Network operation reliability in a Manhattan-like urban system with adaptive traffic lights
Da-Cheng Ao, Rui Jiang, Mao-Bin Hu, Zi-You Gao, and Bin Jia

Abstract—Traffic breakdown to global gridlock occurring in congested traffic network makes the serious traffic congestion even much worse. This paper has proposed to use Network Operation Reliability (NOR) to quantitatively depict the probabilistic feature of traffic breakdown to global gridlock. The Nagel-Schreckenberg cellular automaton model has been used to simulate the traffic flow in a Manhattan-like urban network. A simple adaptive traffic light strategy has been proposed. It has been shown that, by choosing proper parameters, the adaptive traffic signals are able to remarkably enhance the NOR and sometimes the average velocity and the arrival rate as well. The vehicle distribution has been investigated, which has heuristically explained the enhancement of the NOR.

Index Terms—Network operation reliability, adaptive traffic light, traffic breakdown, gridlock.

I. INTRODUCTION

Traffic congestion in transportation network is serious almost all over the world. A series of proposals to lessen congestion have been presented, such as congestion pricing [1,2] that has been adopted in Singapore and London, the travel restriction based on license plate tail number as adopted in Beijing, advanced traveller information system (ATIS) like variable message sign (VMS) board [3,4], the variable speed limit, the ramp metering and so on [5-7].

In network traffic flow, recently it has been found empirically that the average flow and average density are related by a unique, reproducible curve, which is named as Macroscopic Fundamental Diagram (MFD) [8]. Utilizing the MFD, optimal control methodologies such as perimeter and boundary flow control strategies for multi-reservoir networks [9,10] and cooperative traffic control of a mixed network with two urban regions and a freeway have been studied [11].

In urban traffic networks, the traffic signals control the flows of vehicles at signalized intersections. Therefore, the optimization of traffic signal plans is one major target of intelligent transportation system research. The traffic signal systems can be classified into two types: static systems and dynamic ones. In static systems, the pre-timed signal might have been optimized via off-line optimization approaches, using historically measured data. However, the systems are problematic since the historical data do not accurately reflect the current traffic conditions.

In the dynamic systems, the traffic signal timing reacts adaptively to real-time traffic conditions based on a responsive technique in which real-time measured data is used. As a result, it is expected that the adaptive traffic signals are able to enhance the performance of traffic networks.

Researchers have proposed many adaptive traffic signal strategies and tested them at isolated intersection and traffic networks [12-20]. This paper studies the effect of adaptive traffic signal on the performance of traffic network from a different perspective. It has been shown [21,22] that when the network density is large, traffic breakdown to global gridlock with zero flow will occur. Since the traffic flow has a probabilistic nature to breakdown into global gridlock, we propose an index “network operation reliability” (NOR) to quantitatively describe this kind of performance of traffic network. Our studies show that the adaptive traffic signal is able to significantly enhance the NOR.

The paper is organized as follows. In the next section, the simulation model is introduced. In section 3, we have presented the simulation results to show the probabilistic nature of traffic network’s breakdown into global gridlock. The effect of adaptive traffic signal on the NOR is discussed in section 4. Finally, conclusion and future work are given in section 5.

II. MODEL

We study traffic flow in urban traffic network in which all the intersections are signalized. We use the Nagel-Schreckenberg cellular automaton model [23] to simulate the motion of vehicles. For simplicity, we consider a Manhattan-like urban system as shown in Fig.1(a). The network consists of $N \times N$ square lattice. Any two successive intersections are connected by two lanes, one for each direction, which are both divided into cells. Vehicle driving is restricted to the right lane. In each traffic phase, the traffic lights stay green for one ingoing street and red for the other ingoing streets (yellow light is not considered). When the green light is on, vehicles on the corresponding ingoing street can go straight ahead, turn left or right, or make a U-turn.

Vehicles, except for the leading ones in each lane, travel in the system following the Nagel-Schreckenberg rules as follows:
a vehicle reaches its destination, a new destination is chosen randomly from the system.

In some previous simulations, to mimic the origin-destination behavior, it was assumed that each vehicle makes a random choice about which link it wants to turn into at the intersection \[14,21,24\]. This paper considers the route choice behavior of drivers. However, for simplicity, it assumes that vehicles move along the physically shortest path. Since there is usually more than one shortest path between an origin and destination pair (e.g., from A to B as shown in Fig.1(a)), we assume that at the intersections, there is ATIS that provides information of the mean velocity of each street to the drivers, and the drivers would choose an outgoing street with a larger mean velocity. As shown in Li et al. [25], the ATIS could improve the performance of traffic network.

Now we present the adaptive traffic light strategy. The traffic light period at all intersections is set as \( T \), and the traffic lights are switched in a clockwise manner. For each ingoing street, green light period consists of a static period and a dynamic one.

1. Static green light time \( T_{\text{static}} \), which denotes the minimum green time, is the same for all ingoing streets;
2. Dynamic green light time \( T_{\text{dynamic}} \) is set to be proportional to the vehicle density on the street

\[
T_{\text{dynamic},i} = \frac{\rho_i^2}{\sum_{i=1}^{m} \rho_i^2} \times (T - mT_{\text{static}}) \quad (1)
\]

Here \( m \) is the number of ingoing streets at the intersections. \( m = 4 \) for most intersections that are not on the boundary. For the intersections on the boundary, \( m = 3 \). For the intersections on the four corners, \( m = 2 \). \( \rho_i \) is instantaneous vehicle density on the ingoing street \( i \) of the intersection, obtained at time step \( t = 0, T, 2T \ldots \). \( \alpha \) is a tunable parameter. Note that in the special case \( T_{\text{static}} = T/m \), one has \( T_{\text{dynamic}} = 0 \). When \( \alpha = 0 \), \( T_{\text{dynamic}} = T - mT_{\text{static}} \) is fixed. In the two cases, the adaptive traffic light strategy reduces to the fixed strategy as used in Li et al. [25]. Since the green time should be an integer, we choose \( m - 1 \) \( T_{\text{dynamic}} \) randomly at each intersection and roundoff their non-integer part, and let the last \( T_{\text{dynamic}} \) equal to the difference between \( T - mT_{\text{static}} \) and the sum of the \( m - 1 \) \( T_{\text{dynamic}} \).

The dynamic traffic light formula in Eq. (1) is similar to Webster’s formula \[26\]. The difference is that Eq. (1) is based on densities while Webster’s formula is based on saturation levels. However, Webster’s formula is “optimal” in some sense only for isolated junctions and free flow conditions. Consideration for Webster’s formula is questionable for saturated networks.

III. NETWORK OPERATION RELIABILITY

This section presents and discusses the evolution of the average velocity of the system. In the simulation, the system size is set as \( N \times N = 24 \times 24 \) and the length of the street is set as \( L = 100 \). The maximum velocity \( v_{\text{max}} = 3 \) and the
randomization probability \( p = 0.1 \). The traffic light period is set as \( T = 80 \) unless otherwise mentioned.

Fig.2(a) shows a typical evolution of the average velocity of the system at low global density. One can see that the average velocity becomes stationary after a transient time. Moreover, the evolution process is almost the same under different runs, in which different random seeds are used and other configurations are the same.

However, with the increase of the density, the situation becomes different, see Fig.2(b). The average velocity firstly becomes stationary after a transient time as before. Then it begins to drop. This is because the entrance sections of some streets have been jammed. As a result, vehicles cannot enter these streets and thus congestion spills back and gradually propagates to the whole network. If the congestion propagation has led to global gridlock, in which a loop jam structure has formed as shown in Fig.1(b), then the average velocity will finally decrease to zero.

Fig.2(c) shows the evolution of the average velocity at the same global density under several different runs. One can see that maintain time of the stationary state is different. This means that the occurrence of congestion propagation has a probabilistic nature. Nevertheless, even if the average velocity drops due to the congestion propagation, this does not necessarily mean that global gridlock will occur. Fig. 2(d) shows two typical examples that the average velocity recovers after it drops to a very small value, in which the global gridlock has not formed. Finally, when the density is large enough, the traffic breakdown occurs so quickly that the stationary state cannot be observed, see Fig.2(e).

To quantitatively depict this probabilistic nature of global gridlock formation, we define the network operation reliability (NOR) as follows. At given time interval \( T_{\text{interval}} \), the NOR is determined by \( \text{NOR} = 1 - P \). Here \( P \) is the probability that global gridlock has formed, which is calculated by \( P = n_p/n \). Here \( n \) is the total number of runs, and \( n_p \) is the number of runs in which the average velocity becomes zero, which definitely implies that the global gridlock has occurred.

IV. Effect of adaptive traffic signal

In the adaptive traffic signal strategy (1), there are three parameters \( T_{\text{static}}, \alpha \) and \( T \). Moreover, in the calculation of NOR, there is another parameter \( T_{\text{interval}} \). This section studies the effect of the four parameters on the NOR.

A. Effect of \( T_{\text{static}} \)

Fig.3 shows the dependence of NOR on the global density under different values of \( T_{\text{static}} \). The parameters \( \alpha = 1, T_{\text{interval}} = 10^5 \).

Fig. 3. NOR versus the global density under different values of \( T_{\text{static}} \). The parameters \( \alpha = 1, T_{\text{interval}} = 10^5 \).

To understand the change of NOR under adaptive traffic signal, we investigate the vehicle distribution. Fig. 4
Fig. 4. The density distribution of vehicles in the stationary state under different values of $T_{\text{static}}$. The parameter (a) $T_{\text{static}} = 20$, (b) $T_{\text{static}} = 15$, (c) $T_{\text{static}} = 10$, (d) $T_{\text{static}} = 5$. Other parameters $\alpha = 1$, $\rho = 0.22$.

shows the distribution of density on each street in the stationary state under different values of $T_{\text{static}}$. In the fixed strategy (Fig.4(a)), one can see that surprisingly, the density in the center area is not large. The large densities appear on the four corners. With the decrease of $T_{\text{static}}$, on the one hand, the vehicles gradually tend to assemble in the center area. On the other hand, the maximum value of local density decreases. Let $\rho_{\text{max},k}$ denote the maximum value of the density on the streets in the direct $k \in \{\text{eastbound, westbound, northbound, southbound}\}$. Fig.5(a) shows $\rho_{\text{max}}$, the average value of $\rho_{\text{max},k}$ over four directions, which decreases as $T_{\text{static}}$ decreases. This means that decrease of the maximum value of local density leads to the enhancement of NOR, which is because the initialization of congestion spillover and propagation become difficult when the maximum local density decreases.

Fig.6(a) and fig.7(a) show the effect of $T_{\text{static}}$ on the average velocity $\bar{v}$ and the average arrival rate $\bar{q}$ in the stationary state. The latter is defined as $\bar{q} = N_{\text{arr}}/N_{\text{total}}$. Here $N_{\text{arr}}$ denotes the number of vehicles that arrive their destinations per time step and $N_{\text{total}}$ denotes the total number of vehicles. One can see that $\bar{v}$ and $\bar{q}$ behave very similarly. At low global densities, decreasing $T_{\text{static}}$ is able to improve $\bar{v}$ and $\bar{q}$. However, with the increase of global density, decreasing $T_{\text{static}}$ has essentially not improved them.

maximum NOR could be achieved at intermediates value of $\alpha$. Then the NOR begins to decrease with the further increase of $\alpha$. Figs. 6(b) and 7(b) shows that the average velocity and the average arrival rate could also be enhanced at intermediate values of $\alpha$.

Fig.9 shows the density distribution of vehicles under different values of $\alpha$. One can see that when $\alpha$ increases from small value, the density distribution also shrinks toward the center area and $\rho_{\text{max}}$ also decreases, see Fig.5(b). Therefore, the NOR increases. However, when $\alpha$ further increases, $\rho_{\text{max}}$ increases. Thus, the NOR decreases.

B. Effect of $\alpha$

Next we investigate the effect of parameter $\alpha$. Fig.8 shows the dependence of NOR on the parameter $\alpha$. One can see that the NOR firstly increases with the increase of $\alpha$. The

C. Effect of $T$

Now we study the effect of traffic light period $T$. As shown in Fig.10, the NOR is very sensitive to $T$ and exhibits a
Fig. 7. Average arrival rate $\bar{\eta}$ in the stationary state. In (a) and (b), traffic light cycle $T = 80$. In (a) and (c), the parameter $\alpha = 1$. In (b) and (c), the parameter $T_{\text{static}} = 5$.

Fig. 8. NOR versus parameter $\alpha$. The static green time $T_{\text{static}} = 5$.

complicated changing behavior. Two maximum values have observed at $T \approx 77$ and 95, and one minimum value has been observed at $T \approx 85$. Nevertheless, Figs.6(c) and fig.7(c) shows that the average velocity and the average arrival rate are almost independent of $T$.

Fig.11 shows the density distribution of vehicles under different values of $T$, which essentially neither shrinks nor expands. The high density region at the four corners expand with the increase of $T$. Fig.5(c) shows that there is a maximum value of $\hat{\rho}_{\text{max}}$ at $T \approx 85$, which leads to the minimum value of NOR.

To explore the origin of the two maximum value of NOR, we examine the average density $\hat{\rho}$ in the center area with size $5 \times 5$. Fig.5(d) shows that there is a minimum value of $\hat{\rho}$ at $T \approx 77$, which leads to one maximum value of NOR. Moreover, when $T$ is in the range $85 < T < 95$, $\hat{\rho}$ is almost a constant but $\hat{\rho}_{\text{max}}$ decreases. Thus, NOR increases in this range and reaches another maximum value of NOR at $T \approx 95$.

Fig. 9. The density distribution of vehicles in the stationary state under different values of $\alpha$. The parameter (a) $\alpha = 1$, (b) $\alpha = 2$, (c) $\alpha = 3$, (d) $\alpha = 5$. Other parameters $T_{\text{static}} = 5$, $\rho = 0.30$.

Fig. 10. Dependence of NOR on the traffic light cycle $T$. The parameters $T_{\text{static}} = 5$, $\alpha = 1$, $\rho = 0.297$.

D. Effect of $T_{\text{interval}}$

Finally, we investigate the effect of $T_{\text{interval}}$ on the NOR. Fig.12 shows that the NOR is independent of $T_{\text{interval}}$ when the global density is low (e.g. $\rho = 0.26$). When the global density increases (e.g. $\rho = 0.31$ and 0.36), the NOR equals to 1 when $T_{\text{interval}}$ is small. It gradually decreases to zero with the increase of $T_{\text{interval}}$. Nevertheless, the basic conclusion that the adaptive traffic signal could enhance the NOR does not change, as can be seen from Fig.13, which shows the NOR with $T_{\text{interval}} = 8 \times 10^4$, shorter than that shown in Fig.3.

We would like to mention that it would be better if we could obtain the distribution of the network traffic breakdown time. However, since the traffic breakdown takes very long
time, especially when NOR is close to or equals 1, to obtain the distribution is very time consuming. Therefore, we propose to use NOR instead of simulating the distribution of traffic breakdown time.

V. CONCLUSION AND FUTURE WORK

Nowadays, traffic congestion in transportation network is serious and widely observed almost in all large cities. When traffic breakdown to global gridlock occurs in congested traffic network, the network completely collapses. This makes the serious traffic congestion even much worse.

To quantitatively depict the probabilistic nature of traffic breakdown to global gridlock in traffic network, this paper has proposed the index of NOR. We have used the Nagel-Schreckenberg cellular automaton model to simulate the traffic flow in a Manhattan-like urban network. A simple adaptive traffic signal strategy has been proposed. It has been shown that the adaptive traffic signal is able to remarkably enhance the NOR and sometimes the average velocity and the arrival rate as well. The vehicle distribution has been investigated and it is found that the enhancement of NOR is due to a decrease of the maximum value of local density and sometimes depends on the average density in the center area.

Although our model is very simple, it has captured the most important issues in network traffic, i.e., the route choice behavior, the spillover and propagation of traffic congestion, and the formation of gridlock. The simple model in this paper can be extended in several directions in future work. For instance, (i) The route choice behavior [27]. Usually drivers familiar with the traffic situation choose the time-shortest path instead of the physically shortest path. (ii) Heterogeneous traffic signal setup at different intersections. The traffic light period $T$, the static time $T_{\text{static}}$, and the parameter $\alpha$ could be different among the intersections; (iii) Traffic light strategy. We need to study other adaptive traffic signal strategies and check whether they are better than the one proposed in the present paper, or whether better strategy can be designed; (iv) Network structure: a more realistic road network and multilane road sections need to be considered; (v) More realistic origin-destination data need to be deployed; (vi) To consider classifying the system into multi-reservoirs, and to investigate the perimeter and boundary flow control strategy [9,27-29].
REFERENCES


Da-Cheng Ao received the B.E. degree from The University of Science and Technology of China, Hefei, China, in 2011. He is currently a graduate student with the School of Engineering Science, University of Science and Technology of China, Hefei, China, working in the fields of traffic flow theory and intelligent transportation systems.

Rui Jiang received the B.E. degree and the Ph.D. degree from The University of Science and Technology of China, Hefei, China, in 1998 and 2003, respectively. He was an Alexander von Humboldt research fellow from 2005 to 2006, and a Japanese Society for Promotion of Science (JSPS) research fellow from 2008 to 2009. He is currently a Professor with the School of Engineering Science, University of Science and Technology of China, Hefei, China, working in the fields of traffic flow theory and intelligent transportation systems.

Mao-Bin Hu received the B.E. degree and the Ph.D. degree from The University of Science and Technology of China, Hefei, China, in 2000 and 2005, respectively. He is currently an associate Professor with the School of Engineering Science, University of Science and Technology of China, Hefei, China, working in the fields of traffic flow theory and intelligent transportation systems.

Zi-You Gao received the Ph.D. degree in operations research and control theory from the Institute of Applied Mathematics of Academia Sinica, Beijing, China, in 1994. He is currently a Professor with the School of Traffic and Transportation, Beijing Jiaotong University. His current research interests include the modeling of transportation network systems, network design problems, traffic flow theory, and dynamic traffic assignment.

Bin Jia received the Ph.D. degree from The University of Science and Technology of China, Hefei, China, in 2003. He is currently a Professor with the School of Traffic and Transportation, Beijing Jiaotong University. His current research interests include traffic flow theory and intelligent transportation systems.