Design of diffractive phase element for modulating the electric field at the out-of-focus plane in a lens system

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Received 22 March 2012; revised 10 June 2012; accepted 15 June 2012; posted 15 June 2012 (Doc. ID 165243); published 16 July 2012

We demonstrate an efficient method to design the diffractive phase element for modulating the electric field at the out-of-focus plane of a lens system by using an equivalent Fresnel diffraction in free space. In the monochromatic illumination, we show an example to certify the validity of our method experimentally. In the nonmonochromatic illumination, we theoretically display that the spectral beam splitting and highly confined intensity can be obtained simultaneously at the out-of-focus plane, which has the potential in the solar concentrating system and optical encryption. © 2012 Optical Society of America

OCIS codes: 050.1940, 050.1970, 260.1960.

1. Introduction

The diffractive phase element (DPE) has been well known for its capacity of modulating the electric field in the Fresnel or Fraunhofer region of an optical beam after the theoretical and technical development in the past half century. Generally, the DPE is used to control the far-field intensity in the material processing [1] and inertial confinement fusion [2]. For an incident beam parallel to the optical axis, its far-field intensity can be observed in the focal plane of a Fourier lens. In some applications, i.e., three-dimensional (3D) display [3,4] and multiwavelength beam splitting [5], designing the DPE in the Fresnel region is easier than that in the Fraunhofer region. In the Fresnel diffraction, the satisfying results are easily obtained only when the distance between the DPE and the target plane is much larger in comparison with the aperture of the DPE [6]. This induces that the intensity in the target plane cannot be well confined around the optical axis. In the solar concentrating system [5], the spectral beam splitting and a well-confined intensity are pursued synchronously. To solve this problem, we suggest that a Fourier lens should be introduced while the target plane is located at an out-of-focus plane [Fig. 1(a)].

The simplest way to design the DPE in Fig. 1(a) is to consider the DPE and lens as one optical component. To realize a desired field at the target plane, the required phase distribution of this composite component can be found by using any phase retrieval algorithm for Fresnel diffraction. The phase of DPE alone is then obtained by subtracting the lens phase from the phase of the composite component [7]. In this method, the phase modulation range of DPE is 2π, which is very efficient in the monochromatic illumination. However, for the nonmonochromatic illumination, the phase modulation range of DPE is usually larger than 2π [8]. In addition, the phase delay that comes from the lens is strongly dependent on the wavelength of the incident light. For

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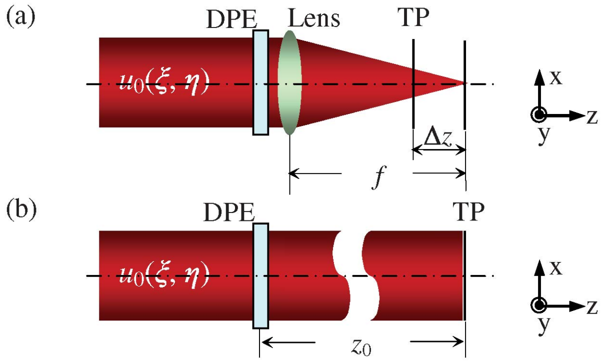

Fig. 1. (Color online) Schematic of modulating the electric fields by a DPE at an out-of-focus plane of a lens (a) and at a plane where $z = z_0$ in the free space (b). The electric field of the incident light is $u_0(\xi,\eta)$. f is the focus of the lens. Δz is the distance between the target plane (TP) and the focal plane.

the multicolor DPE, it is complicated to carry out the operation of subtracting the lens phase. Therefore, this method does not work well in the nonmonochromatic illumination.

We find that the optical system in Fig. 1(a) is equivalent to the Fresnel diffraction in free space in Fig. 1(b). In that case, to design the DPE in Fig. 1(a), one substituted approach can be found by designing the DPE in Fig. 1(b). In Fig. 1(b), any phase retrieval algorithm based on the Fresnel diffraction can be implemented to design the DPE without considering the lens, which is demonstrated in Section 2. To check the validity of our method, we display an example for single-wavelength illumination in Section 3. In Section 4, we investigate the design of multicolor DPE by using the configuration of Fig. 1(a).

2. Theory

A Fraunhofer diffraction pattern, whose observation distance is infinite, can be obtained on the focal plane of a focusing lens. In contrast, a Fresnel diffraction pattern is an image on a plane at a finite distance. Intuitively, the equivalent image should be located at a plane distant from the focal plane of the lens when a focusing lens is used to realize the Fresnel diffraction pattern. Here, we theoretically display the relationship between a Fresnel diffraction pattern in the free space and the equivalent image at the out-of-focus plane of the focusing lens. In Fig. 1(a), we assume that the DPE is located at $z = 0$ and the distance between DPE and lens is ignored. According to the theory of Fourier optics [9], the transmission of a lens is a pure phase factor of $\exp[-ik(x^2 + y^2)/(2f)]$, where $k = 2\pi/\lambda$. The propagation of light from the lens to the target plane can be described by the Fresnel diffraction. Therefore, the electric field at the target plane in Fig. 1(a) is

$$U(x,y) = \frac{e^{ik(f-\Delta z)}}{i\lambda(f-\Delta z)} e^{\frac{ik(x^2+y^2)}{2(f-\Delta z)}} \iint_{-\infty}^{\infty} \Big[u(\xi,\eta) \cdot e^{\frac{-ik(\xi^2+\eta^2)}{2f}} e^{\frac{ik(\xi^2+\eta^2)}{2(f-\Delta z)}} \Big] \times e^{-i\frac{2\pi(x\xi+y\eta)}{\lambda(f-\Delta z)}} \mathrm{d}\xi \mathrm{d}\eta, \tag{1}$$

where $u(\xi,\eta) = u_0(\xi,\eta)\exp[iP(\xi,\eta)]$ and $P(\xi,\eta)$ is the phase of DPE. After substituting $z_0 = f \cdot (f - \Delta z)/\Delta z$, $x' = x \cdot f/\Delta z$, and $y' = y \cdot f/\Delta z$ in Eq. (1), one can easily have

$$\begin{aligned} U(x,y) &= A\frac{f}{\Delta z}\left\{ \frac{e^{ikz_0}}{i\lambda z_0} e^{\frac{ik(x'^2+y'^2)}{2z_0}} \iint_{-\infty}^{\infty} \Big[u(\xi,\eta) \cdot e^{\frac{ik(\xi^2+\eta^2)}{2z_0}} \Big] e^{-i\frac{2\pi(x'\xi+y'\eta)}{\lambda z_0}} \mathrm{d}\xi \mathrm{d}\eta \right\} \\ &= A\frac{f}{\Delta z}\mathrm{FresnelT}[u(\xi,\eta)]\Big|_{z=z_0}, \end{aligned} \tag{2}$$

where A is the phase factor of $\exp\{ikz_0(\Delta z/f - 1)[1 + \frac{1}{2}(x^2 + y^2)/(f - \Delta z)^2]\}$, $\mathrm{FresnelT}[u(\xi,\eta)]|_{z=z_0}$ denotes the Fresnel diffraction of the electric field $u(\xi,\eta)$ at the plane where $z = z_0$, and FresnelT[] stands for the Fresnel transformation. In Eq. (2), one can see that the field $U(x,y)$ at the target plane in Fig. 1(a) can be described by the field $U(x',y')$ at the plane where $z = z_0$ in the free space in Fig. 1(b), where $U(x',y') = \mathrm{FresnelT}[u(\xi,\eta)]|_{z=z_0}$. After the space-coordinate transformation by a scaling factor of $f/\Delta z$ and multiplying the amplitude by $A \cdot f/\Delta z$, the field $U(x',y')$ is equivalent to $U(x,y)$. According to the definition $z_0 = f \cdot (f - \Delta z)/\Delta z$, the distance z_0 is unique when the out-of-focus distance Δz is known with a given focus f. This means that the fields $U(x',y')$ and $U(x,y)$ have a corresponding relationship, as the field on the focal plane of a lens and the Fraunhofer diffraction pattern do. Therefore, one can use an equivalent Fresnel diffraction in Fig. 1(b) to design the DPE in Fig. 1(a). The convenience of this equivalence is that, when the configuration parameters (such as Δz, f, and so on) in Fig. 1(a) are known, an equivalent Fresnel diffraction in the free space without a lens can be reconstructed by the method of scaling and moving the target plane to the plane $z = z_0$. It is worthy to note that the role the lens in Fig. 1(a) has played is substituted by the propagation with a distance z_0 in free space in Fig. 1(b).

Here, we introduce the method of designing DPE by using the technique mentioned above. To obtain a special intensity profile with the field $U_o(x,y)$ at the target plane in Fig. 1(a), one can first change the field $U_o(x,y)$ into $U_o(x',y')$ by the coordinate transformation of $x' = x \cdot f/\Delta z$ and $y' = y \cdot f/\Delta z$. The field $U_o(x',y')$ is considered to be the target field in Fig. 1(b). In the optical system of Fig. 1(b), an iterative algorithm can be used to design the DPE with the target field $U_o(x',y')$. Then, the special intensity profile in Fig. 1(a) can be obtained by using the designed DPE. This is our routine of designing the expected DPE in Fig. 1(a). The key point is to build the iterative algorithm based on the Fresnel diffraction. Some theories are proposed to realize the Fresnel diffraction in the free space [9–11]. Here, we use the fast chirp transform, described by Deng *et al.* [10] and that has been applied to multiwavelength optical interconnects [12], to realize the Fresnel diffraction with high numerical precision. Because the inverse propagation in the Fresnel diffraction can be described by setting a negative propagating distance along the optical axis [13], the reported iterative algorithms can be used to design the DPE [14–16].

3. Experiment

To check the validity of our method, we design a DPE to exhibit a picture at the target plane in Fig. 1(a). We choose $f = 99$ mm and $\Delta z = 2$ mm, having the distance $z_0 = 4.8015$ m. The incident beam has a wavelength of 532 nm, and the electric field $u_i(\xi, \eta) = \exp(-r^2/w_0^2)$ with $w_0 = 4.096$ mm where $r = \sqrt{\xi^2 + \eta^2}$. The sampling number is 1024×1024. Here, the Gerchberg–Saxton algorithm [14] is implemented to design the DPE for exhibiting four characters "USTC" at the target plane. The experimental setup is displayed in Fig. 2(a). A beam with the wavelength of 532 nm is generated in a laser. After being expanded by a beam expander that is composed of the lens L_1 and L_2, this beam is modulated by a phase only spatial light modulation (SLM) with a pixel of 8 μm (PLUTO by Holoeye Photonics AG). This SLM, controlled by a personal computer (PC), is used to realize the phase of designed DPE. The focus of the lens L_3 is 99 mm. We utilize a CCD camera to record the intensity profile at the target plane. Figure 2(b) shows the phase profile of the designed DPE, whose phase has a range from $-\pi$ to π. The simulated and experimental pictures at the target plane are displayed in Figs. 2(c) and 2(d), respectively. Comparing Fig. 2(c) and Fig. 2(d), one can see that the experimental result is well consistent with the simulated picture except a bright spot at the center. Experimentally, because the small gap of about 0.76 μm exists between two adjacent pixels in the SLM, one partial light impinging on these gaps is not modulated by the SLM and has the same phase. Although this partial light has small energy, the intensity at the target plane is strong when focused by a lens. Therefore, the bright spot at the center of Fig. 2(c) is mainly due to these gaps [17,18]. Although we do not give a quantitative comparison between the simulated picture and the experimental result, one can have a conclusion that our approach to design the DPE in Fig. 1(a) is valid.

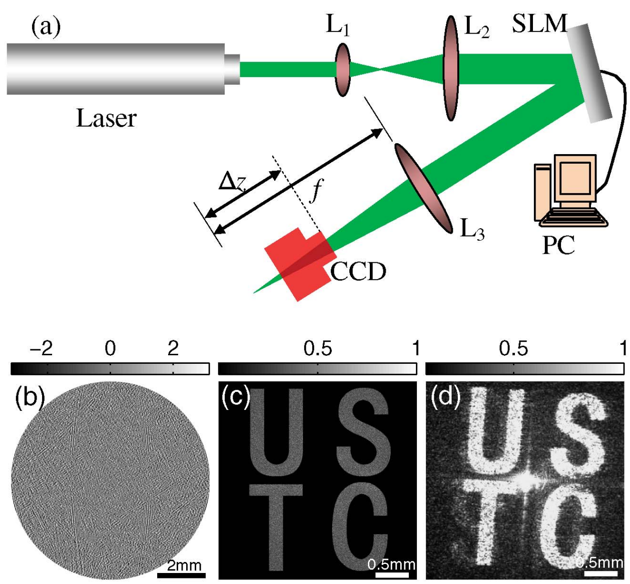

Fig. 2. (Color online) Experimental verification of the DPE design for single wavelength. (a) Experimental setup; (b) the phase profile of designed DPE. The phase in (b) is in units of radian. The simulated (c) and experimental (d) intensity (normalized) at the target plane are also given. The experimental data (d) is obtained with a charge-coupled-device (CCD) camera.

4. Discussion

As mentioned at the beginning, the optical system in Fig. 1(a) is introduced to realize the spectral beam splitting and highly confined intensity at the target plane for the nonmonochromatic illumination. In addition, we have demonstrated theoretically and experimentally that design of DPE in the optical system of Fig. 1(a) is a problem that can be resolved in the Fresnel diffraction. In the nonmonochromatic illumination, the advantage of designing a DPE in the Fresnel region, compared with that in the Fraunhofer region, is that the phase modulation for every wavelength comes from both the DPE and the wavelength-dependence optical path difference between the DPE and the target plane [6]. Although some results have also been reported experimentally for the purpose of realizing the spectral splitting by DPE [6,8], the distance between the DPE and the target plane is much larger in comparison with the aperture of DPE, resulting in the not-well-confined target intensity. A lens exists in the optical system of Fig. 1(a), which makes it possible to realize the spectral beam splitting and highly confined intensity simultaneously. Therefore, to realize this idea, the optical system in Fig. 1(a) can be replaced by that in Fig. 3(a). Here, we theoretically investigate the feasibility of realizing the spectral beam splitting and highly confined intensity using the optical system of Fig. 3(a), which is the same as that in Fig. 1(a) except the nonmonochromatic illumination.

In the optical system with multiple wavelengths, some theories have been provided to design the DPE in the Fresnel region [6,8,12]. Here, we adopt the quite efficient method [12], described by Deng *et al.* and based on the approximation of weak-phase deviations, to design DPE in the optical system of Fig. 3(a). In our simulation, we investigate the optical system of Fig. 3(a) in the illumination with three wavelengths ($\lambda_1 = 0.5$ μm, $\lambda_2 = 0.8$ μm, and $\lambda_3 = 1.2$ μm). We assume that the DPE is transmissive and the DPE material is fused quartz, whose refractive index is, respectively, 1.46233, 1.45332, and 1.44805 at λ_1, λ_2, and λ_3. For the wavelength λ_n, the incident light has the intensity profile with $u_{in}(\xi, \eta) = \exp[-(r/w_0)^{20}]$ ($n = 1$, 2, and 3). The focal length of the lens is $f = 150$ mm, $\Delta z = 1$ mm, and $z_0 = 22.35$ m. The radius of the entrance pupil w_0 is 15 mm, and the sampling number is

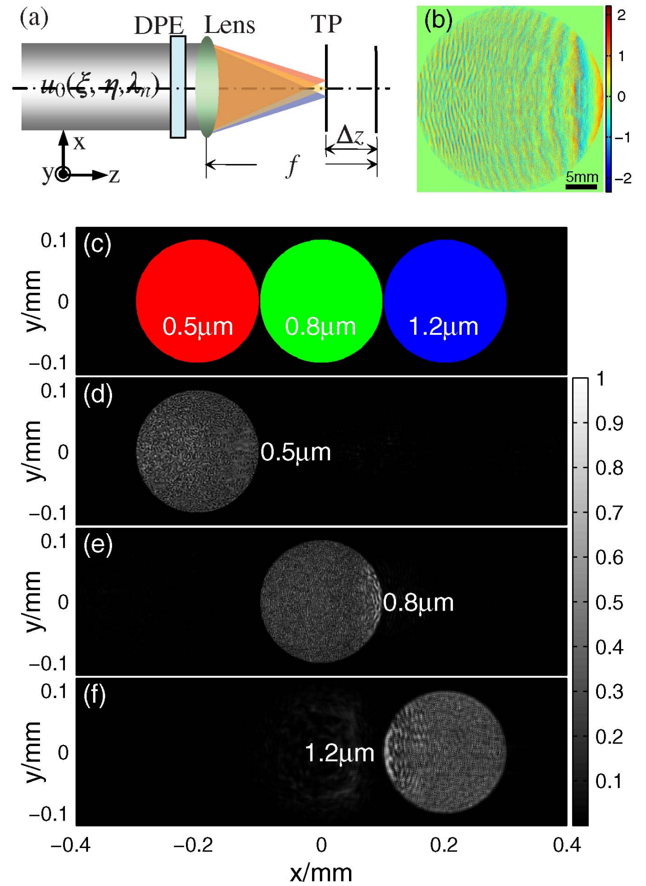

Fig. 3. (Color online) Design of DPE in the nonmonochromatic illumination. (a) Schematic of modulating the electric field by a DPE at the out-of-focus plane of a lens in the nonmonochromatic illumination. (b) The relief profile of the designed DPE. The colorbar is in units of μm. (c) The ideal intensity and position for three wavelengths at the target plane. (d) (f) The normalized intensity profiles for three wavelengths [$\lambda_1 = 0.5\ \mu$m (d), $\lambda_2 = 0.8\ \mu$m (e), and $\lambda_3 = 1.2\ \mu$m (f)] at the target plane using the designed DPE in (b).

1024×1024. Our goal is to design the DPE for realizing three confined spots with different positions for three wavelengths at the target plane, respectively. Figure. 3(c) gives the intensity profile and position of the ideal confined spot (with the radius $R = 0.1$ mm) for every wavelength.

The relief profile of the designed DPE is displayed in Fig. 3(b). The colorbar is in units of micrometers. The range of relief depth is about 4 μm. For the multicolor DPE, the relief depth is usually larger than that of single-color DPE when one purses a satisfactory intensity profile at the target plane. The large relief depth gives more phase selection for every wavelength, which makes the problem of designing the multicolor DPE easier. For a single-color DPE with the wavelength λ_1, the intensity profile at the target plane does not change when the DPE phase is added by $n \cdot 2\pi$ (n is an integer), which means that the relief depth increases $n \cdot \lambda_1$. However, the increment $n \cdot \lambda_1$ of relief depth is corresponding to different phase modulation for another wavelength, which provides the feasibility of modulating the nonmonochromatic light with a phase relief element. Theoretically, a good design of multicolor DPE demands the high relief depth. Technically, it is difficult to fabricate a DPE with high relief depth because the error enlarges with the increase of relief depth. Therefore, it is necessary to weigh the benefit between theory and fabrication when carrying out the design of multicolor DPE.

Using the designed DPE in Fig. 3(b), we give the simulated intensity profile $U_{\mathrm{on}}(x, y)$ for wavelength λ_n ($n = 1, 2$, and 3), which is shown in Figs. 3(d), 3(e), and 3(f), respectively. The intensity confinement factors are $E_{c1} = 0.858$ for λ_1, $E_{c2} = 0.810$ for λ_2, and $E_{c3} = 0.788$ for λ_3. Here, the intensity confinement factor is defined as $E_{\mathrm{cn}} = \iint_{(x-x_n)^2+y^2\le R^2} |U_{\mathrm{on}}|^2 \mathrm{d}x\mathrm{d}y / \iint_\infty |U_{\mathrm{on}}|^2 \mathrm{d}x\mathrm{d}y$, where $x_1 = -0.2$ mm, $x_2 = 0$, and $x_3 = 0.2$ mm. From the values of E_c for three wavelengths, one can see that the well-confined intensity is achieved at the target plane. Certainly, the intensity confinement will be better when the relief depth of DPE increases. Although the lens in the optical system discussed here has a low numerical aperture (NA, about 0.1), a similar result can also be achieved in the case of a higher NA lens. Under the nonmonochromatic illumination, the separation of beam among different wavelengths is easier by using a higher NA lens, because the dependence of the change of complex amplitude during the propagation from DPE to the target plane on the wavelength is strong.

The phase of designed DPE is obtained by the weighting superposition of the phase profiles for three wavelengths. The phase competition among three wavelengths occurs when carrying out the iterative algorithm. As a result, the simulated intensity profiles for three wavelengths at the target plane [Figs. 3(d)–3(f)] have a little difference from each other despite the fact that their ideal intensity profiles are the same [Fig. 3(c)]. In [12], the weighting factor for wavelength λ_n is $\mathrm{RMS}_n / \sum_{n=1}^{N} \mathrm{RMS}_n$, where N is the number of wavelengths, RMS_n is the root-mean-square error between the intensity at the target plane last iteration, and the ideal intensity for wavelength λ_n. Here, we use $\mathrm{RMS}_n^m / \sum_{n=1}^{N} \mathrm{RMS}_n^m$ as the weighting factor for wavelength λ_n, where m is the power exponent. In our design, we choose $m = 4$. For the wavelength λ_n with a large RMS_n in one iteration, its weighting factor next iteration could be larger when $m > 1$ in contrast with the case in [12]. Correspondingly, more phase information for wavelength λ_n can be added into the phase of DPE so that the RMS_n for wavelength λ_n decreases quickly. To some extent, the phase competition can be suppressed by using this method. Nevertheless, the phase competition is still not eliminated to the satisfactory situation. Therefore, more work needs to be done for solving the problem of phase competition among different wavelengths.

In addition to the relief depth and phase competition among different wavelengths, the number of the optimized wavelengths and the interval (or wavelength resolution) among the optimized wavelengths also have an important influence on the result of the designed DPE. Considering that these two factors have been demonstrated in detail in [8], we do not discuss them here.

5. Conclusion

We have demonstrated that modulating the electric field at the out-of-focus plane of a lens system by a DPE is a problem that can be resolved in an equivalent Fresnel diffraction in free space. As a result, the efficient iterative algorithms can be used to design the DPE. Although Δz is positive in two examples here, Δz can be negative, which means that the target plane locates behind the focal plane and the corresponding z_0 is also negative. The negative z_0 means that the image at the target plane in Fig. 1(b) is a virtual image. The real and virtual images are corresponding to the positive and negative Δz, respectively. The positive or negative sign of Δz only means that the position of target plane in Fig. 1(a) is before or after the focal plane. Therefore, the area where the electric field is modulated by the DPE can be extended to the whole place where $z > 0$. The optical system suggested in this paper (Fresnel diffraction) is different from the general optical system whose target plane locates in the focal plane (Fraunhofer diffraction). Because the optical system in the paper exhibits the advantage of realizing the spectral splitting and highly confined intensity, we expect that it could be applied in the solar concentrating system and optical encryption.

The authors thank Dr. Xuegong Deng and Yuxuan Ren for the valuable discussion. This work was supported by the National Natural Science Foundation of China (11074239).

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