

5. the Advanced Encryption Standard (AES)

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Key Points

- **AES** is a **block cipher** intended to **replace DES** for commercial applications. It uses a **128-bit block size** and a **key size of 128, 192, or 256 bits**.
- AES does **not** use a Feistel structure. Instead, each full round consists of **four separate functions**: byte substitution, permutation, arithmetic operations over a finite field, and XOR with a key.

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1. Introduction to AES
2. The AES Structure
3. AES Round Function
4. AES Key Expansion
5. Security Analysis

1. Introduction to AES

Motivation

- Security of DES
 - Brute-force attacks based on 56-bits keys
- A Solution: **3DES**, with 168-bit keys
 - Pros: robust against brute-force attacks
 - Cons: Low computational efficiency
- Another solution?
 - **AES**

1. Introduction to AES

History

- AES is a subset of the Rijndael block cipher developed by two Belgian cryptographers
 - Vincent Rijmen and Joan Daemen, who submitted a proposal to NIST during the AES selection process.
- AES has been adopted by the U.S. government.
- The Advanced Encryption Standard (AES) is defined in each of:
 - FIPS PUB 197: Advanced Encryption Standard (AES)
 - ISO/IEC 18033-3: Block ciphers

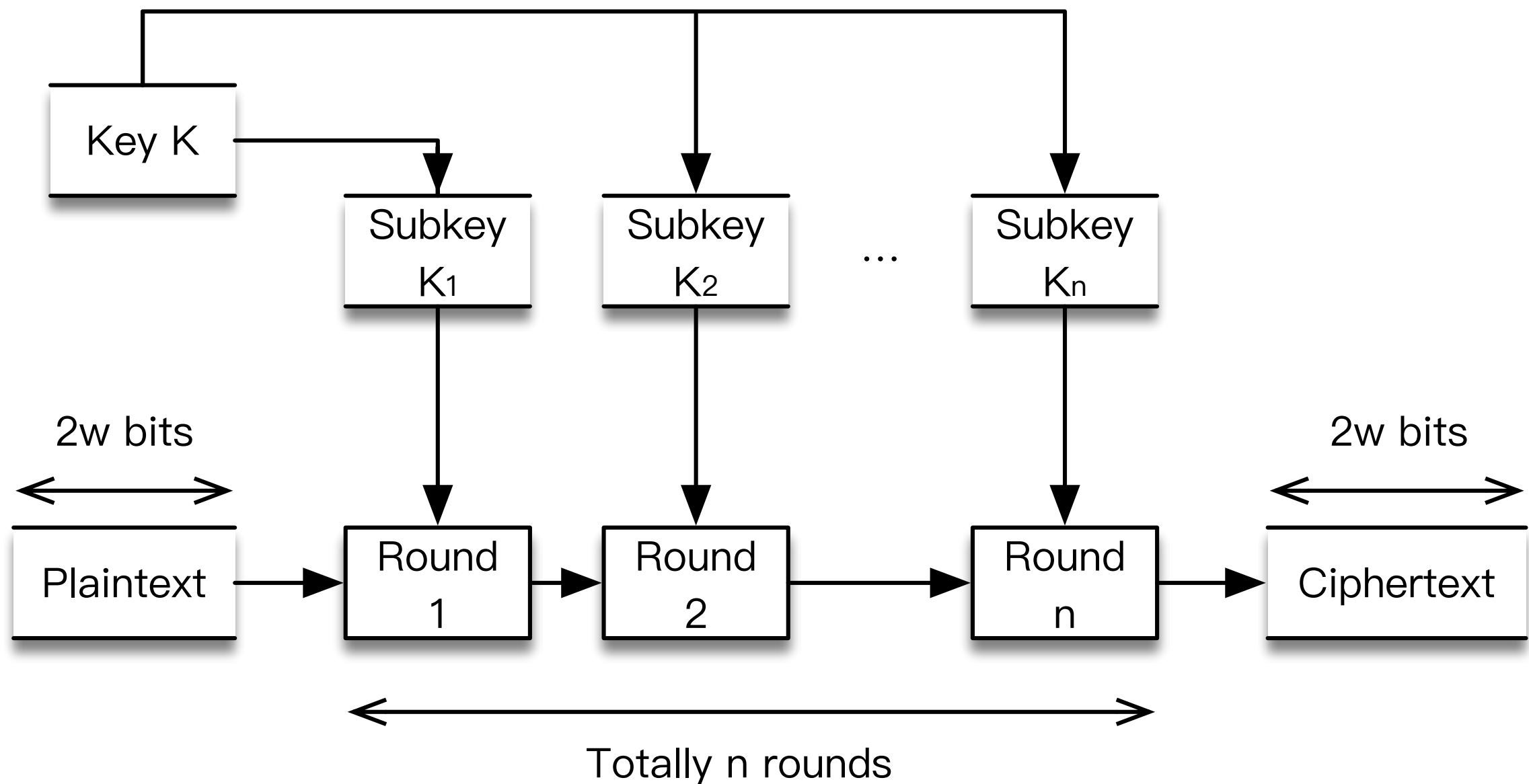
2 The AES Structure

Recall

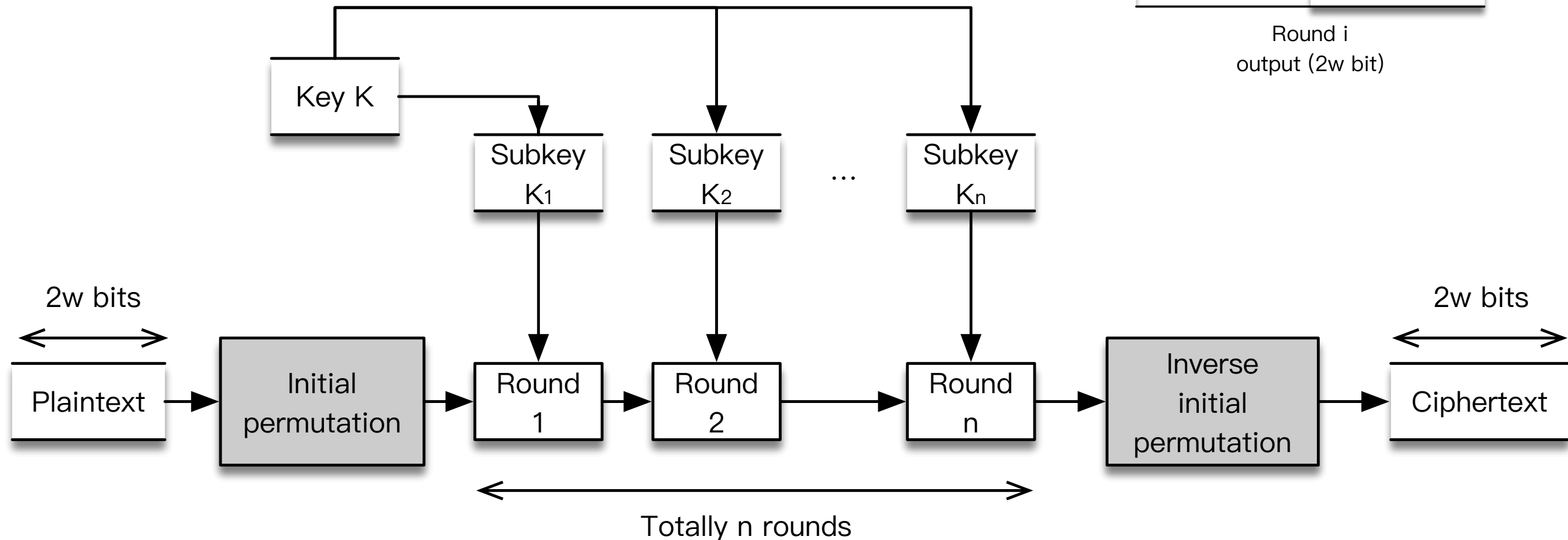
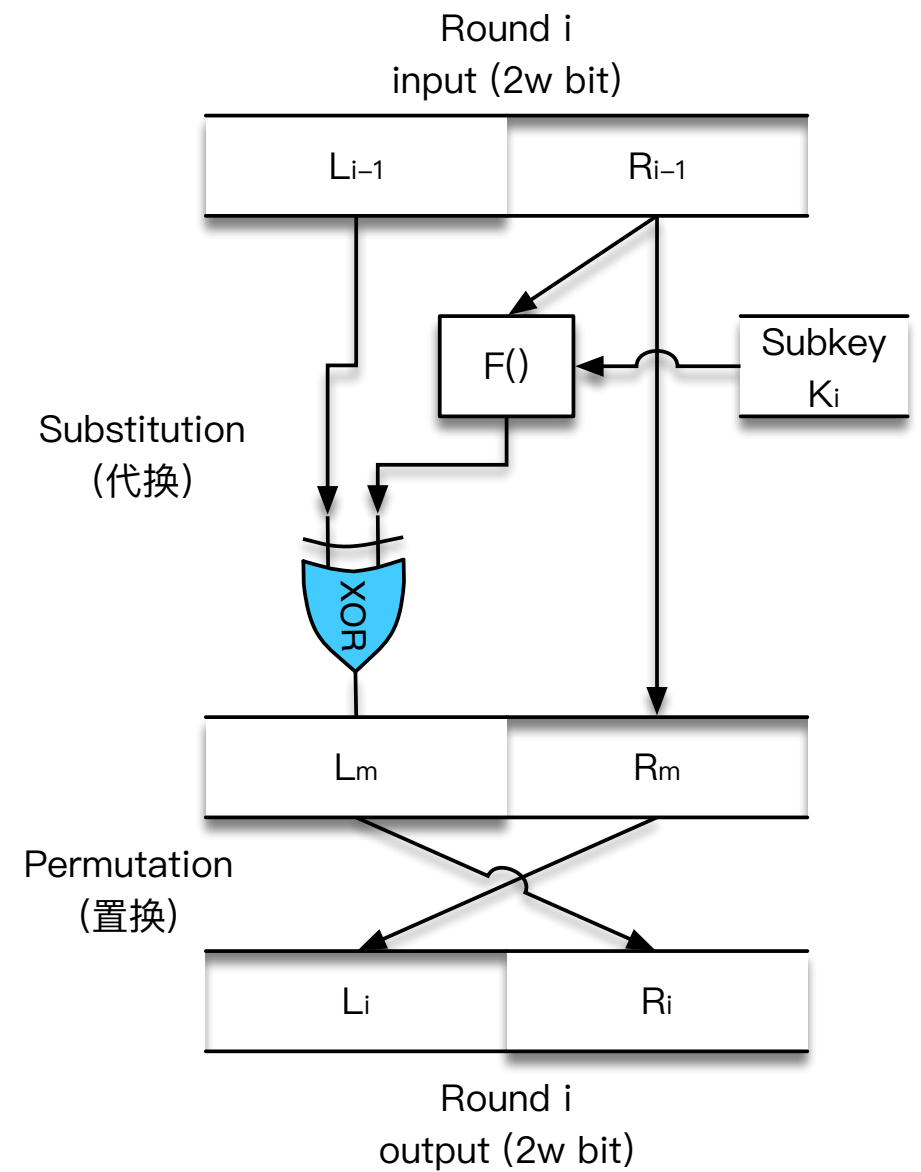
- **Product cipher (乘积密码):**
 - *Definition:* the execution of two or more simple ciphers in sequence
 - *Feature:* the final result or product is cryptographically stronger than any of the component ciphers
- **Substitution:** Each plaintext element or group of elements is uniquely replaced by a corresponding ciphertext element or group of elements
- **Permutations:** A sequence of plaintext elements is replaced by a permutation of that sequence.
 - That is, no elements are added or deleted or replaced in the sequence

2 The AES Structure

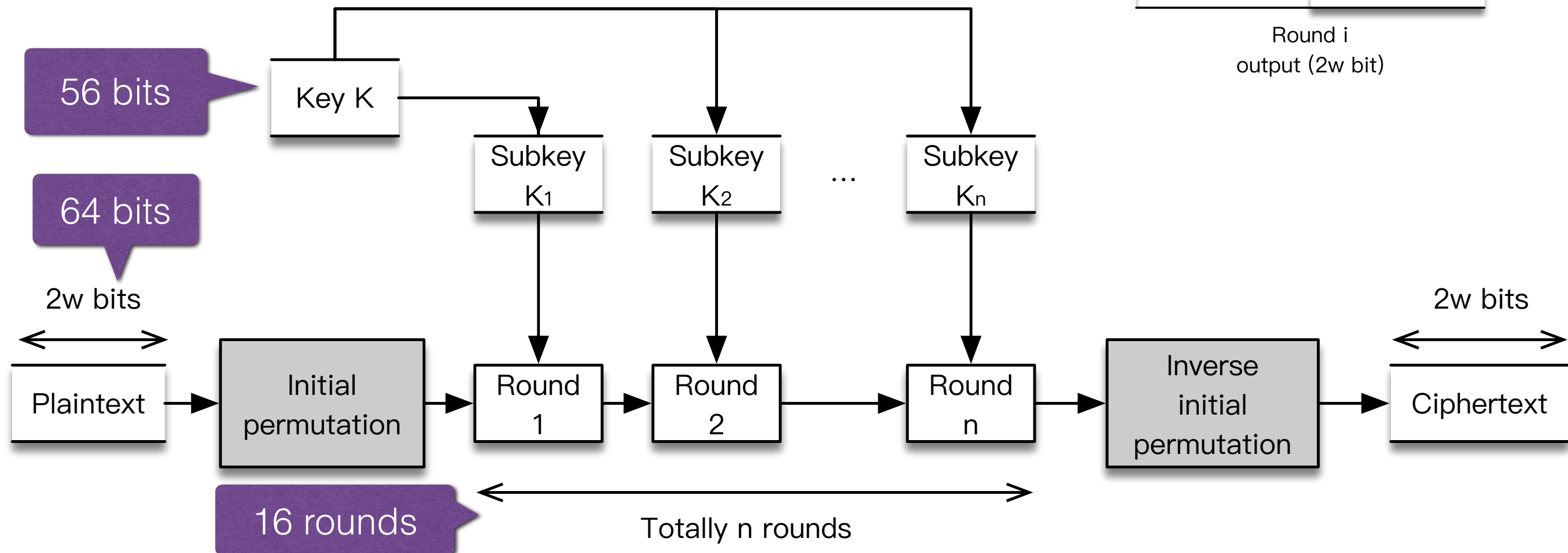
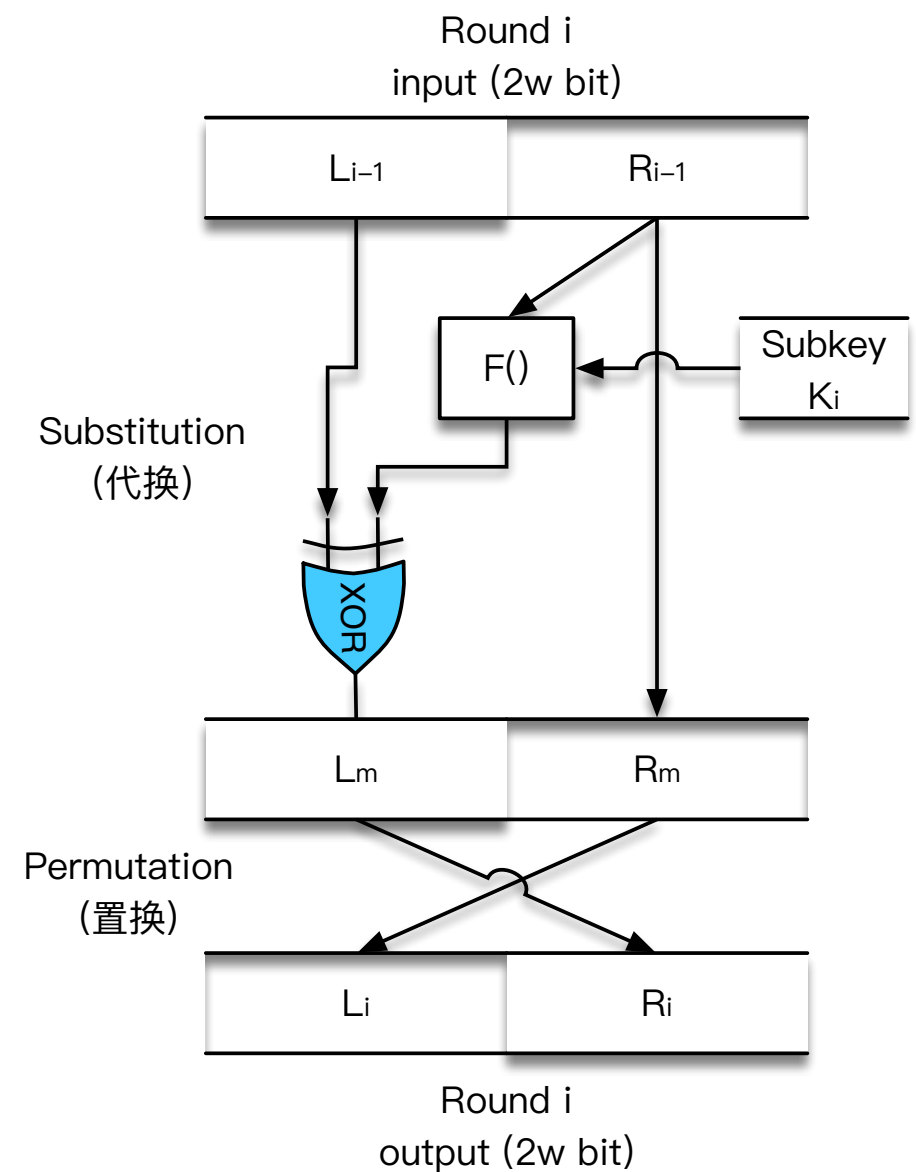
Recall



Recall: DES Implementation



Recall: DES Implementation



Recall: DES Implementation

Round Function F
(Next Page)

Substitution
(代换)

Subkey generation algorithm
(Next Page)

Permutation
(置换)

56 bits

64 bits

2w bits

Plaintext

Initial
permutation

Round
1

Round
2

Round
n

Inverse
initial
permutation

Ciphertext

16 rounds

Totally n rounds

Round i
input (2w bit)

L_{i-1}

R_{i-1}

F()

Subkey
 K_i

XOR

L_m

R_m

L_i

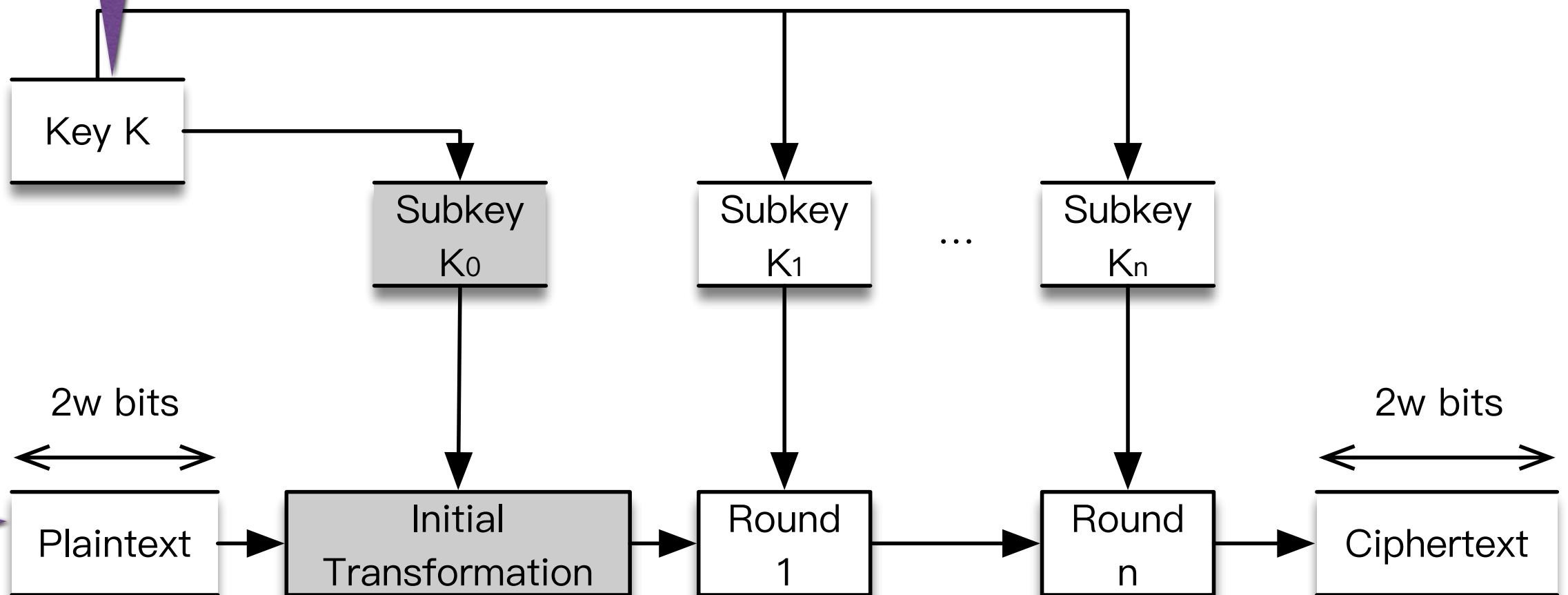
R_i

Round i
output (2w bit)

2 The AES Structure

128, 192, 256 bits
i.e. 16, 24, 32 bytes

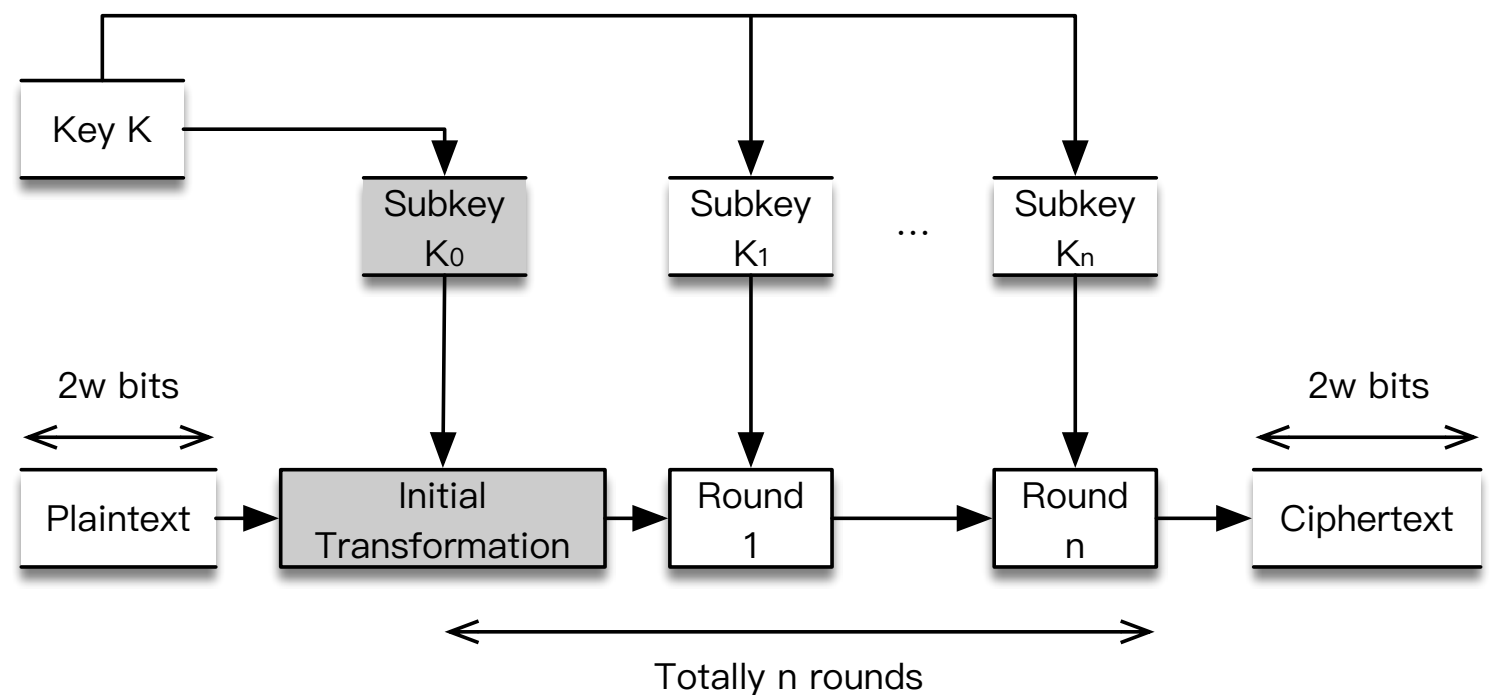
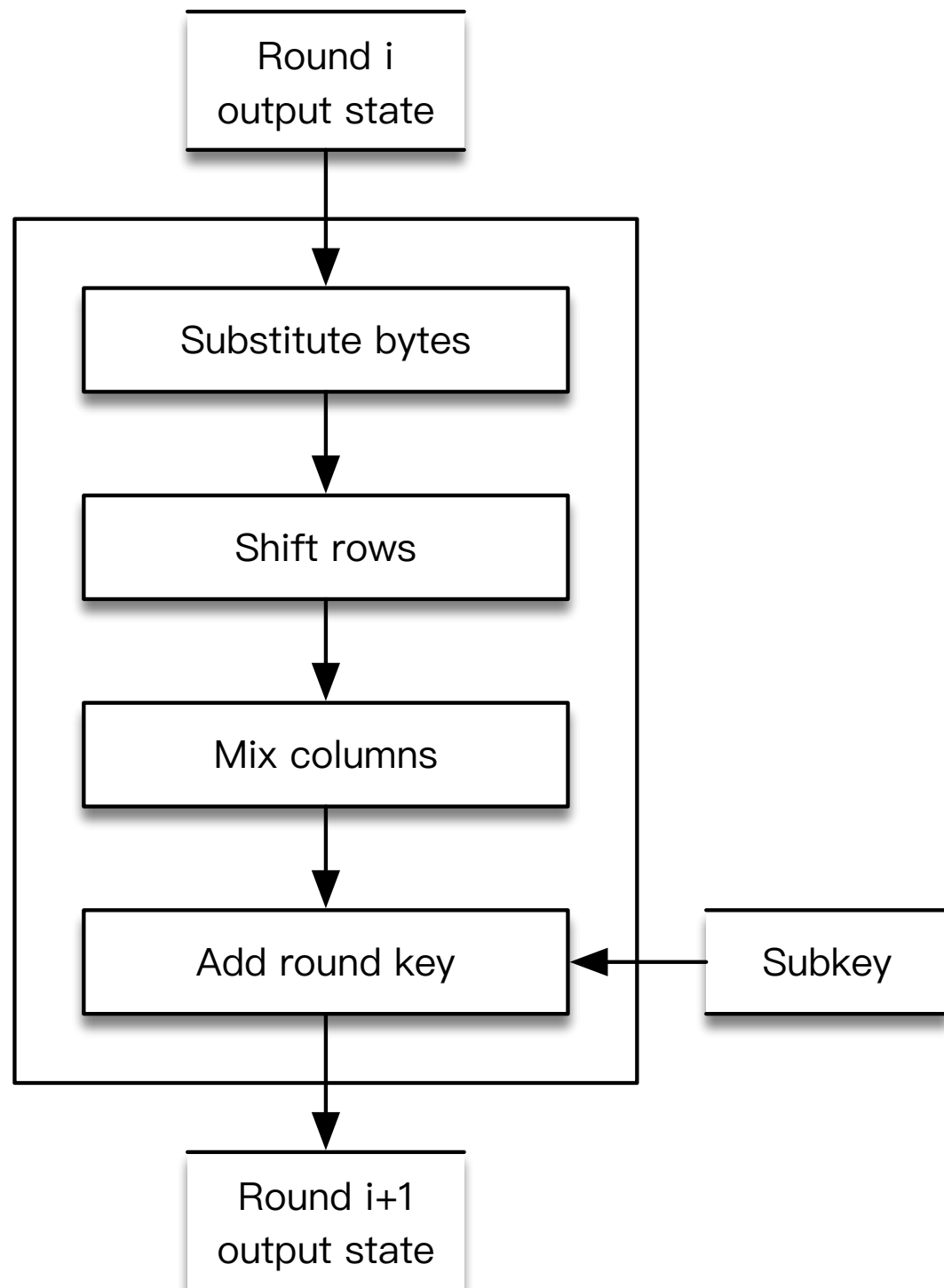
Subkey generation



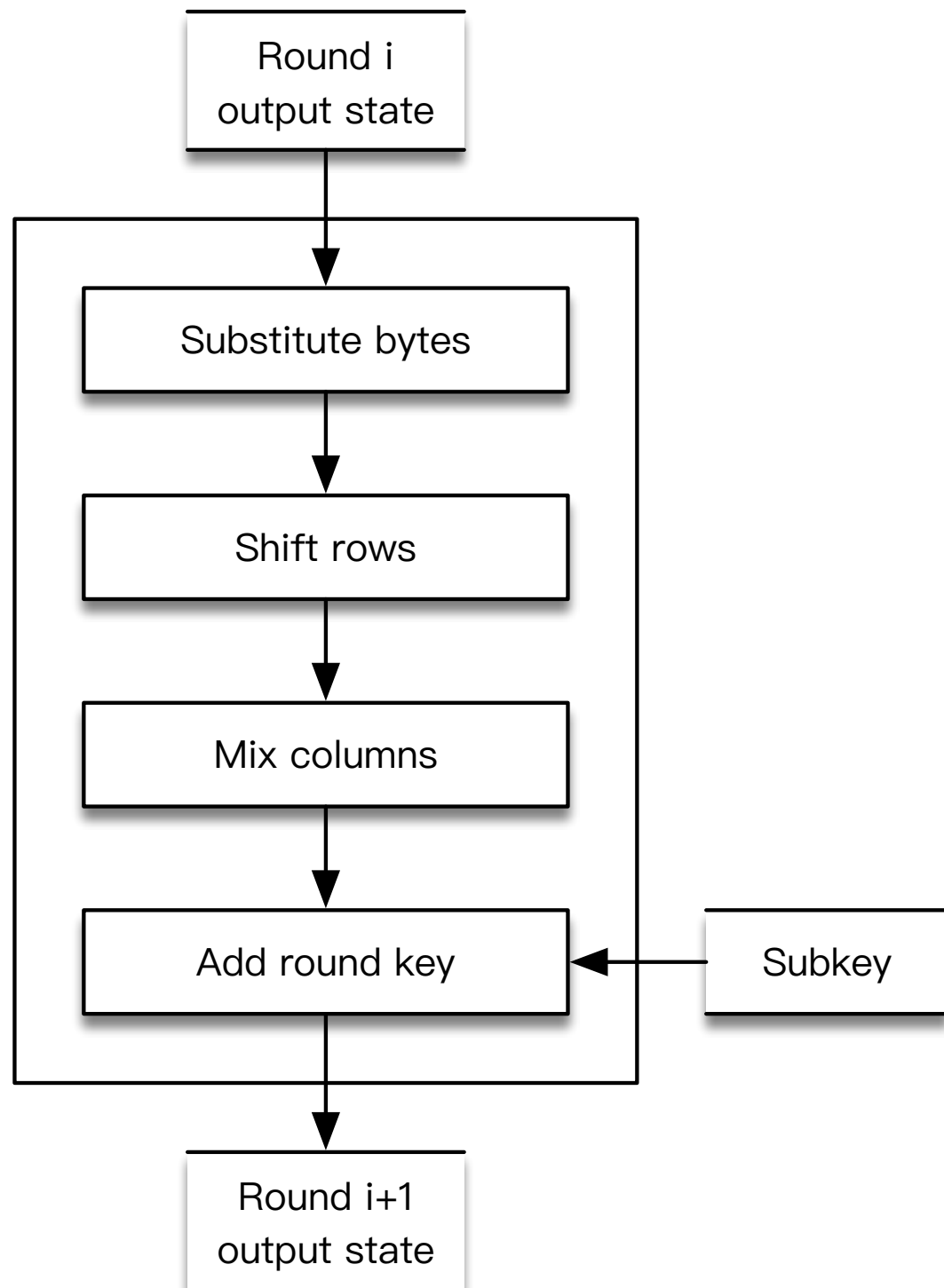
Rounds: 10, 12, 14

Round
function

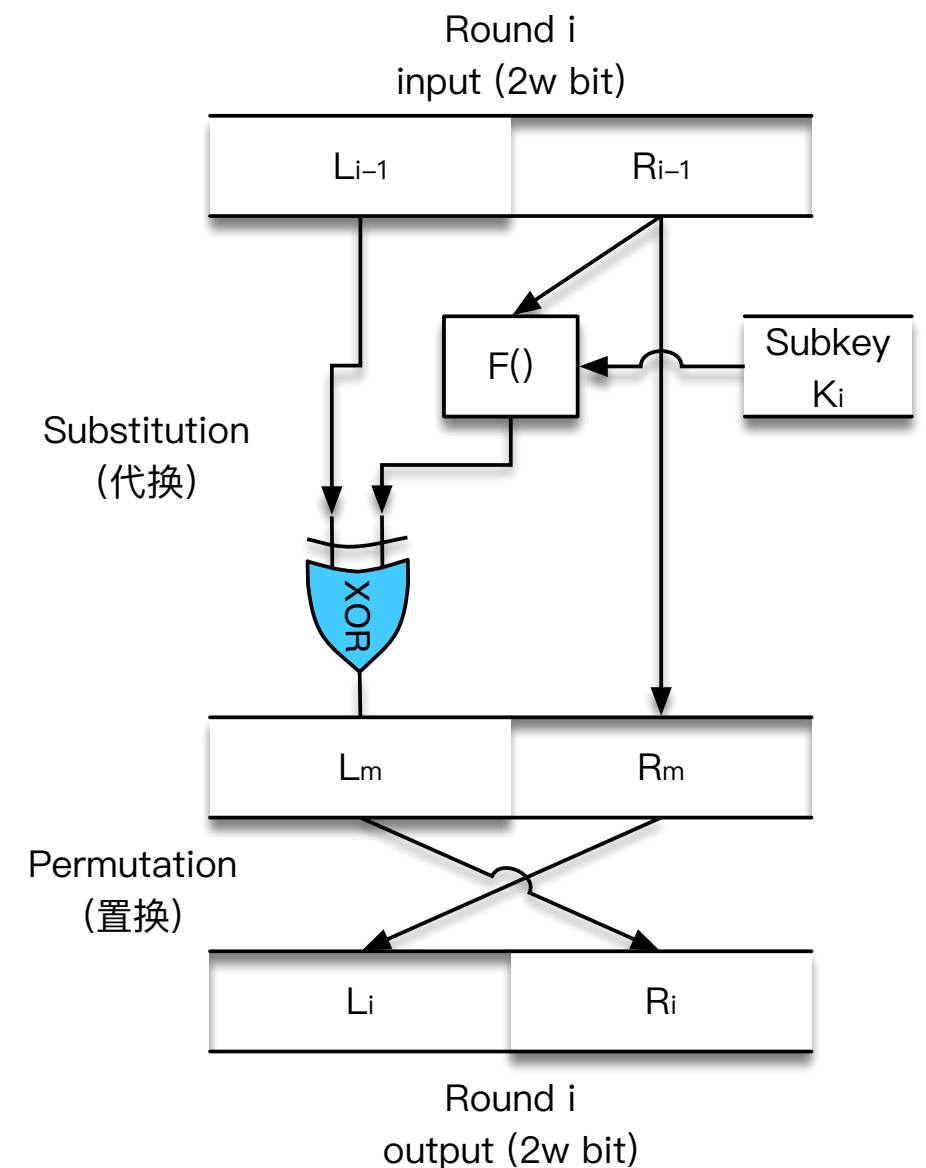
2 The AES Structure



3 AES Round Function



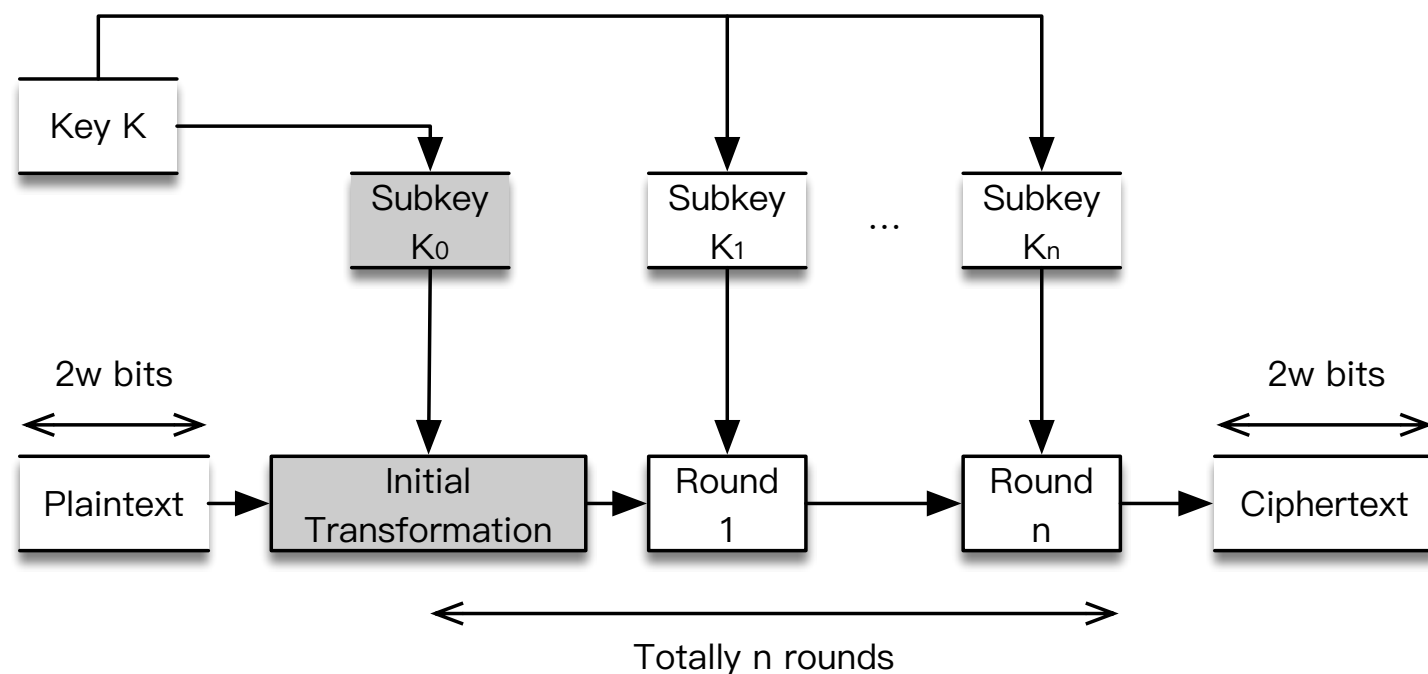
- **No** Feistel structure



3 AES Round Function

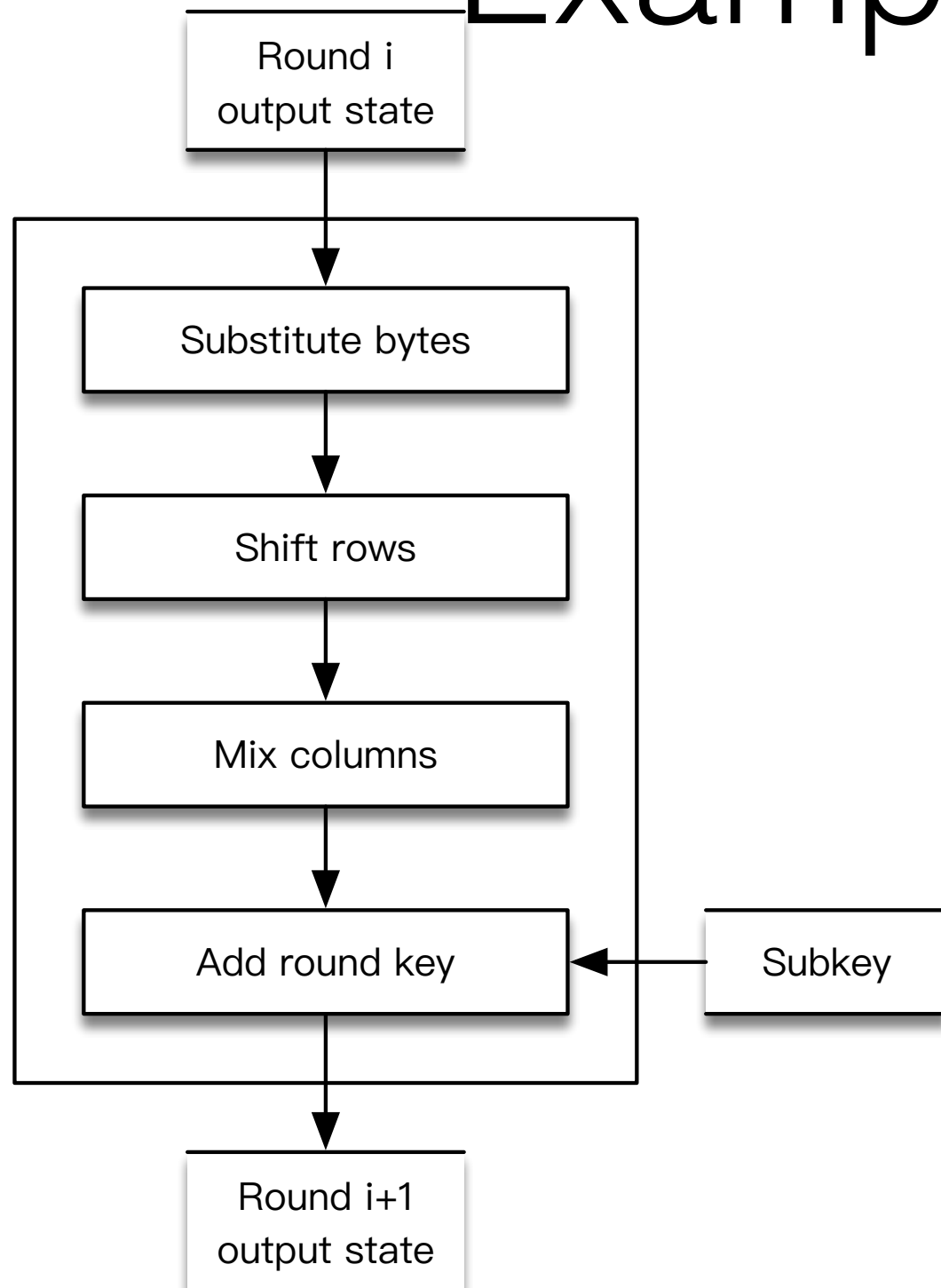
Example: AES-128

- The cipher consists of N rounds
- 10 rounds for a 16-byte key
- 12 rounds for a 24-byte key
- 14 rounds for a 32-byte key
- When $N=10$
 - 16-byte key
 - 11 subkeys are generated



3 AES Round Function

Example: AES-128



- Round function
 - Input
 - state (16 bytes)
 - subkey (16 bytes)
 - Output
 - state (16 bytes)

3 AES Round Function

Recall

Example

The Advanced Encryption Standard (AES) uses arithmetic in the finite field $\text{GF}(2^8)$, with the irreducible polynomial $m(x) = x^8 + x^4 + x^3 + x + 1$. Consider the two polynomials $f(x) = x^6 + x^4 + x^2 + x + 1$ and $g(x) = x^7 + x + 1$. Then

$$\begin{aligned} f(x) \times g(x) &= x^{13} + x^{11} + x^9 + x^8 + x^7 \\ &\quad + x^7 + x^5 + x^3 + x^2 + x \\ &\quad + x^6 + x^4 + x^2 + x + 1 \\ &= x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1 \\ &= x^7 + x^6 + 1 \end{aligned}$$

3 AES Round Function

$\text{GF}(2^8)$

- $\text{GF}(2^8)$
- Irreducible Function $m(x) = x^8 + x^4 + x^3 + x + 1$
- Example: $A = (a_7 a_6 \dots a_1 a_0)$ $B = (b_7 b_6 \dots b_1 b_0)$

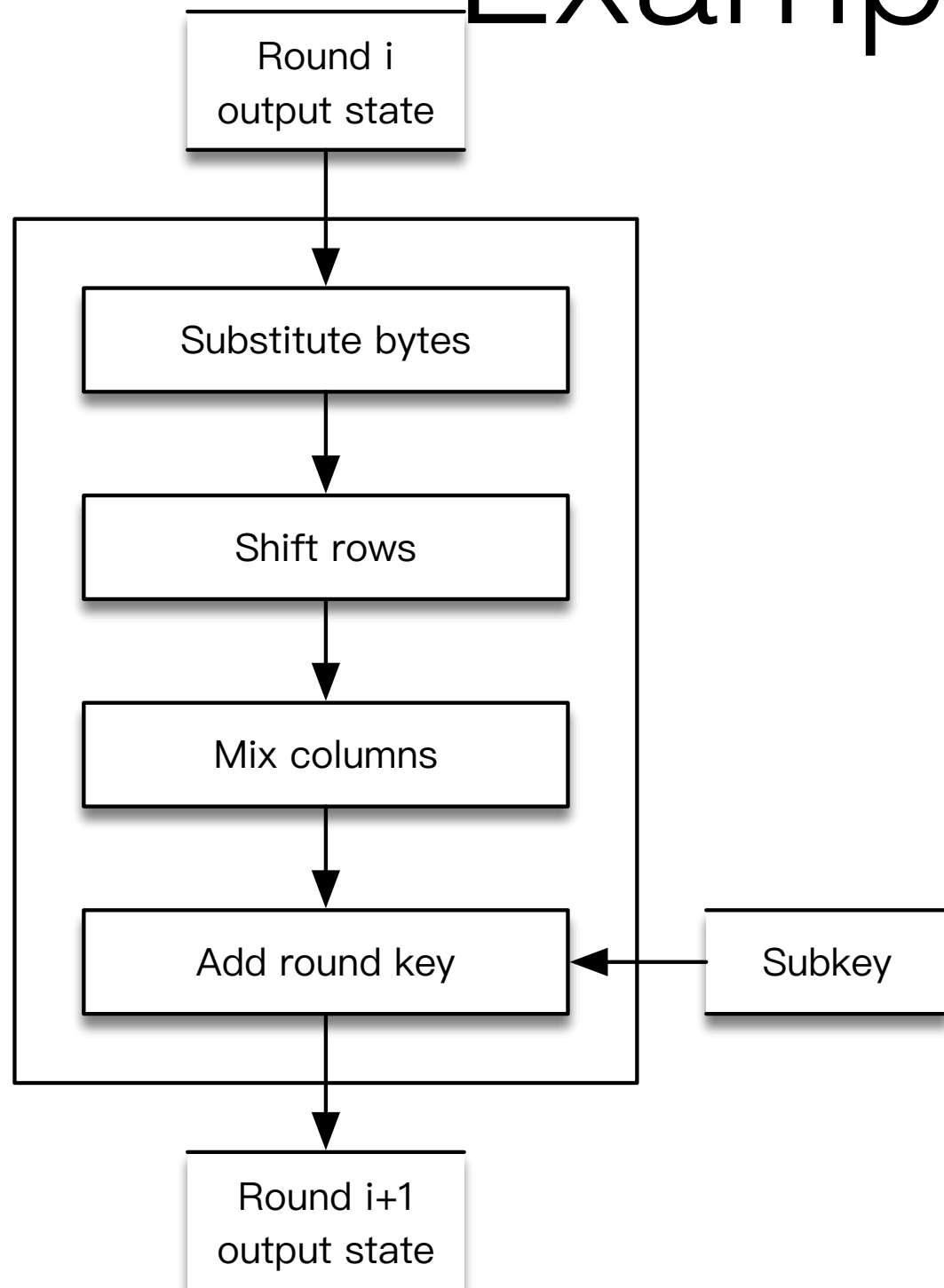
$$A + B = (c_7 c_6 \dots c_1 c_0) \quad c_i = a_i \oplus b_i$$

$$\{02\} \cdot A = (a_6 \dots a_1 a_0 0), \text{ if } a_7 = 0$$

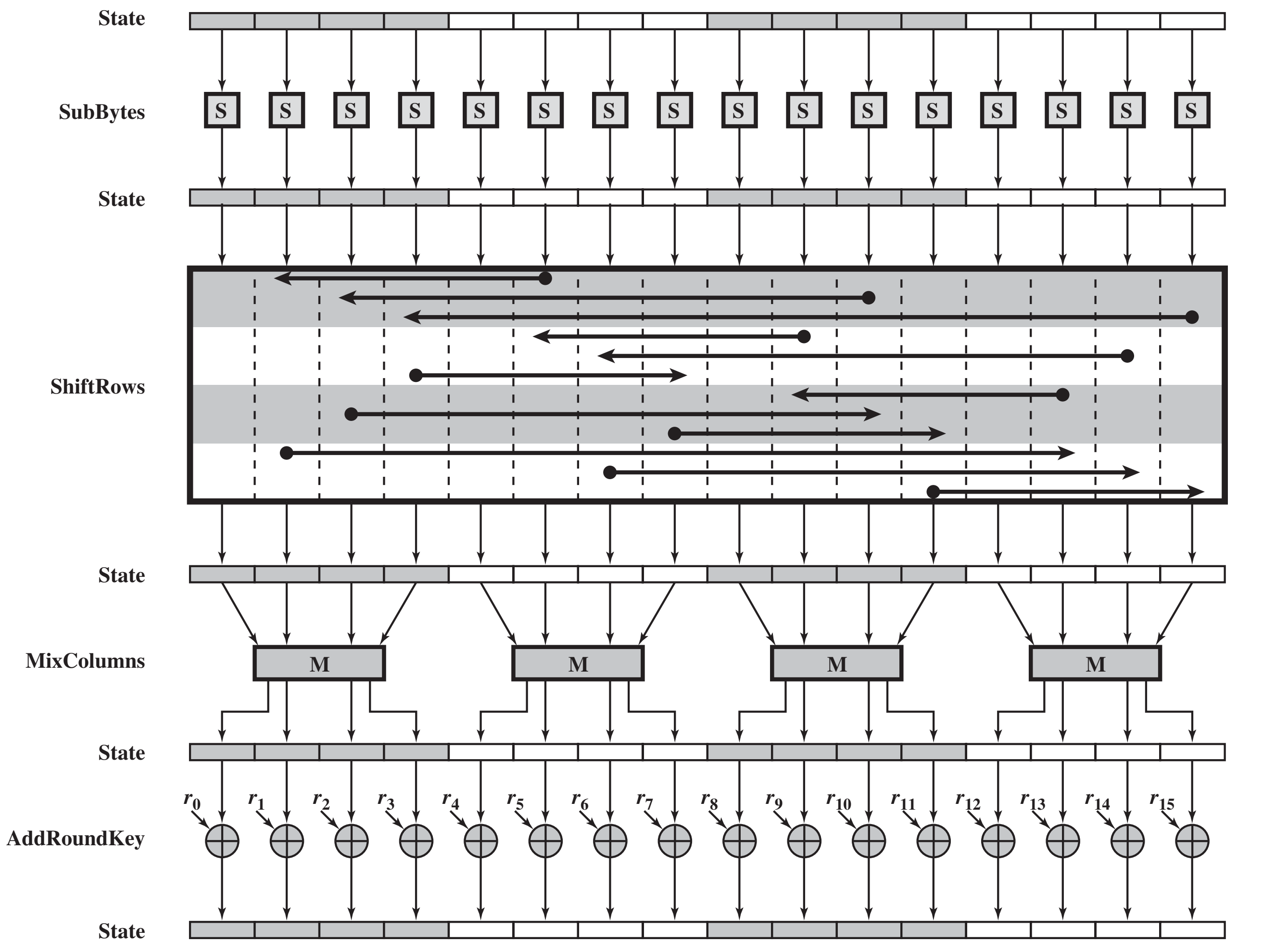
$$\{02\} \cdot A = (a_6 \dots a_1 a_0 0) \oplus (00011011), \text{ if } a_7 = 1$$

3 AES Round Function

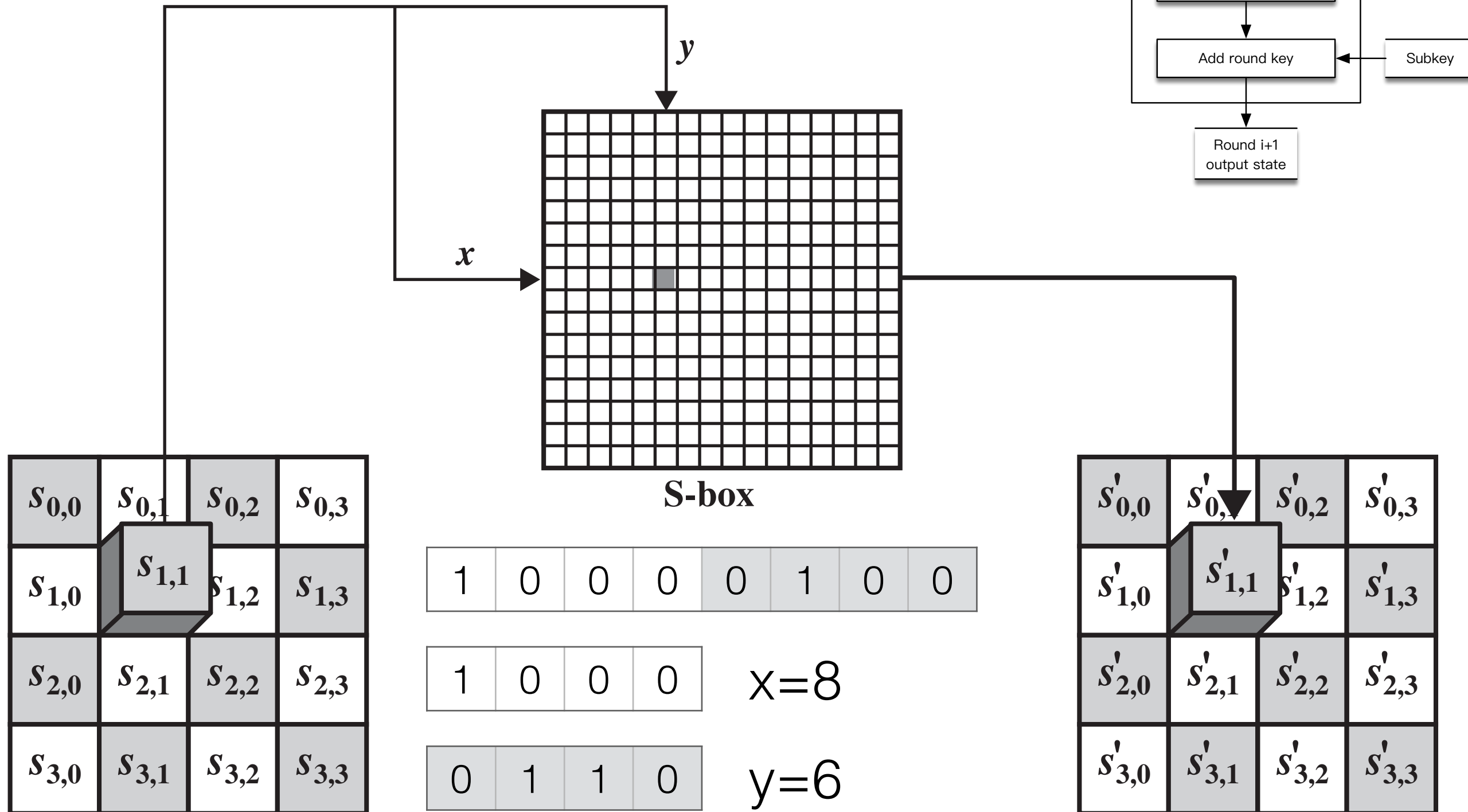
Example: AES-128



- Observation
 - The structure is quite simple
 - The subkey is only used in 4th stage
 - Each stage is easily reversible
 - The decryption algorithm is not identical to the encryption algorithm
 - The final round of both encryption and decryption consists of only three stages



Stage 1: Substitute bytes



1000

x=8

0110

y=6

01000100

s=44

S-box		y															
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x	0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
	1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
	2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
	3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
	4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
	5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
	6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
	7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
	8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
	9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
	A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
	B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
	C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
	D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
	E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
	F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

01000100

s=44

1000

x=8

0110

y=6

Inverse S-box		y															
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
x	0	52	09	6A	D5	30	36	A5	38	BF	40	A3	9E	81	F3	D7	FB
	1	7C	E3	39	82	9B	2F	FF	87	34	8E	43	44	C4	DE	E9	CB
	2	54	7B	94	32	A6	C2	23	3D	EE	4C	95	0B	42	FA	C3	4E
	3	08	2E	A1	66	28	D9	24	B2	76	5B	A2	49	6D	8B	D1	25
	4	72	F8	F6	64	86	68	98	16	D4	A4	5C	CC	5D	65	B6	92
	5	6C	70	48	50	FD	ED	B9	DA	5E	15	46	57	A7	8D	9D	84
	6	90	D8	AB	00	8C	BC	D3	0A	F7	E4	58	05	B8	B3	45	06
	7	D0	2C	1E	8F	CA	3F	0F	02	C1	AF	BD	03	01	13	8A	6B
	8	3A	91	11	41	4F	67	DC	EA	97	F2	CF	CE	F0	B4	E6	73
	9	96	AC	74	22	E7	AD	35	85	E2	F9	37	E8	1C	75	DF	6E
	A	47	F1	1A	71	1D	29	C5	89	6F	B7	62	0E	AA	18	BE	1B
	B	FC	56	3E	4B	C6	D2	79	20	9A	DB	C0	FE	78	CD	5A	F4
	C	1F	DD	A8	33	88	07	C7	31	B1	12	10	59	27	80	EC	5F
	D	60	51	7F	A9	19	B5	4A	0D	2D	E5	7A	9F	93	C9	9C	EF
	E	A0	E0	3B	4D	AE	2A	F5	B0	C8	EB	BB	3C	83	53	99	61
	F	17	2B	04	7E	BA	77	D6	26	E1	69	14	63	55	21	0C	7D

3 AES Round Function

Stage 1: Substitute bytes

- How to build S-box and Inverse S-box?

EA	04	65	85
83	45	5D	96
5C	33	98	B0
F0	2D	AD	C5

→

87	F2	4D	97
EC	6E	4C	90
4A	C3	46	E7
8C	D8	95	A6

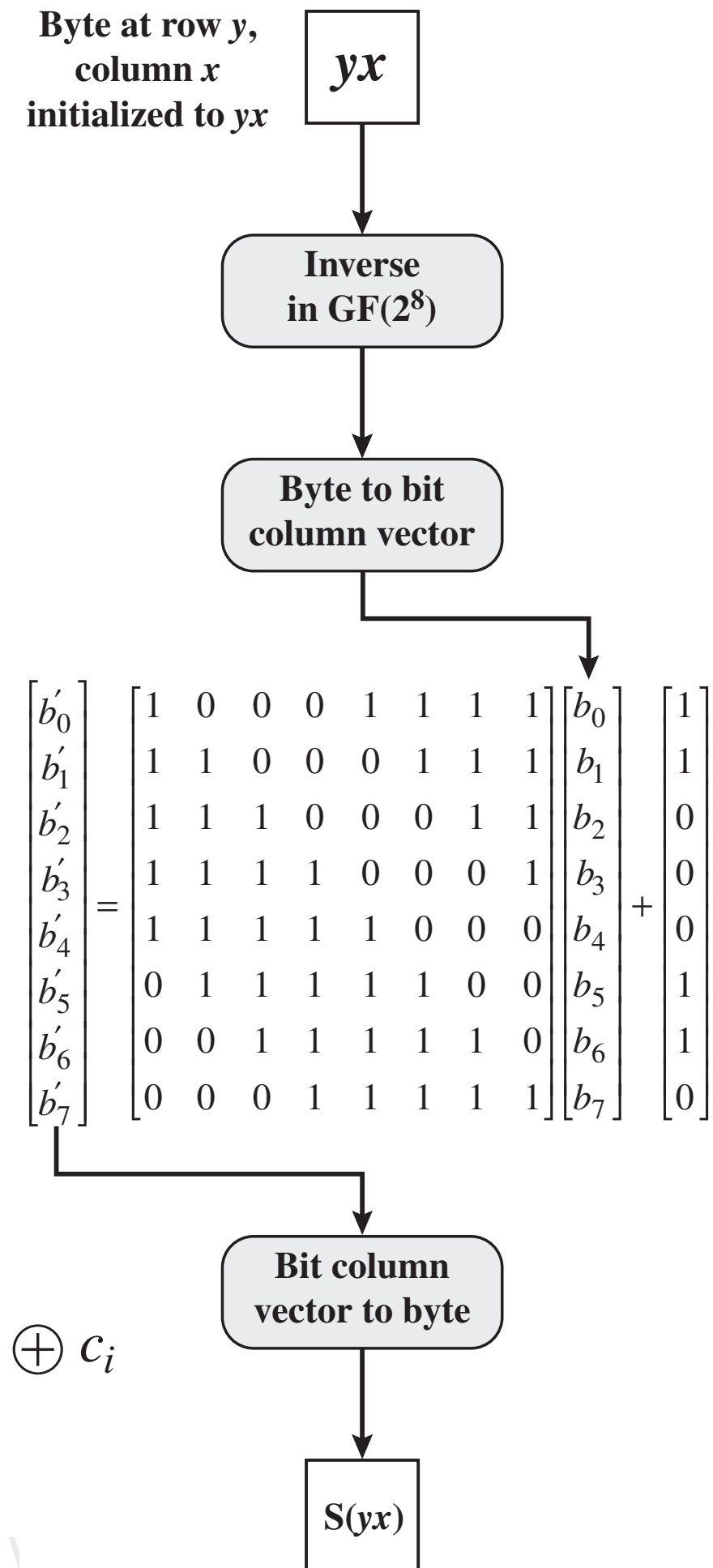
3 AES Round Function

Stage 1: Substitute bytes

1. Initialize the S-box with the **byte values** row by row
 - The first row contains {00}, {01}, {02}, ... , {0F}
 - The second row contains {10}, {11}, etc.
 - the value of the byte at row y , column x is $\{yx\}$
2. Map each byte in the S-box to its **multiplicative inverse** in the finite field $GF(2^8)$
 - Specially, the value {00} is mapped to itself.
3. Transformation: Consider that each byte in the S-box consists of 8 bits labeled $(b_7, b_6, b_5, b_4, b_3, b_2, b_1, b_0)$, compute b'_i

$$b'_i = b_i \oplus b_{(i+4) \bmod 8} \oplus b_{(i+5) \bmod 8} \oplus b_{(i+6) \bmod 8} \oplus b_{(i+7) \bmod 8} \oplus c_i$$

4. Bit column vector to byte into $S(y,x)$

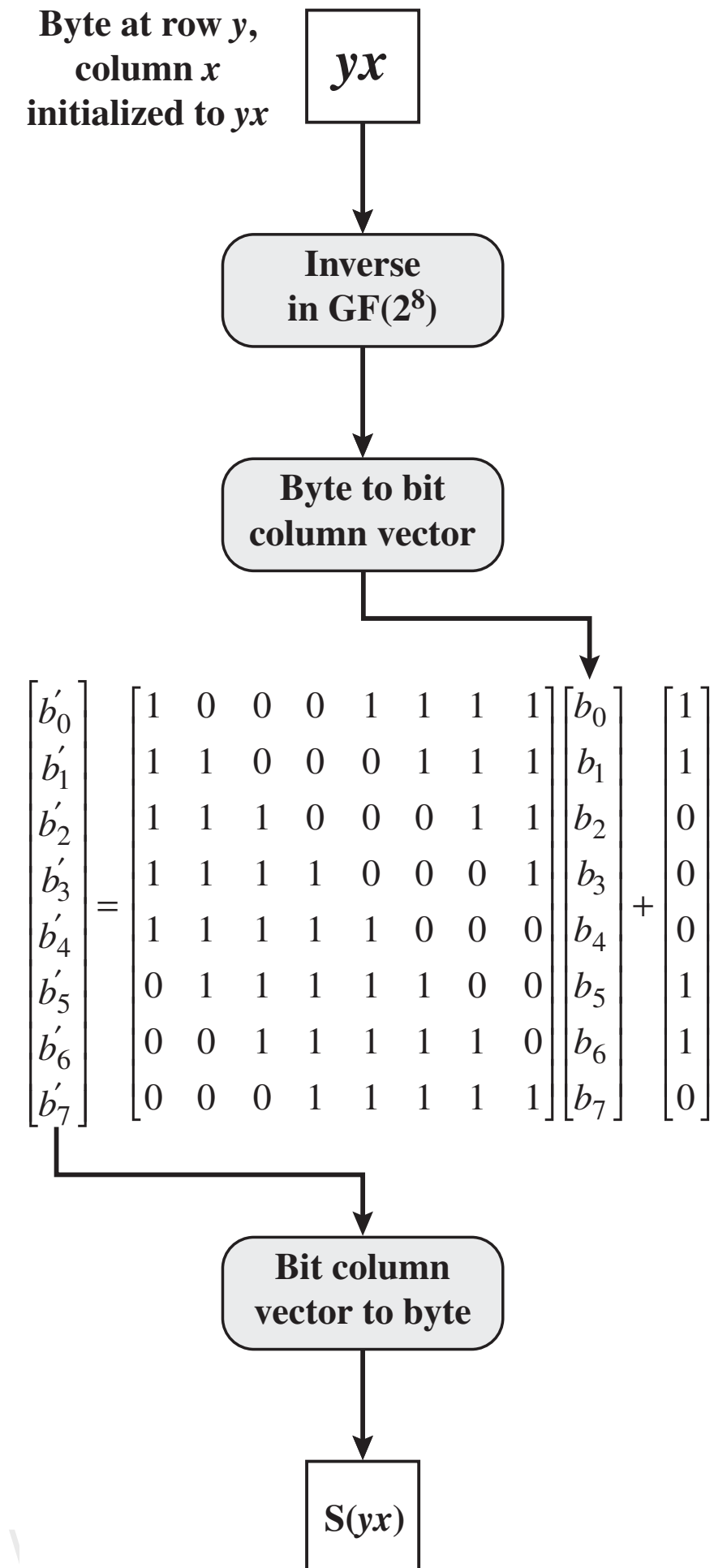


3 AES Round Function

Stage 1: Substitute bytes

Example: input Row 9, Column 5

1. The value of byte: {95}
2. The inverse of {95} is {8A}
= $\{10001010\}$
3. $M \cdot [0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1]' + [1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0]'$
4. $S(95) = \{2A\}$



Byte at row y ,
column x
initialized to yx

yx

Inverse
in $GF(2^8)$

Byte to bit
column vector

$$\begin{bmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Bit column
vector to byte

$S(yx)$

3 AES Round Function Stage 1: Substitute bytes

From S-box to Inverse S-box

$$\mathbf{B}' = \mathbf{XB} \oplus \mathbf{C}$$

$$\mathbf{Y}(\mathbf{XB} \oplus \mathbf{C}) \oplus \mathbf{D} = \mathbf{B}$$

$$\mathbf{YXB} \oplus \mathbf{YC} \oplus \mathbf{D} = \mathbf{B}$$

$$\mathbf{YC} = \mathbf{D}$$

$\mathbf{YX}$ equals the identity matrix

Byte at row y ,
column x
initialized to yx

yx

Byte to bit
column vector

$$\begin{bmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Bit column
vector to byte

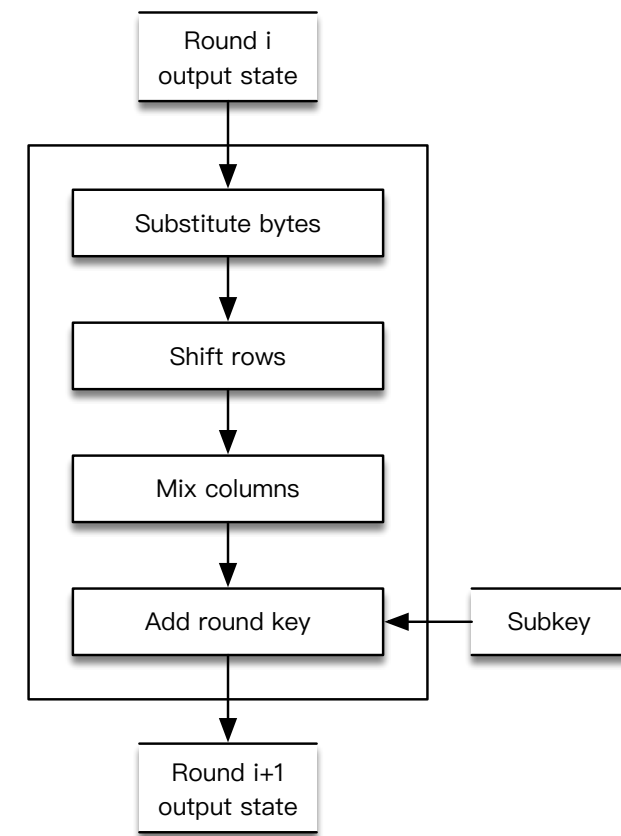
Inverse
in $GF(2^8)$

$IS(yx)$

3 AES Round Function

Stage 1: Substitute bytes

- The Rationale of Stage 1
 - Resistant to known cryptanalytic attacks.
 - low correlation between input bits and output bits

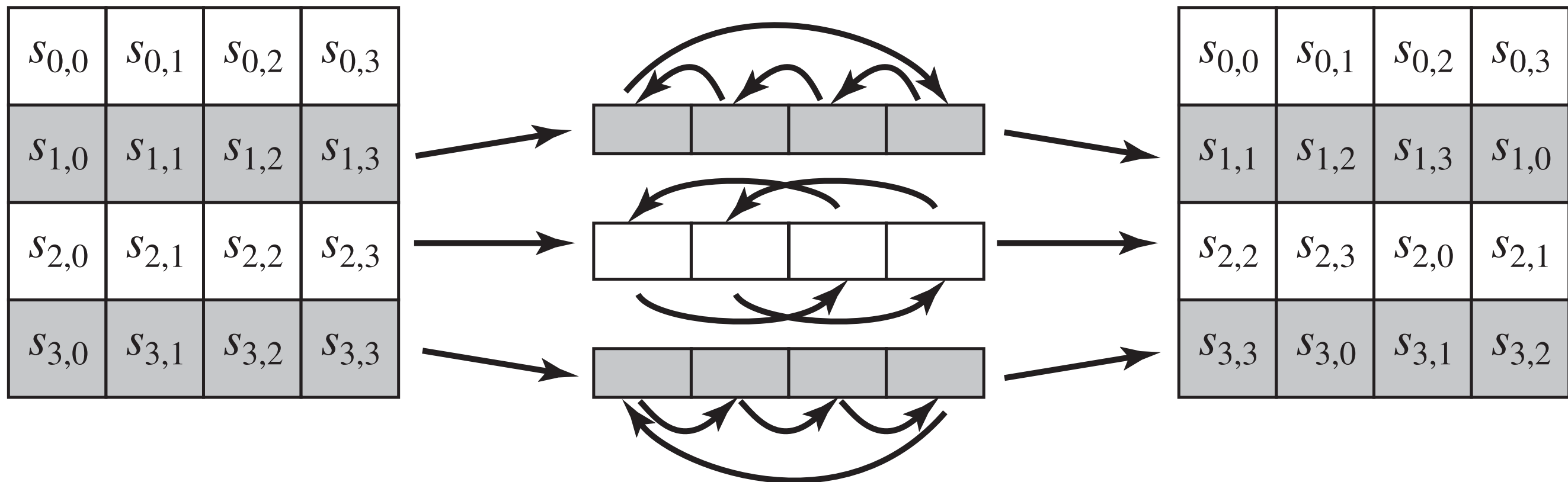
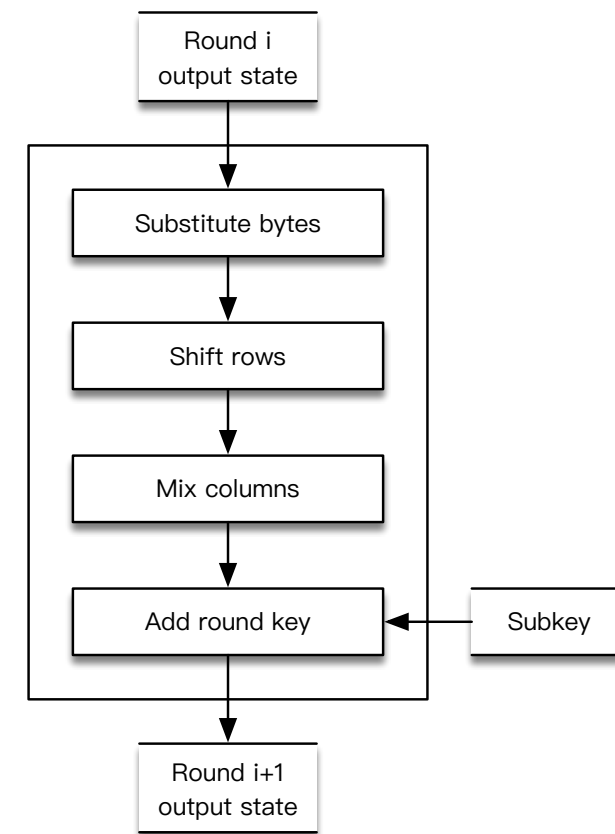


$$b'_i = b_i \oplus b_{(i+4) \bmod 8} \oplus b_{(i+5) \bmod 8} \oplus b_{(i+6) \bmod 8} \oplus b_{(i+7) \bmod 8} \oplus c_i$$

- The output is **not** a **linear** mathematical function of the input
 - The nonlinearity is due to the use of the **multiplicative inverse**
- No fixed points, i.e., $S\text{-box}(a) \neq a$

3 AES Round Function

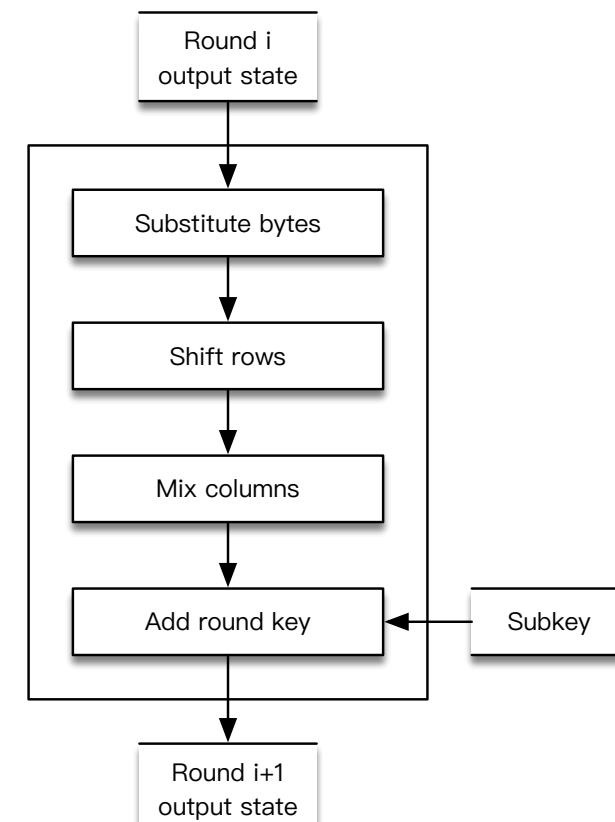
Stage 2: Shift rows



3 AES Round Function

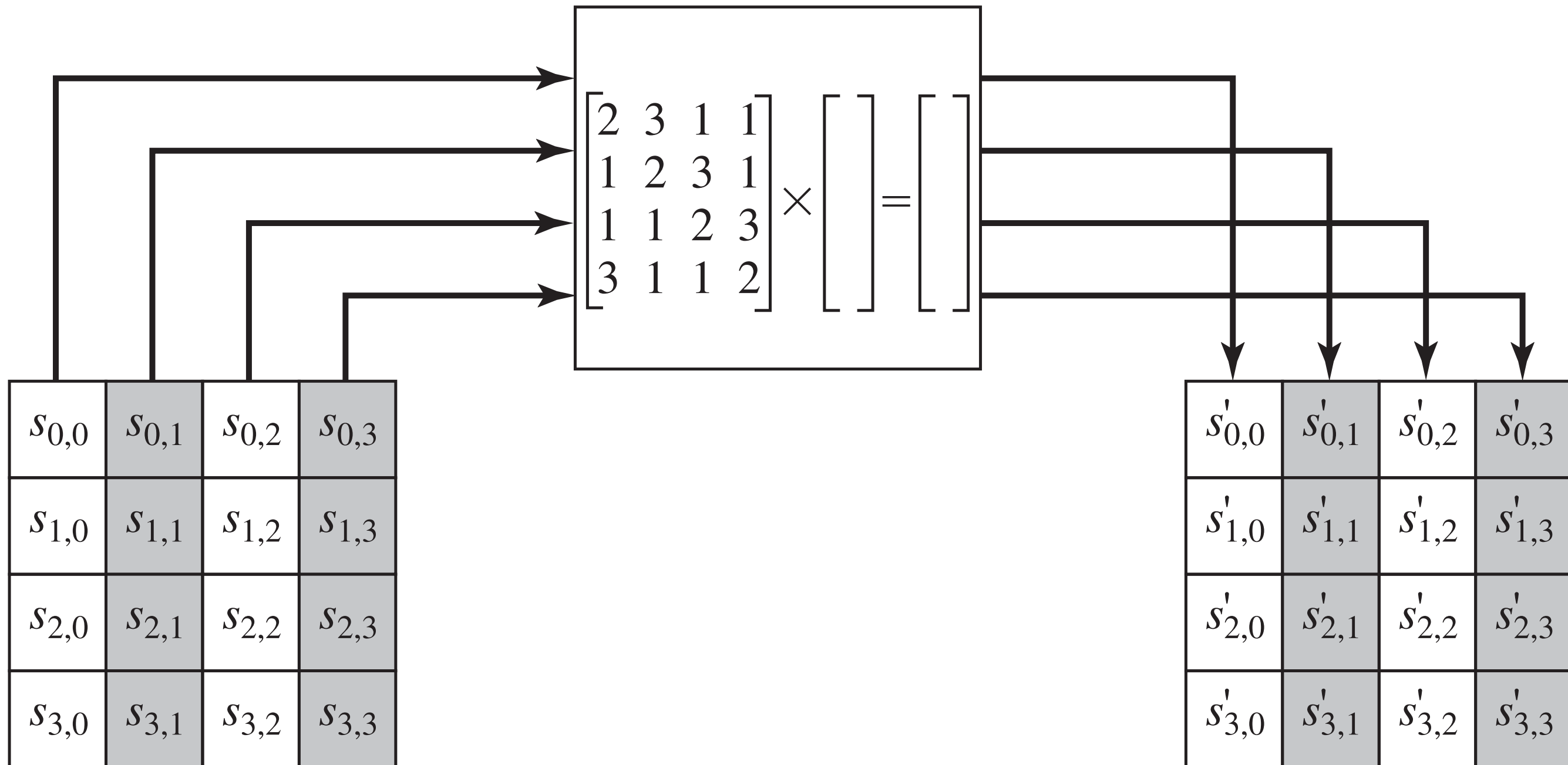
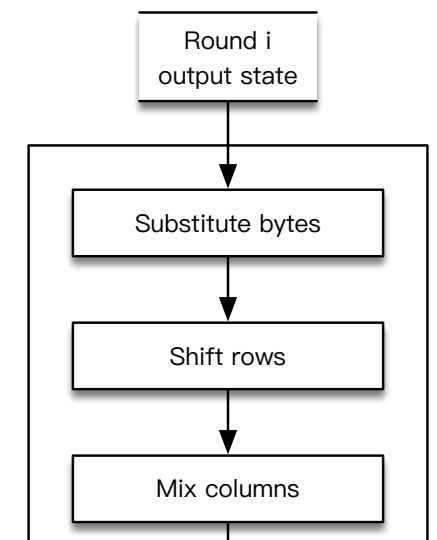
Stage 2: Shift rows

- The Rationale of Stage 2
 - The **State**, as well as the cipher input and output, is treated as an array of four **4-byte columns**.
 - A row shift moves an individual byte from one **column** to another
 - The transformation ensures that the 4 bytes of one column are **spread** out to four **different columns**



3 AES Round Function

Stage 3: Mix Columns



3 AES Round Function

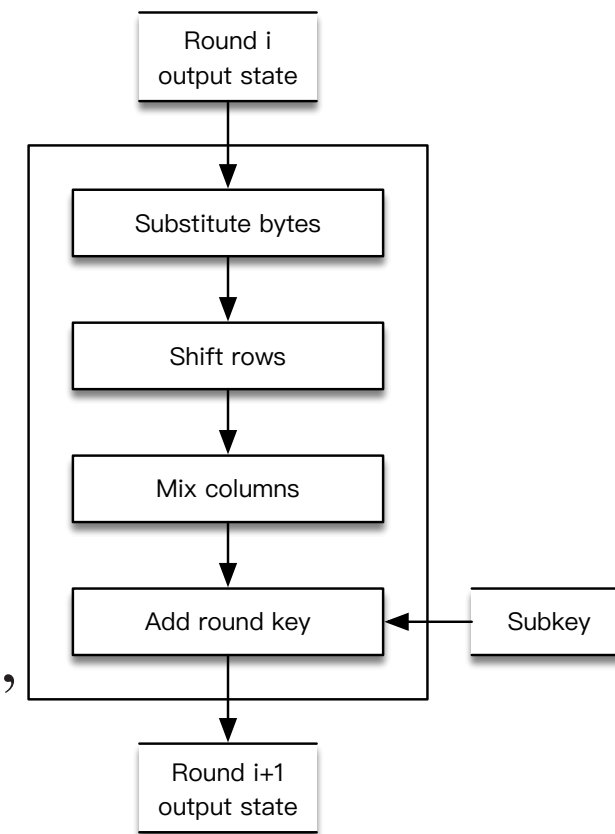
Stage 3: Mix Columns

$$s'_{0,j} = (2 \cdot s_{0,j}) \oplus (3 \cdot s_{1,j}) \oplus s_{2,j} \oplus s_{3,j}$$

$$s'_{1,j} = s_{0,j} \oplus (2 \cdot s_{1,j}) \oplus (3 \cdot s_{2,j}) \oplus s_{3,j}$$

$$s'_{2,j} = s_{0,j} \oplus s_{1,j} \oplus (2 \cdot s_{2,j}) \oplus (3 \cdot s_{3,j})$$

$$s'_{3,j} = (3 \cdot s_{0,j}) \oplus s_{1,j} \oplus s_{2,j} \oplus (2 \cdot s_{3,j})$$

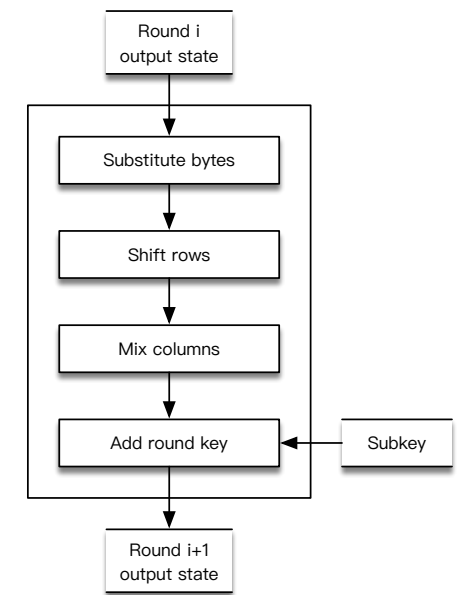


$$\{02\} \cdot A = (a_6 \dots a_1 a_0 0), \text{ if } a_7 = 0$$

$$\{02\} \cdot A = (a_6 \dots a_1 a_0 0) \oplus (00011011), \text{ if } a_7 = 1$$

3 AES Round Function

Stage 3: Mix Columns



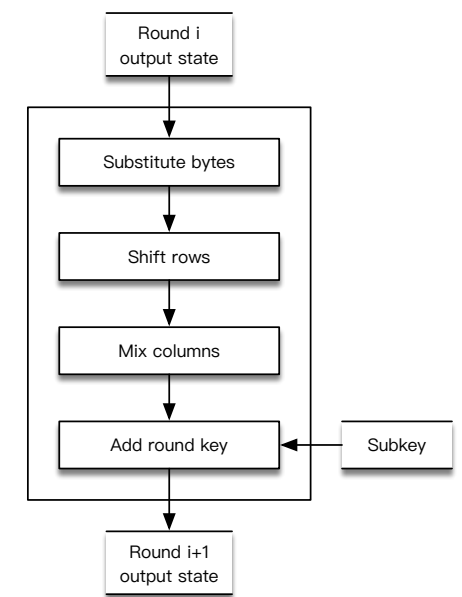
- Inverse mix column transformation

$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix}$$

3 AES Round Function

Stage 3: Mix Columns



- The Rationale of Stage 3
 - The **coefficients** of the matrix are based on a linear code with **maximal distance** between code words
 - which ensures a good **mixing** among the **bytes of each column**
 - The **choice of coefficients** in MixColumns, which are all **{01}, { 02}, or {03}**, was influenced by implementation considerations

3 AES Round Function

Stage 4: Add Round Key

- The first matrix is State
- the second matrix is the round key

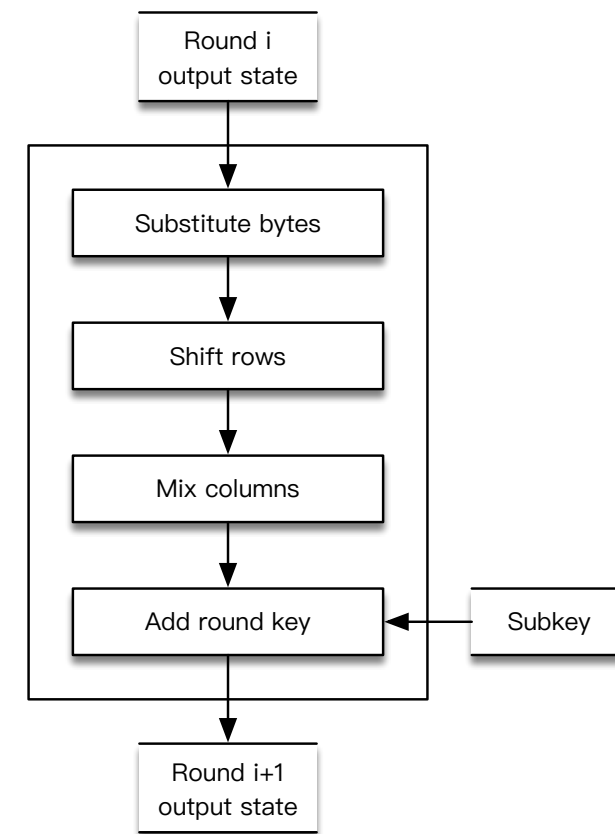
47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

 $\oplus$

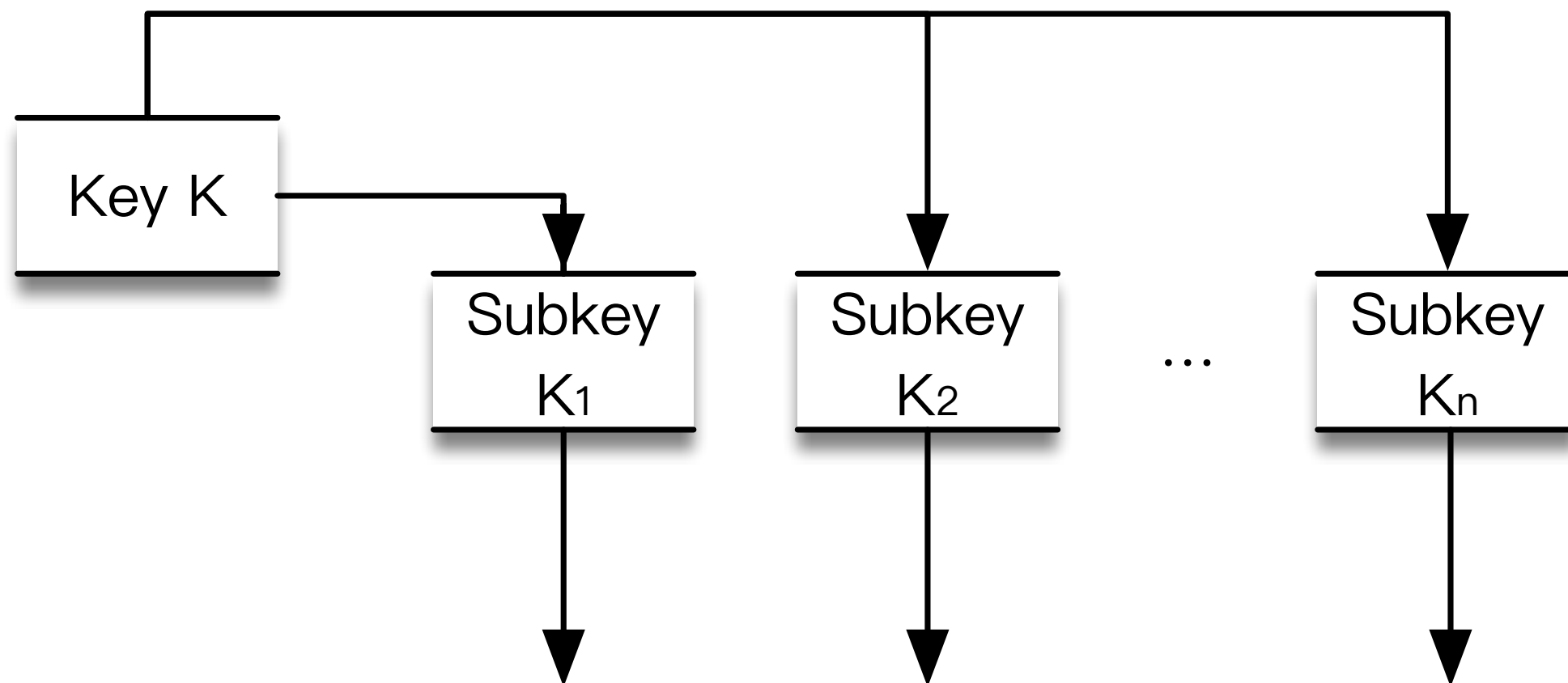
AC	19	28	57
77	FA	D1	5C
66	DC	29	00
F3	21	41	6A

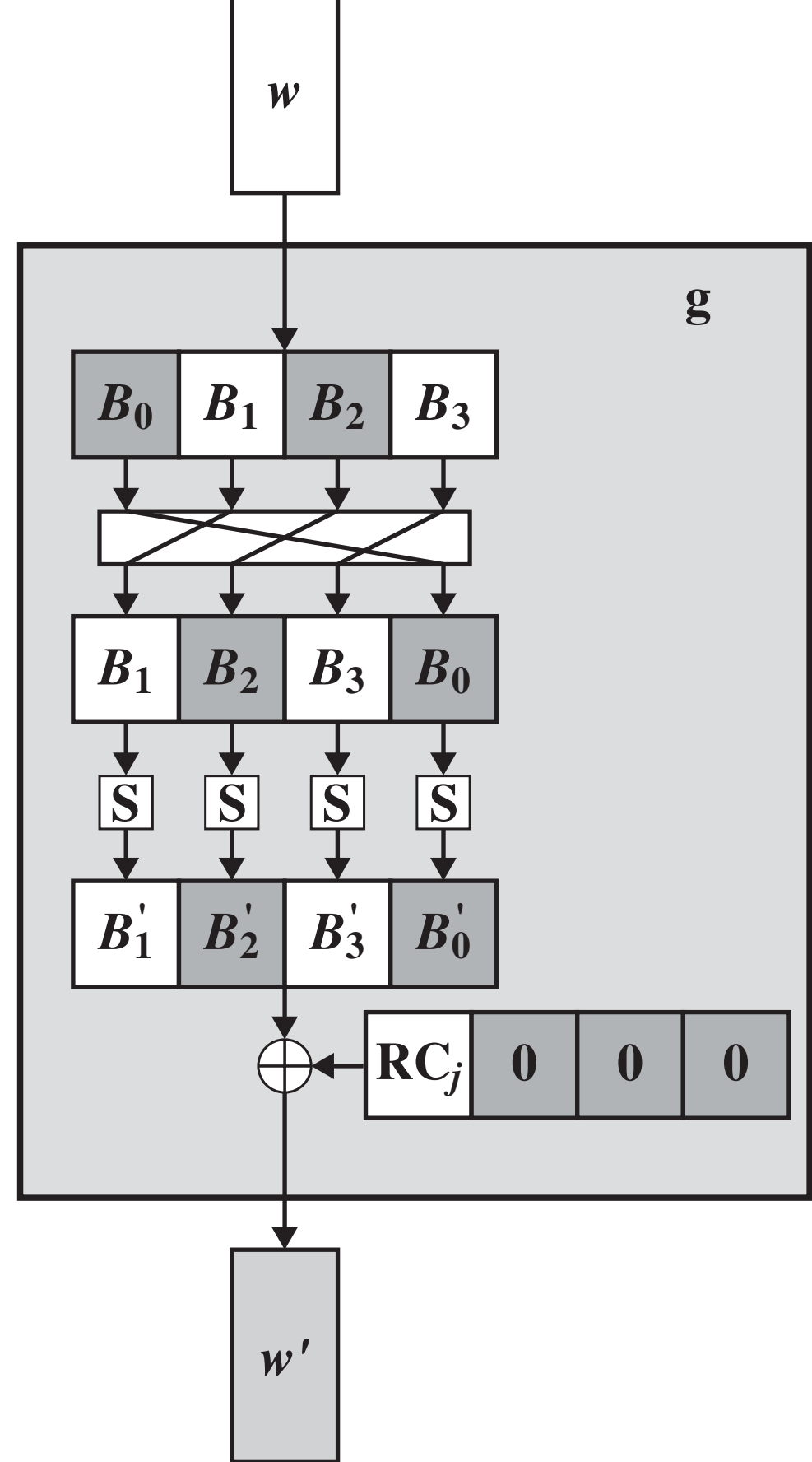
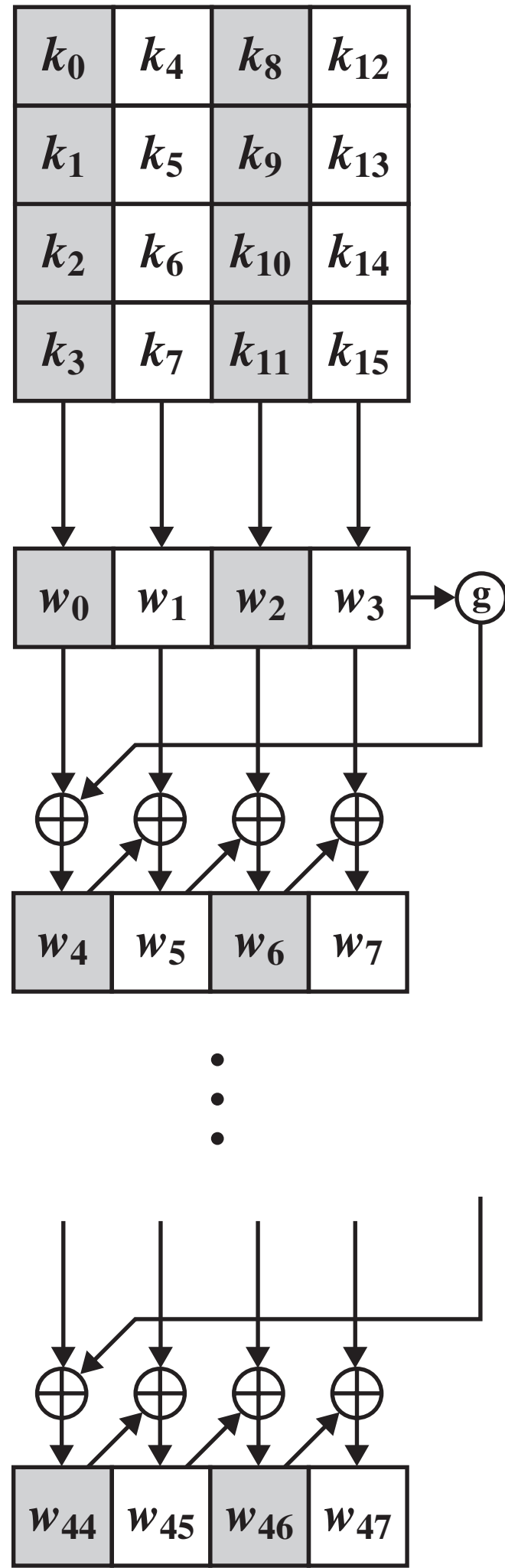
 $=$

EB	59	8B	1B
40	2E	A1	C3
F2	38	13	42
1E	84	E7	D6



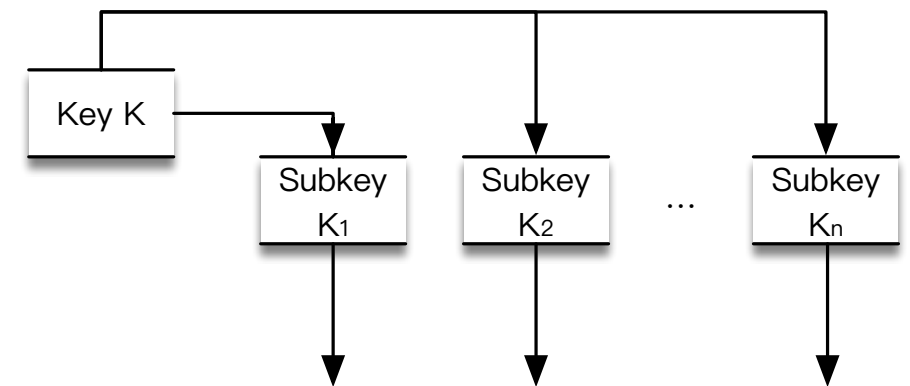
4 AES Key Expansion





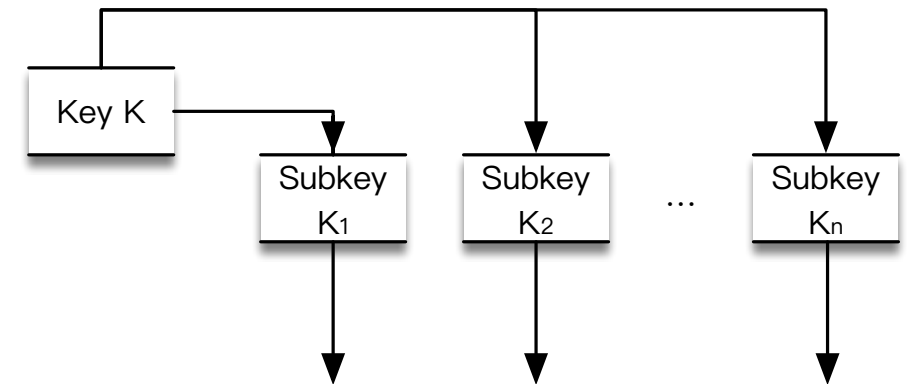
(b) Function g

4 AES Key Expansion



- The rational of AES Key Expansion
 - Knowledge of a part of the cipher key or round key does not enable calculation of many other round-key bits.
 - An invertible transformation
 - knowledge of any N_k consecutive words of the expanded key enables regeneration the entire expanded key (N_k = key size in words)
- Speed on a wide range of processors.

4 AES Key Expansion



- The rational of AES Key Expansion
 - Usage of round constants to eliminate symmetries.
 - Diffusion (扩散) of cipher key differences into the round keys
 - each key bit affects many round key bits.
 - Enough nonlinearity to prohibit the full determination of round key differences from cipher key differences only.
 - Simplicity of description

5 Security Analysis

The Avalanche Effect (雪崩效应)

- (Recall) *Definition*: a change in one bit of the plaintext or one bit of the key should produce a change in many bits of the ciphertext
- If the change were small, this might provide a way to reduce the size of the plaintext or key space to be searched

5 Security Analysis

Cryptanalytic attacks: XSL attack

- **XSL attack**
 - In 2002
 - A theoretical attack
 - Announced by Nicolas Courtois and Josef Pieprzyk
 - A weakness in the AES algorithm
 - partially due to the low complexity of its nonlinear components
- Since then, other papers have shown that the attack, as originally presented, is unworkable

5 Security Analysis

Cryptanalytic attacks: XSL attack

- The process of XSL attack
 - The problem can be **reduced** to another problem
 - Solving multivariate quadratic equations (多元二次方程组)
 - However, solving multivariate quadratic equations (MQ) over a finite set of numbers is an **NP-hard problem** (in the general case) with several applications in cryptography

5 Security Analysis

Cryptanalytic attacks: XSL attack

- Discussions on XSL attack

The method has some merit, and is worth investigating, but it does not break Rijndael as it stands.

The XSL attack is not an attack. It is a dream.

XSL may be a dream. It may also be a very bad dream and turn into a nightmare.

Example: input Row 9,
Column 5

1. The value of byte:
{95}
2. The inverse of {95}
is {8A}={10001010}
3. $M^* \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 \end{bmatrix}' + \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}'$
4. $S(95)=\{2A\}$

5 Security Analysis

Cryptanalytic attacks: XSL attack

Example: input Row 9,
Column 5

- Discussions on XSL attack

We have one criticism of AES: we don't quite trust the security...

What concerns us the most about AES is its simple algebraic structure...

No other block cipher we know of has such a simple algebraic representation.

1. The value of byte:
{95}

2. The inverse of {95}
is {8A}={10001010}

3. $M^* \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 \end{bmatrix}' + \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}'$

4. $S(95)=\{2A\}$

5 Security Analysis

Cryptanalytic attacks

- 2009
 - The complexity of 2^{119}
- 2009.12
 - The complexity of $2^{99.5}$
- All the previous attacks are impractical

5 Security Analysis

Side-channel Attack (旁路攻击)

- 2005.4 Cache-timing attack
 - [OpenSSL](#)'s AES encryption
 - Requirements: 200 million chosen plaintext
- 2005.10 several cache-timing attacks
 - One requirements: 800 operations triggering encryptions
 - 65ms

5 Security Analysis

Side-channel Attack (旁路攻击)

- 2009
 - an attack on some hardware implementations
 - Differential fault analysis
 - The complexity of 2^{32}
- 2010
 - AES-128 implementations that use compression tables, such as OpenSSL
 - A "near real time" recovery of secret keys
 - Without the need for either cipher text or plaintext
 - Requirements: the ability to run unprivileged code on the system performing the AES encryption

5 Security Analysis

Side-channel Attack (旁路攻击)

- 2016
 - Attacks on 128 bit AES
 - a substantial improvement over previous works that require between 100 and a million encryptions
 - Requirements:
 - recover the complete 128-bit AES key in just 6–7 blocks of plaintext/ciphertext
 - standard user privilege
 - key-retrieval algorithms run under a minute

5 Security Analysis

Side-channel Attack (旁路攻击)

- Protections against timing-related side-channel attacks
- Many modern CPUs have built-in [hardware instructions for AES](#)

Homework

- Null