# Design of DOE for generating a needle of a strong longitudinally polarized field 

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#### Abstract

A needle of strong longitudinally polarized field with homogeneous intensity along the optical axis, long depth of focus, and subdiffraction beam size can be generated by focusing a radially polarized light with a high-NA lens and a diffractive optical element (DOE) with belts. A method combining the global-searchoptimization algorithm and the tight focusing properties of the radially polarized light is proposed to design the DOE. Based on the tight focusing properties, the light incident on the lens is divided into two parts: areas A and B. We discover that the longitudinal field in the focal region is mainly dependent on the number of belts in area B but not the total number of belts in the DOE. © 2010 Optical Society of America

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Radially polarized light has been gaining great attention due to its novel properties. The longitudinal field component (LFC) created by focusing a radially polarized light with a high-NA lens in the focal plane is applied in particle acceleration [1], high resolution-near-field optical microscopy [2], second-harmonic generation [3,4], and Raman spectroscopy [5]. Single molecules with fixed absorption dipole moments were used to probe the longitudinal field [6]. A needle of longitudinally polarized light with a homogeneous intensity along the optical axis has recently been achieved by focusing a radially polarized BesselGaussian (BG) beam with a high-NA lens and a diffractive optical element (DOE) [7]. The DOE was introduced to control the light field in the focal region by polarization and phase modulation [8]. The traditional global-search-optimization (GSO) algorithm was implemented to design the DOE [7,9,10]. Combining the GSO algorithm and the focusing properties of the radially polarized light, we propose a method to design a DOE for obtaining a needle of longitudinally polarized field with a strong intensity. For exhibiting the validity of our method, a four-belt DOE is designed to generate a needle of strong longitudinally polarized field whose peak intensity is 1.42 times that in [7].
According to the theory of Richards and Wolf [11], when a radially polarized beam is focused by a high NA lens, the field near the focal plane can be approximated by the vectorial Debye integral. The radially polarized BG beam is one of the vector-beam solutions of the Maxwell's wave equation in the paraxial approximation $[12,13]$.The apodization function $l_{0}(\theta)$ for a radially polarized BG beam [14] with a beam waist in the pupil of the focusing lens is given as

$$
\begin{equation*}
l_{0}(\theta)=J_{1}\left(\frac{2 \beta_{1} \sin \theta}{\sin \alpha}\right) \exp \left[-\left(\frac{\beta_{2} \sin \theta}{\sin \alpha}\right)^{2}\right] \tag{1}
\end{equation*}
$$

where $\beta_{1}$ and $\beta_{2}$ are taken as unity in our design. $\theta$ denotes the angle between the convergent ray and the optical axis; $\alpha=\arcsin (\mathrm{NA} / n)$, with $n$ as the refractive index. In this Letter, the lens' NA is 0.95 and $n=1$.

When a radially polarized beam is focused, the electric field components in the focal plane contain the radial component $E_{r}$ and the longitudinal component $E_{z}$ [14]. Here, the ratio of the longitudinal and radial fields in the focal plane is taken to evaluate a focusing ray's contribution to the longitudinal component. Figure 1 shows the ratio of the longitudinal and radial peak intensities for the radial position $R$ (normalized to the optical aperture $D$ ) in the lens' pupil plane. When $R=R_{0}$, the longitudinal peak intensity equals the radial peak intensity. We divide the lens' pupil into two parts: area A (where $R<R_{0}$ ) and area B (where $R>R_{0}$ ). The rays incident on area A contribute more to the radial component, while those incident on area $B$ contribute more to the longitudinal component $[15,16]$. Therefore, the method of designing DOE in this Letter is based on the idea that the light incident on areas A and B should be disposed separately. In order to obtain a strong longitudinal field, much attention should be paid to the light incident on area B. A DOE with belts can be used to control the field in the focal region [7-10]. For simplicity, we introduce a four-belt DOE with two belts in area A and two in area B (Fig. 2). Since two belts in area A or B supply the phase difference $\Delta \varphi=\pi$, the light incident on area A or B interferes destructively in the fo-


Fig. 1. The ratio of the peak intensity of the longitudinal and radial component field in the focal plane of a 0.95 NA lens illuminated by a radially polarized BG beam (solid curve). The dotted line means $\left|E_{z}\right|_{\text {max }}^{2}=\left|E_{r}\right|_{\text {max }}^{2}$. For a radially polarized BG beam, $\left|E_{z}\right|_{\text {max }}^{2}$ equals $\left|E_{r}\right|_{\text {max }}^{2}$ at $R=R_{0}$ ( $R_{0}=0.5293$ for a 0.95 NA lens). The boundary of areas A and B is the vertical dotted-dashed line $R=R_{0}$.
cal region, decreasing the radial field component. Although the LFC also attenuates due to the destructive interference, it always dominates the focal region. Because $R_{0}$ is fixed for a lens with a given NA, one can optimize the structure of the DOE by changing parameters $R_{1}$ and $R_{2}$ to control the field in the focal region. However, it is critical to modulate the light incident on area B, which will be discussed in detail later. In order to obtain a needle of strong longitudinally polarized field with a long depth of focus and homogeneous intensity along the optical axis, some evaluation parameters must be defined in optimizing the structure of the DOE.

The beam quality in the focal plane is described as $\eta=\phi_{z} /\left(\phi_{r}+\phi_{z}\right)$ [7], $\phi_{r(z)}=2 \pi \int_{0}^{\infty}\left|E_{r(z)}(r, 0)\right|^{2} r d r$. The upper limits of the integrals $\phi_{r(z)}$ are taken as $2 \lambda$ in the optimization because the tiny area off the optical axis in the focal plane suffices the approximate conditions of the Debye integral [11]. The function $\eta$, an important evaluation parameter, demonstrates indirectly the ratio of the longitudinal and radial field compo-


Fig. 2. (a) Schematic of focusing of a radially polarized BG beam with a DOE and high NA lens. The focal plane of the focusing lens is at $z=0$. (b) Phase of a four-belt DOE in the $x-y$ plane. Phases in the white and gray areas are 0 and $\pi$, respectively. The dashed circle with radius $R_{0}$ divides the DOE into two parts: areas A and B.
nents. The large $\eta$ means the strong longitudinal component in the focal plane. The function $\Delta I$
 energy distribution along the optical axis. $\left|E_{z}(0, z)\right|^{2}$ and $\left|E_{\text {ideal }}(0, z)\right|^{2}$ are the phase-modulated and desired electric energy densities of the longitudinal component on the optical axis, respectively. The beam with a good energy homogeneity along the optical axis has small $\Delta I$. In order to attain a needle of strong longitudinally polarized field with homogeneous energy in the focal region, the functions $\eta$ and $\Delta I$ should be considered simultaneously in the optimizing procedure. Here we suggest a set of radii obtained by adopting our optimizing method,

$$
\begin{aligned}
R_{1}= & 0.3420, \quad R_{2}=0.7205 \\
& \text { with fixed } R_{0}=0.5293
\end{aligned}
$$

The electric energy density profiles of the radial, longitudinal, and total fields in the focal region are displayed in Fig. 3. A needle of LFC with a depth of focus of about $3 \lambda$ and homogeneous energy along the optical axis is achieved in the focal region by the phase modulation of the DOE. The ratio of the longitudinal and radial field components is 3.5 with $\eta=0.78$. The size (FWHM) of the total energy density spot in the focal plane is $0.44 \lambda$ in Fig. 3(d), so the spot area is $0.15 \lambda^{2}$. The incoming radially polarized BG beam and the lens in this Letter are identical to those in [7], but the peak intensity of the total field is 1.42 times that in [7].

For a radially polarized beam focused without modulation of the DOE, the LFC in the focal plane $(z=0)$ is a maximum and the LFC in the out-of-focus plane $(z \neq 0)$ reduces as the distance away from the focal plane increases along the optical axis [14]. After the modulation of the DOE, the LFC in the focal plane reduces and the LFC in the out-of-focus plane increases, having the potential in creating a needle of homogeneous LFC. When the distance between the out-of-focus and focal planes is long, the LFC in the out-of-focus plane gains little increment because of


Fig. 3. (Color online) The electric density distributions in the $y-z$ plane after the phase modulation of DOE. (a) Radial component. (b) Longitudinal component. (c) Total electric energy distribution. (d) The radial (dashed curve), longitudinal (dotted-dashed curve), and total (solid curve) electric fields in the focal plane.


Fig. 4. Phase of DOE with belts. Phases in the white and gray areas are 0 and $\pi$, respectively. The dashed circle with radius $R_{0}$ is the boundary of area $\mathrm{A}\left(R<R_{0}\right)$ and area B ( $R>R_{0}$ ). (a) A three-belt DOE with $R_{1}=0.4635$ and $R_{2}$ $=0.6966$. (b) A five-belt DOE with $R_{1}=0.2042, R_{2}=0.3252$, $R_{3}=0.5084$, and $R_{4}=0.7111$. (c) A four-belt DOE with $R_{1}$ $=0.3904, R_{2}=0.5923$, and $R_{3}=0.7682$. The belt ( $R_{1}<R$ $<R_{2}$ ) is divided into two belts by the dashed circle, resulting two belts in area A and three in area B.
the weak modulation by the DOE. Therefore, if one pursues the homogeneous LFC from the out-of-focus plane to the focal plane, the peak intensity of the LFC needle is restricted in the intensity of the LFC in the out-of-focus plane. As a result, the LFC needle with a long depth of focus is at the cost of the low intensity; the LFC needle with a high intensity is at the cost of the short depth of focus. This is the reason why we have higher peak intensity and shorter depth of focus (the depth of focus is $4 \lambda$ in [7]) than those in [7]. In addition, both methods focus on obtaining a purified LFC in the focal region using the DOE to diffract the radial field component away from the optical axis [7]. The total field approaches the LFC in the focal region. The size of the LFC in the focal region depends mainly on the NA of a lens $[15,16]$. Since the focusing lenses have the same NA (0.95), the radial sizes (FWHM) of the needle generated in this Letter and in [7] have little difference $(<0.01 \lambda)$.

As previously discussed, it is critical to modulate the light incident on area B . The longitudinal field in the focal region is mainly dependent on the number of belts in area B but not the total number of belts in the DOE. For example, one can utilize a DOE with three or five belts [Figs. 4(a) and 4(b)] to obtain the LFC needle as depicted in this Letter. As shown in Figs. 4(a) and 4(b), the two DOEs both have two belts in area B. Despite that the light incident on area A contributes less to the LFC in the focal region, the modulation of light incident on area A cannot be neglected completely, which means that at least two belts in area A are required. The LFC in the focal region has a slight improvement in the depth of focus or intensity by modulating the light incident on area A, for which one cannot get the needle depicted in this Letter using a two-belt DOE (with one belt in area A or B). Therefore, the idea that the light inci-
dent on areas A and B should be disposed separately is instructive in designing a DOE with belts. According to the properties (depth of focus and intensity) of the expected longitudinal field, one can choose the appropriate number of belts in area B. Generally, two belts in area A are enough. Following the idea, we suggest a four-belt DOE (with two belts in area A and three in area B) in Fig. 4(c) so as to realize the LFC needle (obtained using a five-belt DOE) in [7].

In conclusion, we have investigated the radially polarized BG light focused by a high-NA lens and divided the lens' aperture into two parts (areas A and B ) with boundary $R=R_{0}$. Modulating the light incident on area B plays a significant part in controlling the field in the focal region, which is instructive in designing a DOE with belts. Compared with the method of using the GSO algorithm solely in [7], we can not only get what their method does by our method but also turn the problem of designing a DOE into modulating the light incident on area B , which immensely reduces the complexity of designing a DOE. A DOE with four belts is introduced to deal with the light incident on areas A and B separately. By the method, we obtain a needle of longitudinally polarized field with a strong intensity, which can be used to accelerate the particles efficiently [1] and improve coupling light to scanning near-field optical microsocpes [2] and focusing in second-harmonic generation polarization microscopy [4].

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