et al., 2017]
Goal

- A cut construction method that satisfies
  - The distortion of a subsequent planar parameterization is low.
  - The cuts are feature-aligned, resulting in visual beauty.
  - The cuts are short.

- It is challenging to satisfy all the above requirements.
Method
Mapping, Parameterization & Distortion

• Distortion metrics
  • Conformal distortion (angle preserving) [Hormann et al., 2000]
    \[ d_i^{\text{conf}} = \frac{1}{2} \left( \frac{\sigma_1}{\sigma_2} + \frac{\sigma_2}{\sigma_1} \right) = \frac{1}{2} \frac{\|J_i\|^2}{\det J_i} \]
  • Areal distortion (area preserving) [Fu et al., 2015]
    \[ d_i^{\text{area}} = \frac{1}{2} \left( \det J_i + (\det J_i)^{-1} \right) \]
  • Isometric distortion (isometry preserving) [Fu et al., 2015]
    \[ d_i^{\text{iso}} = \alpha d_i^{\text{conf}} + (1 - \alpha) d_i^{\text{area}} \]
Key Observation

• The high isometric distortion mainly appears at the extrusive regions when a mesh is parameterized onto a constant curvature domain (such as a sphere or the plane) as conformal as possible.
Pipeline

Input a closed genus-zero triangular mesh

Step 1: parameterize to a sphere ACAP

Step 2: find feature points by hierarchical clustering

Step 3: cut by a minimal spanning tree

Output an open mesh of disk topology

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ACAP Spherical Parameterization

As isometric as possible (AIAP)
AHSP [Hu et al., 2018]

As conformal as possible (ACAP)
Hierarchical Clustering
Three connected regions
Cut Construction

On the mesh

On the sphere
High-Genus Cases

• Cut along handles [Dey et al., 2013] → Fill the holes → Apply our algorithm
Results
AQP [Kovalsky et al., 2016]
Discussions
Different $N_R$

(4.88/1.14/0.12) (1.23%/1.28%) (0.05%N, 31)
(5.03/1.15/0.13) (1.16%/1.22%) (0.15%N, 25)
(5.17/1.15/0.13) (1.12%/1.20%) (0.45%N, 22)
(5.18/1.17/0.18) (1.06%/1.13%) (1.00%N, 16)
VS. Average Filter

Average filter

Median filter
Comparisons
Comparison on a 5140 dataset

<table>
<thead>
<tr>
<th></th>
<th>Geometry</th>
<th>Image</th>
<th>Seamster</th>
<th>Ours (cut on mesh)</th>
<th>Ours (cut on sphere)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum – AQP</td>
<td>97.4</td>
<td>87.4</td>
<td>79.4</td>
<td>78.2</td>
<td>97.5</td>
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<tr>
<td>Average – AQP</td>
<td>8.67</td>
<td>3.85</td>
<td>4.00</td>
<td>6.16</td>
<td>5.49</td>
</tr>
<tr>
<td>Maximum – SA</td>
<td>25.9</td>
<td>4.24</td>
<td>4.08</td>
<td>1.63</td>
<td>13.8</td>
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<tr>
<td>Average – SA</td>
<td>0.028</td>
<td>4.77</td>
<td>3.79</td>
<td>1.18</td>
<td>0.048</td>
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<tr>
<td>Edge number</td>
<td>4.18</td>
<td>2.37%</td>
<td>5.79</td>
<td>95.9</td>
<td>4.09</td>
</tr>
<tr>
<td>Edge length</td>
<td>0.13%</td>
<td>0.94%</td>
<td>1.01%</td>
<td>1.19</td>
<td>0.015</td>
</tr>
</tbody>
</table>

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Comparison with Geometry Image [Gu et al., 2002]

(4.07/1.28/0.37)  (4.78/1.13/0.12)  (4.03/1.11/0.11)
(0.97%/1.09%)    (1.51%/1.40%)    (1.89%/1.74%)
Comparison with Seamster [Shaffer and Hart, 2002]

(3.81/1.09/0.11)  
(1.44%/1.23%)

(4.58/1.05/0.13)  
(1.26%/1.14%)

(4.44/1.04/0.09)  
(1.68%/1.43%)
Comparison with Autocuts [Poranne et al., 2017]

(7.71/1.27/0.52)
(4.20%/4.06%)

(1.95/1.16/0.14)
(3.85%/4.01%)

(1.95/1.15/0.14)
(4.46%/4.38%)
Comparison with Persistence-based Method
[Chazal et al., 2013]
Conclusion

• We present a sphere-based method for constructing high-quality cuts...
  • ACAP spherical parameterization
  • Hierarchical clustering
  • Cut on the sphere
• such that the subsequent planar parameterization can have low isometric distortion.
Limitations and Discussions

- Coupled planar parameterizations
- Domains other than the sphere
- Theoretical guarantees
- Tessellations
- Symmetry
Thank you!