

# Tracking Knowledge Proficiency of Students with Educational Priors

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# Outline

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- ❑ **Backgroud and Related Work**
- ❑ Problem Statement
- ❑ Methodology
- ❑ Experiments
- ❑ Conclusion



# Background

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- Traditional teaching method
  - Classroom Teaching
    - The teacher's energy is limited.
    - The same learning strategy, same exercises, impersonality.
  - Extracurricular Tutorials
    - Teaching quality is difficult to guarantee
    - A higher cost



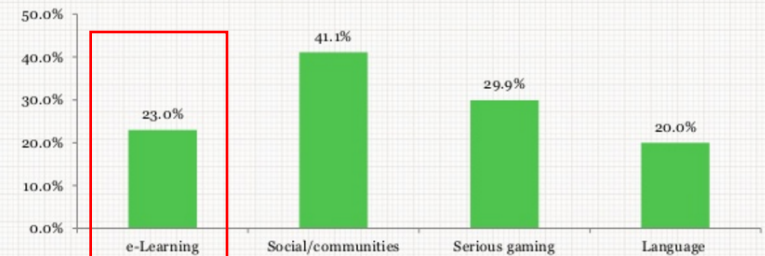
# Background

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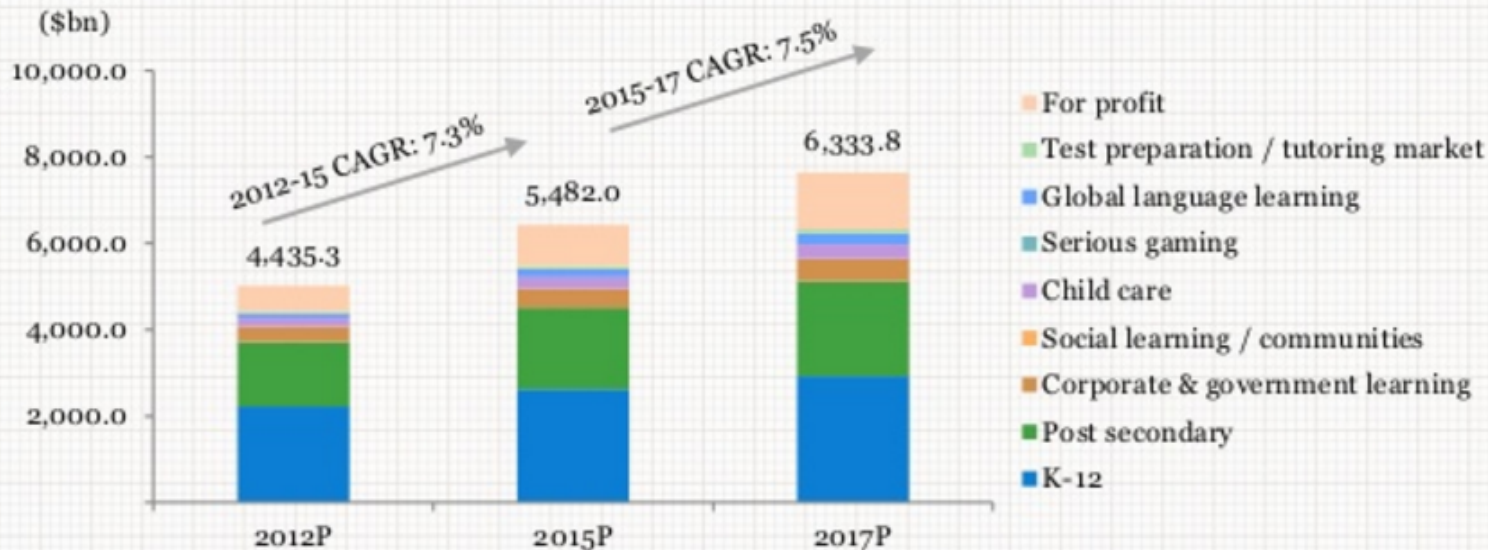
## □ E-Learning(Online learning)

- Knewton
- Cognitive Tutor
- etc

Fast Growing Segments (2012–2017 CAGR)



Global Education Expenditure Forecast by Subsectors



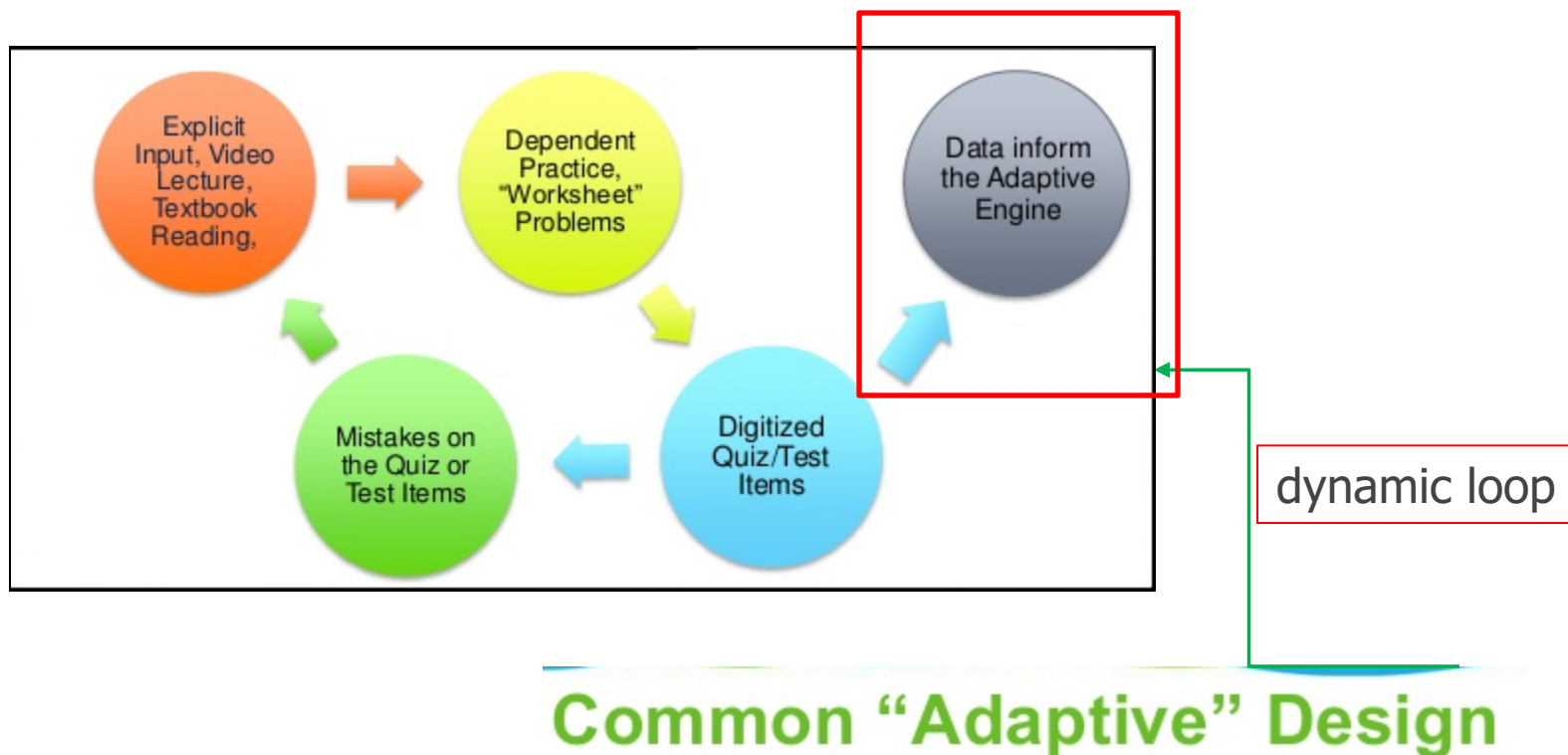


# Background

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## □ Education Service Systems

- Various online tutoring systems allow students to learn and do exercises individually.





# Related work-static

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## IRT

$$P(X_{ij} = 1|\theta_j) = c_i + \frac{1 - c_i}{1 + \exp[-1.7a_i(\theta_j - b_i)]}$$

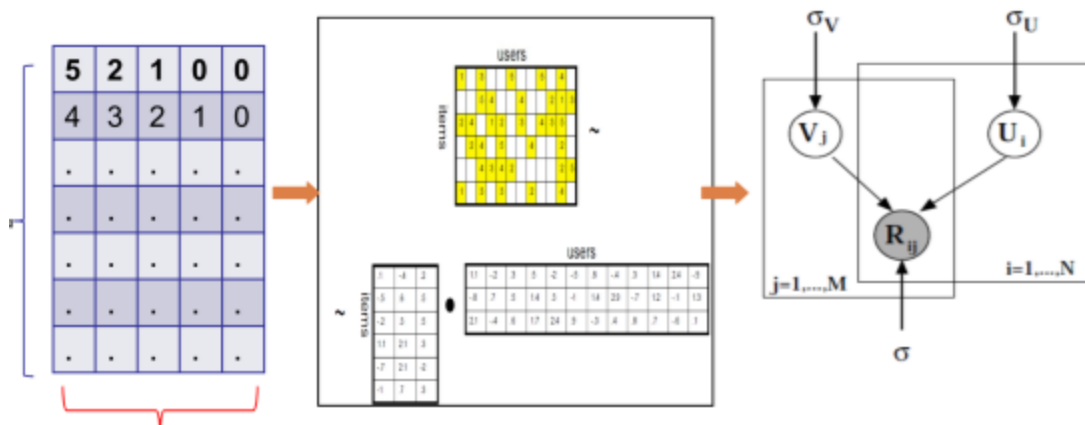
## DINA

$$P_j(\alpha_i) = P(X_{ij} = 1|\alpha_i) = g_j^{1 - \eta_{ij}} (1 - s_j)^{\eta_{ij}}.$$

## PMF

$$p(R|U, V, \sigma^2) = \prod_{i=1}^N \prod_{j=1}^M \left[ \mathcal{N}(R_{ij} | U_i^T V_j, \sigma^2) \right]^{I_{ij}}$$

they are only good at predicting student's proficiency from a **static perspective**.





# Related work-dynamic

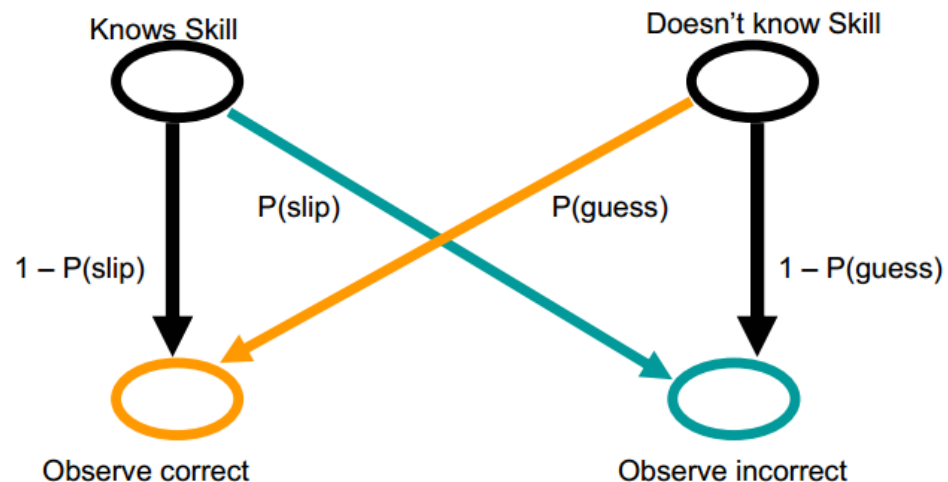
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## □ LFA - one-dimensional

$$m(i, j \in KCs, n) = \alpha_i + \sum_{j \in KCs} (\beta_j + \gamma_j n_{i,j})$$

$$p(m) = \frac{1}{1 + e^{-m}}$$

## □ BKT- binary entities





# Outline

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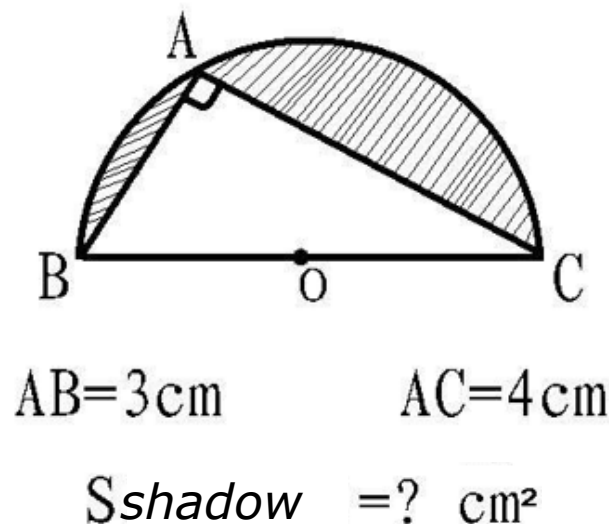
- Background and Related Work
- **Problem Statement**
- Methodology
- Experiments
- Conclusion



# Motivation

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- Problem: How to **track** students' **knowledge proficiency** over time. (TKP task)?
- Opportunity
  - Widely use of **Intelligent tutoring system**
  - Record **exercises logs** and **Q-matrix**
  - **Educational Priors**
- Focus on Math problem





# Problem Statement

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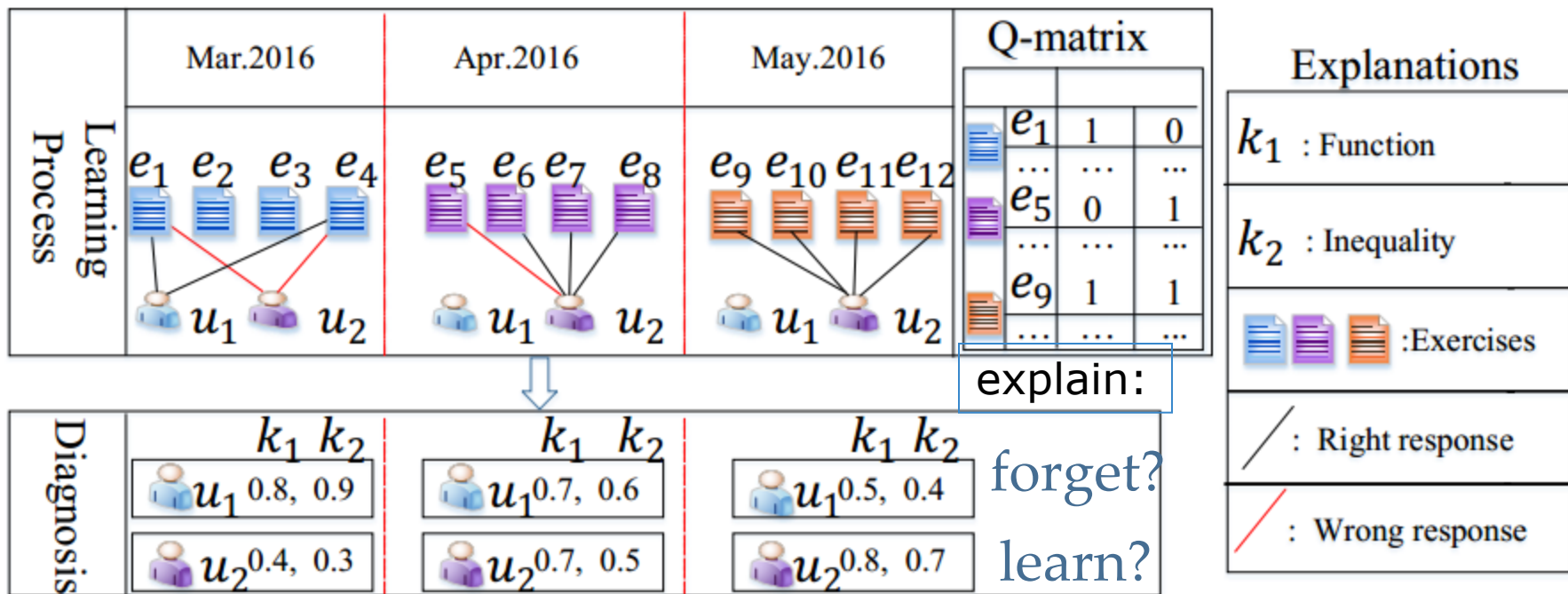
- Given the students' response tensor  $R$  and  $Q$ -matrix labelled by educational experts
- our goal is two-fold:
  - modeling the change of students' knowledge proficiency from time 1 to  $T$ .
  - predicting students' knowledge proficiency and responses in time  $T + 1$ .
- Challenge:
  - 1. How to get a student's knowledge proficiency?
  - 2. How to explain the change of knowledge proficiency over time?



# A toy example

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- A showcase of KPD task on mathematical exercises related to the knowledge points of Function and Inequality





# Outline

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- Background and Related Work
- Problem Statement
- **Methodology**
  - **Probabilistic Modeling with Priors**
  - **Model Learning and Prediction**
- Experiments
- Conclusion



# Framework

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□ KPT: a supervised way

□ Modeling Stage

□ Partial order

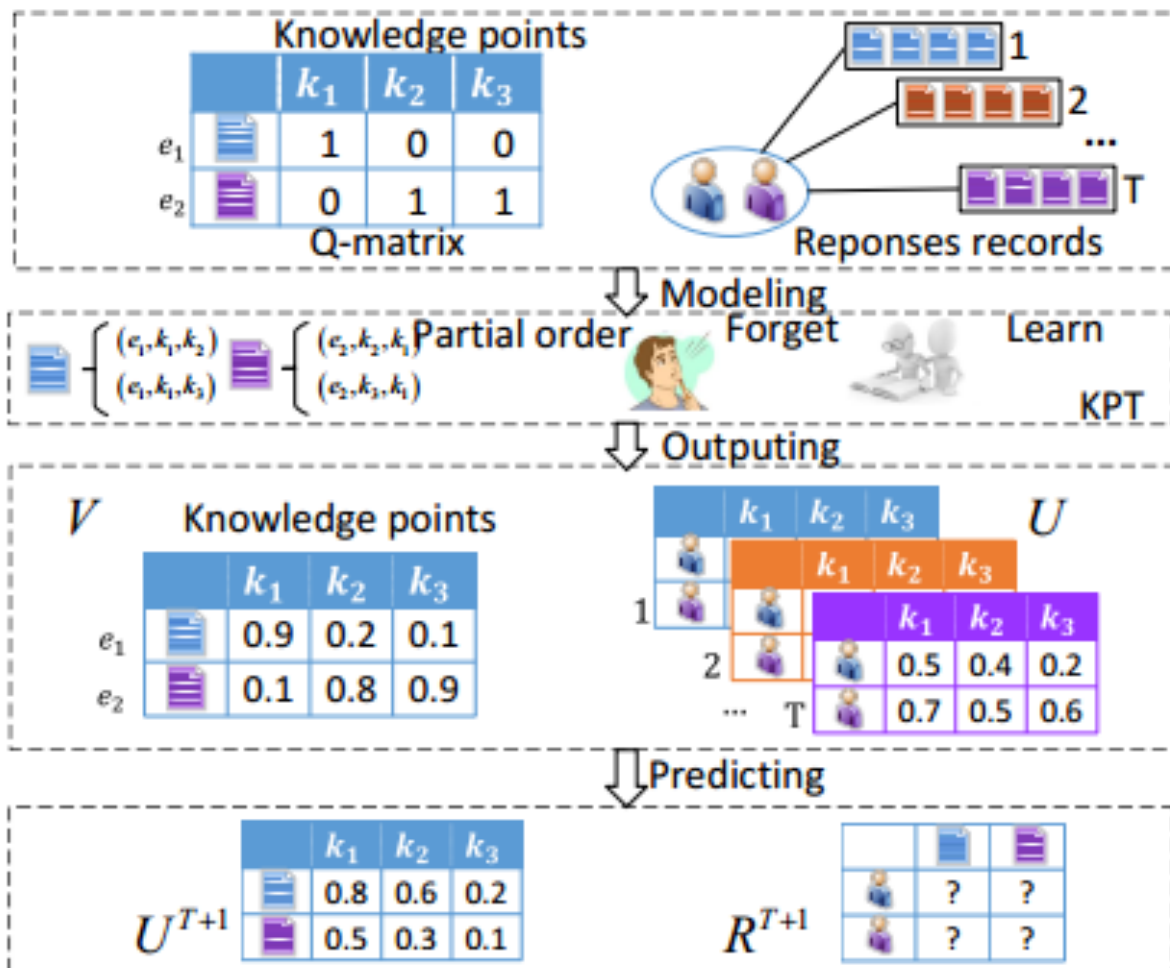
□ Forget

□ Learn

□ Predicting Stage

□ Input:  $U^{1-T}, V$

□ Output:  
 $U^{T+1}, R^{T+1}$





# KPT model

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## □ Probabilistic Modeling with Priors

- for each student and each exercise, we model the response tensor  $R$  as:

$$p(R|U, V, b) = \prod_{t=1}^T \prod_{i=1}^N \prod_{j=1}^M [\mathcal{N}(R_{ij}^t | \langle U_i^t, V_j \rangle - b_j, \sigma_R^2)]^{I_{ij}^t}, \quad (1)$$

- $U_i^t \in \mathbb{R}^{K \times 1}$  is the knowledge proficiency of student  $i$
- $V \in \mathbb{R}^{M \times K}$  denotes the relationship between exercises and knowledge points
- How to establish the corresponding relationship between **students**, **exercises** and **knowledge points**?



# Modeling $V$ with the Q-matrix prior

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## □ Q-matrix

- depicts the knowledge points of the exercises
- each row denotes an exercise
- each column stands for a knowledge point.

|           | Function | Solid Geometry | Arithmetic Progression | Inequation |
|-----------|----------|----------------|------------------------|------------|
| exercise1 | 1        | 0              | 0                      | 0          |
| exercise2 | 1        | 1              | 0                      | 1          |
| exercise3 | 1        | 0              | 1                      | 0          |
| exercise4 | 0        | 1              | 0                      | 0          |

- The sparsity with the binary entities **does not fit** probabilistic modeling well.



# Modeling $V$ with the Q-matrix prior

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- for exercise  $j$ , if a knowledge point  $q$  is marked as 1, then we assume that  $q$  is more relevant to exercise  $j$  than  $p$  with mark 0

$$q >_j^+ p, \text{ if } Q_{jq} = 1 \text{ and } Q_{jp} = 0.$$

Partial order

- After that, we can transform the original Q-matrix into a set of comparability  $D_T \in R^{M \times K \times K}$  by:  $D_T = \{(j, q, p) | q >_j^+ p\}$ .

| exercises | knowledge |    |    |    |
|-----------|-----------|----|----|----|
|           | K1        | K2 | K3 | K4 |
| E1        | 1         | 0  | 1  | 0  |
| E2        | 0         | 1  | 0  | 0  |
| E3        | 0         | 0  | 0  | 1  |

Q-matrix

(E1,K1,K2)  
(E1,K1,K4)  
(E1,K3,K2)  
(E1,K3,K4)  
...  
(E3,K4,K1)  
(E3,K4,K2)  
(E3,K4,K3)



# Modeling $V$ with the Q-matrix prior

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- we define the probability that exercise  $j$  is more relevant to knowledge point  $q$  than knowledge point  $p$  as:

$$p(q >_j^+ p | V_j) = \frac{1}{1 + e^{-(V_{jq} - V_{jp})}}. \quad (6)$$

- the log of the posterior distribution

$$\begin{aligned} \ln p(V | D_T) &= \ln \prod_{(j,q,p) \in D_T} p(>_j^+ | V) p(V) \\ &= \sum_{j=1}^M \sum_{q=1}^K \sum_{p=1}^K I(q >_j^+ p) \ln \frac{1}{1 + e^{-(V_{jq} - V_{jp})}} \\ &\quad - \frac{1}{2\sigma_V^2} \|V\|_F^2. \end{aligned} \quad (7)$$



# Modeling U with learning theories.

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- we assume a student's current knowledge proficiency is mainly influenced by two underlying reasons:
  - She **forgets** her previous knowledge proficiency over time.
  - The **more exercises she does**, the higher level of related knowledge proficiency she will get.
  - We model the two effects of each student's knowledge proficiency in time window  $t = 2; 3; \dots; T$  as:

$$p(U_i^t) = \mathcal{N}(U_i^t | \bar{U}_i^t, \sigma_U^2 \mathbf{I}), \text{ where } \bar{U}_i^t = \{\bar{U}_{i1}^t, \bar{U}_{i2}^t \dots \bar{U}_{iK}^t\}$$
$$\bar{U}_{ik}^t = (1 - \alpha_i) \boxed{f^t(*)} + \alpha_i \boxed{l^t(*)}, s.t. \quad 0 \leq \alpha_i \leq 1, \quad (8)$$

forgetting      learning



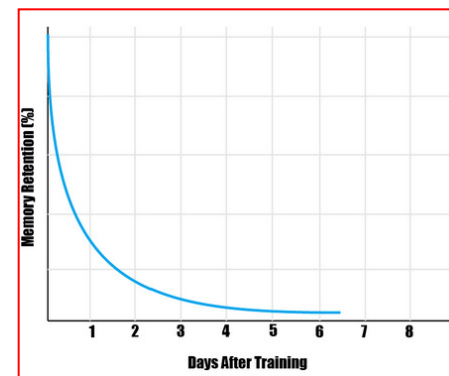
# Modeling U with learning theories.

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- $f^t(*)$  depicts the decline of knowledge over time:

$$f^t(*) = U_{ik}^{t-1} e^{-\frac{\Delta t}{S}}$$

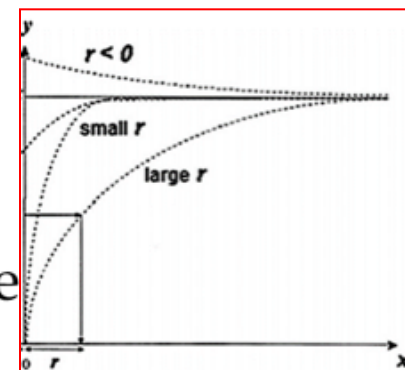
- $\Delta t$  is the time interval
- $S$  denotes the strength of memory.



- $l^t(*)$  captures the growth of knowledge with exercises:

$$l^t(*) = U_{ik}^{t-1} \frac{D f_k^t}{f_k^t + r}$$

- $f_k^t$  denotes the frequency of knowledge  $k$
- $r$  and  $D$  control the magnitude and multiplier of growth respectively.





# Model Learning and Prediction

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- graphical representation of the proposed latent model

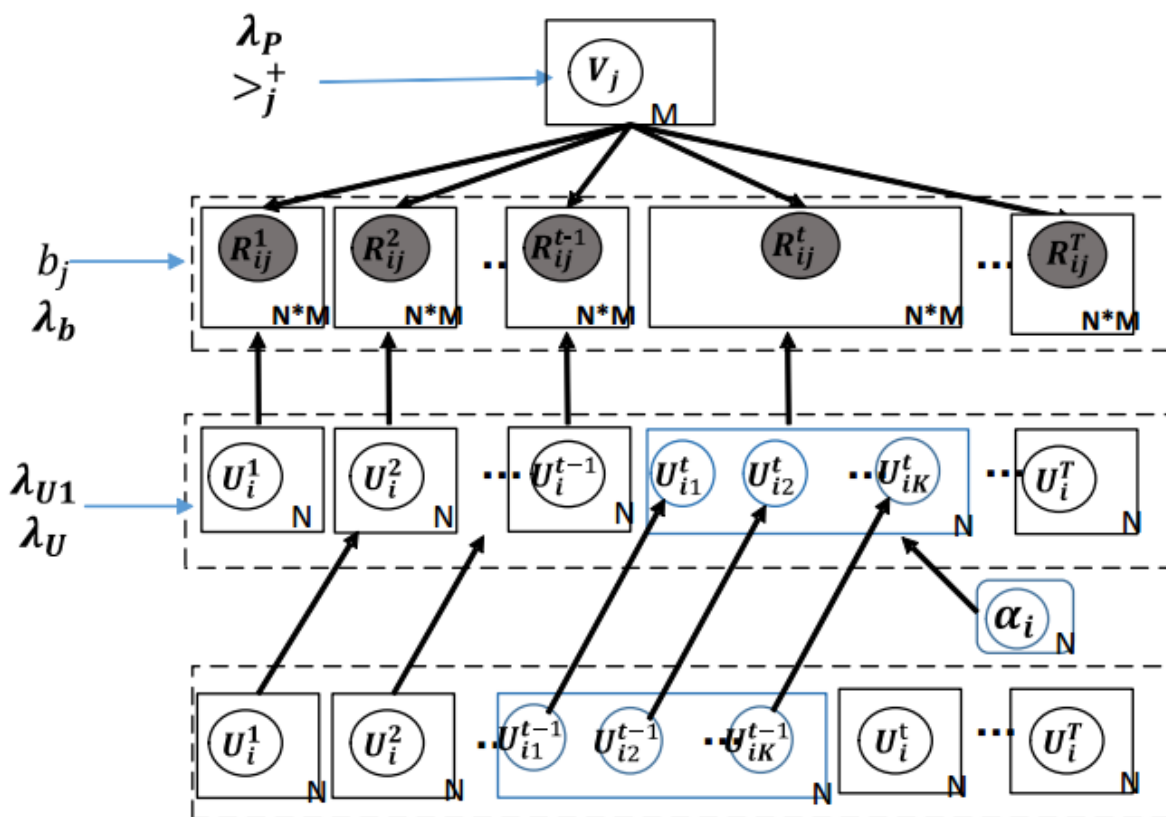


Figure 3: Graphical representation of KPT.



# Model Learning and Prediction

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- our goal is to learn the parameters  $\Phi = [U, V, \alpha, b]$ 
  - Particularly, the posterior distribution over  $\Phi$  is:  

$$p(U, V, \alpha, b | R, D_T) \propto p(R | U, V, \alpha, b) \times p(U | \sigma_U^2, \sigma_{U1}^2) \times p(V | D_T).$$
  - Maximizing the log posterior of the above equation is equivalent to minimize the following objective:

$$\begin{aligned}
 \min_{\Phi} \mathcal{E}(\Phi) = & \frac{1}{2} \sum_{t=1}^T \sum_{i=1}^N \sum_{j=1}^M I_{ij}^t [\hat{R}_{ij}^t - R_{ij}^t]^2 \\
 & - \lambda_P \sum_{j=1}^M \sum_{q=1}^K \sum_{p=1}^K I(q >_j^+ p) \ln \frac{1}{1 + e^{-(V_{jq} - V_{jp})}} + \frac{\lambda_V}{2} \sum_{i=1}^M \|V_i\|_F^2 \\
 & + \frac{\lambda_U}{2} \sum_{t=2}^T \sum_{i=1}^N \|\bar{U}_i^t - U_i^t\|_F^2 + \frac{\lambda_{U1}}{2} \sum_{i=1}^N \|U_i^1\|_F^2,
 \end{aligned} \tag{13}$$



# Model Learning and Prediction

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- With students' knowledge proficiency  $U^1, U^2, \dots, U^T$  and related parameters, students' responses and knowledge proficiency in the next time can be calculated as:

$$U_i^{(T+1)} = \left\{ U_{i1}^{(T+1)}, U_{i2}^{(T+1)}, \dots, U_{iK}^{(T+1)} \right\},$$

$$U_{ik}^{(T+1)} \approx (1 - \alpha_i) U_{ik}^T e^{-\frac{\Delta(T)}{S}} + \alpha_i U_{ik}^T \frac{M f_k^{T+1}}{f_k^{T+1} + r},$$

$$\hat{R}_{ij}^{(T+1)} \approx \langle U_i^{(T+1)}, V_j \rangle - b_j.$$



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- **Experiments**
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# Experiments

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- Dataset
  - *Two private datasets* which are collected from daily exercise records of high school students
  - *ASSIST* is a public dataset *Assistments*<sup>1</sup> 2009-2010 “Non-skill builder”

**Table 4: The statistics of the three datasets.**

| Dataset                                      | Math1   | Math2   | ASSIST |
|----------------------------------------------|---------|---------|--------|
| Training scores logs                         | 521,248 | 347,424 | 13,443 |
| Testing scores logs                          | 74,464  | 18,312  | 1,822  |
| Students                                     | 9,308   | 1,306   | 215    |
| Exercises                                    | 64      | 280     | 71     |
| Time windows                                 | 4       | 10      | 4      |
| Knowledge points                             | 12      | 13      | 7      |
| Average knowledge points<br>of each exercise | 1.15    | 1.3215  | 1.02   |



# Evaluations

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- Two evaluations:
  - evaluate on Students' Responses Prediction.
    - proved the **rationality of three priors for** prediction accuracy
  - evaluate on Knowledge Proficiency Diagnosis.
    - proved that the **effectiveness** of associating each exercise and student with **a knowledge vector** in the same knowledge space .



# Evaluations on Students' Responses Prediction.

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## □ Evaluation Metrics

### □ For Scores prediction task performance

■ RMSE、MAE

### □ baselines:

■ *IRT*

■ *DINA*

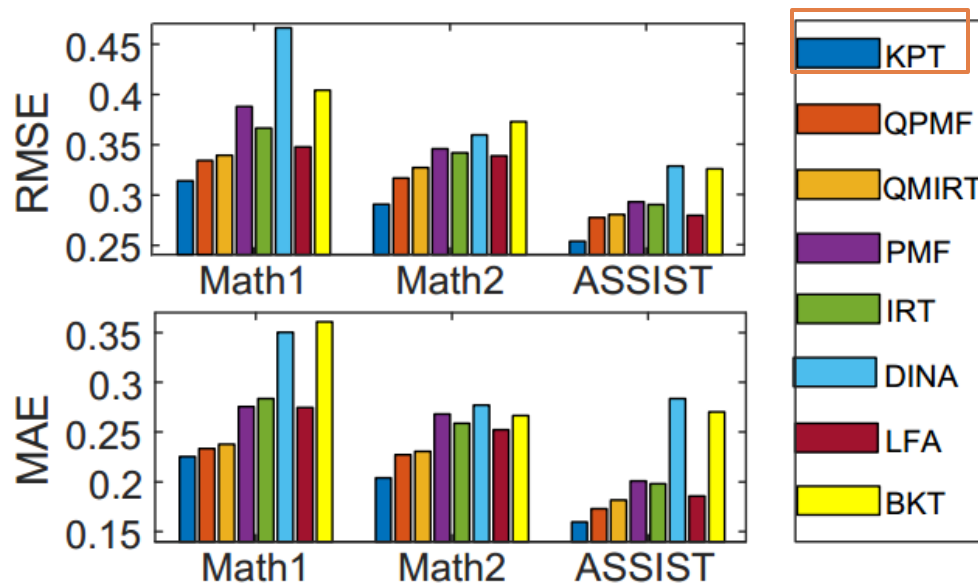
■ *PMF*

■ *LFA*

■ *BKT*

■ *QMIRT* (MIRT+partial order)

■ *QPMF* (PMF+Partial order)



■ KPT performs best on all three datasets.



# Evaluations on Knowledge Proficiency Diagnosis

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- For Knowledge Proficiency Diagnosis
  - DOA-of each specific knowledge point  $k$

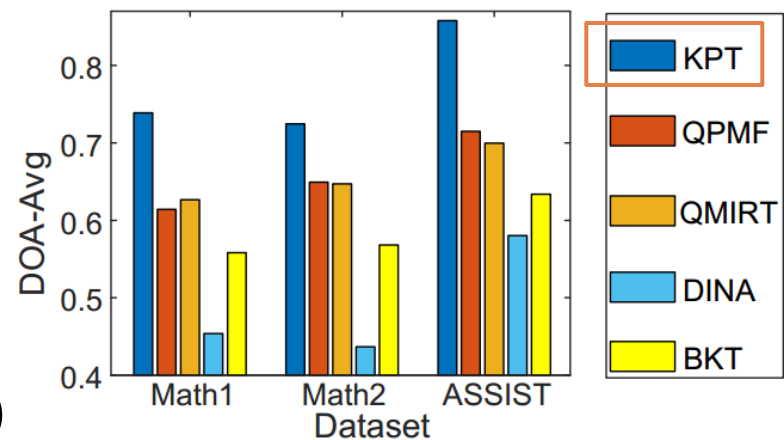
$$DOA(k) = \sum_{j=1}^M I(Q_{jk} = 1) \sum_{u1=1}^N \sum_{u2=1}^N \frac{\delta(U_{u1k}^T - U_{u2k}^T) \cap \delta(R_{u2j}^T - R_{u1j}^T)}{\delta(U_{u1k}^T - U_{u1k}^T)},$$

- DOA-average of all knowledge points

$$DOA - Avg = \frac{1}{K} \sum_{k=1}^K DOA(k)$$

- baselines:

- *DINA*
- *BKT*
- *QMIRT* (MIRT+partial order)
- *QPMF* (PMF+Partial order)





# Evaluations on Knowledge Proficiency Diagnosis

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- KPT performs best on KPD task for all knowledge points, followed by QPMF and QIRT, which indicates that the educational prior of Q-matrix does effectively.

(c) ASSIST

| K  | Baselines    |       |       |       |       |
|----|--------------|-------|-------|-------|-------|
|    | KPT          | QPMF  | QMIRT | DINA  | BKT   |
| K1 | <b>0.793</b> | 0.747 | 0.716 | 0.605 | 0.592 |
| K2 | <b>0.823</b> | 0.653 | 0.673 | 0.593 | 0.672 |
| K3 | <b>0.887</b> | 0.852 | 0.671 | 0.631 | 0.577 |
| K4 | <b>0.792</b> | 0.598 | 0.755 | 0.525 | 0.569 |
| K5 | <b>0.891</b> | 0.576 | 0.672 | 0.511 | 0.624 |
| K6 | <b>0.871</b> | 0.647 | 0.657 | 0.628 | 0.604 |
| K7 | <b>0.901</b> | 0.793 | 0.654 | 0.573 | 0.796 |

(a) Math1

| K   | Baselines    |       |       |       |       |
|-----|--------------|-------|-------|-------|-------|
|     | KPT          | QPMF  | QMIRT | DINA  | BKT   |
| K1  | <b>0.798</b> | 0.565 | 0.595 | 0.524 | 0.558 |
| K2  | <b>0.733</b> | 0.576 | 0.621 | 0.473 | 0.623 |
| K3  | <b>0.827</b> | 0.614 | 0.629 | 0.497 | 0.523 |
| K4  | <b>0.752</b> | 0.581 | 0.675 | 0.486 | 0.565 |
| K5  | <b>0.791</b> | 0.559 | 0.723 | 0.476 | 0.578 |
| K6  | <b>0.838</b> | 0.730 | 0.766 | 0.485 | 0.628 |
| K7  | <b>0.842</b> | 0.697 | 0.634 | 0.520 | 0.697 |
| K8  | <b>0.784</b> | 0.699 | 0.657 | 0.498 | 0.617 |
| K9  | <b>0.771</b> | 0.609 | 0.712 | 0.501 | 0.645 |
| K10 | <b>0.834</b> | 0.597 | 0.515 | 0.489 | 0.503 |
| K11 | <b>0.786</b> | 0.608 | 0.631 | 0.478 | 0.617 |
| K12 | <b>0.842</b> | 0.532 | 0.641 | 0.523 | 0.645 |

(b) Math2

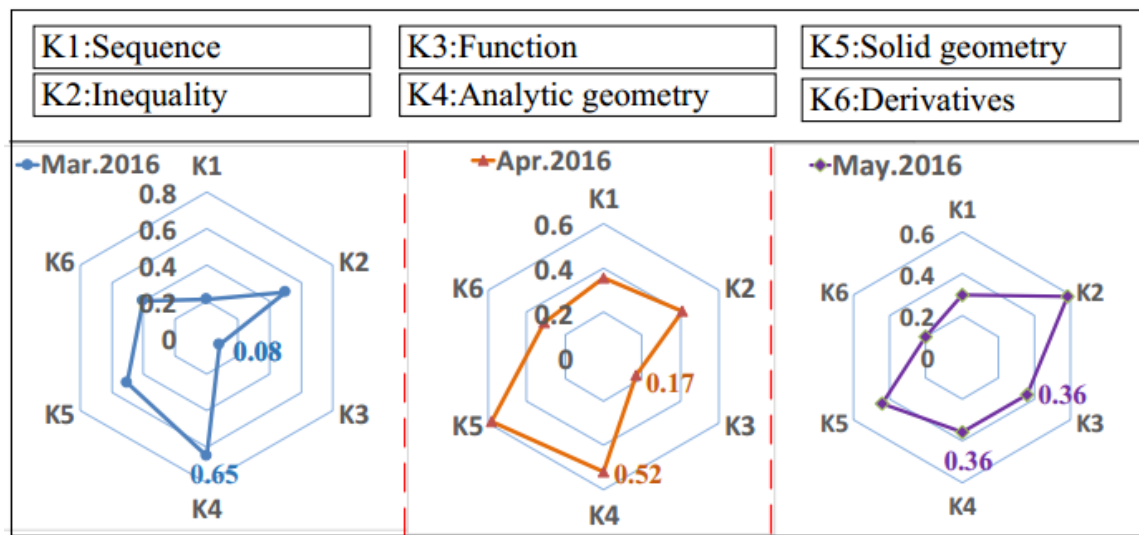
| K   | Baselines    |       |              |       |       |
|-----|--------------|-------|--------------|-------|-------|
|     | KPT          | QPMF  | QMIRT        | DINA  | BKT   |
| K1  | <b>0.804</b> | 0.743 | 0.754        | 0.517 | 0.568 |
| K2  | <b>0.757</b> | 0.632 | 0.659        | 0.534 | 0.753 |
| K3  | <b>0.818</b> | 0.761 | 0.723        | 0.510 | 0.669 |
| K4  | 0.688        | 0.733 | <b>0.734</b> | 0.534 | 0.711 |
| K5  | <b>0.891</b> | 0.703 | 0.668        | 0.474 | 0.553 |
| K6  | <b>0.699</b> | 0.547 | 0.653        | 0.489 | 0.644 |
| K7  | <b>0.791</b> | 0.677 | 0.722        | 0.483 | 0.730 |
| K8  | <b>0.726</b> | 0.722 | 0.659        | 0.523 | 0.668 |
| K9  | <b>0.736</b> | 0.558 | 0.541        | 0.507 | 0.567 |
| K10 | <b>0.652</b> | 0.639 | 0.650        | 0.511 | 0.614 |
| K11 | <b>0.888</b> | 0.836 | 0.692        | 0.522 | 0.630 |
| K12 | <b>0.798</b> | 0.737 | 0.794        | 0.498 | 0.528 |
| K13 | <b>0.813</b> | 0.797 | 0.804        | 0.453 | 0.633 |



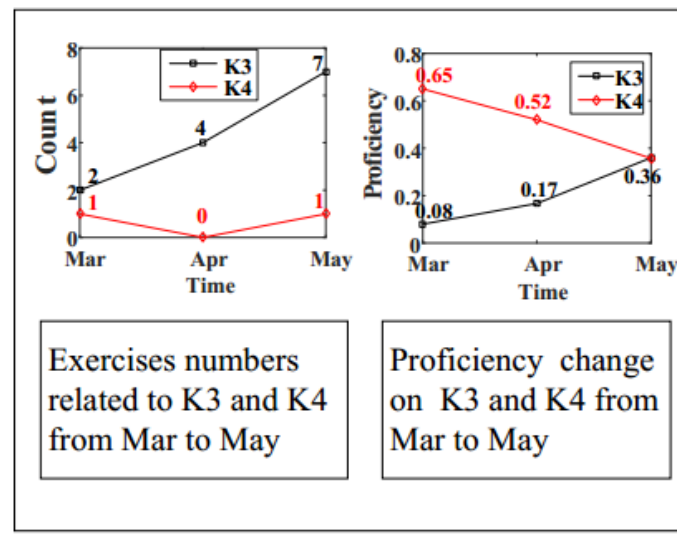
# Case study

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- The diagnosis results of a student on six knowledge points at three particular time in Math2
- It clearly demonstrated the explanatory power of our proposed KPT model



(a) Diagnosis



(b) Analysis



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# Conclusion

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- **Problem:** track students' knowledge proficiency mastery over time
- **Method:** probabilistic model with three educational priors
- **Contributions:**
  - We designed an *explanatory* probabilistic KPT model for solving the TKP task
  - We associated each exercise with a knowledge vector with the *Q-matrix* prior.
  - we embedded the *Learning curve* and *Forgetting curve* as priors to capture the change of each student's proficiency over time.



# Future Work

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- First, we will consider to combine more kinds' of users' behaviors (e.g., reading records) for the TKP task.
- Second, as students may learn difficult knowledge points (e.g., Function) after some basic ones (e.g., Set), it is interesting to take this kind of knowledge relationship into account for TKP



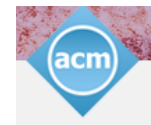
# Acknowledgements

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□ We thanks for:

□ the SIGIR Travel Award

■ url: <http://sigir.org/general-information/travel-grants/>



□ the SIGWEB and US NSF Travel Award

■ url: <https://cmt3.research.microsoft.com/CIKMTA2017>



# Q & A

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# *Thanks !*



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