Totally asymmetric exclusion processes on two intersected lattices with open and periodic boundaries

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Abstract. In this paper, we study TASEPs on two intersected lattices. One TASEP has an open boundary (lattice 1) and the other has a periodic boundary (lattice 2). Extensive Monte Carlo simulations are carried out. It is found that the system phase diagram structure depends on the density $\rho_2$ on lattice 2. Three critical densities $\rho_c = 1/3, 1/2$ and $2/3$ are identified and four phase diagram structures are observed. The density profiles corresponding to all stationary phases and the phase boundaries are calculated using mean-field analysis. The analytic results are in good agreement with the simulation results.

Keywords: phase diagrams (theory), driven diffusive systems (theory), traffic models

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1. Introduction

Since the inception of using asymmetric simple exclusion processes (ASEPs) to describe the kinetics of biopolymerization in 1968 [1], ASEPs have attracted great interest from physicists, biologists and chemists [2]–[10]. Despite its simple rules, an ASEP can reproduce rich non-equilibrium phenomena, such as the boundary-induced phase transition [11], shock formation [12], spontaneous symmetry breaking [13], etc.

The exact solutions for the totally asymmetric simple exclusion process (TASEP), which is the simplest limit of an ASEP describing particles moving in one direction, exist for both open [14,15] and periodic [16,17] boundaries under random sequential update rules. In the case with open boundaries, three stationary phases exist. We denote the injection and extraction rates at the entrance and exit by $\alpha$ and $\beta$, respectively. For $\alpha < \beta$ and $\alpha < 0.5$, the system is in low-density (LD) entry limited phase with the particle current and bulk density

$$J_{LD} = \alpha(1 - \alpha) \quad \text{and} \quad \rho_{bulk,LD} = \alpha.$$ (1)

For $\alpha > \beta$ and $\beta < 0.5$, the system is in the high-density (HD) exit limited phase with the particle current and bulk density

$$J_{HD} = \beta(1 - \beta) \quad \text{and} \quad \rho_{bulk,HD} = \beta.$$ (2)

And for $\alpha > 0.5$ and $\beta > 0.5$, we have a maximal current (MC) phase with

$$J_{MC} = 0.25 \quad \text{and} \quad \rho_{bulk,MC} = 0.5.$$ (3)

Denote the density of the lattice by $\rho$; the exact solutions for the TASEP with a periodic boundary can be written as

$$J = \rho(1 - \rho).$$ (4)

In the real world, ranging from vehicle traffic to intracellular transport, it is very common to find intersections between two roads or microtubules. Therefore, it is necessary to extend the single-lane TASEP to two intersected lanes. In the literature, some work has been done [18]–[22]. For example, Ishibashi and Fukui studied traffic flow on two one-dimensional roads with a crossing under a periodic boundary condition and with parallel update rules, taking different velocities and real time information into account [18].
Foulaadvand et al also studied a similar issue, and they considered traffic light and yielding behavior [19] or single-occupancy restriction at the intersection [20]. The authors studied TASEPs on two one-dimensional lattices with an intersection under an open boundary condition and with random update rules, and spontaneous symmetry breaking is observed [21]. Recently, Embley et al have studied TASEPs on networks [22]. For instance, the ‘figure of eight’ in their work denotes two intersected lattices with periodic boundary. They have presented generic analysis by considering intersections as explicit additional vertices.

To our knowledge, TASEPs on two intersected lattices where one lattice has an open boundary but the other has a periodic boundary have not been studied yet. This situation is often observed in vehicle traffic (i.e., a road crosses a ring road) and might also be relevant with molecular motor motion. In this paper, the issue is investigated. The lattices are sketched in figure 1(a). Two one-dimensional lattices with equal length $L$ intersect at site $c$. Lattice 1 is in the horizontal direction with an open boundary and lattice 2 is in the vertical direction with a periodic boundary. Site 1 and site $L$ on lattice 1 are the entrance and exit respectively. On lattice 2, a particle moving out from site $L$ hops into site 1.

In this paper, the random sequential update rules for TASEPs are adopted. For the initialization, lattice 1 is set empty and $\rho L$ particles are randomly distributed on lattice 2.

Figure 1. (a) Schematic picture of the two one-dimensional lattices with crossing; (b) the system is divided into three segments by the intersection.
Thus the density on lattice 2 is $\rho$. Particles move along the lattice and do not change lane at the intersection. Note that this is different from the rule in [22], in which particles at the intersection choose the outgoing segment with equal or biased probabilities. At the entrance, particles are injected with rate $\alpha$; at the exit, particles are removed with rate $\beta$. In the bulk and at the intersection, particles hop forward with rate 1 provided the target site is empty.

The phase diagrams and density profiles are given from extensive simulations and mean-field analysis. It is found that the system phase diagram structure depends on density $\rho$ on lattice 2. Three critical densities $\rho_c = 1/3, 1/2$ and $2/3$ are identified and four phase diagram structures are observed.

The paper is organized as follows. The simulation results are presented in section 2. Then, the mean-field analysis is given in section 3. Finally, we give conclusions in section 4.

2. Simulation results

In this section, the simulation results are presented and discussed. In the simulation, the system length is set to $L = 2001$. We perform $1 \times 10^9$ Monte Carlo time steps and the previous $2.5 \times 10^8$ Monte Carlo time steps are abandoned in order to obtain stationary states.

We denote the upstream sites of site $c$ in lattice 1 and lattice 2 as site $c_1 - 1$ and site $c_2 - 1$, and the downstream sites of site $c$ as site $c_1 + 1$ and site $c_2 + 1$. Thus, lattice 1 is divided into two segments: segment I is from site 1 to site $c_1 - 1$ and segment II is from site $c_1 + 1$ to site $L$ (figure 1(b)).

First, we present the phase diagrams related to $\alpha$ and $\beta$ for different values of $\rho$, as shown in figure 2. We found that the system has four different phase diagram structures, classified by three critical densities $\rho_c = 1/3, 1/2, 2/3$.

When $0 < \rho < 1/3$, the phase diagram is as shown in figure 2(a), which consists of four regions. For $0 < \alpha < \lambda_2$ and $\alpha < \beta$, the system is in the LL–LD phase, i.e., the two segments of lattice 1 are both in LD and lattice 2 is in LD as well (figure 3(a)). For $0 < \beta < \lambda_1$ and $\beta < \alpha$, the system is in the HH–LH phase, i.e., the two segments of lattice 1 are both in HD and lattice 2 is in the LH phase, and a shock is formed in lattice 2 (figure 3(b)). For $\lambda_1 < \beta < \lambda_2$ and $\beta < \alpha$, the system is in the HH–LD phase (figure 3(c)). Finally, for $\alpha, \beta > \lambda_2$, the system is in the HL–LD phase (figure 3(d)). With increase of $\rho$, $\lambda_2$ decreases and $\lambda_1$ increases. At $\rho = 1/3$, $\lambda_2$ becomes equal to $\lambda_1$, and the HH–LD region disappears.

When $\rho$ exceeds 1/3, HH–LH remains unchanged but the HL–LD region changes into the HL–LH region (figure 3(e)). Moreover, another new region, LL–LH, appears at $\lambda_3 < \alpha < \lambda_4 = 1/3$ and $\alpha < \beta$ (figure 3(f)). An LL–LD region exists at $0 < \alpha < \lambda_3$ and $\alpha < \beta$. The phase diagram is shown in figure 2(b). With increase of $\rho$, $\lambda_3$ decreases. At $\rho = 1/2, \lambda_3$ becomes equal to zero and the LL–LD region disappears.

When $\rho$ exceeds 1/2, the HH–LH region and HL–LH region do not change. However, a third new region, LL–HD, appears at $0 < \alpha < \lambda_5$ and $\alpha < \beta$ (figure 3(g)), and LL–LH exists at $\lambda_5 < \alpha < \lambda_6 = 1/3$ and $\alpha < \beta$. The phase diagram is shown in figure 2(c). With increase of $\rho$, $\lambda_5$ increases and it becomes equal to $\lambda_6$ at $\rho = 2/3$, and the LL–LH region disappears.

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Figure 2. Phase diagram of the system. (a) $0 < \rho < 1/3$; (b) $1/3 \leq \rho < 1/2$; (c) $1/2 \leq \rho < 2/3$; (d) $2/3 \leq \rho < 1$. The expressions in parentheses are the values of the phase boundaries obtained by mean-field analysis. The arrows denote the moving direction of the boundaries with the increase of $\rho$.

Finally, when $\rho$ exceeds $2/3$, HL–LH changes into HL–HD (figure 3(h)) and it exists at $\alpha, \beta > \lambda_7$. The phase diagram is shown in figure 2(d). With increase of $\rho$, $\lambda_7$ decreases. As a result, HL–HD expands and LL–HD and HH–LH shrink until they disappear at $\rho = 1$.

We also would like to point out that the HL state on lattice 1 means that the intersection behaves as a bottleneck for lattice 1, and the LH state on lattice 2 means that the intersection behaves as a bottleneck for lattice 2. Thus, the intersection behaves as a bottleneck for both lattices only in the HL–LH region.

3. Mean-field analysis

This section carries out mean-field analysis of the model by adopting the method of effective rates [23, 24] and explicit vertices [22]. As shown in figure 1(b), the two segments of lattice 1 ($S_1$ and $S_{II}$) are regarded as two one-dimensional TASEPs after introducing the effective injection and extraction rates $\alpha_{eff1}$, $\beta_{eff1}$, $\alpha_{eff2}$ and $\beta_{eff2}$. Lattice 2 ($S_{III}$) is a one-dimensional TASEP with an open boundary (site $c_2 + 1$ as the entrance and site $c_2 − 1$ as the exit) with injection rate $\alpha_{eff3}$ and extraction rate $\beta_{eff3}$. Site $c$ is regarded as an additional explicit vertex.

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Figure 3. The corresponding density profiles: (a) $\alpha = 0.3, \beta = 0.6, \rho = 0.2$ (LL–LD); (b) $\alpha = 0.6, \beta = 0.2, \rho = 0.6$ (HH–LH); (c) $\alpha = 0.6, \beta = 0.3, \rho = 0.2$ (HH–LD); (d) $\alpha = 0.6, \beta = 0.7, \rho = 0.2$ (HL–LD); (e) $\alpha = 0.6, \beta = 0.7, \rho = 0.4$ (HL–LH); (f) $\alpha = 0.3, \beta = 0.6, \rho = 0.4$ (LL–LH); (g) $\alpha = 0.1, \beta = 0.6, \rho = 0.8$ (LL–HD); (h) $\alpha = 0.6, \beta = 0.7, \rho = 0.8$ (HL–HD). $\triangle$, $\bigcirc$ represent the simulation results for lattices 1, 2, respectively; the solid lines represent mean-field analysis results. For lattice 2, site $c$ corresponds to $x = 0$ and site $c_2 - 1$ corresponds to $x = 1$.

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Denote the density on the vertex by $\tilde{\rho}$. Since particles do not change lane at the intersection, there are two contributions to $\tilde{\rho}$: $\tilde{\rho} = \rho_1 + \rho_2$, in which $\rho_1$ is the contribution of particles on lattice 1 and $\rho_2$ is the contribution of particles on lattice 2. For simplicity, we use $\rho_1$ and $\rho_2$ instead of $\tilde{\rho}_1$ and $\tilde{\rho}_2$ in the following text.

Thus the values of the effective rates are written as follows:

\[
\begin{align*}
\alpha_{\text{eff}1} &= \alpha \\
\beta_{\text{eff}1} &= 1 - \rho_1 - \rho_2 \\
\alpha_{\text{eff}2} &= \rho_1 \\
\beta_{\text{eff}2} &= \beta \\
\alpha_{\text{eff}3} &= \rho_2 \\
\beta_{\text{eff}3} &= 1 - \rho_1 - \rho_2.
\end{align*}
\]

(5)

By analyzing the system behavior, we see that lattice 1 can be in any of three phases: (i) the LL phase; (ii) the HH phase; and (iii) the HL phase. Next, we discuss the three situations:

(i) The LL phase:
When $S_I$ and $S_{II}$ are both in the low-density phase, we have

\[
\begin{align*}
\alpha_{\text{eff}1} < \beta_{\text{eff}1} \quad \text{and} \quad \alpha_{\text{eff}1} < 1/2 \\
\alpha_{\text{eff}2} < \beta_{\text{eff}2} \quad \text{and} \quad \alpha_{\text{eff}2} < 1/2.
\end{align*}
\]

(6)

Since the current of lattice 1 is spatially constant, we have $J_{I,LD} = J_{II,LD}$, i.e.,

\[
\alpha_{\text{eff}1}(1 - \alpha_{\text{eff}1}) = \alpha_{\text{eff}2}(1 -\alpha_{\text{eff}2}).
\]

(7)

If $S_{III}$ is also in the LD phase, we have

\[
\alpha_{\text{eff}3} < \beta_{\text{eff}3} \quad \text{and} \quad \alpha_{\text{eff}3} < 1/2.
\]

(8)

Now the conservation of particles is considered on $S_{III}$; we have

\[
\rho_{\text{bulk},III} = \alpha_{\text{eff}3} = \rho.
\]

(9)

Substituting equations (7) and (9) into (5), we can obtain

\[
\begin{align*}
\alpha_{\text{eff}1} &= \alpha_{\text{eff}2} = \rho_1 = \alpha \\
\alpha_{\text{eff}3} &= \rho_2 = \rho \\
\beta_{\text{eff}1} &= \beta_{\text{eff}3} = 1 - \alpha - \rho \\
\beta_{\text{eff}2} &= \beta.
\end{align*}
\]

(10)

Thus, from equations (6), (8) and (10), we have

\[
\alpha < \beta \quad \alpha < 1 - 2\rho \quad \alpha < \frac{1 - \rho}{2}.
\]

(11)

Then, the parameters for the existence of the LL–LD phase can be written as

\[
\begin{align*}
\alpha < \beta \quad \text{and} \quad \alpha < (1 - \rho)/2; \quad (0 < \rho < 1/3); \\
\alpha < \beta \quad \text{and} \quad \alpha < 1 - 2\rho; \quad (1/3 < \rho < 1/2).
\end{align*}
\]

(12)

If $S_{III}$ is in the HD phase, we have

\[
\beta_{\text{eff}3} < \alpha_{\text{eff}3} \quad \text{and} \quad \beta_{\text{eff}3} < 1/2.
\]

(13)

Then we consider the conservation of particles on $S_{III}$; we have

\[
\rho_{\text{bulk},III} = 1 - \beta_{\text{eff}3} = \rho.
\]

(14)
Combining equations (5), (7) and (14), we can obtain
\[
\alpha_{\text{eff}1} = \alpha_{\text{eff}2} = \rho_1 = \alpha \quad \alpha_{\text{eff}3} = \rho_2 = \rho - \alpha \\
\beta_{\text{eff}1} = \beta_{\text{eff}3} = 1 - \rho \quad \beta_{\text{eff}2} = \beta
\]  
and then substituting this into (6) and (13), we can find that the system is in the LL–HD phase when
\[
\alpha < \beta \quad \text{and} \quad \alpha < 2\rho - 1, \quad (1/2 < \rho < 2/3) \\
\alpha < \beta \quad \text{and} \quad \alpha < 1 - \rho, \quad (2/3 < \rho < 1).
\]  
(16)

If \( S_{\text{III}} \) is in the LH phase, we have
\[
\alpha_{\text{eff}3} = \beta_{\text{eff}3} < 1/2.
\]  
(17)

Combining equations (5), (7) and (17), we obtain
\[
\alpha_{\text{eff}1} = \alpha_{\text{eff}2} = \rho_1 = \alpha \quad \alpha_{\text{eff}3} = \beta_{\text{eff}1} = \beta_{\text{eff}2} = \beta = \frac{1 - \alpha}{2} \\
\beta_{\text{eff}1} = \beta_{\text{eff}3} = \rho_2 = \rho - \alpha.
\]  
(18)

Then, we introduce \( x_w = i/L \) as the position of the domain wall (DW), where \( i \) is the site index and \( L \) is the length of the system. Considering the conservation of particles on \( S_{\text{III}} \), we can obtain
\[
\alpha_{\text{eff}3}x_w + (1 - \beta_{\text{eff}3})(1 - x_w) = \rho \times 1.
\]  
(19)

Now, from equations (18) and (19), we can obtain
\[
x_w = \frac{1 + \alpha - 2\rho}{2\alpha}.
\]  
(20)

Since \( S_{\text{III}} \) is in the LH phase, we have \( 0 < x_w < 1 \), i.e.,
\[
\alpha < 1 - 2\rho, \quad (\rho < 1/2); \quad \alpha < 2\rho - 1, \quad (\rho > 1/2).
\]  
(21)

Take equations (6), (17), (18) and (21) into account; we obtain the parameters for the existence of the LL–LH phase:
\[
\alpha < \beta \quad \text{and} \quad 1 - 2\rho < \alpha < 1/3, \quad (1/3 < \rho < 1/2) \\
\alpha < \beta \quad \text{and} \quad 2\rho - 1 < \alpha < 1/3, \quad (1/2 < \rho < 2/3).
\]  
(22)

(ii) The HH phase:

When \( S_1 \) and \( S_{\text{II}} \) are both in the high-density phase, we have
\[
\beta_{\text{eff}1} < \alpha_{\text{eff}1} \quad \text{and} \quad \beta_{\text{eff}1} < 1/2 \\
\beta_{\text{eff}2} < \alpha_{\text{eff}2} \quad \text{and} \quad \beta_{\text{eff}2} < 1/2.
\]  
(23)

Considering the conservation of current in lattice 1, we can obtain \( J_{1,\text{HD}} = J_{1,\text{LH}} \), i.e.,
\[
\beta_{\text{eff}1}(1 - \beta_{\text{eff}1}) = \beta_{\text{eff}2}(1 - \beta_{\text{eff}2}).
\]  
(24)

If \( S_{\text{III}} \) is in the LD phase, we combine the equations (5), (9) and (24), and obtain
\[
\alpha_{\text{eff}1} = \alpha \quad \alpha_{\text{eff}2} = \rho_1 = 1 - \beta - \rho \\
\alpha_{\text{eff}3} = \rho_2 = \rho \quad \beta_{\text{eff}1} = \beta_{\text{eff}2} = \beta.
\]  
(25)
From equations (8), (23) and (25), we can obtain the parameters for the existence of the HH–LD phase:

\[ \beta < \alpha \quad \text{and} \quad \rho < \beta < \frac{1 - \rho}{2}, \quad (0 < \rho < 1/3). \quad (26) \]

If \( S_{III} \) is in the LH phase, we combine equations (5), (17), and (24), and obtain

\[
\begin{align*}
\alpha_{\text{eff}1} &= \alpha \\
\alpha_{\text{eff}2} &= \rho_1 = 1 - 2\beta \\
\alpha_{\text{eff}3} &= \beta_{\text{eff}1} = \beta_{\text{eff}2} = \beta_{\text{eff}3} = \rho_2 = \beta.
\end{align*}
\]

(27)

Combining equations (19) and (27), we can obtain

\[ x_w = \frac{1 - \beta - \rho}{1 - 2\beta}. \quad (28) \]

Since \( 0 < x_w < 1 \), and considering equations (17), (23), (27) and (28), we obtain the parameters for the existence of the HH–LH phase:

\[
\begin{align*}
\beta &< \alpha \quad \text{and} \quad \beta < \rho, \quad (0 < \rho < 1/3) \\
\beta &< \alpha \quad \text{and} \quad \beta < 1/3, \quad (1/3 < \rho < 1/2) \\
\beta &< \alpha \quad \text{and} \quad \beta < 1 - \rho, \quad (2/3 < \rho < 1).
\end{align*}
\]

(iii) The HL phase:

When \( S_I \) is in the HD phase and \( S_{II} \) is in the LD phase, we have

\[
\begin{align*}
\beta_{\text{eff}1} &< \alpha_{\text{eff}1} \quad \text{and} \quad \beta_{\text{eff}1} < 1/2 \\
\alpha_{\text{eff}2} &< \beta_{\text{eff}2} \quad \text{and} \quad \alpha_{\text{eff}2} < 1/2.
\end{align*}
\]

(30)

Considering the conservation of currents of \( S_I \) and \( S_{II} \), we have \( J_I = J_{II} \), i.e.,

\[ \beta_{\text{eff}1}(1 - \beta_{\text{eff}1}) = \alpha_{\text{eff}2}(1 - \alpha_{\text{eff}2}). \quad (31) \]

If \( S_{III} \) is in the LD phase, we combine equations (5), (9) and (31), and obtain

\[
\begin{align*}
\alpha_{\text{eff}1} &= \alpha \\
\alpha_{\text{eff}2} &= \beta_{\text{eff}1} = \beta_{\text{eff}3} = \rho_1 = \frac{1 - \rho}{2} \\
\alpha_{\text{eff}3} &= \rho_2 = \rho = \beta_{\text{eff}2} = \beta.
\end{align*}
\]

(32)

Now, we substitute (32) into (8) and (30), and find that the system is in the HL–LD phase when

\[ \alpha, \beta > \frac{1 - \rho}{2}, \quad (0 < \rho < 1/3). \quad (33) \]

If \( S_{III} \) is in the HD phase, we consider the previous discussion and obtain

\[
\begin{align*}
\alpha_{\text{eff}1} &= \alpha \\
\alpha_{\text{eff}2} &= \beta_{\text{eff}1} = \beta_{\text{eff}3} = \rho_1 = 1 - \rho \\
\alpha_{\text{eff}3} &= \rho_2 = 2\rho - 1 = \beta_{\text{eff}2} = \beta.
\end{align*}
\]

(34)

Now, we can obtain that the system is in the HL–HD phase when

\[ \alpha, \beta > 1 - \rho, \quad (2/3 < \rho < 1). \quad (35) \]

If \( S_{III} \) is in the LH phase, we take equations (5), (17) and (31) into account, and find

\[
\begin{align*}
\alpha_{\text{eff}1} &= \alpha \\
\alpha_{\text{eff}2} &= \alpha_{\text{eff}3} = \beta_{\text{eff}1} = \beta_{\text{eff}3} = \rho_1 = \rho_2 = \frac{1}{3} \\
\beta_{\text{eff}2} &= \beta.
\end{align*}
\]

(36)

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From equations (19) and (36), we find
\[ x_w = 2 - 3\rho. \]  

(37)

Considering \( 0 < x_w < 1 \) and combining equations (17), (30) and (36), we find that the system is in the HL–LH phase when
\[ \alpha, \beta > \frac{1}{3}, \quad \left( \frac{1}{3} < \rho < \frac{2}{3} \right). \]

(38)

From the previous mean-field analysis, we can obtain the phase boundaries, as follows:
\[ \lambda_1 = \rho, \quad \lambda_2 = \frac{1 - \rho}{2}, \quad \lambda_3 = 1 - 2\rho, \]
\[ \lambda_4 = \lambda_6 = \frac{1}{3}, \quad \lambda_5 = 2\rho - 1, \quad \lambda_7 = 1 - \rho. \]

(39)

These analytic results are in good agreement with the simulation results, as shown in figures 2 and 3. Slight deviations appear in HH–LH and HL–HD, in which one lattice is in HD and the other is dominated by the intersection bottleneck. This is due to correlation near the intersection, which also results in spontaneous symmetry breaking as found in [21].

Three critical densities \( \rho_c = 1/3, 1/2, 2/3 \) are identified. The critical density \( \rho_c = 1/2 \) characterizes the maximum current phase of a single-lane TASEP. The other two critical densities \( \rho_c = 1/3 \) and \( 2/3 \) are the same as those previously reported in [21,22].4 This means that the two critical densities are universal, either under open boundary conditions or periodic boundary conditions. Moreover, this remains unchanged under the two different particle motion rules (in this paper and in [22]) at the intersection5.

Finally, we consider the flow rate \( J_1 \) on lattice 1. It is obvious that when lattice 1 is in the LL or HH state, \( J_1 = \rho_{\text{bulk}}(1 - \rho_{\text{bulk}}) \), which is independent of \( \rho \). The bulk density \( \rho_{\text{bulk}} = \alpha \) when lattice 1 is in LL and \( \rho_{\text{bulk}} = 1 - \beta \) when lattice 1 is in HH. On the basis of the mean-field results presented above, we have:

- \( J_1 = (1 - \rho)(1 + \rho)/4 \) in the HL–LD region. In this region, the bulk density equals \( (1 + \rho)/2 \) in segment I and equals \( (1 - \rho)/2 \) in segment II.
- \( J_1 = 2/9 \) in the HL–LH region. In this region, the bulk density equals \( 2/3 \) in segment I and equals \( 1/3 \) in segment II.
- \( J_1 = \rho(1 - \rho) \) in the HL–HD region. In this region, the bulk density equals \( \rho \) in segment I and equals \( 1 - \rho \) in segment II.

4 In model B in [21] and V(2:2) in [22], the critical values are \( \alpha = 1/3 \) (corresponding to \( \rho_c = 1/3 \)) and \( \beta = 1/3 \) (corresponding to \( \rho_c = 2/3 \)).

5 Note that the critical values are \( \alpha = \sqrt{2} - 1 \) and \( \beta = \sqrt{2} - 1 \) in model A in [21], in which the particle motion rules are different from those in this paper and in [22]. Similarly, in the biased case studied in [22], the critical densities are also different.

4. Conclusions

In this paper, we have presented a model for investigating TASEPs on two intersected lattices. In this model, one lattice has an open boundary and the other has a periodic boundary. Extensive Monte Carlo simulations are carried out. Four different phase
diagrams are found with various values of $\rho$ and altogether eight different phases (LL–LD, LL–HD, LL–LH, HH–LD, HH–LH, HL–LD, HL–HD, HL–LH) are identified. We obtain the phase boundaries by using mean-field analysis. The analytic results are in good agreement with simulation results.

In our future work, the studies will be extended to consider a more general ASEP model with intersected lattices, as in [18, 19].

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