Evolutionary prisoner’s dilemma game on weighted scale-free networks

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Abstract

This paper investigates the evolutionary prisoner’s dilemma on weighted scale-free networks. The weighted networks are generated by adopting Barabási–Albert scale-free network and assigning link weight with $w_{ij} = (k_i \times k_j)^\beta$. Simulation results show that the cooperation frequency has a strong dependence on $\beta$. The value of $\beta$ which is associated with the maximal cooperation frequency has been sought out. Moreover, Gini coefficient and Pareto exponent of the system’s wealth distribution are investigated. The inequality of wealth distribution is minimized at $\beta \approx -1$.

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1. Introduction

Cooperative behaviors among selfish individuals are ubiquitous in the real world, ranging from biological to social and economic systems. The prisoner’s dilemma game (PDG)\textsuperscript{[1–4]} provides a useful framework to study these phenomena. By varying the parameters of this simple model, one can understand how the cooperation is affected by the factors, such as the population relation structure, the selection of strategy, extensive noise and so on.

In the original PDG, players can make two choices: cooperate or defect. They get payoff depending on their choices, which can be expressed by a $2 \times 2$ payoff matrix. The players get rewards $R$ or punishment $P$ if both cooperate or defect. If one player cooperates and the other defects, then the cooperator (C) gets the lowest payoff $S$ (sucker’s payoff), and the defector (D) gains the highest payoff $T$ (the temptation to defect). The elements of the payoff matrix satisfy the conditions: $T > R > P > S$ and $2R > (T + S)$. These game rules yield a dilemma for intelligent players who wish to maximize their own income. That is, defection is good for one individual but mutual defection leads to the
second worst result. Consequently, the evolutionary PDG is proposed [3], where the players’ income come from iterated PDGs played with several co-players. In each round of the games, the players can choose a ‘good’ strategy with the knowledge of the co-players’ previous decisions. To enhance the cooperation, several rules for adopting strategies have been suggested, such as “Tit-for-tat” [3,5], “win stay and lose shift” [6], stochastic selection [7] and so on.

Nowak and May [8] showed that the PDG on a simple spatial structure induces emergence and persistence of cooperation even with the co-existence of spatial chaos. Since then, much attention has been given to the PDG on several population structures [9], including on regular networks [7,10–12] and on complex networks [13–21]. These works focus on the effect of population structure on the emergence of cooperation. Especially, in Ref. [18], the authors point out that scale-free networks, compared with regular or random networks, can enhance the cooperation frequency of evolutionary PDG.

However, the previous works are mainly on un-weighted networks, ignoring the difference between links. On the other hand, empirical evidences have shown that most social networks in real world are associated with weighted networks [24–26] and some weighted network models have also been proposed [27–29]. In these networks, weight of links is used to represent the contact frequency between individuals. Thus in this paper, we investigate the effect of link weight on some properties of evolutionary PDG games on a weighted network. The wealth distribution of the system [20,21] is also investigated by the Gini coefficient and the Pareto exponent.

The paper is organized as follows. In Section 2, we describe the evolutionary PDG and the model of weighted scale-free networks in detail. The simulation results and discussions are given in Section 3. The paper is concluded in the last section.

2. The model

In this paper, we adopt a simple weight-configuration mechanism to create a weighted scale-free network based on the well-known Barabási–Albert (BA) scale-free network model [30]. In the original BA model, starting from \( m_0 \) fully connected nodes, a new node is added to the system per step with \( m \) (\( m \leq m_0 \)) edges that link to \( m \) different nodes already existing in the system in such a way that the probability of being connected to the existing node \( i \) is proportional to its degree. After the topology has been generated, the links’ weight are set as \( w_{ij} = (k_i \times k_j)^\beta \), where \( k_i \) and \( k_j \) are the degrees of node \( i \) and \( j \) respectively. The value of \( \beta \) is tunable to control the heterogeneity of the link weights. The larger the \( \beta \), the higher the heterogeneity. This weight-setting mechanism is supported by empirical evidence [25,26].

For the PDG, we firstly rescale the payoff matrix such that there is only one parameter: \( T = b \geq 1 \), \( R = 1 \) and \( P = S = 0 \), where \( 1 \leq b \leq 2 \) represents the advantage of defectors over cooperators [8]. In each step, all pairs of directly connected individuals \( i \) and \( j \) are involved in \( w_{ij} \) rounds of PDG, because the link weight represents the contact frequency. Thus the payoff of the agent is multiplied by the value of link weight. For example, one defector will get \( b \times w_{ij} \) against his cooperative neighbors and two cooperators both get \( w_{ij} \). The total payoff of vertex \( i \) for the step is stored as \( P_i \). Then the strategy of each node is updated in parallel according to the following rule. Whenever a site \( s \) is updated, a neighbor \( t \) is drawn from among all its \( k_s \) neighbors, and \( s \) will adopt the strategy of \( t \) with a probability depending on the difference of payoff [7]:

\[
H_{s\rightarrow t} = \frac{1}{1 + \exp[(P_s - P_t)/\kappa]},
\]

where \( \kappa \) characterizes the noise reflecting irrational choices, thus \( \kappa = 0 \) and \( \kappa \rightarrow \infty \) denote the completely deterministic and completely random selection of the neighbor’s strategy respectively. Any finite positive values of \( \kappa \) mean the uncertainties in the strategy adoption, i.e., there is some probability to select the worse strategy. The effect of noise \( \kappa \) on the cooperation frequency has been studied in detail in Refs. [31,32]. In this paper, we take \( \kappa = 0.1 \) in the following simulation as in other studies.

3. Simulation results and discussion

The simulation is carried out on a weighted scale-free network with different values of \( \beta \). Here, we set \( m = m_0 = 4 \) and network size \( N = 1000 \) for all simulations. And we have found that the size of network will not affect the overall behavior of the system. Initially, strategies are randomly distributed among the population. Equilibrium frequency of
cooperators is obtained by averaging over 3000 generations after a transient time of $10^4$ generations. Each data is obtained by averaging $n = 50$ runs.

Previous research on un-weighted networks has revealed that the heterogeneity of networks can affect cooperation [22,23]. However, they have only investigated the effect of network topology. Fig. 1 shows the relation of cooperator frequency and $b$ for different $\beta$. One can see that the cooperation frequency is highly affected by the value of $\beta$. When $\beta = 0$, the network degenerates to an un-weighted SF network, and high cooperator frequency is obtained for almost the entire range of $b$ as in [18]. When $\beta > 0$, the cooperative behavior no longer dominates the evolution. The cooperation frequency is remarkably depressed as a monotonously decreasing function of $b$. When $\beta < 0$, the behavior of the system becomes a little complicated. When $\beta = -6, -5, -4$, the equilibrium frequency of cooperators is extremely high even when $b$ is large. But for $\beta = -3, -2, -1$, it decreases rapidly as $b$ increases.

Fig. 2 shows the variation of cooperator frequency with $\beta$. One can see that the cooperator frequency follows a non-linear variation with $\beta$. Generally, the frequency of cooperators decreases when $\beta > 0$. When $\beta < 0$, the cooperation frequency decreases first and then increases to a high value again with the increment of the absolute value of $b$.

Then we investigate two important parameters (Gini coefficient and Pareto exponent) for wealth distribution in the population. The Gini coefficient is a measure of inequality of income distribution. It is defined as a ratio with values between 0 and 1: the numerator is the area between the Lorenz curve of the distribution and the uniform distribution line; the denominator is the area under the uniform distribution line. Thus, a low Gini coefficient indicates more
equal income or wealth distribution, while a high Gini coefficient indicates more unequal distribution. 0 corresponds to perfect equality (everyone having exactly the same income) and 1 corresponds to perfect inequality (where one person has all the income, while everyone else has zero income).

The wealth in real world has been found to be distributed according to a power law \[ P(W) \sim W^{-(1+v)} \]. The Pareto exponent is defined as the exponent \((1 + v)\) of the power-law distribution. The values of Gini coefficient and Pareto exponent have important implications for the society.

Fig. 3 shows the variation of Gini coefficient and Pareto exponent with \(\beta\). One can see that the Gini coefficient is minimal and the Pareto exponent is maximal around \(\beta = -1\). Thus the system’s wealth is most equally distributed at around \(\beta = -1\). We note that in the assignment of link weight, \(\beta > 0\) leads to an increase in the contact frequency of high-degree nodes and an increase in the payoff of high-degree nodes, while \(\beta < 0\) will decrease the contact frequency of high-degree nodes. Thus \(\beta\) is also reflecting the encouragement of high-degree nodes or low-degree nodes. If high-degree nodes represent powerful individuals or large companies in the society and low-degree nodes represent weak individuals: One can easily conclude from our simulation result that the government should encourage the weak individuals in the economy system in order to maintain a low Gini coefficient. One can also see that \(\beta > 0\) and a large \(\beta\) value are no good for the cooperation frequency. The system will be more unstable and the wealth will be more unequally distributed when \(\beta > 0\).

4. Conclusion

To summarize, we have investigated the effect of edge-weight’s heterogeneity on the cooperation frequency, Gini coefficient and Pareto exponent of evolutionary prisoner’s dilemma taking place in a weighted scale-free network. The setting of the edge-weight’s heterogeneity depends on only one parameter \(\beta\) as \(w_{ij} = (k_i \times k_j)^\beta\). The cooperation frequency, Gini coefficient and Pareto exponent show strong dependence on the value of \(\beta\). The system’s cooperation frequency will be maximized when \(\beta = 0\) and when \(\beta < 0\) and the absolute value of \(\beta\) is large. Moreover, the system’s wealth will be more equally distributed when \(\beta \approx -1\).

Our study shows that the link weight of networks has an important effect on the evolutionary game. Also in order to decrease the Gini coefficient, we need to increase the income of low-degree nodes and restrain the income of high-degree nodes in the game.

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