7.2 湍流Reynolds时均方程

工程中应用最广泛的湍流计算方法

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7.2.1 湍流中的'平均'概念

- 时间平均：

$$\overline{\phi_t}(x) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\phi(x,t)dt$$

- 空间平均：

$$\overline{\phi_s}(t) = \lim_{X\to\infty}\frac{1}{2X}\int_{-X}^{X}\phi(x,t)dx$$

- 系综平均：

$$\overline{\phi_e}(x,t) = \frac{1}{N}\sum_{n=1}^{N}\phi_n(x,t)$$

'时间平均'

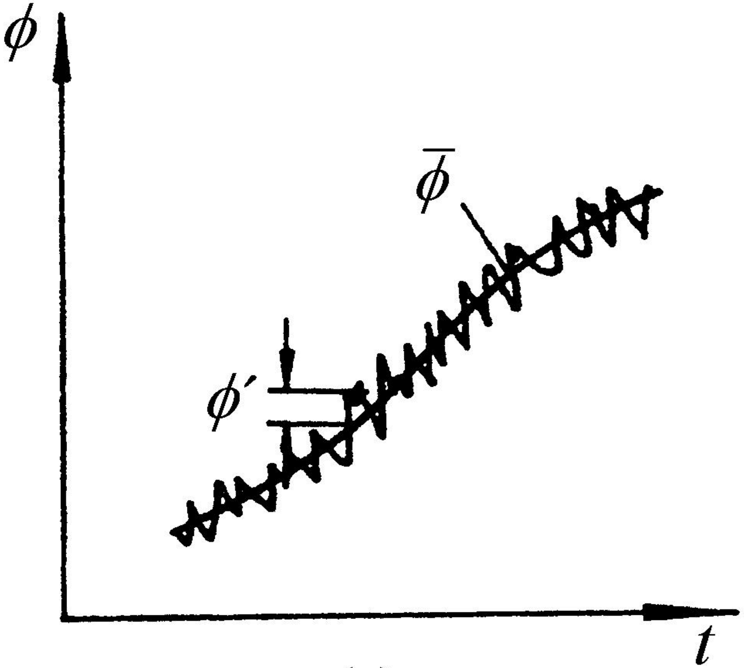
- 时间平均：

$$\overline{\phi}_t(x,t)=\frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}}\phi(x,\tau)d\tau,\qquad T_1<<T<<T_2$$

- 性质：

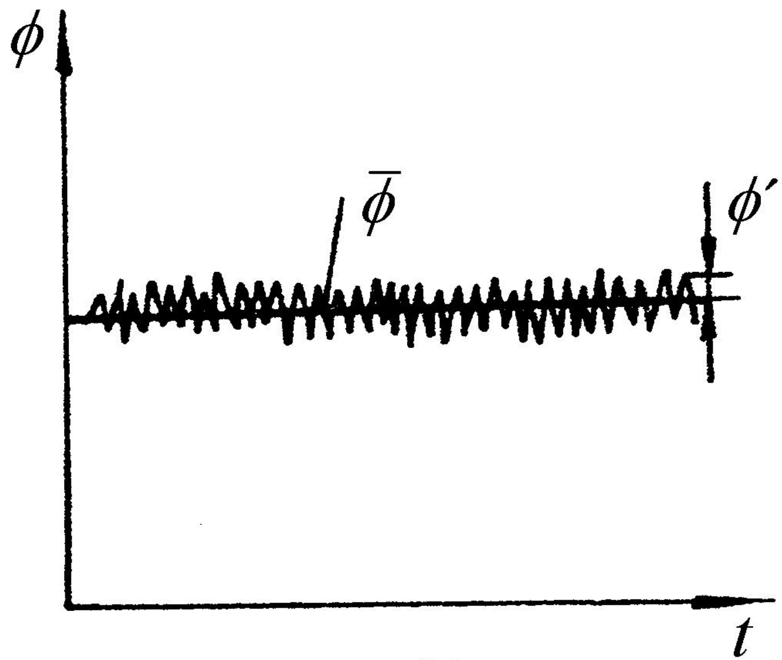
$$\begin{cases}\overline{\overline{\phi}}=\overline{\phi};\quad \overline{\phi'}=0;\quad \overline{\overline{\phi}+\phi'}=\overline{\phi}\\ \overline{\overline{\phi}\,\overline{\psi}}=\overline{\phi}\,\overline{\psi};\quad \overline{\phi\psi}=\overline{\phi}\,\overline{\psi}+\overline{\phi'\psi'}\\ \overline{\frac{\partial\phi}{\partial x}}=\frac{\partial\overline{\phi}}{\partial x};\quad \overline{\frac{\partial\phi}{\partial t}}=\frac{\partial\overline{\phi}}{\partial t}\end{cases}$$

$$\phi = \overline{\phi} + \phi'$$



(a)

非稳态湍流



(b)

准稳态湍流

7.2.2 Reynolds时均方程

- 不计质量力，不可压流体湍流瞬时运动控制方程：

$$\operatorname{div}\boldsymbol{U}=0$$

$$\frac{\partial u}{\partial t}+\operatorname{div}(u\boldsymbol{U})=\nu\operatorname{div}(\operatorname{grad}u)-\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t}+\operatorname{div}(v\boldsymbol{U})=\nu\operatorname{div}(\operatorname{grad}v)-\frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{\partial w}{\partial t}+\operatorname{div}(w\boldsymbol{U})=\nu\operatorname{div}(\operatorname{grad}w)-\frac{1}{\rho}\frac{\partial p}{\partial z}$$

$$\frac{\partial T}{\partial t}+\operatorname{div}(\boldsymbol{U}T)=\operatorname{div}(a\operatorname{grad}T)+\frac{S_T}{\rho}$$

$$\boldsymbol{U}=\overline{\boldsymbol{U}}+\boldsymbol{U}';\;\; u=\overline{u}+u';\;\; v=\overline{v}+v';\;\; w=\overline{w}+w';\;\; p=\overline{p}+p';\;\; T=\overline{T}+T'$$

湍流Reynolds时均控制方程组

- 二阶关联量

湍流应力

$$\operatorname{div}\overline{\boldsymbol{U}}=0$$

$$\frac{\partial\bar{u}}{\partial t}+\operatorname{div}(\bar{u}\overline{\boldsymbol{U}})=\nu\operatorname{div}(\operatorname{grad}\bar{u})+\boxed{\left[-\frac{\partial\overline{u'^2}}{\partial x}-\frac{\partial\overline{u'v'}}{\partial y}-\frac{\partial\overline{u'w'}}{\partial z}\right]}-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial x}$$

$$\frac{\partial\bar{v}}{\partial t}+\operatorname{div}(\bar{v}\overline{\boldsymbol{U}})=\nu\operatorname{div}(\operatorname{grad}\bar{v})+\boxed{\left[-\frac{\partial\overline{u'v'}}{\partial x}-\frac{\partial\overline{v'^2}}{\partial y}-\frac{\partial\overline{v'w'}}{\partial z}\right]}-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial y}$$

$$\frac{\partial\bar{w}}{\partial t}+\operatorname{div}(\bar{w}\overline{\boldsymbol{U}})=\nu\operatorname{div}(\operatorname{grad}\bar{w})+\boxed{\left[-\frac{\partial\overline{u'w'}}{\partial x}-\frac{\partial\overline{v'w'}}{\partial y}-\frac{\partial\overline{w'^2}}{\partial z}\right]}-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial z}$$

$$\frac{\partial\bar{T}}{\partial t}+\operatorname{div}(\bar{T}\overline{\boldsymbol{U}})=\operatorname{div}(a\operatorname{grad}\bar{T})+\left[-\frac{\partial\overline{u'T'}}{\partial x}-\frac{\partial\overline{v'T'}}{\partial y}-\frac{\partial\overline{w'T'}}{\partial z}\right]+\frac{\overline{S_T}}{\rho}$$

湍流热流率

通用形式

$$\frac{\partial \rho \bar{\phi}}{\partial t}+\frac{\partial(\rho \overline{u_j}\,\bar{\phi})}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\Gamma \frac{\partial \bar{\phi}}{\partial x_j}-\rho \overline{u'_j \phi'}\right)+\overline{S_\phi}$$

- 去掉上标横线：

$$\frac{\partial \rho \phi}{\partial t}+\frac{\partial(\rho u_j \phi)}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\Gamma \frac{\partial \phi}{\partial x_j}-\rho \overline{u'_j \phi'}\right)+S_\phi$$

Reynolds 应力

$$(-\rho \overline{u'_i u'_j})$$

湍流热流率

$$(-\rho \overline{u'_j T'})$$

湍流时均方程封闭性问题

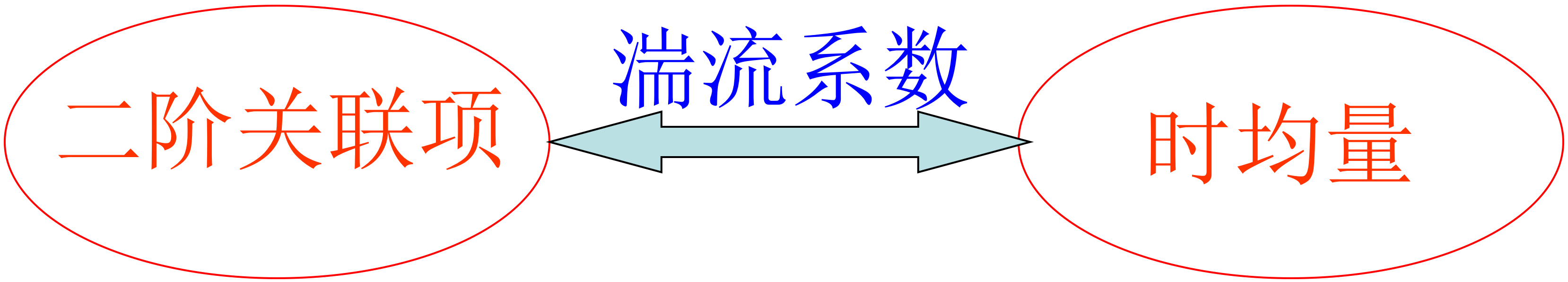
- 脉动关联项的出现，使未知数个数多于方程数，湍流控制方程组不封闭
- 需要找出确定这些附加项的关系式，并不再引入新的未知变量
- 湍流模型：使湍流方程组封闭的模型。常是一些把脉动关联值附加项与时均值联系起来的特定关系式

湍流模型

- 湍流系数（涡粘系数）法
  (1) 零方程；(2)单方程；(3) 双方程

- Reynolds应力方程法
  (1) Reynolds应力方程模型 （七方程）
  (2) 代数应力方程模型

湍流系数（涡粘系数）法



(1) 类比引入湍流粘性系数（2维）

- 分子热运动：

层流粘性系数

$$\tau_L = \mu \frac{\partial \bar{u}}{\partial y}$$

- 湍流涡旋的脉动：

$$-\rho \overline{u'v'} = \mu_t \frac{\partial \bar{u}}{\partial y}$$

涡粘系数

- 零方程模型：混合长度模型

(1) 类比引入湍流粘性系数（3维）

- 分子热运动：

层流粘性系数

$$\tau_{i,j} = -p\delta_{i,j} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu\delta_{i,j}\frac{\partial u_k}{\partial x_k}$$

- 湍流涡旋的脉动：

$$\left(\tau_{i,j}\right)_t = -\rho\overline{u'_i u'_j} = -p_t\delta_{i,j} + \mu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu_t\delta_{i,j}\frac{\partial u_k}{\partial x_k}$$

湍流压力

$$p_t = \frac{1}{3}\rho(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}) = \frac{2}{3}\rho k$$

涡粘系数：非物性参数，而是空间坐标的函数，取决于流动状态

湍动能 $k = \overline{u'_i u'_i}/2 = (\overline{u'^2} + \overline{v'^2} + \overline{w'^2})/2$

(2) 引入湍流扩散系数

- 脉动关联项与时均量：

湍流扩散系数

$$-\rho\overline{u'_j\phi'} = \Gamma_t \frac{\partial \phi}{\partial x_j}$$

- 广义Prandtl数：

$$\sigma = \mu_t / \Gamma_t \approx const$$

湍流粘性系数

湍流扩散系数

(3) 时均值形式的通用对流扩散方程

广义扩散系数 $\Gamma_{eff} = \mu_{eff} = \mu + \mu_t$

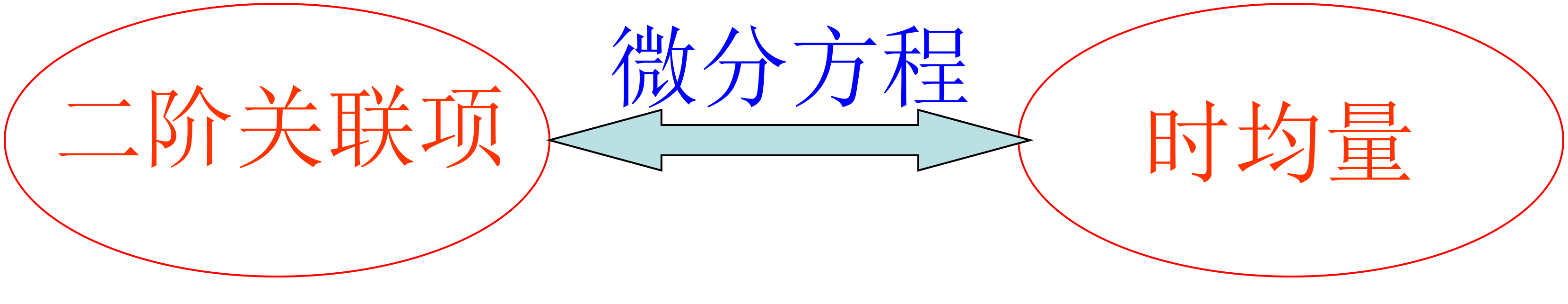
$$\frac{\partial \rho\phi}{\partial t} + \frac{\partial(\rho u_j \phi)}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\Gamma_{eff}\frac{\partial \phi}{\partial x_j}\right) + S_\phi$$

广义源项

$$S_{u_i} = -\frac{\partial p_{eff}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu_{eff}\frac{\partial u_j}{\partial x_i}\right)$$

$$p_{eff} = p + p_t = p + \frac{2}{3}\rho k$$

Reynolds应力方程法（RSM）



(1) Reynolds应力方程模型（七方程）

(2) 代数应力方程模型