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Focusing Properties of Hypergeometric Gaussian Beam through a High Numerical-Aperture Objective

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Abstract—The focusing properties of radially polarized hypergeometric Gaussian beam are studied using the Richards-Wolf vectorial diffraction model. Such a polarized beam is decomposed into radial and longitudinal polarization. With a proper combination of the beam order, beam size and imaginary parameter variables, the adjustably confined flat-topped focus and focal hole can be obtained in the focal region. Moreover, we got originality characteristic for the axial intensity distribution of two shaped symmetric light spots. The tight focusing of a hypergeometric Gaussian beam may find applications in data storage, laser drilling, optical trapping, etc.

1. INTRODUCTION

It is well known that increased induced field intensity in a localized area is required in a variety of applications in optical data storage, microscopy, material processing, micro-particle trapping manipulation, etc. [1–5]. There are many ways to realize increased induced field intensity in a localized area. For example, field enhancement of incident near-infrared light can be achieved by using the exhibited surface plasmon polariton from erbium ions in a golden film [6]. A grounded dielectric slab using a single supersrate layer to achieve field enhancement of Gaussian beams is also investigated in [7–9]. In addition, based on Richards-Wolf vectorial diffraction model [10, 11], it is predicted that strong electric field and a smaller spot are achievable by tightly focusing a radially polarized beam [12, 13]. If the intensity pattern of the radially polarized beam is like an annular, the spot size will be much smaller and the field intensity stronger [14].

Defined by a separable-variable-based solution of the paraxial parabolic equation in cylindrical coordinate systems, a novel family of hypergeometric beams was firstly introduced by Kotlyar et al. [15]. A fully coherent hypergeometric beam would exhibit non-diffractive behavior upon propagation in free space. But these modes have an infinite energy. One way of achieving a practically realizable beam is to modulate the hypergeometric beam with a Gaussian exponential. In this way, we obtain the hypergeometric Gaussian beam. Generation and particular cases of the intensity and diffraction plots for hypergeometric Gaussian beam were discussed and analyzed [16, 17]. Near- and far-field profiles of the hypergeometric Gaussian beam were considered in [18]. However, to the best of our knowledge, there are no papers studying the focusing of the hypergeometric Gaussian beam through a high NA objective.

In this paper, we study the focus-shaping technique of hypergeometric Gaussian beam by a high NA lens. Numerical simulation shows that total intensity distribution in the vicinity of the focus is dependent not only on the beam order and the beam size, but also on the imaginary parameter variables γ. The adjustably confined flat-topped focus and focal hole can be obtained by choosing a proper combination of parameters. Specifically, the distribution of the axial intensity shaped two

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symmetric light spots about the focus can be achieved for different γ. Finally, we discuss the full-width at half-maximum (FWHM) and beam quality on the focal plane. The results show that subwavelength focusing and high beam quality are achieved. Hence an added advantage of using this technique is that it allows us to open up new applications of such small flat-topped focus and focal holes.

2. RICHARDS-WOLF DIFFRACTION THEORY

For a high NA objective lens, the electric field of a hypergeometric Gaussian incident beam at a pupil can be defined as follow [16, 19].

$$E(\theta) = \left(\frac{f \sin\theta}{w_0}\right)^{|m|+i\gamma} \exp\left(-\frac{f^2 \sin^2\theta}{w_0^2}\right) \tag{1}$$

where m is an integer, and it is the order of hypergeometric Gaussian beam. w_0 denotes the source beam size of a fundamental Gaussian beam. However, a new kind of hypergeometric Gaussian beam is obtained if m is lager than 1. f is the focal length of the lens and γ the real parameters. The value of θ is related to the NA of objective lens and $0 \le \theta \le \arcsin(\mathrm{NA}/n)$. $\alpha = \arcsin(\mathrm{NA}/n)$ is the maximal NA angle, n the refractive index between the objective lens and the sample, and $n = 1$ the index of refraction of free space. Obviously, as defined in Eq. (1), the amplitude distribution of hypergeometric Gaussian beam is determined by m and w_0.

The analysis was performed on the basis of Richards-Wolf vectorial diffraction method widely used for high NA focusing systems at arbitrary incident polarization. In the case of a radially polarized hypergeometric Gaussian beam, the focusing field is simplified as [20, 21].

$$E_r(r,z) = A \int_0^{\alpha} \sin(2\theta) \sqrt{\cos\theta}\, E(\theta)\, J_1(knr\sin\theta) \exp(iknz\cos\theta)\, d\theta \tag{2}$$

$$E_z(r,z) = -2iA \int_0^{\alpha} \sin^2\theta \sqrt{\cos\theta}\, E(\theta)\, J_0(knr\sin\theta) \exp(iknz\cos\theta)\, d\theta \tag{3}$$

where $E_r(r,z)$ and $E_z(r,z)$ are radial and longitudinal electric field components near the focal plane, respectively. The azimuthal component $E_\varphi(r,z)$ of the electric field is zero everywhere in the diffraction field and $A = E_0\pi f n/\lambda$, E_0 a constant relating to the power of the incident beam, f the focal length, $k = 2\pi/\lambda$ the wave number in the vacuum. $J_0(x)$ and $J_1(x)$ denote Bessel functions of zero and first order, and $E(\theta)$ is the amplitude of the electric field in front of the pupil. Considering that $E_r(r,z)$ and $E_z(r,z)$ are orthogonal, the total intensity is given as $E_{total}^2 = E_r^2(r,z) + E_z^2(r,z)$.

In order to describe the focusing property, the FWHM on focal plane will be investigated. Meanwhile, in order to describe longitudinal component of the focused field, the beam quality η is defined as: $\eta = \Phi_z/(\Phi_r + \Phi_z)$. Where $\Phi_j = 2\pi \int_0^{r_0} |E_j(r,0)|^2 r dr (j = r \text{ or } z)$, here r_0 is the first zero point in the distribution of focusing electric density on the focal plane. Through numerical calculation, the size of η has a close relationship with the values of m and w_0. However, in order to include the total energy of incident beam, the upper range of integration is taken as 4λ, which is larger than r_0.

3. FOCUSING PROPERTY

In order to obtain the focusing property of a radially polarized hypergeometric Gaussian beam. Based on Eq. (2) and Eq. (3), the focusing performance of the radially polarized hypergeometric Gaussian beam is shown in Figure 1 and Figure 2 for $\gamma = 0$ and $\gamma \neq 0$, respectively. Here the numerical aperture of the focusing lens is considered as NA $= 0.95$ and $f = 1$ cm. $n = 1$ is the index of refraction of free space. The laser wavelength is 632.8 nm and the power of laser 100 mW.

Figures 1(a), 1(e) and 1(i) display the lateral normalized intensity distribution on the focal plane. The total intensity distribution of the incident radially polarized hypergeometric Gaussian beam focused by high NA objective on the focal plane is described as black curves in Figures 1(a), 1(e) and 1(i). Obviously, the radius of incident hypergeometric Gaussian beam increases gradually from Figures 1(a),

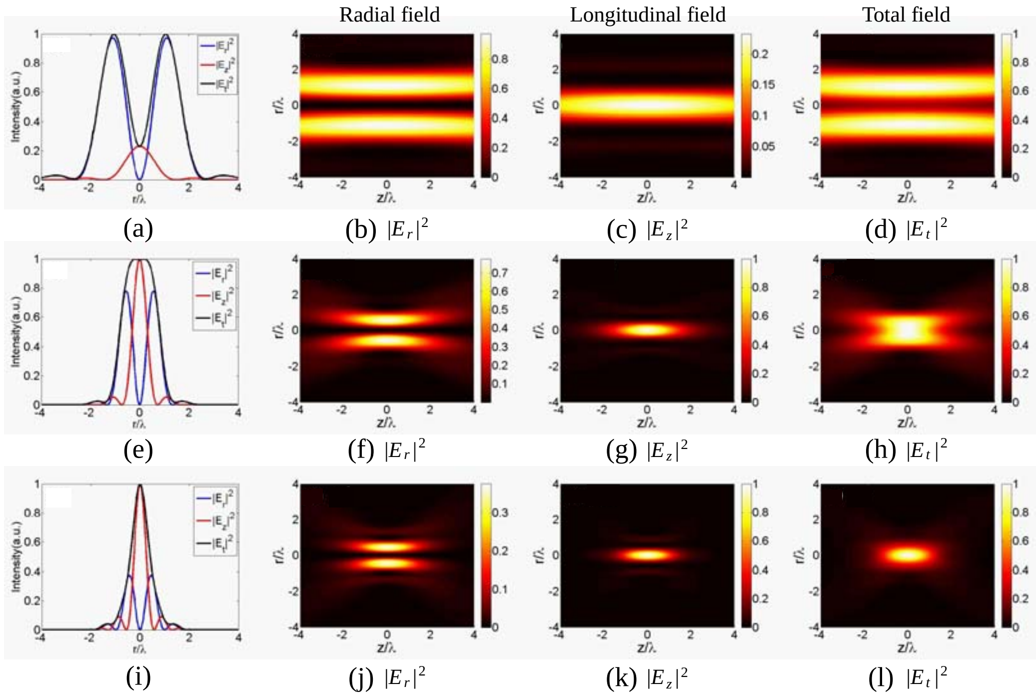

Figure 1. Focusing performance of radially polarized Hypergeometric Gaussian beam with different values of parameter (m, w_0) and $\gamma = 0$, (a)–(d) for $m = 4$, $w_0 = 1.5\,\mathrm{mm}$, (e)–(h) for $m = 4$, $w_0 = 3\,\mathrm{mm}$ and (i)–(l) for $m = 8$, $w_0 = 3\,\mathrm{mm}$, respectively.

to 1(e) and to 1(i). Evidently, for larger m and w_0, the circular focusing spot appears, and the lateral size of the focusing spot decreases. One can see that focusing is achieved in Figures 1(e) and 1(i). Moreover, Figure 1(e) shows flat topped distribution of the amplitude, which can be regarded as the critical condition for obtaining focusing. So when $m = 4$ and $w_0 = 1.5\,\mathrm{mm}$, the incident radially polarized hypergeometric Gaussian beam cannot achieve focusing on the focal plane. As shown in Figures 1(b)–1(d), 1(f)–1(h), and 1(j)–1(l), one can easily know that the total focused field is determined by radial and longitudinal components. The lateral intensity distribution of radial electric field component forms a hollow distribution on the axial or on the focal plane. Furthermore, the position of the maximum of the radial component is most near the axis. But the longitudinal component has an intensity maximum on the axis precisely. Therefore, The maximum intensity of radial and longitudinal electric fields can directly decide the total intensity distribution of the focal plane and size of the focal spot. As shown in the blue curve in Figure 1(a) or contour plot of Figure 1(d), if the maximum radial electric field component is greater than the maximum value of the longitudinal field component, the incident radially polarized hypergeometric Gaussian beam cannot realize focusing on the focal plane. On the contrary, in the vicinity of the focus, focusing spot can be achieved.

According to Eq. (2), because of $J_1(0) = 0$, the intensity of radial component in the longitudinal direction at focal spot is zero, which is shown as the blue curves in Figures 1(a), 1(e) and 1(i). If the maximal intensity is not located on the focal plane, the size of focal spot will be enlarged. Meanwhile, in Eq. (3), $E_z(0, 0) \neq 0$. Therefore, the focusing is determined by the weight of longitudinal component. Obviously, numerical results indicate that the weight of longitudinal electric field component at focal spot will increase for large value of parameters m and w_0. Actually, if large value of w_0 (or m) is selected, for fixed value of m (or w_0) the focused field and subwavelength focal spot can be achieved. However, extremely large values of m and w_0 are not recommended based on the numerical results. It is worth noticing that the incident energy should be confined in the pupil of objective lens when one determines the value of parameter of a hypergeometric Gaussian beam. Therefore, considering the previous designed annual filter, most incident energy of a hypergeometric Gaussian beam lies in the region near the edge of the objective lens.

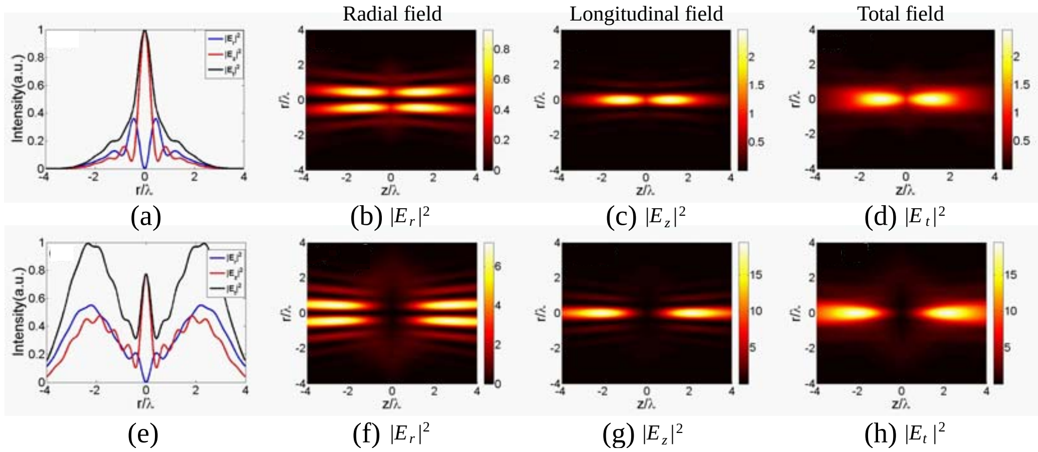

Figure 2. Focusing performance of radially polarized Hypergeometric Gaussian beam with different values of parameter (m, w_0) and $\gamma \neq 0$, (a)–(d) for $m = 8$, $w_0 = 3$ mm and $\gamma = 5$; (e)–(h) for $m = 8$, $w_0 = 3$ mm and $\gamma = 10$, respectively.

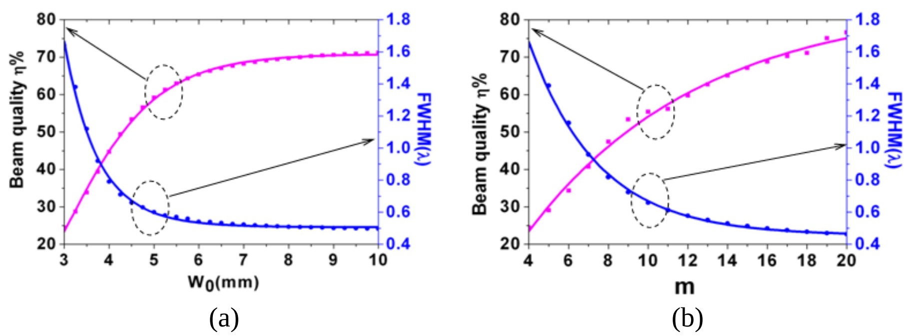

Figure 3. The beam quality and FWHM of the radially polarized Hypergeometric Gaussian beam on the focus plane for different values of parameter m and w_0, (a) for $m = 4$ and (b) for $w_0 = 3$ mm.

Figures 2(a) and 2(e) display the lateral normalized intensity distribution on the focal plane. It is worth noticing that the most difference from Figure 1 is that the parameter of γ is nonzero in Eq. (1). According to Eq. (1), γ is the imaginary part. Therefore, it has no effect on the amplitude distribution of the incident hypergeometric Gaussian beam at the pupil. However, there is a remarkable effect of the intensity distribution of electric field on the focal plane. Comparing Figure 2(d) with Figure 1(l), obviously, it is no longer focused on the focal plane, but the formation of two symmetric focal spots near the focus. Meanwhile, from Figure 2(d) to Figure 2(h), it is easy to see that the distance between two symmetric focal spot increases with an increasing γ, and the focusing field exhibits large lateral width on the focal plane, which may be regraded as the focal spot shifted on the focal plane.

The focusing characteristics, such as FWHM and beam quality η, are shown in Figure 3 for different values of m and w_0. From the numerical results it can be obtained that when $m = 4$, $w_0 = 1.5$ mm and $m = 2$, $w_0 = 3$ mm, the maximal amplitude of incident hypergeometric Gaussian beam is near the origin of coordinate, and the focal spot vanishes on the focal plane. Therefore, in this case, the FWHM of the focal spot cannot be defined. On the other hand, for fixed value of m (or w_0), the beam quality η increases with an increasing w_0 (or m). Therefore, in order to obtain longitudinal polarized focusing larger m or w_0 is essential.

Summarizing the above content, the focusing performance of incident radially polarized hypergeometric Gaussian beam is discussed with different values of m, w_0 and γ. Moreover, in previous researches, the intensity distributions on the focal plane of radially polarized Bessel, Bessel-Gaussian and Gaussian were investigated, and the FWHMs were 0.59λ, 0.68λ and 0.78λ, respectively [22]. Although the FWHM is 0.82λ for the radially polarized hypergeometric Gaussian beam with $m = 8$, $w_0 = 3\,\mathrm{mm}$ in Figure 1(i), smaller FWHM can be achieved with large value of $(m,\, w_0)$ in Figure 3. For example, when $m = 4$, $w_0 = 8\,\mathrm{mm}$, focusing spot appears at focal spot with FWHM of 0.51λ. Therefore, the radially polarized hypergeometric Gaussian beam shows the superiority in focusing with superresolution.

4. CONCLUSION

In summary, based on the Richards-Wolf vectorial diffraction theory, we have clarified the focusing performance of incident radially polarized hypergeometric Gaussian beam by objective lens with large NA. The FWHM and beam quality are analyzed for different values of parameters. It is shown that for small values of $(m,\, w_0)$, their focusing characteristics are similar to that in focusing system with low NA. With a proper combination of the beam order, beam size and imaginary parameter variables, the adjustably confined flat-topped focus and focal hole can be obtained in the focal region. Moreover, the distributions of the axial intensity shaped two symmetric light spots about the focus for different imaginary parameter variables are achieved. Therefore, radially polarized hypergeometric Gaussian beam can be applied in the focusing system with high numerical aperture to achieve focusing with superresolution and can be widely used in applications such as data storage, laser drilling and optical trapping.

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REFERENCES

1. Neves, A. R., A. Fontes, L. Y. De Pozzo, et al., "Electromagnetic forces for an arbitrary optical trapping of a spherical dielectric," *Optics Express*, Vol. 14, No. 26, 13101–13106, 2006.
2. Li, X., Y. Cao, and M. Gu, "Superresolution-focal-volume induced 3.0 Tbytes/disk capacity by focusing a radially polarized beam," *Opt. Lett.*, Vol. 36, 2510–2512, 2011.
3. Jeffries, G. D. M., J. S. Edgar, Y. Zhao, J. P. Shelby, C. Fong, and D. T. Chiu, "Using polarization-shaped optical vortex traps for single-cell nanosurgery," *Nano. Lett.*, Vol. 7, 415–420, 2007.
4. Zhao, Y., J. S. Edgar, G. D. M. Jeffries, D. McGloin, and D. T. Chiu, "Spin-to-orbital angular momentum conversion in a strongly focused optical beam," *Phys. Rev. Lett.*, Vol. 99, 073901, 2007.
5. Davis, J. A., D. E. McNamara, D. M. Cottrell, and J. Campos, "Image processing with the radial Hilbert transform: Theory and experiments," *Opt. Lett.*, Vol. 25, 99–101, 2000.
6. Verhagen, E., L. Kuipers, and A. Polman, "Field enhancement in metallic subwavelength aperture arrays probed by erbium upconversion luminescence," *Optics Express*, Vol. 17, No. 17, 14586–14598, 2009.
7. Valagiannopulos, C. A. and N. L. Tsitsas, "Field enhancement in a grounded dielectric slab by using a single superstrate layer," *Advances in OptoElectronics*, Vol. 10, 1–9, 2012.
8. Tsitsas, N. L. and C. A. Valagiannopoulos, "On centrating the electromagnetic power in a grounded dielectric slab excited by an external gaussian beam," *International Conference on Mathematical Methods in Electromagnetic Theory*, 304–307, 2012.
9. Valagiannopulos, C. A., "Electromagnetic absorption of gaussian beams by a grounded layered structure," *Radioengineering*, Vol. 22, No. 1, 333–340, 2013.
10. Wolf, E., "Electromagnetic diffraction in optical systems. I. An integral representation of the image field," *Proc. R. Soc. A*, Vol. 253, 349–357, 1959.
11. Richards, B. and E. Wolf, "Electromagnetic diffraction in optical systems. II. Structure of the image field in an aplanatic system," *Proc. R. Soc. A*, Vol. 253, 358–379, 1959.

12. Dorn, R., S. Quabis, and G. Leuchs, "Sharper focus for a radially polarized light beam," *Phys. Rev. Lett.*, Vol. 91, 233901, 2003.
13. Yang, L., X. Xie, S. Wang, and J. Zhou, "Minimized spot of annular radially polarized focusing beam," *Opt. Lett.*, Vol. 38, 1331–1333, 2013.
14. Kitamura, K., K. Sakai, and S. Noda, "Sub-wavelength focal spot with long depth of focus generated by radially polarized, narrow-width annular beam," *Optics Express*, Vol. 18, 4518–4525, 2010.
15. Kotlyar, V. V., R. V. Skidanov, S. N. Khonina, and V. A. Soifer, "Hypergeometric modes," *Opt. Lett.*, Vol. 32, 742–744, 2007.
16. Karimi, E., G. Zito, B. Piccirillo, L. Marrucci, and E. Santamato, "Hypergeometric-Gaussian modes," *Opt. Lett.*, Vol. 32, 3053–3055, 2007.
17. Kotlyar, V. V., A. A. Kovalev, R. V. Skidanov, S. N. Khonina, and J. Turunen, "Generating hypergeometric laser beams with a diffractive optical element," *Appl. Opt.*, Vol. 47, 6124–6133, 2008.
18. Kotlyar, V. V. and A. A. Kovalev, "Family of Hypergeometric laser beams," *J. Opt. Soc. Am. A*, Vol. 25, 262–270, 2008.
19. Eyyuboğlu, H. T. and Y. Cai, "Hypergeometric Gaussian beam and its propagation in turbulence," *Opt. Commun.*, Vol. 285, 4194–4199, 2012.
20. Youngworth, K. and T. Brown, "Focusing of high numerical aperture cylindrical-vector beams," *Optics Express*, Vol. 7, 77–87, 2000.
21. Zhan, Q. and J. Leger, "Focus shaping using cylindrical vector beams," *Optics Express*, Vol. 10, 324–331, 2002.
22. Lin, J., Y. Ma, P. Jin, G. Davies, and J. Tan, "Longitudinal polarized focusing of radially polarized sinh-Gaussian beam," *Optics Express*, Vol. 21, 13193–13198, 2013.