A new mechanism for metastability of under-saturated traffic responsible for time-delayed traffic breakdown at the signal

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In this paper we introduce a new mechanism of metastability of under-saturated traffic at the signal that is responsible for a time-delayed traffic breakdown revealed by Kerner (2011). In our model, we assume that the metastability of under-saturated traffic at the signal is caused by a dependence of the mean time of driver acceleration from a queue at the signal on the driver’s stopped time within the queue. With the use of Nagel–Schreckenberg model, we demonstrate that this mechanism of the metastability of city traffic can lead to the time-delayed traffic breakdown at the signal.

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Vehicular flow is a many-particle system far from equilibrium and it exhibits diverse interesting non-equilibrium features. For instance, capacity drop phenomenon is widely observed, which is believed to be related to first-order phase transition from free flow to congested flow. Phantom jam is another interesting phenomenon observed in real traffic [1–7]. However, there is controversy about whether phantom jam is usually formed in a sequence free flow → synchronized flow → jam (F→S→J transitions) (as stated in Kerner’s three-phase theory [9]) or from free flow directly (F→J transition) (as stated in two-phase traffic flow theories [1–4, 8, 11–15]). The mechanism of wide scattering of statistical data in congested flow in the flow rate density plane is not clear, either [8–13].

To explain the features of vehicular flow, many traffic flow models have been proposed, which can be roughly classified into three types. In Lighthill–Whitham–Richards model, the traffic flow is always stable [14,15]. It is believed that complex traffic flow phenomena are induced by external factors (e.g., traffic bottlenecks, lane changing behaviors) [8]. On the other hand, the two-phase traffic theory (see [1,2,11–13] and references therein) and Kerner’s three-phase traffic theory [9] assume that traffic flow is not always stable, and the complex traffic phenomena are related to traffic instability. Nevertheless, there are several heated debates of the two theories about whether a unique relationship between flow rate and density exists or a two-dimensional region exists in the steady state, and traffic breakdown is associated with F→S transition [9] or F→J transition [1,2,11–13]. The latter point is the most important one. This is because if the F→S transition determines traffic breakdown at highway bottlenecks as stated in Kerner’s three-phase theory [9], then the reliable application of all classical methods for traffic control and optimization in transportation networks is a very questionable one as explained in a recent review [10].

The abovementioned controversy about traffic flow mainly arises in uninterrupted flow observed on highways, freeways or expressways. In city traffic, the Biham–Middleton–Levin model is the first city traffic model in the community of traffic physics [16]. Although the model is extremely simple, it displays some interesting behaviors, e.g., self-organization of alternating stripes in free flow. Since its proposal, many extensive studies have been carried out based on the model (see e.g. Refs. [17–19]).

Since vehicles have to stop and start frequently at the traffic lights, it is generally believed that whether to consider instabilities or not is not important. However, recently, Kerner showed that, in undersaturated traffic, spontaneous traffic breakdown can occur at the light signal after a random time delay [20]. It has been argued that this traffic breakdown is initiated by a phase transition from free flow to synchronized flow (in the three-phase traffic model) or by a phase transition from free flow to jam (in the two-phase model) occurring upstream of the queue. Kerner further studied the green-wave breakdown at traffic signal [21–23]. He proposed

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that the under-saturated traffic is metastable when
\[ C_{\text{min}} < \bar{q}_{\text{in}} < C_{\text{max}} \]  
and traffic breakdown could occur in the metastable traffic with a probabilistic nature. Here, \( \bar{q}_{\text{in}} \) is the average arrival flow rate at the signal in under-saturated traffic, \( C_{\text{min}} \) is a minimum signal capacity that is equal to the classical signal capacity, \( C_{\text{max}} \) is the maximum capacity. Kermer has proposed several mechanisms for the metastability of under-saturated traffic such as (i) the arrival flow rate can be considerably larger than the saturation flow rate, (ii) the effect of the finite length of upstream queue front [21–23].

In addition to mechanisms of the metastability of under-saturated traffic assumed by Kermer, in this paper we present a new mechanism assuming that the metastability of under-saturated traffic is caused by a dependence of the mean time of driver acceleration from a queue at the signal on the driver's stopped time within the queue. To clarify our argument, we first present a queue analysis.

Fig. 1(a) shows the situation in undersaturated traffic, in which the queue will be dissipated in the green light period. The number of vehicles passing during the green light period is
\[ N_1 = (T_g - T^* - T'')q_{\text{out}} + T''q_{\text{out}}' + Tq_{\text{in}}. \]

Here \( T_g \) is the green light period, \( T'' \) denotes the time period that the incoming vehicles have to decelerate but do not need to come to a complete stop, \( T' \) denotes the time period in which the incoming vehicles do not need to decelerate. Therefore, \( T_g - T' - T'' \)
denotes the dissolution time of the queue, as shown in Fig. 1(a). \( q_{\text{in}} \) is the inflow rate which is assumed to be constant and time independent as in Ref. [20], \( q_{\text{out}} \) is the average discharge flow rate of the dissipated queue, \( q_{\text{out}}' \) is the flow rate in the period \( T'' \). Fig. 1(b) shows the situation in oversaturated traffic. The number of vehicles passing during the green light period is
\[ N_2 = T_g q_{\text{out}}'. \]

Here \( q_{\text{out}}' \) is the average discharge flow rate of the queue, which does not dissipate. Under-saturated traffic is metastable with respect to traffic breakdown when the following condition is satisfied:
\[ N_1 > N_2. \]

Note that \( N_1 \) corresponds to \( q_{\text{in}}(T_g + T_f) \) and \( N_2 \) corresponds to \( C_{\text{min}}(T_g + T_f) \) in Eq. (1). Here \( T_r \) is the red light period.

In undersaturated traffic, the two speeds \( s_1 \) and \( s_2 \) can be calculated:
\[ s_1 = \frac{q_{\text{in}}}{\bar{v}_{\text{in}} - \rho_{\text{jam}}}, \]
\[ s_2 = \frac{q_{\text{out}}}{\bar{v}_{\text{out}} - \rho_{\text{jam}}}. \]

Here, \( \bar{v}_{\text{in}} \) and \( \bar{v}_{\text{out}} \) are average inflow velocity and discharge flow velocity, respectively. \( \rho_{\text{jam}} \) is jam density in the queue. Assume that both inflow and discharge flow are free flow, one has
\[ \bar{v}_{\text{in}} = \bar{v}_{\text{out}} = \bar{v}_{\text{free}}. \]

Moreover, since traffic is undersaturated, one has
\[ s_1 < s_2. \]

Substituting (7) and (8) into (5) and (6) yields
\[ q_{\text{out}} > q_{\text{in}}. \]

Substituting (9) into (2), one has
\[ N_1 < (T_g - T^*)q_{\text{out}} + T''q_{\text{out}}'. \]

- If \( q_{\text{out}}' > q_{\text{out}} \), then we define the generalized discharged flow rate as
  \[ q_{\text{out},g} = \frac{(T_g - T' - T'')q_{\text{out}} + T''q_{\text{out}}'}{T_g - T'}. \]

  Thus Eq. (2) becomes
  \[ N_1 = (T_g - T'')q_{\text{out},g} + T'q_{\text{in}}. \]

  Since \( q_{\text{out}}' > q_{\text{out}} > q_{\text{in}} \), one has
  \[ N_1 < T_g q_{\text{out},g}. \]

  When \( q_{\text{out}}' > q_{\text{out},g} \), \( N_1 \) will be always larger than \( N_1 \), thus spontaneous traffic breakdown will never happen. On the other hand, when \( q_{\text{out}}' < q_{\text{out},g} \), \( N_2 \) might be smaller than \( N_1 \), and spontaneous traffic breakdown might occur.

- If \( q_{\text{out}}' < q_{\text{out}} \), then Eq. (10) leads to
  \[ N_1 < T_g q_{\text{out}}. \]

Similarly, when \( q_{\text{out}}' > q_{\text{out}} \), \( N_2 \) will be always larger than \( N_1 \), thus spontaneous traffic breakdown will never happen. On the other hand, when \( q_{\text{out}}' < q_{\text{out}} \), \( N_2 \) might be smaller than \( N_1 \), and spontaneous traffic breakdown might occur.

To demonstrate this issue, next we perform traffic simulation by using a modified Nagel–Schreckenberg model with a stopped-time-dependent randomization [24,25]. Specifically, when a vehicle is moving, its randomization probability is set to \( p_r = 0.01 \). In contrast, for a stopped vehicle, the randomization probability depends on its stopped time \( T_{\text{stop}} \). In our simulations, two types of
functions are adopted:

- An increasing one $p_1 = \min(0.5, 0.1 + 0.01 T_{\text{stop}})$;
- A decreasing one $p_2 = \max(0.1, 0.5 - 0.01 T_{\text{stop}})$.

In our simulation, the maximum velocity is set to $v_{\text{free}} = 3$, and each time step corresponds to 1 s. The system size is set to $L = 1000$, a traffic light is in the middle of the system with traffic light period $T = T_g + T_r$. Initially, the system is set to empty.

Fig. 2(a) shows a typical spatiotemporal pattern of traffic flow. The inflow rate $q_{\text{in}} = 900$ vehicles/h, the traffic light period $T = 80$ and the green light period $T_g = 39$. Here the increasing randomization function $p_1$ is used. As a result, one can expect that $q_{\text{out}}'$ is notably larger than $q_{\text{out}}$ and/or $q_{\text{out},g}$. The spontaneous traffic breakdown is observed at $t \approx 1000$. Before the breakdown, traffic flow is undersaturated and after the breakdown, traffic flow transits into oversaturated traffic. Fig. 3(a) shows the number of vehicles passing in each traffic light cycle. In the undersaturated traffic, the fluctuation is weak and the average number is 20, which corresponds to the flow rate 900 vehicles/h. In the oversaturated traffic, the number fluctuates very strongly and its average value decreases to 17.5.

We have performed the simulations many times and it is found that the transition to oversaturated traffic occurs after a random delay, as pointed out by Kerner [20]. Figs. 2(b) and 3(b) show another realization of spontaneous traffic breakdown. Note that Figs. 2 and 3 are qualitatively the same as those found by Kerner with the use of other mechanisms for metastability of undersaturated traffic, see e.g., Figs. 8 and 19(d) in Ref. [23].

On the other hand, when the decreasing randomization function $p_2$ is used, the traffic breakdown cannot be observed because $q_{\text{out}}'$ is notably smaller than $q_{\text{out}}$ and/or $q_{\text{out},g}$. Fig. 4 shows the average number of vehicles discharged in one traffic signal cycle versus the signal green time. The cycle period $T = 80$, the inflow rate $q_{\text{in}} = 900$ vehicles/h.

$N_{\text{discharge}}$ is a constant, which means that the traffic flow is undersaturated. Fig. 5(a) shows a typical spatiotemporal pattern of the undersaturated traffic, in which the queue length does not increase with time. However, when $T_g < 29$, the traffic flow transits into oversaturated traffic and $N_{\text{discharge}}$ begins to decrease with the decrease of $T_g$. Fig. 5(b) shows a typical spatiotemporal pattern of the oversaturated traffic, in which the queue length increases with

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Fig. 2. (Color online) Two different realizations of spontaneous traffic breakdown. The parameters are the same in the two cases.

Fig. 3. Number of vehicles passing traffic light in each traffic light cycle. (a) corresponds to Fig. 2(a), (b) corresponds to Fig. 2(b).

Fig. 4. The average number of vehicles discharged in one traffic signal cycle versus the signal green time. The cycle period $T = 80$, the inflow rate $q_{\text{in}} = 900$ vehicles/h.
Fig. 5. Typical spatiotemporal pattern in (a) undersaturated traffic, (b) oversaturated traffic. $T_g = (a) 29, (b) 28$.

time. This is exactly the same as in classical traffic theory at traffic signal.

To summarize, we have introduced a new mechanism of metastability of under-saturated traffic at the signal that is responsible for a time-delayed traffic breakdown, i.e., a dependence of the mean time of driver acceleration from a queue at the signal on the driver’s stopped time within the queue. With the use of Nagel–Schreckenberg model with a stopped-time-dependent-randomization, we have demonstrated that this mechanism of the metastability of city traffic can lead to the time-delayed traffic breakdown at the signal. However, since up to now the traffic breakdown is observed only in simulations, in the future work, empirical or experimental data need to be collected to validate or invalidate the existence of the spontaneous traffic breakdown phenomenon at traffic signal.

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