Ring-like size segregation in vibrated cylinder with a bottleneck

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Abstract
In this Letter, a ring-like segregation pattern of bi-dispersed granular material in a vibrated bottleneck-cylinder is presented. The driving frequency can greatly affect the strength and structure of the convection roll and segregation pattern. The position and height of the ring (cluster of big beads) can be adjusted by altering the vibration frequency. And a heuristic theory is developed to interpret the ring’s position dependence on driving frequency.

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1. Introduction
Granular materials are ubiquitous and their dynamics are of central importance to many industrial processes [1], in which vibration-induced segregation in granular beds has been a widely studied topic. Granular segregation was first reported in 1939 by Brown [2] and brought in 1987 to the physics field with the name of the “Brazil nut problem” (BNP) [3]. Afterward, both theoretical and experimental studies have been concentrated on it to try to understand the underlying mechanism behind it, such as, “void filling” [3], global convection [4], arching [5] and inertia [6]. Furthermore, the reverse BNP is also predicted by simulation [7] and observed in experiments [8]. Even “sandwich” segregation is observed [9]. Unlike the “layered cake” structures mentioned above, a radial segregation pattern [10] resulting from heap formation, convection and cascading, and a horizontal size segregation [11] due to stratified flows in opposite directions at different heights within the granular layer, are also observed in vibrated
granular materials. Experiments [4,12] and computer simulations [13] showed that the shape of the container can affect the strength and direction of the convection roll. However, the highly complex, ordered or disordered structure, nonlinear internal friction and the presence of internal air effects [14] have so far prevented a comprehensive understanding of granular properties. Based on the debates above, we denote that more research is needed to reveal the physics mechanism of these fascinating granular problems.

In this Letter, two previously unreported experimental observations are communicated: (i) a stable ring-like size segregation pattern is achieved in the granular bed; (ii) the position and height of the ring can be adjusted by altering the vibration frequency. Large particle can congregate to form a cluster at the outer ring of the granular upper surface or in the middle of the granular bed. The vibration frequency can greatly affect the convective rolls and thus affect the ring’s position and height in the granular bed.

2. Segregation pattern

In the experiments for this Letter, the granular bed consists of glass beads of two different diameters \(d_s = 0.154-0.28\ mm\) and \(d_l = 3.2-4.0\ mm\). And two glass containers were used, an ordinary cylinder and a cylinder with a bottleneck, with one end open to the air and the other end fixed to a sinusoidal vibrator. The container base radius \(R = 40\ mm\), and the container height \(H = 130\ mm\). The bottleneck-cylinder has a bottleneck with radius \(r = 32\ mm\) at the height \(H_b = 65\ mm\). The granular bed was prepared by placing a 80 mm layer of small glass beads on the top of a 10 mm layer of large glass beads. In our experiments, the granular bed is operated with a fixed acceleration \(a = 30\ m/s^2\), and the frequency \(f\) ranging from 25 Hz to 65 Hz. The acceleration \(a = A(2\pi f)^2\), where \(A\) is the amplitude of vibration. The dimensionless acceleration \(\Gamma = a/g \approx 3.0\), where \(g\) is the gravity acceleration.

First, the system of a bottleneck-cylinder was vibrated at \(f = 25\ Hz\) for enough time to achieve a fully developed state. Under such conditions, vertical vibration of the granular bed led to the formation of a ring at the bed’s upper surface, with large beads surrounding small beads forming a slightly rippled surface as shown in Fig. 1(a). Large beads immersed in the bed to certain depth, which could be viewed in side view of granular bed (Fig. 1(b)). Convective flow was directly observed at the upper exposed surface, with particles surging vertically up to the top surface, where they moved towards the sidewall of the container. No heap was formed. On the exposed surface, beads behaved calmly, without any “gasification” of beads. And the process of segregation could be directly observed on the bed’s top surface. Once reaching the top ripple surface, big beads rapidly moved in all directions to the outer wall, re-entering the bed near the lateral walls but with more difficulties than small beads because of the small beads’ percolations. Small beads moved much more slowly to the outer wall but re-entered the bed at
all locations along the outer downward convection zone (the zone of the downward stream near the lateral walls). The small beads’ percolation occurred in the outer downward convection zone. This process produced a ring-like segregation pattern that was distinct from the “layered cake” structure and radial structure reported previously.

When the frequency was ranging from 30 Hz to 35 Hz, an obvious increment of the convective strength was observed: the more frequency increased, the more deeply large beads immersed in the bed near the lateral walls as shown in Fig. 2(b), which meant that the height of the ring increased and the thickness of the ring decreased. And the maxium height of the ring occured at 35 Hz. With the increasing of the frequency, the “gasification” of beads appeared on the exposed surface and grew more and more vehement, reaching the most vehement strength at 35 Hz with small beads jetting out near the lateral walls. The height of the jet was about 8 mm, and sometimes there were about 5 to 10 jets around the wall at the same time. As the frequency increased in this range, more and more large beads stayed below the bottleneck. A segregation process could be directly observed around the lateral walls. When large beads below the bottleneck wanted to move upward through the bottleneck, once reaching the bottleneck, partial large beads dropped suddenly downward, leaving a large void area that was quickly filled by small beads.

When the frequency was tuned to 40 Hz, the height of the ring decreased sharply and the entire ring immersed in the bed below the bottleneck. The segregation process was similar to that of \( f = 35 \text{ Hz} \) but less vehement. The small beads’ jets decreased both in strength and in quantity: the small beads occasionally jetted out with a height of about 5 mm near the lateral walls. Large beads were retained below the bottleneck, slowly rising up and dropping down. The strength of the convection decreased and the large beads moved with more difficulties. When the frequency was tuned to 45 Hz, convective flow was also apparent but in a more calm status. No heap developed under these conditions, the exposed surface of the bed remained mostly flat, and there is no small beads’ jets any more. The position and the length of the ring hardly changed.

When the frequency was tuned from 50 Hz to 65 Hz, the structure of the convective flow changed completely. Two downward zones, named outer downward zone and inner downward zone (the zone of the downward stream at the center of the granular bed), and an annulus upward zone (the zone of the upward stream between two downward zones) appeared in the granular bed. An annulus heap was formed, which could be directly observed at the exposed surface, showed in Fig. 3. As a direct proof, we put 4 large beads at the center of the exposed surface, which was at the inner downward zone, where they could not re-enter the bed and the percolation effect was apparent. Nevertheless, when we put large beads near the lateral walls, they could re-enter the bed. During this range of frequency, the ring’s position slowly rose with the increasing of frequency, and the annulus heap grew
higher. After long-time vibration, the heap was stable and the convective motion of the granular bed was coherent, but no cascading was observed. Again, small beads’ percolation could be observed in the downward zones and it was apparent around the bottleneck but less intense than that of the 35–45 Hz. Compared with the strength of the downward convection, however, the percolation could raise the ring higher than that in the 35–45 Hz. As a result, the location of the ring started to rise at 50 Hz, which could be seen at Fig. 2(b).

Completely different behavior was observed when the cylinder container was used, under the same vibration conditions and initial beads’ distributions. When the frequency varied in the range of 25–35 Hz, convective flow was apparent by direct observation, which led to the formation of a well-defined heap at the center of the bed’s surface. Cascading flow was observed at the top surface, with large beads cascading to the bottom of the heap and then re-entering the bed. However, no radial segregation pattern as Brone and Muzzio [10] revealed was observed. Large beads could stay at the upper layer of the bed. Due to the heap and the cascading flow, at the exposed surface, an annulus of large beads surrounded the central small beads. Therefore, we also noted that this pattern was a ring-like segregation pattern. Nevertheless, when the frequency varied in the range of 40–65 Hz, segregation disappeared. No stable heap was formed. Convective flow was apparent, with occasional ripples on the exposed surface. The bed developed into a mixed state. Again, based on the experimental observations, the most vehement “gasification” of the granular bed occurs at 35 Hz. At \( f = 45 \) Hz, compared with any other vibration frequency, through the lateral walls we observed that relatively more large beads stayed in the deeper layer of the granular bed in the mixed state. So large beads immersed in the deepest position in the granular bed at 45 Hz.

3. Analysis and result

Based on our experiments of bottleneck cylinder, at relatively low frequencies, 25–35 Hz, the ring can be seen on the exposed surface. From 40 Hz to 45 Hz, the ring immerses into the granular bed. This effect is a consequence of relatively strong and deep convection rolls. A possible explanation for this phenomenon seems to be that, as frequency increases till to 45 Hz, the granular bed is less compact than before. However, as the frequency
studies of inelastic bouncing ball [18], when large beads immersed in the bed. And the deepest immersing position of large beads occurs at the frequency varied in the range of 40–65 Hz, the ring disappeared in the exposed surface, which meant that large beads immersed in the bed. And the deepest immersing position of large beads occurs at \( f = 45 \) Hz. Those phenomena are also related to the fact that the granular bed is less or more compact.

Based on that granular media typically collides inelastically with container walls [15,16], a simple model, assuming the granular bed as a partially inelastic mass, could help to estimate the resulting motion of the granular assembly as an approximation. The interaction between the base and the mass is characterized by a coefficient \( \varepsilon \) of restitution of collisions between them. It is defined as the ratio of the relative velocity between the mass and the base after the collision to the relative velocity prior to the collision, and it also indicates the properties of energy dissipation of the mass-base collisions. So \( 0 < \varepsilon < 1 \) and the mass-base collision is responsible for all energy dissipation of the granular bed. When the vibrated granular bed under a frequency achieves a fully developed stationary state, the energy inserted into the system is equal to the energy dissipation of the system. In this case, to simply the analysis of the macro-motion of the granular bed, we assume that the kinetic energy of the mass increases linearly with the kinetic energy of the granular system, which means the mass’ velocity \( v \) (absolute value) obeys

\[
v \sim \sqrt{\sum_{i=1}^{N} m_i c_i^2 / M},
\]

where \( c_i \) is each particle’s velocity, \( m_i \) is each particle’s mass and \( M = \sum_{i=1}^{N} m_i \) is the mass’ mass. We can see that \( v \) will achieve its maximum when the kinetic energy of the granular system reaches its maximum, which means that the most vehement “gasification” of the granular system is achieved then \( v \) achieves its maximum.

The mass supported by the base will become airborne when the base downward acceleration exceeds gravity \( g \), that is, after each collision the mass leaves the base and obeys a ballistic trajectory [17]. Based on the previous studies of inelastic bouncing ball [18], when \( \Gamma < \Gamma_0 \) (here \( \Gamma_0 = (\pi (1-\varepsilon)/(1+\varepsilon))^2 + (2(1+\varepsilon^2)/(1+\varepsilon)^2)^2 \) is the critical or bifurcation value [19]), the particle bed collides with the container base after a flight time \( \Delta t \). At a fixed \( \Gamma \), \( \Delta t \) does not vary from cycle to cycle and so the bulk motion of the bed repeats every oscillation cycle. When \( \Gamma > \Gamma_0 \), two flight times, \( \Delta t_1 \) and \( \Delta t_2 \), was observed, moreover, \( \Delta t_1 > T, \Delta t_2 < T \) and \( \Delta t_1 + \Delta t_1 < 2T \), where \( T \) is the vibration period. Furthermore, as \( \Gamma \) increases, \( \Delta t_1 \) increases while \( \Delta t_2 \) decreases. As a result, when \( \Gamma \) is near the critical value \( \Gamma_0 \), there is a transition from one flight time to two flight times, which means that, when \( \Gamma \) is not far from \( \Gamma_0 \), we can still make the assumption that \( \Delta t_1 \approx \Delta t_2 \approx T \). Wassgren [19] gave a measurement for an effective coefficient of restitution for a simulated bed with \( \varepsilon \approx 0.11 \). So \( \Gamma_0 \approx 3.01 \). In our experiments, \( \Gamma \) is near \( \Gamma_0 \). As a result, we can assume that the mass obeys such a motion (see Fig. 4): the periodic fixed point trajectory is apparent where the mass motion repeats after every collision. In the laboratory reference frame, the absolute value of the mass postcollisional velocity approximately equals to that of the mass precollisional velocity. We denote “\( v^- \)” as the mass precollisional velocity, “\( v^+ \)” as the mass postcollisional velocity and \( v_{\text{base}} \) as the base velocity in the laboratory reference frame, then we have (here all velocities are used as absolute value)

\[
v^- \approx v^+.
\]

From the definition of the restitution coefficient \( \varepsilon \) we get the mass postcollisional velocity as being

\[
v^+ = \varepsilon (v^- + v_{\text{base}}) + v_{\text{base}}.
\]

Because \( v_{\text{base}} \propto 1/f \) under the condition of fixed vibration acceleration, we denote \( c/f \) (\( c > 0 \)) for the base velocity of collision point. Then from (2) and (3) we get

\[
v(f) = v^- = v^+ = c(1+\varepsilon)/\left[ f(1-\varepsilon) \right].
\]
then (here $v$ and $\varepsilon$ are the functions of the frequency $f$ ($25 \leq f \leq 65$))

$$v'(f) = c \cdot \frac{2f \varepsilon' - (1 - \varepsilon^2)}{f^2(1 - \varepsilon)^2},$$

$$v''(f) = 2c \cdot \frac{[(1 + \varepsilon)\varepsilon' + f\varepsilon''][f^2(1 - \varepsilon)^2] - [2f \varepsilon' - (1 - \varepsilon^2)]f(1 - \varepsilon)^2 - f^2 \varepsilon'(1 - \varepsilon)]}{f^4(1 - \varepsilon)^4}.$$  

Based on observations of our experiments, the most vehement “gasification” of the system occurs at $f = 35$ Hz. So we denote that a maximum $v$ occurs at $f_0$, near 35 Hz, then

$$v'(f_0) = 0, \quad v''(f_0) < 0, \quad v'(f) > 0 \ (f < f_0), \quad v'(f) < 0 \ (f > f_0).$$

As a result, we can get

$$\varepsilon'(f_0) = \frac{1 - \varepsilon^2}{2f_0^2}, \quad \varepsilon''(f_0) < \frac{(1 + \varepsilon)^2(1 - \varepsilon)}{2f_0^4}, \quad \varepsilon'(f) > \frac{1 - \varepsilon^2}{2f^2} \ (f < f_0), \quad \varepsilon'(f) < \frac{1 - \varepsilon^2}{2f^2} \ (f > f_0).$$

Given that $0 < \varepsilon < 1$, we get $\varepsilon''(f_0) < 0$ and $\varepsilon'(f) > 0$ when $f < f_0$. We conclude that a maximum $\varepsilon$ will occur at $f_1$ ($> f_0$). Before $f_1$, $\varepsilon$ increases as the frequency increases. Based on the definition of $\varepsilon$, the larger $\varepsilon$ is, the less energy dissipation of the vibrated granular system is. As we know, energy dissipation through particle–particle collisions is the most dominating dissipation in vibrated granular bed system. Therefore, when $\varepsilon$ increases the average clearance between particles increases and the rate of the particle–particle collisions decrease. So the bed becomes less compact. Deep convection rolls are developed gradually. Large beads can immerse more deeply as the frequency increases. But after $f_1$, the situation is reversed and the granular bed becomes more compact with the increasing frequency. Convection rolls become weaker and large beads rise up. From Fig. 2(b), we note that the ring immerses in the deepest position at $f = 40$ Hz in the bottleneck-cylinder. In a cylinder, large beads immerse in the deepest position at $f = 45$ Hz. So, as for the bottleneck-cylinder, $f_1$ is approximate 40 Hz; as for the cylinder, $f_1$ is approximate 45 Hz. Both of them are in agreement with the prediction of our simple model.

The difference of a bottleneck-cylinder and an ordinary cylinder was that the outer downward convection zone was narrowed by the bottleneck. When the frequency increased, the large particles were trapped around the bottleneck, dissipating the energy of the passing-by particles. Whereas for an ordinary cylinder, this process could not happen and a mixing state was observed.
4. Conclusion

In summary, a kind of ring-like size segregation patterns can be produced when a bottleneck cylinder granular bed is subjected to continuous sinusoidal vibration. The ring’s height and location can be adjusted by altering the frequency of the vibration. This behavior is a consequence of the bottleneck, which affects the downward zone of the granular bed and balances the competition between the percolation and the downward convective flow. The bottleneck makes the convective pattern change with vibration frequency: at low frequency, there is one upward zone and one downward zone; at high frequency, there is one upward zone and two downward zones. And also, it is clear that vibration frequency is a significant parameter affecting many aspects of the problem, such as large particles immersing in the granular bed when the frequency is relatively higher. We developed a simple model to analyse the phenomenon and obtained that the maximum kinetic energy of the system occurs before the maximum restitution coefficient of the system under the condition of fixed vibration acceleration. In our experiments, heap formation is no longer an important factor for developing the segregation patterns, as proposed by Brone and Muzzio [10], so that more detailed studies are needed to reveal the most relevant mechanism.

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