Propagation factors of cosine-Gaussian-correlated Schell-model beams in non-Kolmogorov turbulence

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Abstract: Based on the extended Huygens-Fresnel principle and second-order moments of the Wigner distribution function (WDF), we have studied the relative root-mean-square (rms) angular width and the propagation factor of cosine-Gaussian-correlated Schell-model (CGSM) beams propagating in non-Kolmogorov turbulence. It has been found that the CGSM beam has advantage over the Gaussian Schell-model (GSM) beam for reducing the turbulence-induced degradation, and this advantage will be more obvious for the beams with larger parameter n and spatial coherence δ or under the condition of stronger fluctuation of turbulence. The CGSM beam with larger parameter n or smaller spatial coherence δ will be less affected by the turbulence. In addition, the effects of the slope-parameter α, inner and outer scale and the refractive-index structure constant of the non-Kolmogorov's power spectrum on the propagation factor are also analyzed in detailed.

OCIS codes: (010.1300) Atmospheric propagation; (010.1330) Atmospheric turbulence; (030.0030) Coherence and statistical optics.

References and links

1. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University, 1995).
2. J. C. Ricklin and F. M. Davidson, "Atmospheric turbulence effects on a partially coherent Gaussian beam: implications for free-space laser communication," J. Opt. Soc. Am. A **19**(9), 1794–1802 (2002).
3. T. Van Dijk, D. G. Fischer, T. D. Visser, and E. Wolf, "Effects of spatial coherence on the angular distribution of radiant intensity generated by scattering on a sphere," Phys. Rev. Lett. **104**(17), 173902 (2010).
4. Y. Cai and S. Y. Zhu, "Ghost imaging with incoherent and partially coherent light radiation," Phys. Rev. E Stat. Nonlin. Soft Matter Phys. **71**(5), 056607 (2005).
5. O. Korotkova, R. L. Phillips, and L. C. Andrews, "Model for a partially coherent Gaussian beam in atmospheric turbulence with application in Lasercom," Opt. Eng. **43**(2), 330–341 (2004).
6. A. Dogariu and S. Amarande, "Propagation of partially coherent beams: turbulence-induced degradation," Opt. Lett. **28**(1), 10–12 (2003).
7. C. Zhao, Y. Cai, X. Lu, and H. T. Eyyuboğlu, "Radiation force of coherent and partially coherent flat-topped beams on a Rayleigh particle," Opt. Express **17**(3), 1753–1765 (2009).
8. C. Zhao and Y. Cai, "Trapping two types of particles using a focused partially coherent elegant Laguerre-Gaussian beam," Opt. Lett. **36**(12), 2251–2253 (2011).
9. F. Gori and M. Santarsiero, "Devising genuine spatial correlation functions," Opt. Lett. **32**(24), 3531–3533 (2007).
10. F. Gori, V. Ramírez-Sánchez, M. Santarsiero, and T. Shirai, "On genuine cross-spectral density matrices," J. Opt. A, Pure Appl. Opt. **11**(8), 085706 (2009).
11. Z. Mei, "Light sources generating self-focusing beams of variable focal length," Opt. Lett. **39**(2), 347–350 (2014).
12. H. Lajunen and T. Saastamoinen, "Propagation characteristics of partially coherent beams with spatially varying correlations," Opt. Lett. **36**(20), 4104–4106 (2011).
13. Z. Tong and O. Korotkova, "Electromagnetic nonuniformly correlated beams," J. Opt. Soc. Am. A **29**(10), 2154–2158 (2012).

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14. Z. Tong and O. Korotkova, "Nonuniformly correlated light beams in uniformly correlated media," Opt. Lett. **37**(15), 3240–3242 (2012).
15. Y. Gu and G. Gbur, "Scintillation of nonuniformly correlated beams in atmospheric turbulence," Opt. Lett. **38**(9), 1395–1397 (2013).
16. S. Cui, Z. Chen, L. Zhang, and J. Pu, "Experimental generation of nonuniformly correlated partially coherent light beams," Opt. Lett. **38**(22), 4821–4824 (2013).
17. S. Sahin and O. Korotkova, "Light sources generating far fields with tunable flat profiles," Opt. Lett. **37**(14), 2970–2972 (2012).
18. O. Korotkova, S. Sahin, and E. Shchepakina, "Multi-Gaussian Schell-model beams," J. Opt. Soc. Am. A **29**(10), 2159–2164 (2012).
19. Y. Zhang, L. Liu, C. Zhao, and Y. Cai, "Multi-Gaussian Schell-model vortex beam," Phys. Lett. A **378**(9), 750–754 (2014).
20. O. Korotkova and E. Shchepakina, "Rectangular Multi-Gaussian Schell-Model beams in atmospheric turbulence," J. Opt. **16**(4), 045704 (2014).
21. Y. Yuan, X. Liu, F. Wang, Y. Chen, Y. Cai, J. Qu, and H. T. Eyyuboğlu, "Scintillation index of a multi-Gaussian Schell-model beam in turbulent atmosphere," Opt. Commun. **305**, 57–65 (2013).
22. S. Du, Y. Yuan, C. Liang, and Y. Cai, "Second-order moments of a multi-Gaussian Schell-model beam in a turbulent atmosphere," Opt. Laser Technol. **50**, 14–19 (2013).
23. Z. Mei and O. Korotkova, "Random sources generating ring-shaped beams," Opt. Lett. **38**(2), 91–93 (2013).
24. J. Cang, P. Xiu, and X. Liu, "Propagation of Laguerre-Gaussian and Bessel–Gaussian Schell-model beams through paraxial optical systems in turbulent atmosphere," Opt. Laser Technol. **54**, 35–41 (2013).
25. R. Chen, L. Liu, S. Zhu, G. Wu, F. Wang, and Y. Cai, "Statistical properties of a Laguerre-Gaussian Schell-model beam in turbulent atmosphere," Opt. Express **22**(2), 1871–1883 (2014).
26. Y. Chen and Y. Cai, "Generation of a controllable optical cage by focusing a Laguerre-Gaussian correlated Schell-model beam," Opt. Lett. **39**(9), 2549–2552 (2014).
27. Y. Chen, F. Wang, C. Zhao, and Y. Cai, "Experimental demonstration of a Laguerre-Gaussian correlated Schell-model vortex beam," Opt. Express **22**(5), 5826–5838 (2014).
28. Y. Chen, L. Liu, F. Wang, C. Zhao, and Y. Cai, "Elliptical Laguerre-Gaussian correlated Schell-model beam," Opt. Express **22**(11), 13975–13987 (2014).
29. F. Wang, X. Liu, Y. Yuan, and Y. Cai, "Experimental generation of partially coherent beams with different complex degrees of coherence," Opt. Lett. **38**(11), 1814–1816 (2013).
30. Z. Mei and O. Korotkova, "Cosine-Gaussian Schell-model sources," Opt. Lett. **38**(14), 2578–2580 (2013).
31. Z. Mei and O. Korotkova, "Electromagnetic cosine-Gaussian Schell-model beams in free space and atmospheric turbulence," Opt. Express **21**(22), 27246–27259 (2013).
32. C. Liang, F. Wang, X. Liu, Y. Cai, and O. Korotkova, "Experimental generation of cosine-Gaussian-correlated Schell-model beams with rectangular symmetry," Opt. Lett. **39**(4), 769–772 (2014).
33. L. Pan, C. Ding, and H. Wang, "Diffraction of cosine-Gaussian-correlated Schell-model beams," Opt. Express **22**(10), 11670–11679 (2014).
34. Z. Mei, "Light sources generating self-splitting beams and their propagation in non-Kolmogorov turbulence," Opt. Express **22**(11), 13029–13040 (2014).
35. Y. Chen, F. Wang, L. Liu, C. Zhao, Y. Cai, and O. Korotkova, "Generation and propagation of a partially coherent vector beam with special correlation functions," Phys. Rev. A **89**(1), 013801 (2014).
36. M. Santarsiero, F. Gori, R. Borghi, G. Cincotti, and P. Vahimaa, "Spreading properties of beams radiated by partially coherent Schell-model sources," J. Opt. Soc. Am. A **16**(1), 106–112 (1999).
37. T. Shirai, A. Dogariu, and E. Wolf, "Directionality of Gaussian Schell-model beams propagating in atmospheric turbulence," Opt. Lett. **28**(8), 610–612 (2003).
38. G. Wu, H. Guo, S. Yu, and B. Luo, "Spreading and direction of Gaussian-Schell model beam through a non-Kolmogorov turbulence," Opt. Lett. **35**(5), 715–717 (2010).
39. G. Zhou and X. Chu, "Propagation of a partially coherent cosine-Gaussian beam through an ABCD optical system in turbulent atmosphere," Opt. Express **17**(13), 10529–10534 (2009).
40. P. Zhou, Z. Liu, X. Xu, and X. Chu, "Propagation of phase-locked partially coherent flattened beam array in turbulent atmosphere," Opt. Lasers Eng. **47**(11), 1254–1258 (2009).
41. Y. Dan and B. Zhang, "Beam propagation factor of partially coherent flat-topped beams in a turbulent atmosphere," Opt. Express **16**(20), 15563–15575 (2008).
42. Y. Yuan, Y. Cai, J. Qu, H. T. Eyyuboğlu, Y. Baykal, and O. Korotkova, "M^2-factor of coherent and partially coherent dark hollow beams propagating in turbulent atmosphere," Opt. Express **17**(20), 17344–17356 (2009).
43. W. Cheng, J. W. Haus, and Q. Zhan, "Propagation of vector vortex beams through a turbulent atmosphere," Opt. Express **17**(20), 17829–17836 (2009).
44. H. Xu, Z. Cui, and J. Qu, "Propagation of elegant Laguerre-Gaussian beam in non-Kolmogorov turbulence," Opt. Express **19**(22), 21163–21173 (2011).
45. F. Wang, Y. Cai, H. T. Eyyuboğlu, and Y. Baykal, "Twist phase-induced reduction in scintillation of a partially coherent beam in turbulent atmosphere," Opt. Lett. **37**(2), 184–186 (2012).
46. Y. Dong, Y. Cai, C. Zhao, and M. Yao, "Statistics properties of a cylindrical vector partially coherent beam," Opt. Express **19**(7), 5979–5992 (2011).
47. A. E. Siegman, "New developments in laser resonators," Proc. SPIE **1224**, 2–14 (1990).
48. E. Shchepakina and O. Korotkova, "Second-order statistics of stochastic electromagnetic beams propagating through non-Kolmogorov turbulence," Opt. Express **18**(10), 10650–10658 (2010).

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49. P. Zhou, Y. Ma, X. Wang, H. Zhao, and Z. Liu, "Average spreading of a Gaussian beam array in non-Kolmogorov turbulence," Opt. Lett. **35**(7), 1043–1045 (2010).
50. I. Toselli, L. C. Andrews, R. L. Phillips, and V. Ferrero, "Free-space optical system performance for laser beam propagation through non-Kolmogorov turbulence," Opt. Eng. **47**(2), 026003 (2008).

1. Introduction

During the past several decades, partially coherent beams with conventional correlation functions (i.e., Gaussian correlated Schell-model functions) have been studied extensively due to their wide applications in free-space optical communications, remote sensing, optical imaging and optical trapping and so on [1–8]. More recently, since the sufficient condition for devising the genuine correlation function of a scalar or electromagnetic partially coherent beam was established by Gori and collaborators [9, 10], partially coherent beams with nonconventional correlation functions, whose degree of coherence is modulated by special correlation functions, have attracted a considerable amount of attention [11–35]. So far, a variety of special model sources have been introduced and studied both theoretically and experimentally, such as nonuniformly correlated (NUC) beam [11–16], multi-Gaussian Schell-model (MGSM) beam [17–22], Laguerre-Gaussian Schell-model (LGSM) and Bessel-Gaussian Schell-mode (BGSM) beam [23–28], cosine-Gaussian Schell-model (CGSM) beam [30–34], special correlated partially coherent vector beam [35], etc. The experimental generation of such random beams can be realized by using the nematic spatial light modulator [16, 26–29, 32, 35]. More interestingly, it has been revealed that those beams with special correlation function exhibit extraordinary propagation characteristics and unique focusing properties, for instance, the NUC beam can display a peculiar self-focusing effect and a lateral shift the intensity maximum on propagation [11–14]; the MGSM beam can generate far fields with tunable flat profiles, whether circular or rectangular [17–20]; the BGSM beam, the LGSM beam and the CGSM beam are capable of generating far fields with ring-shaped intensity distributions [23, 24, 30]; the focused LGSM beam can generate a controllable optical cage by choosing suitable value of the spatial coherence near the focal plane [26]. Therefore, it is possible to generate prescribed far-field intensity distributions by choosing specified correlation properties for the source field.

For a long time, studies on the propagation characteristics of laser beams in free-space or turbulent atmosphere are of vital importance due to their practical applications in optical communication and remote sensing [1, 2, 4–6]. The random fluctuations of the turbulence will affect inevitably the evolution properties of laser beams upon propagation. Thus, it is necessary and meaningful to find suitable ways to overcome or reduce the destructive effect of the atmospheric turbulence. It is well known that partially coherent beams are less affected by turbulence than fully coherent beams, while they will face fast spreading and low Signal to Noise Ratio (SNR) due to the strong fluctuation of the source for small spatial coherence [2, 6, 36–39]. Another effective approach to reducing the negative influence of turbulence is to use the beams with special beam profile, phase or polarization [39–46]. In general, the propagation factor proposed by Siegman can be used for characterizing the beam quality factor of laser beams on propagation [47]. Up to now, much work has been carried out concerning the propagation factors of various beams with classical correlation functions through a turbulent atmosphere [41, 42, 48]. However, to the best of our knowledge, only few papers have been concerned on the propagation factors of the beams with special correlation functions in the turbulence [22, 25]. It has been shown that a LGSM or a MGSM beam has advantage over a GSM beam for reducing the turbulence-induced degradation [22, 25]. Thus it is meaningful for us to explore the statistical properties of the special correlated beams through the turbulent atmosphere.

Just recently, a new kind of partially coherent sources of Schell type with cosine-Gaussian degree of coherence called CGSM beam was proposed in theory and generated in experiment [30–34]. It should be clearly pointed out that the cosine function is employed for modulating the source degree of coherence rather than the spectral density, which is quite different from the partially coherent cosine Gaussian beams [39]. Accordingly, the evolution of the spectral density and transverse spectral degree of coherence of the CGSM beam has been analyzed

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both in free-space and in a turbulent atmosphere [30, 31, 33, 34]. The main purpose of this paper is to investigate the propagation factor of CGSM beam on its atmospheric propagation. In order to cover a wide scope of atmospheric conditions, the non-Kolmogorov atmospheric spectrum is employed. We have derived the explicit expressions for the propagation factors of the CGSM beam in the turbulence by use of the extended Huygens-Fresnel principle and the definition of the Wingner distribution function (WDF). The evolution properties of the propagation factor of the CGSM beams in the non-Kolmogorov turbulence are illustrated with numerical examples. Some interesting and useful results have been found.

2. Second-order moments of CGSM beams in non-Kolmogorov turbulence

The cross-spectral density (CSD) of the CGSM beam at the source plane can be expressed as [30, 31, 34]

$$W^{(0)}(\mathbf{r}_1,\mathbf{r}_2)=\exp\left(-\frac{|\mathbf{r}_1|^2+|\mathbf{r}_2|^2}{4\sigma^2}\right)\cos\left[\frac{n\sqrt{2\pi}(\mathbf{r}_2-\mathbf{r}_1)}{\delta}\right]\exp\left[-\frac{|\mathbf{r}_2-\mathbf{r}_1|^2}{2\delta^2}\right] \tag{1}$$

where $\mathbf{r}_1\equiv(x_1,y_1)$ and $\mathbf{r}_2\equiv(x_2,y_2)$ are two arbitrary transverse position vectors, σ and δ are the transverse beam width and the transverse coherence width of the CGSM beam, respectively. The source degree of coherence of the CGSM beam at $z=0$ takes the following form [30, 34]:

$$\mu^{(0)}(\mathbf{r}_2-\mathbf{r}_1)=\frac{W^{(0)}(\mathbf{r}_1,\mathbf{r}_2)}{\sqrt{W^{(0)}(\mathbf{r}_1,\mathbf{r}_1)}\sqrt{W^{(0)}(\mathbf{r}_2,\mathbf{r}_2)}}=\exp\left[-\frac{|\mathbf{r}_2-\mathbf{r}_1|^2}{2\delta^2}\right]\cos\left[\frac{n\sqrt{2\pi}(\mathbf{r}_2-\mathbf{r}_1)}{\delta}\right] \tag{2}$$

It is clear from Eq. (2) that the source degree of coherence of the CGSM beam reduces to that of the conventional GSM sources for $n=0$, while for $n\neq 0$ the model is modulated by the cosine function. The index n controls the number of oscillations of the source degree of coherence on the half-axis. The beam condition for the CGSM source is the same as that for the classic GSM source [30, 34]

$$\frac{1}{4\sigma^2}+\frac{1}{\delta^2}\ll\frac{2\pi^2}{\lambda^2} \tag{3}$$

Within the validity of the paraxial approximation, the propagation of the CSD of partially coherent beams in a turbulent atmosphere can be studied with the help of the following generalized Huygens-Fresnel integral [25, 41, 42, 48]

$$\begin{aligned}W(\boldsymbol{\rho},\boldsymbol{\rho}_d,z)=&(k/2\pi z)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W^{(0)}(\mathbf{r},\mathbf{r}_d,0)\\&\times\exp\left[\frac{ik}{z}(\boldsymbol{\rho}-\mathbf{r})(\boldsymbol{\rho}_d-\mathbf{r}_d)-\boldsymbol{H}(\boldsymbol{\rho}_d,\mathbf{r}_d;z)\right]d^2\mathbf{r}d^2\mathbf{r}_d\end{aligned} \tag{4}$$

where $k=2\pi/\lambda$ is wave number with λ being the wavelength. In the above equation, we have used the following sum and difference vector notation

$$\mathbf{r}=\frac{(\mathbf{r}_1+\mathbf{r}_2)}{2},\mathbf{r}_d=\mathbf{r}_1-\mathbf{r}_2,\boldsymbol{\rho}=\frac{(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)}{2},\boldsymbol{\rho}_d=\boldsymbol{\rho}_1-\boldsymbol{\rho}_2, \tag{5}$$

where $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_2$ are two arbitrary points in the receiver plane, perpendicular to the direction of propagation of the beam. The CSD of the CGSM beam in the source plane can be expressed as follows

$$W^{(0)}(\mathbf{r},\mathbf{r}_d,0)=W^{(0)}(\mathbf{r}_1,\mathbf{r}_2,0)=W^{(0)}\left(\mathbf{r}+\frac{\mathbf{r}_d}{2},\mathbf{r}-\frac{\mathbf{r}_d}{2},0\right) \tag{6}$$

For the convenience of operations, we use the coordinate transformation as $\mathbf{r}\to\mathbf{r}',\mathbf{r}_d=\boldsymbol{\rho}_d+\frac{z}{k}\boldsymbol{\kappa}_d$. After some operations as shown in [41], on the basis of inverse Fourier

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transform of the Dirac delta function and its property of even function, the CSD of the source of the CGSM beam can be expressed as follows

$$\begin{aligned} &W^{(\mathbf{0})}\left(\mathbf{r}',\boldsymbol{\rho}_d+\frac{z}{k}\boldsymbol{\kappa}_d,0\right) \\ &=\exp\left[-\frac{1}{2\sigma^2}\mathbf{r}'^2-\left(\frac{1}{8\sigma^2}+\frac{1}{2\delta^2}\right)\left(\boldsymbol{\rho}_d+\frac{z}{k}\boldsymbol{\kappa}_d\right)^2\right]\cos\left[\frac{n\sqrt{2\pi}}{\delta}\left(\boldsymbol{\rho}_d+\frac{z}{k}\boldsymbol{\kappa}_d\right)\right] \end{aligned} \tag{7}$$

where $\boldsymbol{\kappa}_d\equiv\left(\kappa_{dx},\kappa_{dy}\right)$ is the position vector in the spatial frequency domain. Similarly, Eq. (4) can be expressed in the following alternative form

$$\begin{aligned} W(\boldsymbol{\rho},\boldsymbol{\rho}_d,z)=&\left(\frac{1}{2\pi}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W^{(0)}\left(\mathbf{r}',\boldsymbol{\rho}_d+\frac{z}{k}\boldsymbol{\kappa}_d,0\right) \\ &\times\exp\left[-i\boldsymbol{\rho}\boldsymbol{\kappa}_d+i\mathbf{r}'\boldsymbol{\kappa}_d-\boldsymbol{H}\left(\boldsymbol{\rho}_d,\boldsymbol{\rho}_d+\frac{z}{k}\boldsymbol{\kappa}_d,z\right)\right]d^2\mathbf{r}'d^2\boldsymbol{\kappa}_d \end{aligned} \tag{8}$$

The term $\exp\left[-\boldsymbol{H}\left(\boldsymbol{\rho}_d,\boldsymbol{\rho}_d+\frac{z}{k}\boldsymbol{\kappa}_d,z\right)\right]$ in Eq. (8) represents the effect of the turbulence defined as

$$\boldsymbol{H}\left(\boldsymbol{\rho}_d,\boldsymbol{\rho}_d+\frac{z}{k}\boldsymbol{\kappa}_d;z\right)=\frac{\pi^2k^2z}{3}\left(3\boldsymbol{\rho}_d^2+3\frac{z}{k}\boldsymbol{\rho}_d\boldsymbol{\kappa}_d+\frac{z^2}{k^2}\boldsymbol{\kappa}_d^2\right)\int_0^{\infty}\boldsymbol{\kappa}^3\boldsymbol{\Phi}_n(\boldsymbol{\kappa})d\boldsymbol{\kappa} \tag{9}$$

where $\boldsymbol{\Phi}_n(\boldsymbol{\kappa})$ is the spatial power spectrum of the refractive-index fluctuations of the isotropic turbulent medium and κ is the magnitude of two-dimensional spatial frequency. A generalized model for the power spectrum valid in non-Kolmogorov turbulence can be characterized as [34, 38, 44, 48–50]

$$\boldsymbol{\Phi}_n(\kappa,\alpha)=A(\alpha)\tilde{C}_n^2\exp\left[-\left(\kappa^2/\kappa_m^2\right)\right]/\left(\kappa^2+\kappa_0^2\right)^{\alpha/2},0\le\kappa<\infty,3<\alpha<4 \tag{10}$$

where $\kappa_0=2\pi/L_0$ with L_0 being the outer scale parameter and $\kappa_m=c(\alpha)/l_0$ with l_0 being the inner scale parameter, the parameter α is the power-law exponent, and

$$c(\alpha)=\left[\Gamma(5-\alpha/2)\cdot A(\alpha)\cdot 2\pi/3\right]^{[1/(\alpha-5)]},A(\alpha)=\Gamma(\alpha-1)\cdot\cos(\alpha\pi/2)/\left(4\pi^2\right) \tag{11}$$

with $\Gamma(\cdot)$ being the Gamma function. The term $\tilde{C}_n^2$ in Eq. (10) is the generalized refractive-index structure parameter with units $\mathrm{m}^{3-\alpha}$. With the power spectrum in Eq. (10) the integral in Eq. (9) becomes [34, 38, 44]

$$\mathrm{I}=\int_0^{\infty}\boldsymbol{\kappa}^3\boldsymbol{\Phi}_n(\boldsymbol{\kappa})d\boldsymbol{\kappa}=\frac{A(\alpha)\tilde{C}_n^2}{2(\alpha-2)}\left[\kappa_m^{2-\alpha}\beta\exp\left(\frac{\kappa_0^2}{\kappa_m^2}\right)\Gamma\left(2-\frac{\alpha}{2},\frac{\kappa_0^2}{\kappa_m^2}\right)-2\kappa_0^{4-\alpha}\right] \tag{12}$$

where $\beta=2\kappa_0^2-2\kappa_m^2+\alpha\kappa_m^2$ and $\Gamma(\cdot,\cdot)$ denotes the incomplete Gamma function.

It is well known that the WDF is the space domain and frequency domain function, which is especially suitable for the treatment of partially coherent beams, and the WDF can be expressed in terms of the CSD as follows [22, 25, 41, 42, 48]

$$h(\boldsymbol{\rho},\boldsymbol{\theta};z)=\left(\frac{k}{2\pi}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W(\boldsymbol{\rho},\boldsymbol{\rho}_d;z)\exp(-ik\boldsymbol{\theta}\cdot\boldsymbol{\rho}_d)d^2\boldsymbol{\rho}_d \tag{13}$$

where $\boldsymbol{\theta}\equiv\left(\theta_x,\theta_y\right)$ denotes an angle which the vector of interest makes with the z-direction, $k\theta_x$ and $k\theta_x$ are the wave vector components along the x-axis and y-axis, respectively.

Applying Eqs. (7), (8) and (13), we obtain the following expression for the WDF of the CGSM beam in turbulent atmosphere

$$h\left(\boldsymbol{\rho},\boldsymbol{\theta},z\right)=\frac{k^2}{16\pi^4}2\pi\sigma^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left[-a\boldsymbol{\rho}_d^2-b\boldsymbol{\kappa}_d^2-c\boldsymbol{\rho}_d\boldsymbol{\kappa}_d-ik\boldsymbol{\theta}\cdot\boldsymbol{\rho}_d-i\boldsymbol{\rho}\cdot\boldsymbol{\kappa}_d\right.$$
$$\left.-\boldsymbol{H}\left(\boldsymbol{\rho}_d,\boldsymbol{\rho}_d+\frac{z}{k}\boldsymbol{\kappa}_d,z\right)\right]\cos\left[\frac{n\sqrt{2\pi}}{\delta}\left(\boldsymbol{\rho}_d+\frac{z}{k}\boldsymbol{\kappa}_d\right)\right]d^2\boldsymbol{\rho}_d d^2\boldsymbol{\kappa}_d \tag{14}$$

where

$$\frac{1}{\varepsilon^2}=\frac{1}{4\sigma^2}+\frac{1}{\delta^2},a=\frac{1}{2\varepsilon^2},b=\frac{1}{2\varepsilon^2}\frac{z^2}{k^2}+\frac{\sigma^2}{2},c=\frac{1}{\varepsilon^2}\frac{z}{k} \tag{15}$$

The moments of order $n_1+n_2+m_1+m_2$ the WDF for a laser beam is defined as [25, 41, 42]

$$<x^{n_1}y^{n_2}\theta_x^{m_1}\theta_y^{m_2}>=\frac{1}{P}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x^{n_1}y^{n_2}\theta_x^{m_1}\theta_y^{m_2}h\left(\boldsymbol{\rho},\boldsymbol{\theta},z\right)d^2\boldsymbol{\rho}d^2\boldsymbol{\theta} \tag{16}$$

where

$$P=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h\left(\boldsymbol{\rho},\boldsymbol{\theta},z\right)d^2\boldsymbol{\rho}d^2\boldsymbol{\theta} \tag{17}$$

Substituting Eq. (14) into Eqs. (16) and (17), we obtain (after tedious integration) the following expressions for the second-order moments of WDF of the CGSM beam in a turbulent atmosphere

$$<\boldsymbol{\rho}^2>=\left(\frac{2}{\varepsilon^2}+\frac{4\pi n^2}{\delta^2}\right)\frac{z^2}{k^2}+2\sigma^2+\frac{4}{3}\pi^2z^3\mathrm{I} \tag{18}$$

$$<\boldsymbol{\theta}^2>=\frac{1}{k^2}\left(\frac{2}{\varepsilon^2}+\frac{4\pi n^2}{\delta^2}\right)+4\pi^2z\mathrm{I} \tag{19}$$

$$<\boldsymbol{\rho}\cdot\boldsymbol{\theta}>=\left(\frac{2}{\varepsilon^2}+\frac{4\pi n^2}{\delta^2}\right)\frac{z}{k^2}+2\pi^2z^2\mathrm{I} \tag{20}$$

According to the definition of rms spatial and angular width of light beams [41], the rms spatial and angular width of CGSM beams in the turbulence can be expressed as

$$w_N\left(z\right)\equiv\left(<\left|\boldsymbol{\rho}-<\boldsymbol{\rho}>\right|^2>\right)^{1/2}=\left(<\boldsymbol{\rho}^2>\right)^{1/2}$$
$$=\left[\left(\frac{2}{\varepsilon^2}+\frac{4\pi n^2}{\delta^2}\right)\frac{z^2}{k^2}+2\sigma^2+\frac{4}{3}\pi^2z^3\mathrm{I}\right]^{1/2} \tag{21}$$

$$\theta_N\left(z\right)\equiv\left(<\left|\boldsymbol{\theta}-<\boldsymbol{\theta}>\right|^2>\right)^{1/2}=\left(<\boldsymbol{\theta}^2>\right)^{1/2}$$
$$=\left[\frac{1}{k^2}\left(\frac{2}{\varepsilon^2}+\frac{4\pi n^2}{\delta^2}\right)+4\pi^2z\mathrm{I}\right]^{1/2} \tag{22}$$

For comparison, the normalized rms angular width, which represents the relative angular spreading of the beams, can be expressed as

$$\theta_{rN}\left(z\right)\equiv\frac{\theta_N\left(z\right)}{\theta_N\left(0\right)}=\left[1+4\pi^2k^2\left(\frac{2}{\varepsilon^2}+\frac{4\pi n^2}{\delta^2}\right)^{-1}z\mathrm{I}\right]^{1/2} \tag{23}$$

The propagation factor of a partially coherent beam in the turbulent atmosphere can be defined in terms of the second-order moments of WDF as follows [25, 41, 42, 47]

$$M^2(z)=k(<\boldsymbol{\rho}^2><\boldsymbol{\theta}^2>-<\boldsymbol{\rho}\cdot\boldsymbol{\theta}>^2)^{1/2}$$
$$=k\left[(<x^2>+<y^2>)(<\theta_x^2>+<\theta_y^2>)-(<x\theta_x>+<y\theta_y>)^2\right]^{1/2} \tag{24}$$

On substituting from Eqs. (18)–(20) into Eq. (24), we obtain the following explicit expression for the propagation factor of the CGSM beam in the turbulence

$$M^2(z)=\left\{\left[\left(\frac{2}{\varepsilon^2}+\frac{4\pi n^2}{\delta^2}\right)\frac{z^2}{k^2}+2\sigma^2+\frac{4}{3}\pi^2 z^3 \mathrm{I}\right]\left(\frac{2}{\varepsilon^2}+\frac{4\pi n^2}{\delta^2}+4\pi^2 k^2 z \mathrm{I}\right)\right.$$
$$\left.-\left[\left(\frac{2}{\varepsilon^2}+\frac{4\pi n^2}{\delta^2}\right)\frac{z}{k}+2\pi^2 k z^2 \mathrm{I}\right]^2\right\}^{1/2} \tag{25}$$

Under the condition of $\Phi_n(\kappa)=0$, Eq. (25) reduces to the expression for the propagation factor of the CGSM beam in free space

$$M^2(z)=\left[1+\frac{4\sigma^2}{\delta^2}\left(1+2\pi n^2\right)\right]^{1/2} \tag{26}$$

We can see that the propagation factor of the CGSM beam in free space is independent of the propagation distance as expected [41, 42], and its value increases as the parameter n increases. In addition, Eq. (26) reduces to the propagation factor of a GSM beam in free space for $n=0$, which is agreement with previous result of [36]

$$M^2(z)=\left(1+4\sigma^2/\delta^2\right)^{1/2} \tag{27}$$

3. Numerical calculation results and analysis

In this section, we will study the statistical properties of the CGSM beams on propagation in non-Kolmogorov turbulence based on the analytical formulas derived in the previous section. For the convenience of the following analysis, the calculation parameters of the source are chosen to be $n=2$, $\sigma=0.01\mathrm{m}$, $\delta=0.02\mathrm{m}$, $\lambda=632.8\mathrm{nm}$ and of the turbulence are $\alpha=3.67$, $L_0=1\mathrm{m}$, $l_0=0.001\mathrm{m}$, $\tilde{C}_n^2=10^{-15}\mathrm{m}^{3-\alpha}$ unless other variable parameters are specified in each figure.

Figure 1 shows the normalized rms angular width of CGSM beams on propagation in non-Kolmogorov turbulence for different values of parameter n and spatial coherence δ, respectively. It can be seen that the relative rms angular width of the CGSM beams increases with the propagation distance in the turbulence, and it spreads slower for the beams with larger parameter n or smaller spatial coherence, i.e., the relative angular spreading is less affected by the turbulence.

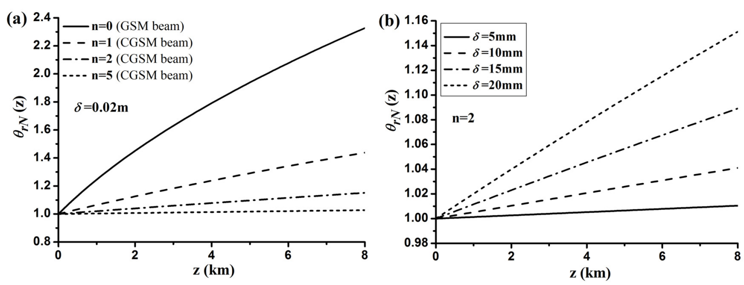

Fig. 1. Normalized rms angular width of the CGSM beams on propagation in non-Kolmogorov turbulence for different values of (a) parameter n with $\delta=0.02\mathrm{m}$ and (b) spatial coherence δ with $n=2$, respectively.

Figure 2 shows the normalized propagation factor of CGSM beams versus the propagation distance in non-Kolmogorov turbulence for different values of parameter n and spatial coherence δ, respectively. For comparison, the case of GSM beam ($n=0$) is also shown under the same conditions. One clearly sees that the normalized propagation factor of the CGSM beams increases monotonically as the propagation distance increases in the presence of atmospheric turbulence, which means that the beam quality is degraded by the turbulence. This behavior is quite different from that in free space, where the propagation factor is independent of the propagation distance. Furthermore, for very small spatial coherence δ, there is almost no distinct difference between the normalized propagation factor of the CGSM beams with different parameter n and the GSM beam. However, as the spatial coherence increases, the difference becomes more apparent. The normalized propagation factor of the CGSM beams with larger parameter n increases much slower than that of the CGSM beams with smaller parameter n or the GSM beam on propagation, which means that the CGSM beams with larger parameter n are less affected by the turbulence. We also find from Fig. 3 that normalized propagation factor spreads more rapidly for the CGSM beams with larger spatial coherence or smaller beam waist width. Figure 4 shows the dependence of the normalized propagation factor of the CGSM beams for several values of parameter n on spatial coherence δ at propagation distance $z = 5\text{km}$ in the turbulence. As shown in Fig. 4, the normalized propagation factor of the CGSM beams for all values of parameter n increases with the increase of spatial coherence and then tends to a definite value when the spatial coherence is very large, for this case, the CGSM beam turns into the coherent Gaussian beam. The normalized propagation factor of the CGSM beams increases more slowly than that of the GSM beam, and this tendency becomes more obvious for the CGSM beams with larger parameter n.

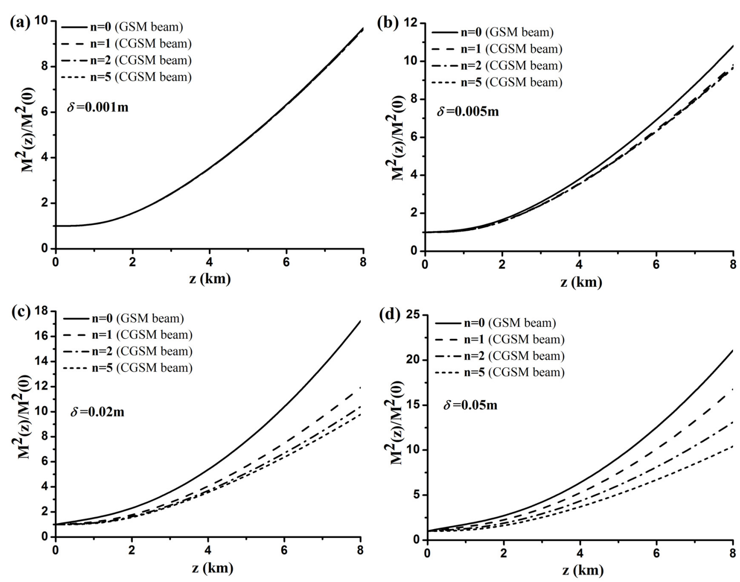

Fig. 2. Normalized propagation factor of CGSM beams versus the propagation distance in non-Kolmogorov turbulence for several values of parameter n and spatial coherence δ, respectively.

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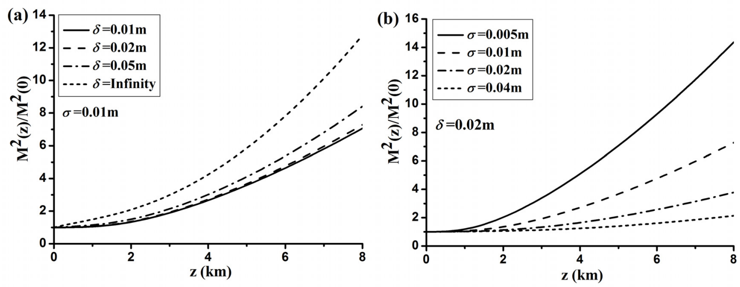

Fig. 3. Normalized propagation factor of CGSM beams with parameter $n = 2$ versus the propagation distance in non-Kolmogorov turbulence for different values of (a) spatial coherence δ with $\sigma = 0.01\text{m}$ and (b) beam waist width σ with $\delta = 0.02\text{m}$, respectively.

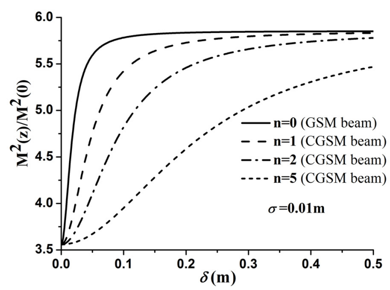

Fig. 4. The dependence of the normalized propagation factor of the CGSM beams for several values of parameter n on spatial coherence δ at propagation distance $z = 5\text{km}$ in non-Kolmogorov turbulence.

We will now analyze the effects of the turbulence on the propagation factor of CGSM beams. We calculate in Fig. 5 the normalized propagation factor of the CGSM beams for several values of parameter n on propagation in non-Kolmogorov turbulence with different structure constant $\tilde{C}_n^2$, respectively. One finds from Fig. 5 that, in weak turbulence (i.e., small structure constant), there is little difference between the normalized propagation factor of the CGSM beams and that of the GSM beam. However, as the structure constant increases, the difference becomes apparent and the advantage of the CGSM beams over the GSM beam for overcoming the negative effect of the turbulence is gradually revealed. Figure 6 shows the normalized propagation factor of the CGSM beam on propagation in non-Kolmogorov turbulence for different slope-parameter α , inner scale l_0 , outer scale L_0 and structure constant $\tilde{C}_n^2$, respectively. It can be seen that the normalized propagation factor of the CGSM beam spreads more rapidly in the turbulence with smaller inner scale, larger outer scale and larger structure constant, this indicates that the beam is more affected by the turbulence under this conditions. Figure 7 shows the dependence of the normalized propagation factor of the CGSM beams for several values of parameter n and spatial coherence δ on slope-parameter α at propagation distance $z = 5\text{km}$ in non-Kolmogorov turbulence. As is shown in Fig. 7, the dependence of the normalized propagation factor of the CGSM beams on slope-parameter α is non-monotonic, it will first increase quickly as α increases until it reaches the maximum point (about at $\alpha = 3.067$), then it decreases gradually with the further increasing of α . We may explain that there exists a singular point and the beam quality is degraded the most at this point. In addition, the CGSM beams with larger parameter n or smaller spatial coherence δ are less affected by the turbulence.

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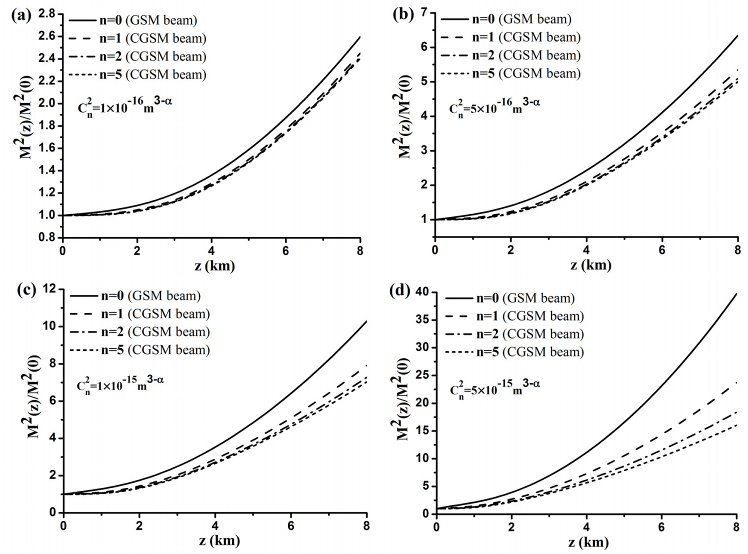

Fig. 5. Normalized propagation factor of CGSM beams with $\delta = 0.02\text{m}$ for several values of parameter n versus the propagation distance in non-Kolmogorov turbulence with different structure constant $\tilde{C}_n^2$, respectively.

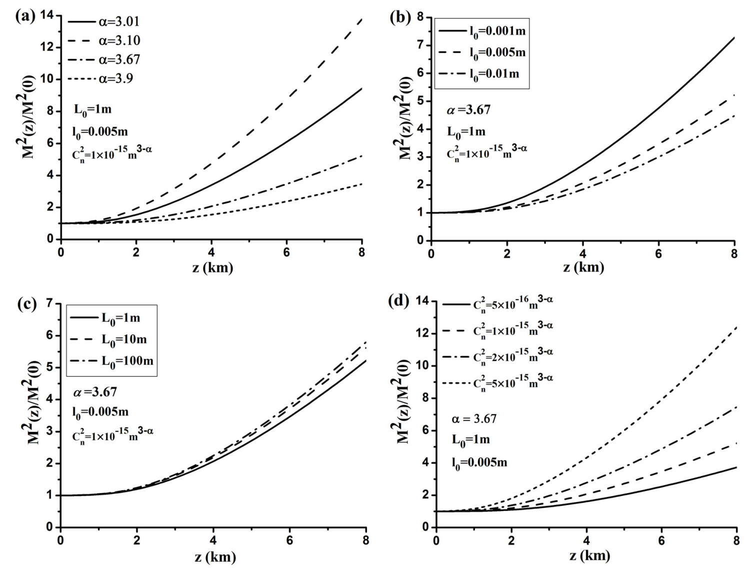

Fig. 6. Normalized propagation factor of the CGSM beam with $n = 2$ and $\delta = 0.02\text{m}$ versus the propagation distance in non-Kolmogorov turbulence for different values of slope-parameter α, inner scale l_0, outer scale L_0 and structure constant $\tilde{C}_n^2$, respectively. The other turbulence parameters are shown in each figure.

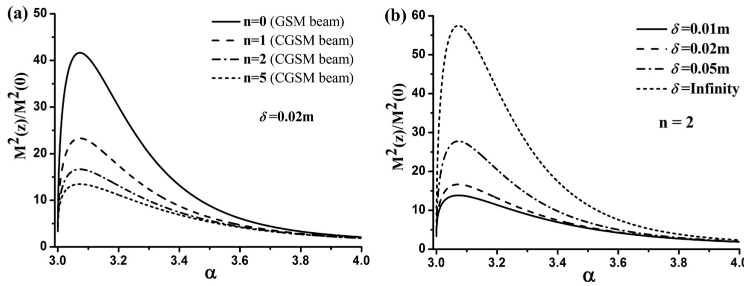

Fig. 7. The dependence of the normalized propagation factor of the CGSM beams on slope-parameter α at propagation distance $z = 5\text{km}$ in non-Kolmogorov turbulence for several values of (a) parameter n with $\delta = 0.02\text{m}$ and (b) spatial coherence δ with $n = 2$.

4. Summary

In conclusion, the analytical expressions for the propagation factor of the CGSM beams in turbulent atmosphere have been derived in terms of the extended Huygens-Fresnel principle and second-order moments of the WDF. We have numerically investigated the relative rms angular width and the normalized propagation factor of CGSM beams for different beam and turbulence parameters on propagation in non-Kolmogorov turbulence in detail. Numerical results have been shown that the CGSM beam has advantage over the GSM beam for reducing the turbulence-induced degradation, and this advantage will be more obvious for the beams with larger parameter n and spatial coherence δ or under the condition of stronger fluctuation of turbulence. It is indicated that partially coherent beams with special correlation function have the potential ability to further improve the performance of laser beam propagation. What's more, the CGSM beam with larger parameter n or smaller spatial coherence δ will be less affected by the turbulence. The beam will be more affected by the turbulence under the conditions of larger inner scale and structure constant or smaller outer scale. In addition, the propagation properties are closely related to slope-parameter α of the turbulence's power spectrum. Our results will be useful in long-distance free-space optical communications.

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