Knowledge Based Topic Model for Multi-modal Social Event Analysis

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Abstract—With the accumulation of data on the Internet and progress in representation learning techniques, knowledge priors learned from a large-scale knowledge base has been increasingly used in probabilistic topic models. However, it is challenging to learn interpretable topics and a discriminative event representation based on multi-modal information. To address these issues, we propose a knowledge priors- and max-margin-based topic model for multi-modal social event analysis, called the KGE-MMSLDA, in which feature representation and knowledge priors are jointly learned. Our model has three main advantages over current methods: (1) It integrates additional knowledge from external knowledge base into a unified topic model in which the max-margin classifier, and multi-modal information are exploited to increase the number of event descriptions obtained. (2) We mined knowledge priors from over 74,000 web documents. Multi-modal data with these knowledge priors are then incorporated into the topic model to increase the number of coherent topics learned. (3) A large-scale multi-modal dataset (containing 10 events, where each event contained approximately 7,000 Flickr pages) was collected and has been released publicly for event topic mining and classification research. In comparative experiments, the proposed method outperformed state-of-the-art models on topic coherence, and obtained a classification accuracy of 85.1%.

Index Terms—Knowledge Embedding, Multi-Modal, Topic Coherence, Event Classification.

I. INTRODUCTION

With the widespread popularity of social networking sites, social media data are growing rapidly. The development of mobile Internet and digital photography has made it more convenient for people to report events and express their opinions anytime and anywhere, and this has led to an explosion of data on social media sites. When an event occurs (e.g., a football match, an accident on a highway, or an Apple release conference), an increasing amount of multimedia content (e.g., text, image, and video) is uploaded to social media sites by users. Because social event analysis is a type of data-driven process, the huge user-contributed data are extremely helpful and important for obtaining accurate analysis results. However, as users have become the producers and disseminators of data, user-contributed data have become unstructured and noisy, and it is difficult for researchers to use them for social events analysis. Organizing numerous social media data and analyzing social hotspot events automatically are particularly helpful for improving event analysis.

Recently, social event analysis has garnered the interest of researchers in multi-media analysis, such as for social event detection [1], [2], [3], [4], event tracking [4], and event mining [5]. There are three main challenges in social event analysis.

(1) A social event is multi-modal. Traditional social event analysis tasks, such as event tracking and topic mining, involve a single modality, and do not make full use of multi-modal information (e.g., text descriptions and images). On the Internet, social media consists of rich unstructured data with multiple modalities that can complement one another. They help express the complete meaning of social event analysis [6], [7], [8]. For instance, when a new event appears in the web society, different media sites report it from different perspectives. The report text description may be different, but the visual information is often similar.

Considering that past studies have focused on single-modal-based social event analysis [9], [10], [11], it is useful to analyze multi-modal event data in a uniform model.

(2) A social event involves prior knowledge. Many entities of social events are freely available in general knowledge bases, and social event data typically contain some important
entities, such as people and locations, that may be encoded in these knowledge bases, such as Wikipedia and Freebase. For example, the text in Fig. 1 contains entities (e.g., al-Assad, Muslims of Syria, horizontal triple jump, and athletics) that can be represented as entity pairs in a knowledge base (e.g., a Wikipedia dataset). We consider the must-link relations between these entities in knowledge base as prior knowledge. A knowledge graph is embedded into a low-dimensional continuous vector space in which the inherent structure and certain properties of a large-scale knowledge base are preserved. Specifically, the points indicate entities in vector space, and a single edge for a given pair of entities indicates their relation. Knowledge entity pairs can directly capture the structural information of the original knowledge base, which can be used as a kind of knowledge prior for social event analysis.

(3) A social event has the multi-class property. For example, Fig. 1 shows two multi-modal social events containing titles, descriptions, tags, and images. Class label information can be used for discriminative feature representation to analyze social events. Therefore, exploiting a model that fuses multi-modal and multi-category information with prior knowledge can help with social event analysis. Many recent studies have explored multi-modal information on social media, and proposed methods based on the topic model for social event analysis. For example, Corr-LDA is pioneering research on using relations images of between events and their text descriptions at the topic level. However, these approaches use unsupervised topic models, which cannot use labels to obtain discriminative representations for social event analysis. To address this limitation, some researchers have integrated feature representation learning with discriminative classifiers to obtain a unified classification model. A supervised LDA model was proposed in 

However, the above studies use only softmax regression to relate supervised information with data representations, which yields suboptimal classification results. Moreover, some traditional methods without prior knowledge often generate topics that are difficult to interpret. Topic model-based knowledge is a new research area that has potential for use in event mining. The relevant methods exploit the low-dimensional continuous vector of entities in the knowledge graph, which helps retain the internal structure and certain properties of a large-scale knowledge base. Andrzejewski et al. proposed topic-in-set knowledge to add partial supervision to latent Dirichlet allocation (LDA). The authors of Ref. proposed a fold-all framework to extend topic-in-set knowledge in which first-order logic is used to specify general knowledge. In , human knowledge and the topic model were integrated, and a Probase-LDA on probabilistic knowledge was designed to boost topic coherence performance. Although current models combine prior knowledge and the topic model using different approaches, they do not consider large-scale triple-oriented knowledge graphs.

We propose a knowledge and max-margin based topic model (KGE-MMSLDA) for multi-modal event classification and analysis, where the topic model and knowledge entity priors are combined. Our proposed KGE-MMSLDA explicitly models multimedia documents with knowledge that is automatically learned from a pre-existing knowledge graph. Our experimental results show that KGE-MMSLDA extracts more coherent topics than other compared methods. Moreover, we introduce the max-margin classifier as a regularization term to the topic model. Specifically, the max-margin classifier and KGE-MMSLDA model are integrated into a united generative model that is more conducive to the optimizing model of event analysis. For feature representation, we design multi-modal feature representation where prior knowledge is integrated into the training process of the latent topic relevance. In the classifier design, the max-margin classifier is used for social event classification because it is a powerful discriminant classifier. Specifically, Gibbs sampling is used in the training of the KGE-MMSLDA. We empirically evaluated our proposed model through topic coherence and social event classification on an empirical dataset. Both qualitative and quantitative results show its effectiveness. Our study makes the following contributions:

- A unified framework (KGE-MMSLDA) is proposed where knowledge priors and a max-margin topic model are integrated for multi-modal social event analysis.
- The KGE-MMSLDA model uses prior knowledge mined from external knowledge base and supervised information as part of the regularization to output a reasonable representation and coherently mined topics for social event classification.
- No public dataset is available for multi-media analysis research. We collected a large-scale multi-media dataset called HUTF-mmdata for public social event analysis research. We conducted experiments on it to verify that our KGE-MMSLDA outperforms current topic models.

The remainder of this paper is organized as follows: We first discuss related work in Section II and present the details of KGE-MMSLDA and its optimization process in Section III. We report the experimental results in Section IV. In Section V, we offer the conclusions of this study and highlight avenues for future work in the area.

II. RELATED WORK

Social Event Analysis: Social event analysis has attracted considerable attention from researchers in multimedia analysis. A well-designed event classification model can help boost the performance of event classification tasks. In general, a social event analysis algorithm consists of two processes: feature representation and classifier training. Early event classification methods considered these processes as separate . Blei et al. used the LDA to learn the text representation

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1https://dumps.wikimedia.org/
2https://en.wikipedia.org/wiki/Freebase
3http://scholarhub.cn/ScholarHubProject/MMTM/HFUT-mmdata.zip, HFUT stands for HeFei University of Technology
for text classification. Qian et al. proposed a multi-modal event topic model, that models social media documents and learns text image correlations to separate topics in text representation from those not in it [25]. Gao et al. [26] proposed a deep learning-based approach to perform for event classification using microblogs. The above methods mainly focused on event feature design, that is used to train the classifier separately. However, they do not integrate event feature representation and classifier training into a unified model. Many supervised models have been proposed to improve social event classification by introducing a classifier to the topic model [15], [16], [18], [27].

Simon et al. [16] proposed a model called the DiscLDA that applies a class-related linear transformation to mixed topic proportions. Blei et al. [18] introduced a supervised LDA (sLDA) to associate categorical information with each document that jointly models documents and supervised information. In [28], two supervised topic models were proposed to solve event classification and regression problems. They account for heterogeneity and biases among different annotators that are encountered in practice when learning from crowds.

The above supervised methods calculate the probability that a document belongs to each event class using maximum likelihood estimation, and the classification is regarded as a process of maximum voting that ignores the powerful discriminative classification algorithm. Recently, researchers have noticed this problem, and some powerful discriminative classifiers have been exploited for social event classification [19], [29], [30]. Wang et al. [19] improved the LDA model using a max-margin classifier for the image classification task. Zhu et al. [29] proposed an alternative approach by considering a new max-margin loss to jointly model the max-margin classifier and feature representation learning for multi-class and multi-label text classification. However, the max-margin methods in [19], [29] focus on text or images separately, where multi-modal information in social media data is often ignored. Unlike these models, in this study, supervised information and multi-modal information of social events is unified in a framework in which multi-modal event representation is learned using a discriminative max-margin classifier.

**Topic Model:** Topic models have been researched for many years, and are proved to be effective for modeling social events.

By learning document representation, the traditional LDA [23] solves the problem of topic mining and text categorization. Many improvements on the proposal in [23] have been made, such as [13], [17], [18], [31] and [32]. Bao et al. [17] proposed a partial sLDA model for cross-domain learning, and in [33], a hierarchical sLDA was presented for a hierarchical multi-media data structure.

The above methods are useful for mining document information for social events. However, only text information is considered in these methods, and other modal information has been ignored. To address the above issues, researchers have proposed methods that make full use of the multi-modal properties of documents [34], [35]. Prabhuvesai et al. proposed an extended LDA topic model where the topics are represented using the Gaussian mixture models [36]. In [34], an improved multi-modal LDA model using a topic regression technique was designed to capture correlations between visual features and annotation text for image and video annotation tasks. In [37], Qian et al. proposed a multi-modal sLDA (MMSLDA) for social event classification that can jointly learn textual and visual topics across multi-modal social media data.

Recently, researchers have attended to topic models with domain knowledge to clarify the interpretation of the generated topics. The work in Ref. [38] develops a knowledge-based topic model by incorporating knowledge graph embedding into it for topic mining and document classification. In a similar spirit, our KGE-MMSLDA model combines knowledge entity priors and the multi-modal topic model to jointly learn event representation and the classifier in a unified framework. The work in Ref. [20] first combines domain knowledge with the LDA model and then uses the Dirichlet forest prior to encoding the two basic types of domain knowledge (must-links and cannot-links). Knowledge graph embedding is a new research area in which the entities and relationships of a knowledge graph are encoded into continuous vectors [39], [40], [41], [42], [43]. To automatically learn word correlation knowledge topic modeling with automatically generated must-links and cannot-links (AMC) was proposed in [44] to improve topic modeling in each domain. To achieve the goal of interpretation, [45] proposed a hierarchical topic model, called the graph-sparse LDA model using “controlled structured vocabularies.” The work in [22] designed an integrated model, called Probase-LDA, where probabilistic knowledge and the topic model are combined to mine useful topics from multi-media documents. In [46], an extended multi-nomial model was proposed to improve performance, where latent embedding vectors were applied to a large corpus. By replacing the traditional multi-nomial distribution of the LDA model with the Gaussian distribution, [47] used Euclidean distance to calculate the similarity among word vectors to capture semantic regularities in language.

**Visual Feature Extraction:** The development of deep neural networks (DNNs) has made them prominent in computer vision. Krizhevsky et al. [48] proposed a large and deep convolutional neural network to classify images in the ImageNet dataset and won first place in the ImageNet LSVC-2012 contest. In [49], an architecture with very small convolution filters and very deep networks was designed, where this demonstrated the importance of depth in visual representations. Recently, most researchers have focused on ways to increase the depth of convolution networks. However, the increasing depth of networks leads to gradients vanishing or exploding, which makes the networks difficult to train. In [50], He et al. proposed a deep residual learning framework with a depth of up to 152 layers that was easy to optimize and achieved good performance. Because of the efficiency of the residual connection, Gao et al. [51] introduced direct connections to every layer in the network and proposed an architecture called DenseNet that can achieve state-of-the-art performance with few parameters and little computation. The work in [52] focused on the relations between feature maps and proposed the “squeeze-and-excitation” block to capture the interdependencies between channels that can be simply
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Fig. 2. Framework of the KGE-MMSLDA model for social event classification.

integrated into CNN architectures to yield better performance.

Our KGE-MMSLDA model is a generic framework that incorporates the aforementioned visual feature extraction techniques. In our implementation, for computational efficiency, AlexNet in [48] is used to generate visual feature vectors.

III. PROPOSED MODEL

In this section, we define the problem to tackle here and describe the training of our proposed model.

A. Problem Definition

It is important to be able to automatically analyze a social event because multi-media data on the Internet are massive in size and confusing in their variety. During the collection of our HFUT-mmdata dataset, a number of social events are first defined, and images and text belonging to each event were downloaded from well-known websites, such as Flickr. We consider the image and its text description to be a document. In this study, two modalities, image and text, were used for learning.

To depict multi-media information, we used a traditional bag-of-words model to represent the text [23], and a similar data structure was used for images.

A document presentation was converted into a word count vector, where the word order and its context cues were ignored. Let \( D = \{ (d_1, y_1), (d_2, y_2), \ldots, (d_M, y_M) \} \) be a set of multi-modal documents, where \( d \) denotes each document that consists of two modalities \( d = \{ w, v \} \), where \( w \) and \( v \) represent the text and image inputs, respectively, and \( y_i \) denotes the event class number of a given document. Its value is in the range \([1, L]\), where \( L \) denotes the number of event classes. The proposed KGE-MMSLDA is a general framework that can be expanded conveniently to exploit data from more modalities. Moreover, the knowledge base \( G \) is introduced to the KGE-MMSLDA a priori to improve performance.

We detail the framework of the KGE-MMSLDA model in the following subsection:

B. Unified Framework of KGE-MMSLDA

The framework of the proposed KGE-MMSLDA model has two aspects: (1) a knowledge-based topic model that learns a probabilistic distribution over latent topics for the given documents with prior knowledge, and (2) a delicate classifier in which max-margin theory is used and supervised information is considered during the training phase.

1) KGE-MMSLDA Model: We propose a knowledge-based topic model that is designed to mine latent topics from multi-modal data and learn the representation of a multi-media event using probabilistic distributions over the found topics. In the KGE-MMSLDA, knowledge entity pairs and multiple modal information are incorporated into a unified model. All text words, image words, and entities are sampled in the same topic space. Specifically, documents that consist of multi-modal data and knowledge entity priors are mined from a large-scale knowledge base.

Typically, most documents on the Internet consist of both visual content and corresponding text. To better represent documents and learn text image correlations, we propose a multi-modal topic model that mines correlations between visual and textual contents. Moreover, the proposed model incorporates knowledge priors into multi-modal topic modeling by taking knowledge entities as another input to the model to guide the traditional topic model to discover more coherent topics.

Knowledge entity embedding is a useful method to embed entities and relations inside the knowledge base into continuous vectors where original knowledge is preserved. Each entity is considered as a high-dimensional point in the vector space and each relation as an operation over the entities. When sampling an entity, the model automatically searches for the entity from the knowledge base. For example, when the word “Trump” is sampled, the model obtains the corresponding entity embedding from the knowledge base in which this entity may be spatially similar to the topic “politics.” Recent advances in modeling continuous entity embeddings [38] have shown that entity embedding lies on a unit sphere and vMF distribution to model it instead of a multivariate Gaussian distribution [53] makes the inference much more efficient.

The probability density function of the vMF distribution is

\[
 f(x|\mu, \kappa) = C_l(\kappa) \exp \left( \kappa \mu^T x \right) \frac{\kappa^{0.5l-1}}{(2\pi)^{0.5l} I_{0.5l-1}(\kappa)} \exp \left( \kappa \mu^T x \right)
\]

where \( x \in \mathbb{R}^l \) lies on an \( l-1 \) dimensional sphere, \( u \) is the average vector and \( \kappa \) denotes the inverse of the variance of the training set. \( I_a(b) \) denotes the modified Bessel function of the first type for order \( a \) and argument \( b \).

In this article, the result of our knowledge mining is a set of word pairs \(<w_1, w_2>\), which are frequently co-occurred. During the sampling phase of textual words, we simply set the topic assignment of \( w_1 \) to \( w_2 \) since the two words often appear together.

Fig. 2 shows the graphical representation of the KGE-MMSLDA, where the shaded nodes are observed variables and unshaded nodes are latent variables. Let \( E = \{ d_1, d_2, \ldots, d_M, G \} \) be a collection of event documents, where \( M \) is the number of input documents. For each multi-media document \( d = \{ w_d, v_d \}, w_d \), and \( v_d \) denote textual and visual words, respectively. \( G \) = \{ \( e_1, e_2, \ldots, e_{NG} \} \) denotes
the set of knowledge entities, \( c_{d,m} \) denotes the embedding of the \( m \)-th entity in document \( d \), and \( N^G \) denotes the number of entities. The KGE-MMSLDA can be considered a two-layer model with a topic layer and a word layer. In the topic layer, topic \( k \) is considered a \( K \)-dimensional multi-nomial distribution of document \( d \). In the word-layer, textual word \( w \) and visual word \( v \) are considered \( D_w \)-dimensional and \( D_v \)-dimensional multi-nomial distributions, respectively, of topic \( k \), where \( D_w \) and \( D_v \) are the respective sizes of the textual and visual vocabulary.

Accordingly, in the KGE-MMSLDA, we generate document \( d \) using the following steps:

1. For document \( d \), draw \( \theta_d \sim \text{Dir}(\alpha_{\theta}) \).
2. For visual topic \( k \in \{1,2,...,K\} \), draw \( \phi^v_k|\alpha_{\phi^v} \sim \text{Dir}(\alpha_{\phi^v}) \).
3. For each textual topic \( k \in \{1,2,...,K\} \),
   (1) draw \( \phi^t_k|\alpha_{\phi^t} \sim \text{Dir}(\alpha_{\phi^t}) ; 
   (2) draw \( \mu_k \sim \text{vMF}(\mu_0, C_0) \); and
   (3) draw \( \kappa_k \sim \log \text{Normal}(m, \sigma^2) \).
4. For visual word \( v_d \) in document \( d \),
   (1) draw a topic \( z^v_d|\theta_d \sim \text{Mult}(\theta_d) \); and
   (2) draw a word \( v_d|z^v_d, \phi^v \sim \text{Mult}(\phi^{v}_{z^v_d}) \).
5. For textual word \( w_d \) in document \( d \),
   (1) draw a topic \( z^t_d|\theta_d \sim \text{Mult}(\theta_d) \); and
   (2) draw a word \( w_d|z^t_d, \phi^t \sim \text{Mult}(\phi^{t}_{z^t_d}) \).
6. Draw class label \( y_d|z_d \sim \max \text{–} \text{margin}(\bar{z}_d, \eta) \)

In the above, \( \theta_d \) denotes the distribution of the topics in the documents, \( \phi^v_k \) and \( \phi^t_k \) denote the multi-nomial distributions of topic-specific \( k \) over visual word \( v \) and textual word \( w \). \( \eta \) denotes class coefficients learned by the max-margin classifier, \( \bar{z}_d \) denotes the empirical ratio of topics that appear in the textual or visual modality to the total number of topics in the given document \( d \).

As shown in Fig. 2, textual topic \( z^t_d \) and visual topic \( z^v_d \) are sampled from the same topic distribution \( \theta(d) \). In the generative process of a document, text and image information are processed independently, and the topic distribution(\( \theta \)) and word distribution(\( \phi \)) are updated consequently. For the event class label, we use the max-margin classifier in the KGE-MMSLDA to jointly learn feature representation.

For textual input, our KGE-MMSLDA model considers the learning phase of the parameters in event class \( e \) as a single task; thus, there are \( L \) tasks, where \( L \) is the number of event types. Similarly, there are \( L \) tasks for the visual input. Consequently, there are \( I = 2 \cdot L \) tasks in the KGE-MMSLDA, and the linear discriminative function of each task \( i, i \in I \) is defined as:

\[
F_i(\eta, \bar{z}; w_i, v_i) = \eta_i^T \bar{z}
\]

where \( \bar{z} \) represents feature vectors of the topic assignment used to couple the topic model and max-margin classifier.

The probability of the max-margin classifier in the topic model is similar to that in [29]. As in that method, we use \( \eta \) as random variables to perform Bayesian estimation by transforming the classifier into a probabilistic distribution, which is formulated as:

\[
\phi_i(y_d|z_d, \eta) = \exp\left(-2\epsilon \max\left(0, T - y_d^T \eta_d^T \bar{z}_d\right)\right)
= \int_0^\infty \frac{1}{\sqrt{2\pi\lambda_d}} \exp\left(-\frac{(\lambda_d^i + c_{z_d}^i)^2}{2\lambda_d^i}\right)d\lambda_d^i
\]

where \( \lambda \) denotes the augmented variables and \( i \) is the index of the learning task. According to the above definition, the normalized posterior distribution of the KGE-MMSLDA is formulated as:

\[
q (\eta, \lambda, z, \theta, \phi_w) = \frac{p_0 (\eta, z, \theta, \phi_w) p(w, v, e|z, \theta, \phi_w) \phi(y, \lambda|z, \eta)}{\psi(y, w, v, e)}
\]

where \( \psi(y, w, v, e) \) is a normalized constant. \( p_0 (\eta, z, \theta, \phi_w) \) is the prior distribution, \( p(w, v, e|z, \theta, \phi_w) \) is the conditional probability of the generation process, and \( \phi(y, \lambda|z, \eta) \) is the posterior distribution that indicates the category information.

2) Model inference: In this part, collapsed Gibbs sampling is used to infer the joint posterior probability to sample the latent topic assignments \( z^t_d \) and \( z^v_d \) in the KGE-MMSLDA. The basic idea of collapsed Gibbs sampling is to integrate out the model parameters as a joint probability distribution. During sampling process, the new values of the latent variables are iteratively updated according to the previous states of the conditional distributions.

The conditional probabilities of latent variables \( z^t_d \), \( z^v_d \), and \( \lambda \) are formulated as below and the derivations of \( z^t_d \) and \( z^v_d \) are detailed in the Appendix.

\[
\begin{align*}
\bar{z}_d^t &\sim \text{Mult}(\lambda) \\
\bar{z}_d^v &\sim \text{Mult}(\lambda)
\end{align*}
\]

\[
\begin{align*}
\bar{z}_d^t &\sim \text{Mult}(\lambda) \\
\bar{z}_d^v &\sim \text{Mult}(\lambda)
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\begin{align*}
\bar{z}_d^t &\sim \text{Mult}(\lambda) \\
\bar{z}_d^v &\sim \text{Mult}(\lambda)
\end{align*}
\]

where \( \lambda \) denotes the augmented variables and \( i \) is the index of the learning task. According to the above definition, the normalized posterior distribution of the KGE-MMSLDA is formulated as:

\[
\phi_i(y_d|z_d, \eta) = \exp\left(-2\epsilon \max\left(0, T - y_d^T \eta_d^T \bar{z}_d\right)\right)
= \int_0^\infty \frac{1}{\sqrt{2\pi\lambda_d}} \exp\left(-\frac{(\lambda_d^i + c_{z_d}^i)^2}{2\lambda_d^i}\right)d\lambda_d^i
\]

where \( \lambda \) denotes the augmented variables and \( i \) is the index of the learning task. According to the above definition, the normalized posterior distribution of the KGE-MMSLDA is formulated as:

\[
\phi_i(y_d|z_d, \eta) = \exp\left(-2\epsilon \max\left(0, T - y_d^T \eta_d^T \bar{z}_d\right)\right)
= \int_0^\infty \frac{1}{\sqrt{2\pi\lambda_d}} \exp\left(-\frac{(\lambda_d^i + c_{z_d}^i)^2}{2\lambda_d^i}\right)d\lambda_d^i
\]

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\phi_i(y_d|z_d, \eta) = \exp\left(-2\epsilon \max\left(0, T - y_d^T \eta_d^T \bar{z}_d\right)\right)
= \int_0^\infty \frac{1}{\sqrt{2\pi\lambda_d}} \exp\left(-\frac{(\lambda_d^i + c_{z_d}^i)^2}{2\lambda_d^i}\right)d\lambda_d^i
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**sampling process:**

\[ p \left( z_{dn} = k \mid e_{-(dn)}, w, v, e, \eta, \lambda \right) \]

\[ \propto \frac{L^v_p}{n_{d,k}} C_L \left( \left\| \kappa \sum_{i \neq d, n, k} e_i + C_0 \mu_0 \right\| \right) \]

\[ C_L \left( \left\| \kappa \sum_{i \neq d, n, k} e_i + C_0 \mu_0 \right\| \right) \]

\[ p \left( \kappa | \kappa_{-k}, \ldots \right) \]

\[ \propto \frac{C_L \left( C_0 \right) C_L \left( \kappa_{-k} \right) \kappa^q_{-k}}{C_L \left( \left\| \kappa \sum_{i \neq d, n, k} e_i + C_0 \mu_0 \right\| \right)} \cdot \log \text{Normal} \left( \kappa | m, \sigma^2 \right) \]

where \( e_{-(dn)} \) denotes the topic assignments for all entities except \( e_{dn} \); \( e_{-(dn)} \) denotes the embedding of all entities, except \( e_{dn} \); \( n_{d,k} \) denotes the number of topics assigned to latent topic \( k \) except the current entity \( e \), and \( \kappa_k \) is drawn from the log-normal distribution. We first sample \( \kappa_k \) from samples from \( \log \text{Normal} \left( \kappa_k | m, \sigma^2 \right) \), and then sample the final \( \kappa_k \) from them. \( \mu_0 \), \( C_0 \), \( m \), and \( \sigma \) denote the hyper-parameters that control the corresponding vMF parameters \( \mu_k \) and \( \kappa_k \).

**η**

**sampling process:**

\[ p \left( \eta | z, \lambda \right) = \prod_{i=1}^{l} p \left( \eta_i | z_i, \lambda_i \right) = \prod_{i=1}^{l} N \left( \eta_i; \mu_i^n, \Sigma_i^n \right) \]

where the mean matrix and covariance matrix are written as follows:

\[ \mu_i^n = \Sigma_i^n \left( c \sum_{d=1}^{D} \lambda_{d,i}^j + cT \right) \]

\[ \Sigma_i^n = \left( \frac{1}{\sigma^2} I + c^2 \sum_{d} \frac{Z_d z_d^{j(T)}}{\lambda_d} \right)^{-1} \]

**λ**

**sampling process:**

\[ p \left( \lambda_{d,j}^n | z_j, \eta \right) \approx \frac{1}{\sqrt{2\pi \lambda}} \exp \left( - \frac{\left( \lambda_{d,j}^n + c \eta \right)^2}{2 \lambda} \right) \]

\[ = GIG \left( \lambda_{d,j}^n; \frac{1}{2}, c^2 \left( \eta^2 \right)^{-2} \right) \]

where \( GIG \left( x; p, a, b \right) \) is the generalized inverse Gaussian distribution, and \( GIG \left( x; p, a, b \right) = C \left( p, a, b \right) x^{p-1} \exp \left( - \frac{1}{2} \left( \frac{x}{a} + \frac{a}{x} \right) \right) \).

### C. Classification of a Multi-media Social Event

Following Gibbs sampling, we can estimate the parameters \( \phi^w, \phi^v \), and \( \theta_d \) of the textual and visual modalities as in [54].

\[ \phi_{d,j}^w = \frac{n_{d,j}^w + \beta_w}{\sum_{p=1}^{n_p} \left( n_{p,j}^w + \beta_w \right)} \]

\[ \phi_{d,j}^v = \frac{n_{d,j}^v + \beta_v}{\sum_{p=1}^{n_p} \left( n_{p,j}^v + \beta_v \right)} \]

\[ \theta_{d,k}^w = \frac{n_{d,k}^w + \alpha}{\sum_{k=1}^{K} \left( n_{d,k}^w + \alpha \right)} \]

\[ \theta_{d,k}^v = \frac{n_{d,k}^v + \alpha}{\sum_{k=1}^{K} \left( n_{d,k}^v + \alpha \right)} \]

As formulated above, we can obtain updated parameters \( \phi^w, \phi^v, \theta_{d,k}^w, \theta_{d,k}^v \), and \( \eta \). Given a new event document \( d_{new} \) composed of textual words \( w_{new} \) and visual words \( v_{new} \), we first sample topic distributions \( \theta_{d,k}^w \) and \( \theta_{d,k}^v \) of the document. We then obtain the average topic assignment vector of the new document \( z_{new} \) and parameters \( \eta \) after the Gibbs sampling phase. Ultimately, the category of the new document is predicted based on (2). Based on these predictions, we use a maximum majority voting to obtain the final category of the new document.

### IV. Experimental Results

We performed experiments on a publicly accessible dataset to verify the effectiveness of our proposed method. We first describe the dataset structure, detail the feature extraction process, and finally present the results and analysis.

#### A. Experimental Settings

Considering that the social analysis community does not have a mature multi-modal public dataset for topic mining and event classification, we created a dataset called HFUT-mmdata for social event analysis research using content from Flickr. We exploited the official API provided by Flickr to crawl images and the corresponding textual information, such as titles, descriptions and tags, for predefined events. The dataset contained 74,364 documents belonging to 10 types of events with approximately 7,000 to 9,000 documents inside each event. Each event, contained two types of modal information: text and images. The dataset was thus suitable for multi-modal social event analysis. It contained unrelated
C. Results and Analysis

In this section, we first present the evaluation of our method and the baselines on topic mining in terms of the topic coherence index and then assess their performance in terms of event classification. Finally, we present a qualitative evaluation of the multi-modal topics mined by our model.

1) Coherence of Topic Mining: Topic coherence has been regarded as an efficient measurement of the human ability to interpret the mined topics [56], [57]. To evaluate the quality of the topics mined by each model, we used pointwise mutual information (PMI) [58] to measure topic coherence.

We used the PMI [58] to assess the topic coherence of each model:

\[
PMI(k) = \sum_{j=1}^{N} \log \frac{p(w_j)}{p(w_i) p(w_j)}
\]

where \(N\) denotes the top word number of topic \(k\) and \(p(w_i)\) denotes the probability that word \(w_i\) occurs in a document. \(p(w_i, w_j)\) denotes the probability that \(w_i\) and \(w_j\) occur in the same document. Similar to most methods [38], a higher PMI score in ours signified a more coherent topic. Therefore, the top 10 words of each topic were used to calculate the PMI score. Moreover, 4,776,093 English Wikipedia articles [4] were used to compute the PMI score.

The number of output topics \(K\) was a hyperparameter. To choose the best value of \(K\) in the experiment, we calculated the PMI scores of the KGE-MMSLDA from \(K = 10\) to \(K = 100\). From Fig. 4, we see that the PMI scores increased rapidly from \(K = 10\) to \(K = 40\), and the growth rate slowed and eventually stabilized from \(K = 50\) to \(K = 100\). In general, a larger value of \(K\) leads to longer training, whereas a smaller value of \(K\) worsens performance. Therefore, we fixed \(K = 40\) to compare the PMI scores of the baselines. For other datasets, the balance point of \(K\) should be obtained experimentally.

Table 1 shows the PMI scores of all models run on our HFUT-mmdata dataset. Based on these results, we can draw the following conclusions:

1. The MMLDA outperformed the traditional LDA, which shows that multi-modal information can improve the interpretation of the topics.
2. Link-KGE-LDA and Corr-KGE-LDA were better than traditional LDA and MMLDA, which means that the topic model with knowledge priors improved topic interpretability significantly.

\[\text{https://wordnet.princeton.edu/}\]
Our KGE-MMSLDA outperformed traditional LDA and MMLDA model by 9-11 points. The main conclusion is that our KGE-MMSLDA modeled the two modalities' information and knowledge prior information in a unified topic model. Meanwhile, KGE-MMSLDA performed better than Link-KGE-LDA and Corr-KGE-LDA, which means that incorporating supervised information into the topic model using the max-margin classifier and multimodal information was helpful for mining interpretable topics.

Table I shows textual topics together with their PMI topic coherence scores mined from our collected dataset using the LDA and the KGE-MMSLDA models. We selected the five best-matched topics from the two models. For closely related topics, the topic words learned by the models were similar. The following conclusions can be drawn from the table as well, however: First, for the same topic, the PMI score calculated using our topic model was higher than that obtained by the traditional LDA. Second, our model mined unique words in the same topic. For the first topic—“Wall Street”—the KGE-MMSLDA found “square,” which was not discovered by the traditional LDA. Similarly, for several other topics, such as “Maritime Dispute,” “World Cup,” “Politics,” and “Military,” the KGE-MMSLDA found “Bali,” “Brasil,” “peace,” and “veterans,” which were missed by the LDA model.

2) Event Classification Evaluation: To verify the performance of the KGE-MMSLDA, we used the libSVM classifier to train event features. We compared the KGE-MMSLDA with several related baseline methods. For each, we set the symmetric Dirichlet priors to \(\alpha = 1, \beta_w = 0.01\), and \(\beta_v = 0.01\). We conducted a series of experiments with different numbers of topics and used the best classification accuracy of each method for comparison.

Social Event Classification: In the classification phase, the training and testing sets were setup using the following strategy: We split the dataset into five subsets and then iteratively ran all the methods on them using five-fold cross-validation. In each run of each algorithm, we iteratively selected one subset as the testing set and remaining four as training sets. Table III shows the average accuracy of the methods over the five subsets. To verify the performance of the KGE-MMSLDA for event classification, we added the results for a state-of-the-art supervised topic model (MMSLDA) to Table III for comparison. The LDA delivered the worst performance and the proposed KGE-MMSLDA the best of all methods. Link-KGE-LDA and Corr-KGE-LDA performed better than the traditional LDA, which means that using entity embedding learned from a large-scale knowledge base helped them implement discriminating event representation. The MMLDA outperformed the LDA because the modeling of text and images yield better results of feature representation than modeling text only. The KGE-MMSLDA outperformed the Corr-KGE-LDA, which means that supervised information was useful for social event classification. It also outperformed the MMSLDA and the other four methods, which shows that integrating a max-margin classifier and multi-modal feature representation into a unified topic model significantly helped improve classification performance, and knowledge embedding helped with event classification.

Number of Topics \(K\): We explored the impact of the number of topics \(K\) on event classification performance. An appropriate number of topics \(K\) can help us determine a balance between the training time of the topic model and event classification performance. We evaluated KGE-MMSLDA’s accuracy together with that of mainstream methods using different numbers of topics. Each model’s accuracy is shown in Fig. 5 where the value of parameter \(K\) was changed from 10 to 50. As shown in the figure, our proposed KGE-MMSLDA model stably outperformed the other methods. Note that its classification accuracy increased rapidly when \(K\) was changed from 1 to 25, and remained stable when \(K\)’s was in the range of 30–50. The recommended value for \(K\) is 40 for our dataset.

3) Multi-Modal Topic Mining: In this study, textual information and visual information were modeled using a unified KGE-MMSLDA. To show the multi-modal topic mining capability of the KGE-MMSLDA, we qualitatively show its mined topics in text and images in Fig. 6.

In a qualitative experiment, we set \(K\) to 40 and selected four topics for analysis and visualization. We first sorted the top textual words and image topic words by \(p(w|z)\) and \(p(v|z)\), respectively, and list the top six topics and their corresponding images for comparison. From Fig. 6, most learned textual words and images were interpretable, such as Adele, Syria, ISIS, NSA, and surveillance, and were clearly related to certain events. For example, the textual words in topic 10 were Adele,
TABLE II
LEARNED TOPICS OF TRADITIONAL LDA AND KGE-MMSLDA MODEL.

<table>
<thead>
<tr>
<th>LDA</th>
<th>KGE-MMSLDA</th>
</tr>
</thead>
<tbody>
<tr>
<td>street</td>
<td>sea</td>
</tr>
<tr>
<td>occupy</td>
<td>tune</td>
</tr>
<tr>
<td>day</td>
<td>island</td>
</tr>
<tr>
<td>photos</td>
<td>hotels</td>
</tr>
<tr>
<td>march</td>
<td>coney</td>
</tr>
<tr>
<td>ows</td>
<td>dr</td>
</tr>
<tr>
<td>2011</td>
<td>hotel</td>
</tr>
<tr>
<td>109,7</td>
<td>61.23</td>
</tr>
</tbody>
</table>

TABLE III
CLASSIFICATION ACCURACY OF SOME MAINSTREAM METHOD.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDA</td>
<td>0.682</td>
</tr>
<tr>
<td>MMLDA</td>
<td>0.725</td>
</tr>
<tr>
<td>Link-KGE-LDA</td>
<td>0.713</td>
</tr>
<tr>
<td>Corr-KGE-LDA</td>
<td>0.710</td>
</tr>
<tr>
<td>MMSLDA</td>
<td>0.763</td>
</tr>
<tr>
<td>KGE-MMSLDA</td>
<td>0.851</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

In this paper, we proposed a knowledge embedding-based max-margin topic model for multi-modal event analysis called the KGE-MMSLDA that combines knowledge embedding with the multi-modal topic model to jointly learn an event’s representation and the classifier in a unified framework. The KGE-MMSLDA not only exploits multi-media data with prior knowledge to mine more coherent and meaningful topics with good representation, but also integrates supervised information into the multi-modal topic model as a regularization term to obtain discriminative representation for social event classification. The results of experiments on a large multi-modal dataset demonstrated that the KGE-MMSLDA outperforms mainstream approaches for social event analysis and classification. Despite its effectiveness, the time complexity of the proposed method is high, because it requires three sampling processes for text, image, and knowledge. Two possible improvements should be explored in future work: (1) optimizing the sampling strategy to boost sampling speed; and (2) determining other kind of information to incorporate into the model to improve accuracy.

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REFERENCES


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VI. APPENDIX: INFERENCE FOR CONDITIONAL DISTRIBUTION OF TEXTUAL, VISUAL AND KNOWLEDGE EMBEDDING TOPIC ASSIGNMENT VARIABLE \( Z^w_d, Z^v_d \)

According to the (4) in this paper, the normalized posterior distribution of KGE-MMSLDA can be formulated as:

\[
q(\eta, \lambda, Z, \Theta, \Phi_w, \Phi_v) \propto p_0(\eta, \lambda) \cdot p(w, v, e | Z, \Phi_w, \Phi_v) \Phi(y, \lambda | Z, \eta) = p_0(\eta) \cdot \frac{p(Z | w, v) \cdot p(w)}{p(Z)} \prod_{i=1}^{L} \Phi(y_i, \lambda_i | Z, \eta) \tag{19}
\]

Based on the data augment formula of Gibbs MedLDA in (29), by integrating out the Dirichlet variables \((\Theta, \Phi_w, \Phi_v)\), (19) can be simplified as the collapsed posterior distribution as (20).

\[
q(\eta, \lambda, Z) \propto p_0(\eta) \cdot \prod_{i=1}^{L} \Phi(y_i, \lambda_i | Z, \eta) = p_0(\eta) \cdot \prod_{i=1}^{L} \Phi(y_i, \lambda_i | Z, \eta) \tag{20}
\]

where \(w, v, e\) represent textual, visual and knowledge embedding input, respectively; \(\eta\) denotes the weights of classifier, \(\lambda\) denotes the augmented variable, \(y_i\) is a real label, \(L\) is the number of classifiers. As \(p_0(\eta)\) is a known prior distribution, we ignore this term and (20) can be written as:

\[
p(Z^w_d = k | Z^w_{-d}, w, v, e, \eta, \lambda) \propto p(Z^w_d = k | Z^{w}_{-d}, w) \cdot \prod_{i=1}^{L} \Phi(y_i, \lambda_i | Z, \eta) \tag{21}
\]

where \(p(Z^w_d = k | Z^{w}_{-d}, w)\) is the product of two parts: LDA model and Gibbs regression model. In the following, we will derive the two parts respectively. As image input \(v\) and knowledge entity input \(e\) are irrelevant to the conditional distribution of \(Z^v_d\), the first part in the right of (21) can be rewritten as:

\[
p(Z^w_d = k | Z^{w}_{-d}, w) = \frac{p(w | Z^w_d = k | Z^{w}_{-d}, w) \cdot p(w)}{p(w)} = \frac{\prod_{i=1}^{L} \Phi(y_i, \lambda_i | Z, \eta)}{p(w)} \tag{22}
\]

where

\[
\Gamma(\cdot) \text{ is the Gamma function, } n_{d,k}^{(w)} \text{ is the number of textual words assigned to the latent topic } k \text{ of document } d, \text{ except current textual word } w, \ n_{d,k}^{(w)} \text{ is the number of textual words assigned to the latent topic } k \text{ of document } d, \ n_{k,t}^{(w)} \text{ is the number of textual word } t \text{ of topic } k \text{ that appear in all documents, except the current textual word } w, \ n_{d,t} \text{ is the number of textual words } t \text{ of topic } k \text{ that appear in all documents; } \alpha \text{ and } \beta_w \text{ are the priors of the Dirichlet distribution.}
\]

In fact, the second part of the right in (21) can be seen as a cumulative product of a series of classifiers since our model is used for \(L\) classification tasks.

As defined in the Lemma 2(Scale Mixture Representation) of (29), the individual distribution of a single classifier can be expressed as

\[
\phi_i(y_i, \lambda_i | Z, \eta) = \frac{1}{\sqrt{2\pi \lambda_d}} \exp\left(-\frac{(\lambda_d y_i + c \eta_i)^2}{2 \lambda_d}\right)
\]

where \(\lambda_d = \frac{1}{\sum_{d=1}^{D} \tau_d^2} \sum_{d=1}^{D} \tau_d\), \(\tau_d\) denotes the cost of making a wrong prediction, \(y_i\) is a real label, \(\eta_i\) is the weight of the classifier, \(\lambda\) denotes the augmented variable, \(\tau_d = \frac{1}{N} \sum_{n=1}^{N} Z_{n,d}\) denotes average topic assignment of document \(d\), \(\eta\) is the posterior mean, which can be calculated by using following formulation. Readers are referred to the Section 4 of (29) for details:

\[
\epsilon_i = \frac{1}{v_i + c^2} \sum_{d=1}^{D} \tau_d \cdot \tau_d^T \lambda_d^{-1}
\]

\[
\mu_i = \epsilon_i (c \cdot \lambda_d + c \cdot \tau_d \lambda_d^{-1})
\]

According to (23), (24) and (25), Gibbs regression model can be expressed as

\[
\phi_i(y_i, \lambda_i | Z, \eta) = \exp\left(-\frac{\lambda_d y_i + c \eta_i}{\lambda_d} - \frac{2v_i \eta_i}{2\eta_i} N_d^\eta \lambda_d \eta_i \sum_{k=1}^{K} \sum_{d=1}^{D} \sum_{w=1}^{W} \sum_{n=1}^{N} \frac{n_{d,k}^{(w)}}{\lambda_d} \right)
\]

where \(N_d^\eta \lambda_d \eta_i \sum_{k=1}^{K} \sum_{d=1}^{D} \sum_{w=1}^{W} \sum_{n=1}^{N} \frac{n_{d,k}^{(w)}}{\lambda_d} \) denotes the discriminant function value of the words except the current word \(w\), \(N_d^\eta \) is the number of textual words for document \(d\), \(\eta\) denotes the classifier weights, \(\lambda\) denotes the augmented variable, \(y_i\) is a real label.

Thus, by combining the derivation formulas of the above two parts, we can get the conditional distribution of \(Z^v_d\).

\[
p(Z^v_d = k | Z^{v}_{-d}, w, v, e, \eta, \lambda) \propto \frac{n_{d,k}^{(w)} + \alpha}{\sum_{k=1}^{K} \sum_{d=1}^{D} \sum_{n=1}^{N} n_{d,k}^{(w)} + \alpha} \cdot \exp\left(-\frac{c(\lambda_d y_i + c \eta_i)^2}{2 \lambda_d} \eta_i \sum_{k=1}^{K} \sum_{d=1}^{D} \sum_{w=1}^{W} \sum_{n=1}^{N} \frac{n_{d,k}^{(w)}}{\lambda_d} \right)
\]
which is a product of LDA model and Gibbs regression model according to (20).

\[
p(Z_{d}^{v} = k | Z_{(d)}^{v}, w, v, e, \eta, \lambda) \\
\propto p(Z_{d}^{v} = k | Z_{(d)}^{v}, v) \prod_{i=1}^{L} \varphi_i(y_i, \lambda_i | Z, \eta) \tag{28}
\]

Similar to (22) and (23), \(p(Z_{d}^{v} = k | Z_{(d)}^{v}, v)\) and \(\varphi_i(y_i, \lambda_i | Z, \eta)\) can be defined as

\[
p(Z_{d}^{v} = k | Z_{(d)}^{v}, v) \propto \frac{n_{k,l}^{(v)} + \beta_v}{\sum_{d=1}^{D_v} (n_{k,l} + \beta_v)} \cdot \frac{n_{d,k}^{(v)} + \alpha}{\sum_{k=1}^{K} (n_{d,k}^{(v)} + \alpha)} \tag{29}
\]

\[
\varphi_i(y_i, \lambda_i | Z, \eta) \propto \exp\left(\frac{\gamma_{d}^{(v)T}(e \lambda_d^{(v)} + \lambda_i^{(v)}) \eta_{i,k} - \frac{2N_{d}^{(v)-2}}{N_{d}^{(v)}} \Lambda_{d,n}^{(v)} \eta_{i,k} - \frac{\gamma^2}{2} \eta_{i,k}^2}{N_{d}^{(v)} \lambda_d^{(v)}}\right) \tag{30}
\]

where \(N_{d}^{(v)}\) is the number of visual words for document \(d\). \(\eta\) denotes the classifier weights, \(\lambda\) denotes the augmented variable, \(y_i\) is a real label. \(n_{d,k}^{(v)}\) is the number of visual words assigned to the latent topic \(k\) of document \(d\), except current visual word \(v\). \(n_{k,l}^{(v)}\) is the number of visual words assigned to latent topic \(k\) of document \(d\); \(n_{k,l}^{(v)}\) is the number of visual words of topic \(k\) that appear in all documents, except the current visual word \(v\); \(n_{k,l}\) is the number of visual words of topic \(k\) that appear in all documents; and \(\alpha\) and \(\beta_v\) are the priors of the Dirichlet distribution.

We can derive the conditional distribution equation of \(Z_{d}^{v}\) by combining (29) and (30).

\[
p(Z_{d}^{v} = k | Z_{(d)}^{v}, w, v, e, \eta, \lambda) \propto \frac{n_{k,l}^{(v)} + \beta_v}{\sum_{d=1}^{D_v} (n_{k,l} + \beta_v)} \cdot \frac{n_{d,k}^{(v)} + \alpha}{\sum_{k=1}^{K} (n_{d,k}^{(v)} + \alpha)} \prod_{i=1}^{L} \exp\left(\frac{\gamma_{d}^{(v)T}(e \lambda_d^{(v)} + \lambda_i^{(v)}) \eta_{i,k} - \frac{2N_{d}^{(v)-2}}{N_{d}^{(v)}} \Lambda_{d,n}^{(v)} \eta_{i,k} - \frac{\gamma^2}{2} \eta_{i,k}^2}{N_{d}^{(v)} \lambda_d^{(v)}}\right) \tag{31}
\]