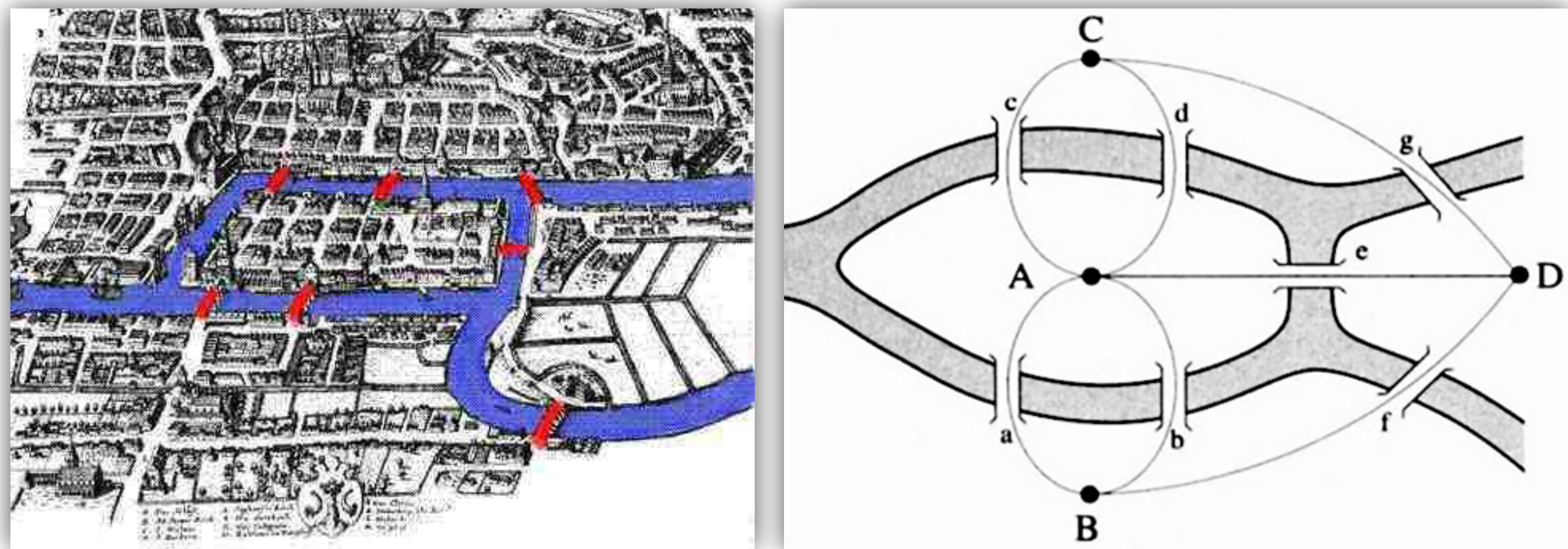
# Learning on Partial-Order Hypergraphs

Fuli Feng[+], Xiangnan He[+], Yiqun Liu[*], Liqiang Nie[#], Tat-Seng Chua[+]
National University of Singapore[+], Tsinghua University[*], Shandong University[#]

25/Apr/2018

# Simple Graph

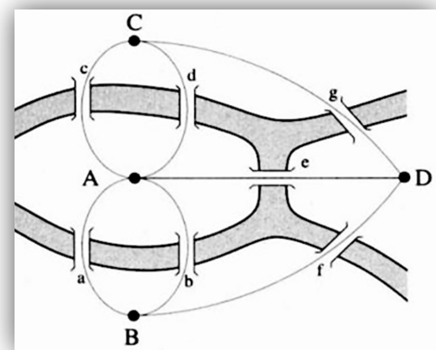



**Solution for the Konigsburg Bridge problem proposed by Euler in 1736.**

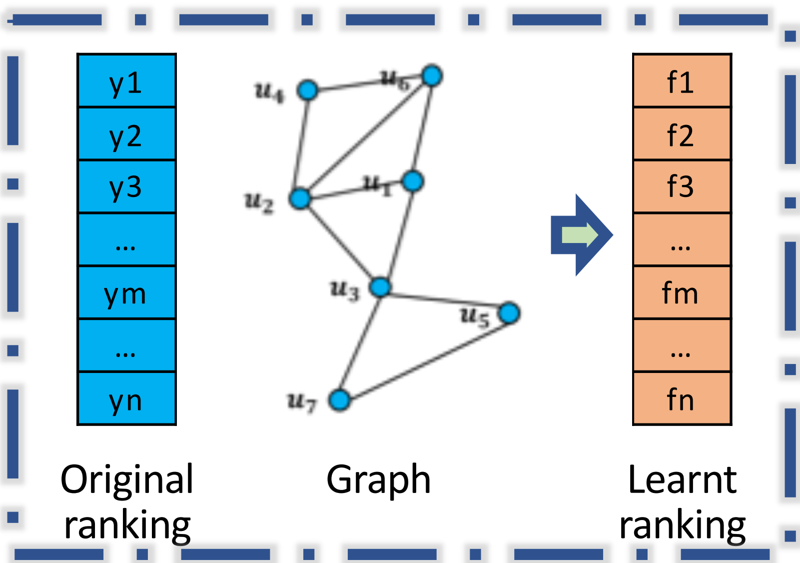
- World Wide Web; social networks; structured knowledge; user-product interactions; supply chains; cash transaction flow; Co-authorship networks; etc.

# Learning on Simple Graphs

- World Wide Web → Find important Web pages
- Social networks → Target potential customers
- User-product interactions → Recommend products
- ……




**Unsupervised**, **semi-supervised** or **supervised** learning




$$\Gamma = \mathcal{L} + \lambda \mathcal{G}$$

Squared loss

$$\mathcal{L} = \sum_{i=1} (y_i - f_i)^2$$

Graph regularizer

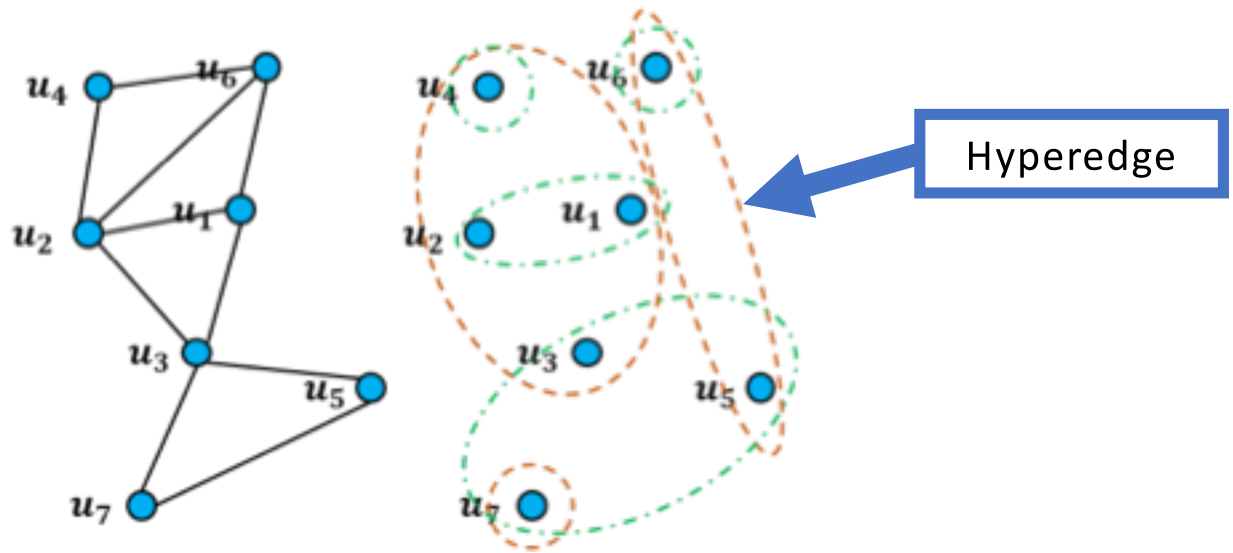
$$\mathcal{G} = \sum_{i,j=1} a_{ij} (f_i - f_j)^2$$

Adjacency

Smoothness

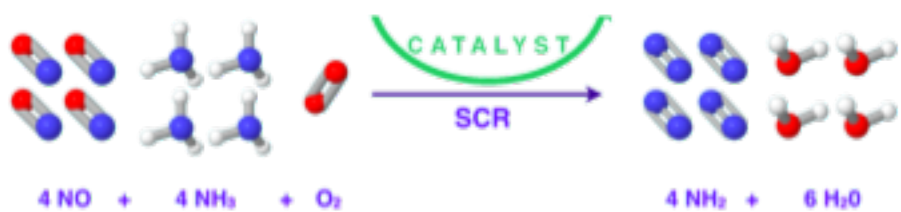
# Hypergraph

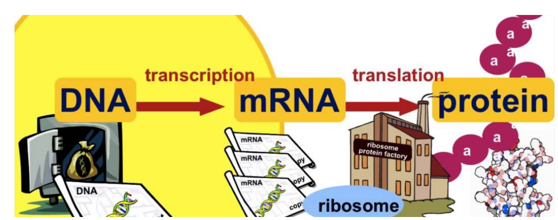
| Uni | City | Sal |
|---|---|---|
| $u_1$ | $c_1$ | $s_2$ |
| $u_2$ | $c_1$ | $s_2$ |
| $u_3$ | $c_1$ | $s_1$ |
| $u_4$ | $c_1$ | $s_4$ |
| $u_5$ | $c_2$ | $s_1$ |
| $u_6$ | $c_2$ | $s_3$ |
| $u_7$ | $c_3$ | $s_1$ |




(a) Data (b) Simple Graph (c) Hypergraph (d)

- Hypergraph VS. Simple Graph: captures the **higher-order relations** among vertices.



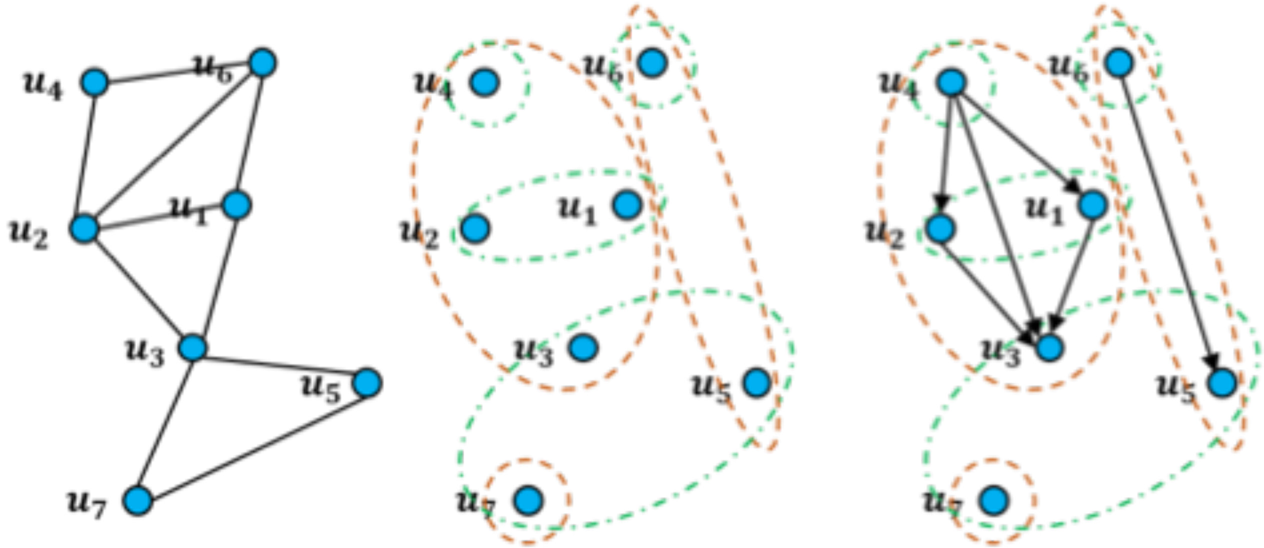



# Outline

- Background
- **Methodology**
- Experiment

# Partial-Order Hypergraph

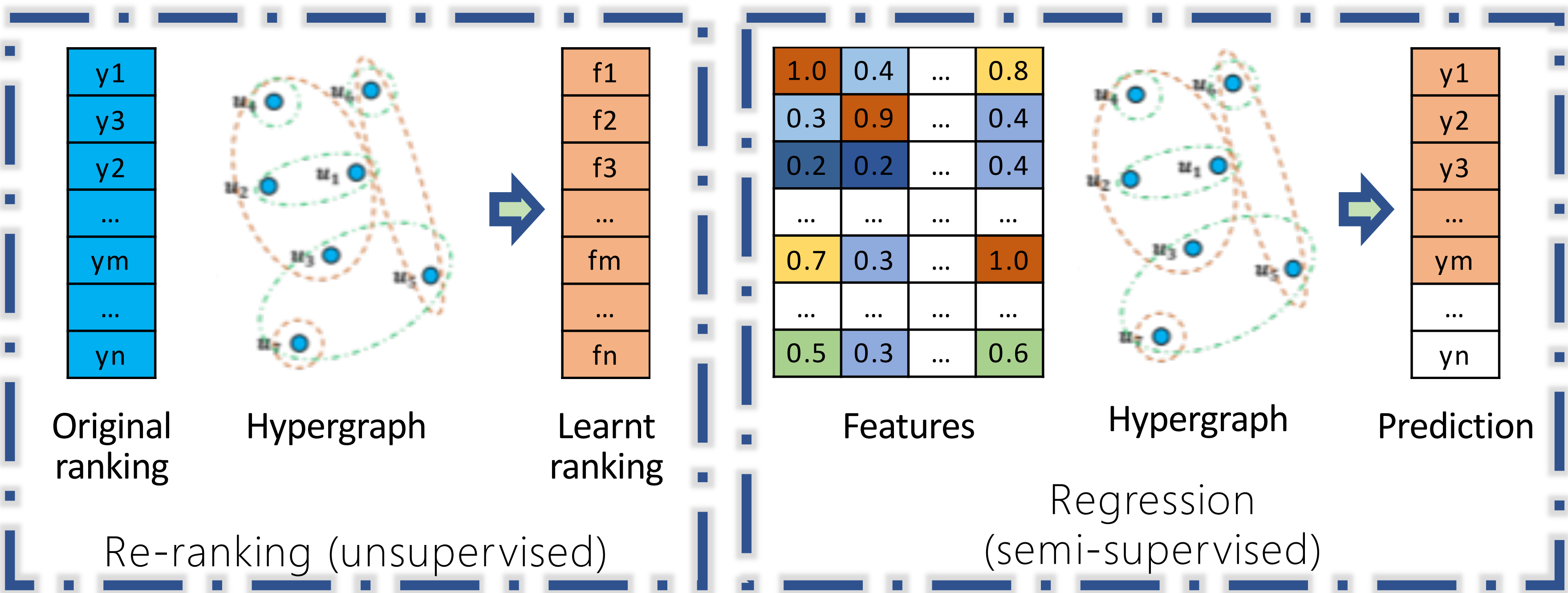
| Uni | City | Sal |
|---|---|---|
| $u_1$ | $c_1$ | $s_2$ |
| $u_2$ | $c_1$ | $s_2$ |
| $u_3$ | $c_1$ | $s_1$ |
| $u_4$ | $c_1$ | $s_4$ |
| $u_5$ | $c_2$ | $s_1$ |
| $u_6$ | $c_2$ | $s_3$ |
| $u_7$ | $c_3$ | $s_1$ |




(a) Data (b) Simple Graph (c) Hypergraph (d) Partial-Order Hypergraph

- Salary level (universities); Number of clicks (Web page); Number of purchases (Products)
- POH VS. Hypergraph: captures the ordering relationships commonly exist, such as in graded categorical features and numerical features.

# Learning on Hypergraphs

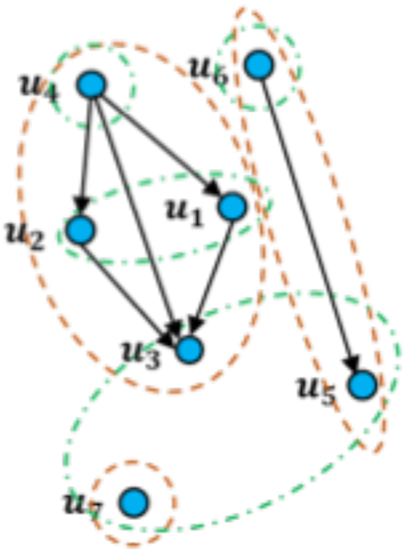



$$\Gamma = \mathcal{L} + \lambda \mathcal{G}$$

- Task-specific loss $\mathcal{L}$ and hypergraph regularization term $\mathcal{G}$ (similar vertices get similar predictions).

$$\mathcal{G} = \sum_{i=1}^{N} \sum_{j=1}^{N} \underbrace{g(\mathbf{x_i}, \mathbf{x_j}) \sum_{k=1}^{M} H_{ik} H_{jk}}_{\text{strength of smoothness}} \underbrace{\left\| f(\mathbf{x_i}) - f(\mathbf{x_j}) \right\|^2}_{\text{smoothness}} .$$

# Learning on POH




Partial-Order Hypergraph

- $\mathcal{L}$ and $\mathcal{G}$ follows the same formulation in hypergraph-based learning

- Task-specific and prior knowledge driven partial-order relations

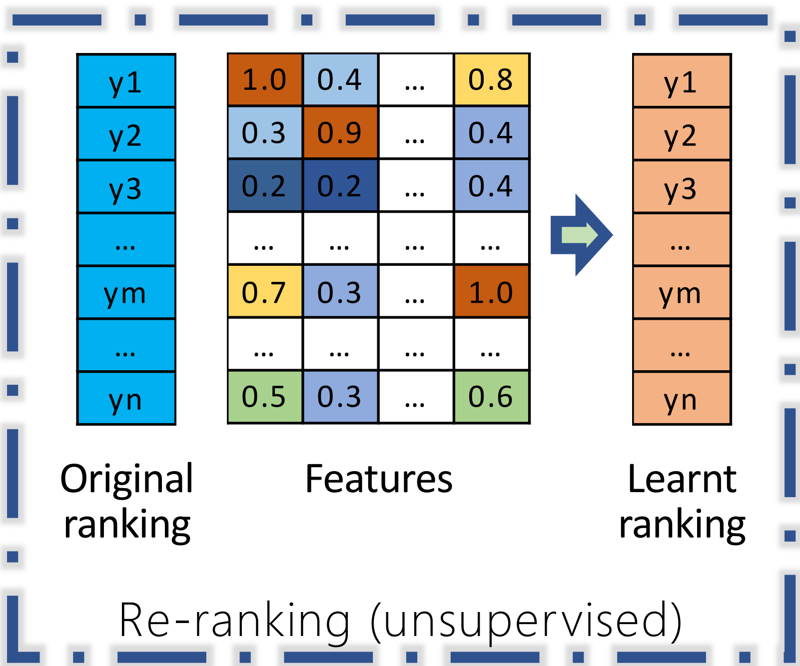
$$\{p^r(\mathbf{x_i}, \mathbf{x_j}) \rightarrow q^r(f(\mathbf{x_i}), f(\mathbf{x_j})) \mid r = 1, 2, \cdots, R\}$$

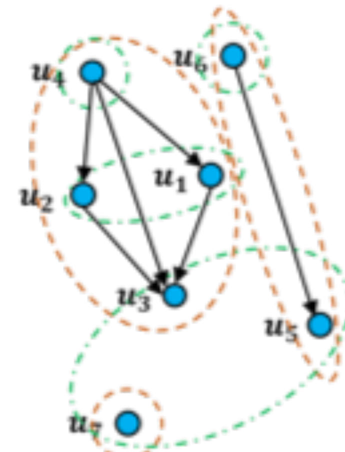

$$\Gamma = \mathcal{L} + \lambda\mathcal{G} + \beta\mathcal{P}$$

- $\mathcal{P}$ is a regularization term to encode partial-order rules:

$$\mathcal{P}_1 = \sum_{r=1}^{R} \frac{a_r}{|\mathbf{H^r}|} \sum_{\{i,j|H_{ij}^r \neq 0\}} (1 - q^r(\hat{\mathbf{y}}_\mathbf{i}, \hat{\mathbf{y}}_\mathbf{j}))H_{ij}^r,$$

# Learning on POH: University Ranking




- **Salary**: $salary_{>}(\boldsymbol{x_i}, \boldsymbol{x_j}) \rightarrow rank_{<}(\boldsymbol{x_i}, \boldsymbol{x_j})$;
- **NCEE**: $(NCEE_{>}(\boldsymbol{x_i}, \boldsymbol{x_j}) \rightarrow rank_{<}(\boldsymbol{x_i}, \boldsymbol{x_j}))$;

$$\Gamma = \mathcal{L} + \lambda\mathcal{G} + \beta\mathcal{P}$$

Squared loss

$$\mathcal{L} = \sum_{i=1}(y_i - f_i)^2$$

Hypergraph regularizer

$$\mathcal{G} = \sum_{i=1}^{N}\sum_{j=1}^{N} \underbrace{g(\mathrm{x_i}, \mathrm{x_j}) \sum_{k=1}^{M} H_{ik}H_{jk}}_{\text{strength of smoothness}} \underbrace{\|f(\mathrm{x_i}) - f(\mathrm{x_j})\|^2}_{\text{smoothness}}.$$

Partial-order regularizer

$$\mathcal{P} = \sum_{r=1}^{R} \frac{a_r}{|\mathrm{H^r}|} \sum_{\{i,j|H_{ij}^r \neq 0\}} ReLU((f_i - f_j)H_{ij}^r)$$

# Outline

- Background
- Methodology
- Experiment

# Experiment: University Ranking

- **Chinese university ranking dataset**:

  ***Ground truth & Original ranking:*** average fusion of traditional rankings in 2016 and 2015, respectively.

  ***Features:*** official platforms, mass media, social media, et al.

- **Evaluation metrics**: Mean absolute error (MAE), Kendall's Tau, and Spearman rank.

- **Compared methods**:

  *Baselines:* Simple Graph; Hypergraph; GMR.

  *POH methods:* POH-Salary ($salary_{>}(\boldsymbol{x_i}, \boldsymbol{x_j}) \rightarrow rank_{<}(\boldsymbol{x_i}, \boldsymbol{x_j})$);
  POH-NCEE ($NCEE_{>}(\boldsymbol{x_i}, \boldsymbol{x_j}) \rightarrow rank_{<}(\boldsymbol{x_i}, \boldsymbol{x_j})$);
  POH-All.

Feng, Fuli, et al. "Computational social indicators: a case study of chinese university ranking." *ACM SIGIR*. 2017.

# Experiment: University Ranking

- Hypergraph > Simple Graph;
- POH-based methods >>> Baselines;
- POH-All > POH-Salary & POH-NCEE

**Table 1: Performance comparison among our methods and baselines.**

| Methods | MAE | Kendall's Tau | Spearman |
|---|---|---|---|
| Simple Graph | $0.074 \pm$ 9e-3 | $0.870 \pm$ 2e-2 | $0.970 \pm$ 8e-3 |
| Hypergraph | $0.067 \pm$ 7e-3 | $0.876 \pm$ 9e-3 | $0.974 \pm$ 5e-3 |
| GMR | $0.065 \pm$ 7e-3 | $0.871 \pm$ 3e-2 | $0.970 \pm$ 1e-2 |
| POH-Salary | $0.054 \pm$ 1e-2* | $0.892 \pm$ 1e-2* | $0.979 \pm$ 5e-3* |
| POH-NCEE | $0.055 \pm$ 1e-2* | $0.893 \pm$ 9e-3* | $0.978 \pm$ 5e-3* |
| POH-All | **$0.053 \pm$ 1e-2*** | **$0.898 \pm$ 1e-2**** | **$0.980 \pm$ 6e-3**** |

$*$ and $**$ denote that the corresponding performance is significantly better ($p$-value < 0.05) than all baselines and all other methods, respectively.

# Experiment: Popularity Prediction

- **Micro-video dataset**: 9,719 micro-videos from Vine.

  ***Features:*** user activities, object distribution, aesthetic description, sentence embedding, etc.

- **Evaluation metrics**: Kendall's Tau, and Spearman rank.

- **Compared methods**:

  *Baselines:* Simple Graph; Hypergraph; GCN.
  *POH methods:*
  - POH-Follow ($followers_{>}(\boldsymbol{x}_i, \boldsymbol{x}_j) \rightarrow popularity_{>}(\boldsymbol{x}_i, \boldsymbol{x}_j)$);
  - POH-Loop ($loops_{>}(\boldsymbol{x}_i, \boldsymbol{x}_j) \rightarrow popularity_{>}(\boldsymbol{x}_i, \boldsymbol{x}_j)$);
  - POH-All.

Chen, Jingyuan, et al. "Micro tells macro: Predicting the popularity of micro-videos via a transductive model." *ACM MM*. 2016.

# Experiment: Popularity Prediction

- **Hypergraph** > **Simple Graph**;
- POH-based methods >>> **Hypergraph**;
- POH-based methods >>> **GCN** > **Hypergraph**,

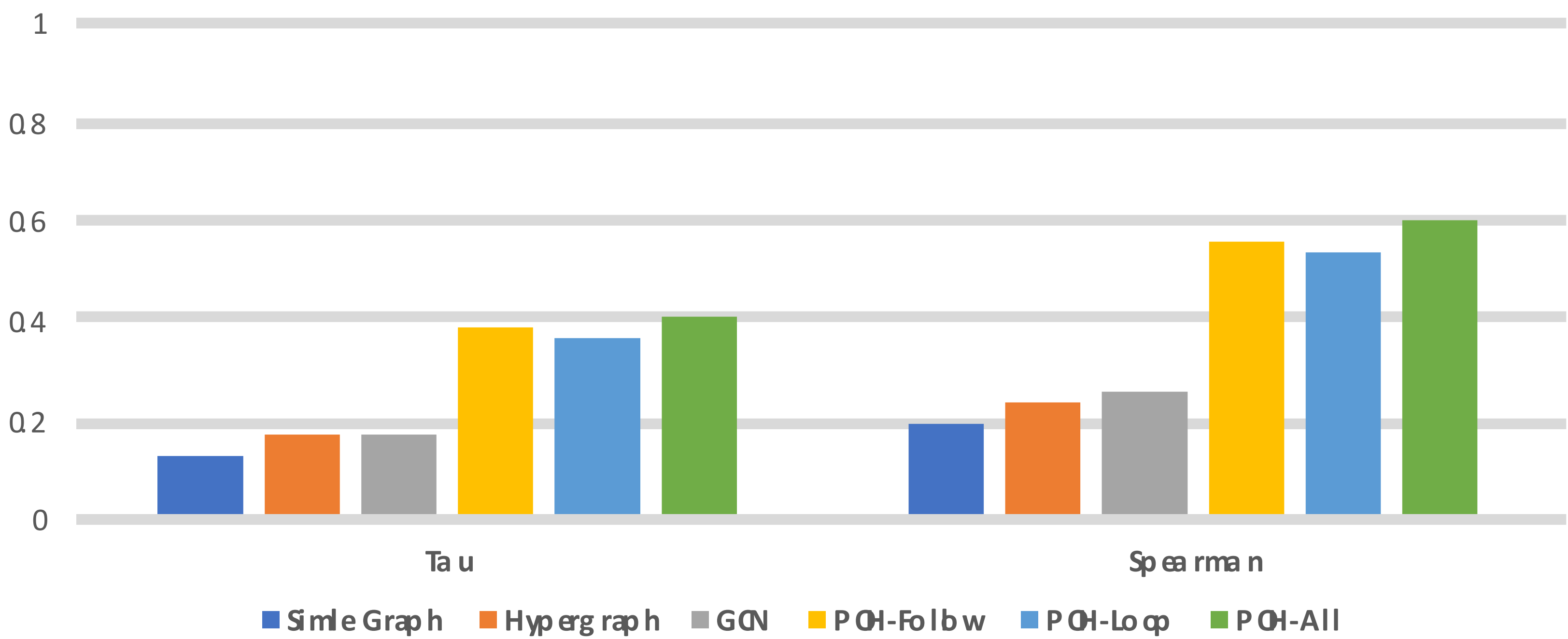



**Figure 1: Performance comparison among our methods and baselines.**

# Conclusion & Future Work

- We proposed a novel **partial-order hypergraph** that enhances the conventional hypergraph.
- We **generalized** the existing **graph-based learning** methods to partial-order hypergraphs.
- The proposed POH-based learning **significantly outperforms** conventional graph-based learning methods.

- We would optimize the spatial complexity of POH via **discrete hashing** techniques.
- We plan to explore **automatic extraction** of partial-order relations and construction of POH.

Thanks