Defect-induced transitions in synchronous asymmetric exclusion processes

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Article history:
Received 27 May 2008
Received in revised form 10 October 2008
Accepted 12 November 2008
Available online 19 November 2008
Communicated by R. Wu

PACS:
05.70.Ln
02.50.Ey
05.60.Cd

Abstract

The effects of a single local defect in synchronous asymmetric exclusion processes are investigated via theoretical analysis and Monte Carlo simulations. Our theoretical analysis shows that there are four possible stationary phases, i.e., the (low density, low density), (low density, high density), (high density, low density) and (high density, high density) in the system. In the (high density, low density) phase, the system can reach a maximal current which is determined by the local defect, but independent of boundary conditions. A phenomenological domain wall approach is developed to predict dynamic behavior at phase boundaries. The effects of defective hopping probability p on density profiles and currents are investigated. Our investigation shows that the value of p determines phase transitions when entrance rate α and exit rate β are fixed. Density profiles and currents obtained from theoretical calculations are in agreement with Monte Carlo simulations.

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1. Introduction

Traffic phenomena (e.g., intracellular transport and vehicular traffic) have received much attention since the early 1970s [1]. One important issue in understanding such traffic is to characterise stationary phases and transitions between them, using theoretical analysis. There are also a great number of efforts to integrate traffic theory with empirical observations in order to describe traffic more realistically [2–5]. These achievements deepen the understanding of the collective dynamics of self-driven interacting particles such as cars and molecular motors (e.g., kinesins, dyneins and myosins).

Totally asymmetric simple exclusion process (TASEP) has been widely accepted as a powerful tool in studying traffic, which is far from equilibrium, and it has been successfully applied to describe stochastic dynamics of multi-particle interactions in chemistry, physics and biology [6–8] such as protein synthesis [9,10], mRNA translation phenomena [11], and motion of molecular motors along the cytoskeletal filaments [12].

The effects of local defects on TASEP have been extensively investigated in recent years [13–24]. The TASEP with a local defect with periodic boundary conditions was studied firstly, such as in Refs. [13–15]. Then Kolomeisky [16] developed an approximate solution for TASEP with a local defect with open boundary conditions in random update. The proposed system is represented as two homogeneous ones connected by the local defect. More recently, this approach has been widely adopted in, e.g., Refs. [18, 20, 23,24]. Refs. [19,20] report that a maximal current may exist in their systems. This current is dictated by the local defect, not by the boundary conditions. In reality, local defects may be involved in many biological transport processes [25–28] as well as in vehicular and pedestrian traffic flow [29,30]. For instance, the local defects of immunoreactivity may lead to a high susceptibility to respiratory infection [28], while a high-density (e.g., jammed or synchronized) traffic can be attributed to local defects, e.g., on-ramps, lane reductions or temporary road works [29].

Most TASEP models are implemented in random update, which shows the weakest spatial correlation as indicated in [31]. Conversely, the synchronous/parallel update [31–36] exhibits the strongest interaction between particles, which has been typically adopted in modeling vehicular and pedestrian traffic [2,37]. In addition, the usual TASEP with different update procedures (e.g., random, sequential, sublattice-parallel, and parallel) has been also compared and analysed in Ref. [31]. The comparative study has shown that the change of update dynamics can have drastic effects. Obviously, our model may be more suitable for car traffic, but less suitable for molecular motors or other biological transport.

The focus of this Letter is on developing the theoretical and computational descriptions of a TASEP with a single local defect in parallel update. The phase diagram is obtained using a simple
approximate theory, which gives an integrated picture of the traffic dynamics of the system and covers the full parameter space. Extensive computer simulations are carried out to verify our theoretical analysis.

The Letter is organised as follows. In Section 2, we give a description of a one-dimensional TASEP model with a local defect in parallel update. In Section 3, a mean-field approximation is developed, followed by a phenomenological domain wall approach in Section 4. In Section 5, we present and discuss the results of theoretical analysis and computer simulations. Finally, conclusions are given in Section 6.

2. The model

Our model is defined in a one-dimensional lattice of $N$ sites ($N$ is an even number). Particles are assumed to go through the system from the left to the right. Site 1 ($N$) defines the left (right) boundary, while a set of sites 2,...,$N−1$ is referred to as the bulk (see Fig. 1(a)). We assume that the link between site $k$ and site $(k+1)$ is defective and other links are normal. In other words, the hopping probability at the defective link is $p$, while the hopping probabilities of other normal links are 1. The defective link can be in any position of the lattice; however, we expect that the phase diagram and density profiles will be qualitatively the same, provided the defective link is far away from the boundaries. For simplicity, we assume $k=N/2$ in this Letter. The following rules are applied to all sites in parallel.

- Entrance: A particle can enter the system to site 1 with rate $\alpha$ if the site is empty.
- Exit: A particle can leave the system from site $N$ with rate $\beta$.
- Movement: A particle can move into site $(i+1)$ from site $i$ with probability $1$, provided $i\neq k$. If $i=k$, it moves to site $(i+1)$ with probability $p$; or if site $(i+1)$ is occupied by another particle, the particle stays at site $i$.

A one-dimensional synchronous TASEP with a local defect can be seen as two homogeneous one-dimensional subsystems connected by a defective link between sites $k$ and $(k+1)$ (see Figs. 1(b) and (c)). The left subsystem is a homogeneous TASEP with entrance rate $\alpha$ at site 1 and exit rate $\beta$ at site $k$. Similarly, the right subsystem is also a homogeneous TASEP with entrance rate $\alpha_{\text{eff}}$ at site $(k+1)$ and exit rate $\beta$ at site $N$.

In this Letter, we only consider $0 < p < 1$ since the dynamics of such system can be analysed and simulated in parallel update. Obviously, when $p = 1$, the system is a TASEP without local defect. The exact solutions for this case can be found in Refs. [32,38]. For $p > 1$, the system is dominated by all the other normal sites with the hopping probability equal to 1. The local defect site has no effect on phase changing. In other words, the system phases (e.g., high, low or maximum current) will not be determined by the value of $p$ when $p > 1$. A similar conclusion has been made in Ref. [16] with random update.

Also, note that we have not found a proper way to get theoretical solution and how to use Monte Carlo to simulate the system with $p > 1$ in parallel update. We noticed that when the parameters, $\alpha$, $\beta$ and $p$ (assuming no defects are in the system and all links have the same hopping probability $p$), are decreased or increased with the same proportion in random update, the system can still have the same currents and bulk densities. For example, the system with $\alpha = 0.4$, $\beta = 1$ and $p = 1$, and the system with $\alpha = 0.2$, $\beta = 0.5$ and $p = 0.5$, have the same density profiles and currents. However, such property does not exist in systems with parallel update. Therefore, we cannot deal with a local defect $p > 1$ in a system with parallel update using the same approach as we deal with such situations with random update. We believe how to deal with $p > 1$ in a system with parallel update is an interesting topic for future work.

3. Mean-field approximation

Here we briefly recall the results of a synchronous TASEP without local defect, which has been presented in Refs. [32,38]. There are three phases (low density, high density and maximal current) in that system. The maximum current (MC), $J = 0.5$, can only be reached at $\alpha = \beta = 1$. When $\alpha < \beta$, a low-density (LD) phase is obtained with

$$J = \rho, \quad \rho = \rho_1, \quad \rho_1 = \frac{\alpha}{1 + \alpha}, \quad \rho_N = \frac{\alpha}{\beta(1 + \alpha)},$$

where $J$ is the system current; $\rho$ is the bulk density; $\rho_1$ ($\rho_N$) is the particle density at the first (last) site of the system. The condition, $\alpha > \beta$, corresponds to a high-density (HD) phase, where

$$J = 1 - \rho, \quad \rho = \frac{1}{1 + \beta}, \quad \rho_1 = 1 - \frac{\beta}{\alpha(1 + \beta)}, \quad \rho_N = \rho.$$

The currents in the left subsystem ($J_L$), the right subsystem ($J_R$) and the local defect ($J_{\text{local}}$) should be equal in a stationary state, that is

$$J_L = J_R = J_{\text{local}}.$$  

(3)

$J_{\text{local}}$ can be written as follows [32]

$$J_{\text{local}} = \frac{1}{2}(1 - \sqrt{1 - 4p\rho_k(1 - \rho_{k+1})}).$$

(4)

The overall phases in the system can be obtained from the combinations of phases in the two subsystems. The possible stationary phases of the system are the (LD, LD), (LD, HD), (HD, LD), (HD, HD), (LD, MC), (HD, MC), (MC, LD), (MC, HD) and (MC, MC) phases as there are three possible phases (LD, HD and MC) in each subsystem. As $p < 1$, it cannot guarantee $\beta_{\text{eff}}$ equal to 1, thus, it is impossible for the MC phase to exist in the left subsystem. As a consequence, $\alpha_{\text{eff}}$ is less than 1. In this condition, the MC phase would not exist in the right subsystem either. That is, the last five phases could not exist in the system.

For the (LD, LD) phase, the following conditions should be satisfied

$$\alpha < \beta_{\text{eff}}, \quad \alpha_{\text{eff}} < \beta.$$  

(5)

According to Eqs. (1), (3) and (4), one obtains

$$\frac{\alpha}{1 + \alpha} = \frac{\alpha_{\text{eff}}}{1 + \alpha_{\text{eff}}} = \frac{1}{2}\left(1 - \frac{1}{1 + \frac{4p\rho_k(1 - \rho_{k+1})}{\beta_{\text{eff}}(1 + \alpha)}}\right).$$

Then, we get $\alpha_{\text{eff}} = \alpha$ and $\beta_{\text{eff}} = p$. Thus, the system is in the (LD, LD) phase when

$$\alpha < \beta, \quad \alpha < p.$$  

(7)
The corresponding current and density profiles in this phase are
\[ J = \rho, \quad \rho = \rho_1, \quad \rho_1 = \frac{\alpha}{1 + \alpha}, \quad \rho_0 = \frac{\alpha}{p(1 + \alpha)}, \]
\[ \rho_{k+1} = \frac{\alpha}{1 + \alpha}, \quad \rho_N = \frac{\alpha}{\beta(1 + \alpha)}. \quad (8) \]

The (LD, HD) phase corresponds to \( \alpha < \beta_{\text{eff}} \). \( \beta_{\text{eff}} > \beta \).

As the current is conserved in the system, that is,
\[ \frac{\alpha}{1 + \alpha} = \frac{\beta}{1 + \beta} = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{4p\alpha}{\beta(1 + \alpha)\alpha_{\text{eff}}(1 + \beta)} + \frac{\beta}{1 + \beta} } \right). \quad (10) \]

which means \( \alpha = \beta \) and \( \alpha_{\text{eff}} = \beta_{\text{eff}} = p \). As \( \alpha_{\text{eff}} > \beta \) and \( \beta_{\text{eff}} > \alpha \), \( \alpha = \beta < p \). This phase corresponds to a transition line between the (LD, LD) and (HD, HD) phases in the system. We will study the density profiles of this phase using the domain wall theory as shown in Section 4.

The (HD, LD) phase corresponds to the following conditions
\[ \alpha > \beta_{\text{eff}} \quad \alpha_{\text{eff}} < \beta. \quad (11) \]

According to Eqs. (1)–(4), we have
\[ \frac{\beta_{\text{eff}}}{1 + \beta_{\text{eff}}} = \frac{\alpha_{\text{eff}}}{1 + \alpha_{\text{eff}}} = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{4p\alpha}{\beta(1 + \alpha)\alpha_{\text{eff}}(1 + \beta)} + \frac{\beta}{1 + \beta} } \right). \quad (12) \]

where we obtain \( \alpha_{\text{eff}} = \beta_{\text{eff}} = p \). Thus, the system is in the (HD, LD) phase when \( \alpha > p, \quad \beta > p \).

The current and density profiles in this phase are given by
\[ J = \frac{p}{1 + p}, \quad \rho_1 = \frac{1}{1 + p}, \quad \rho_1 = 1 - \frac{p}{\alpha(1 + \beta)}, \quad \rho_0 = \frac{1}{1 + p}, \]
\[ \rho_{k+1} = \frac{1}{1 + p}, \quad \rho_N = \frac{p}{\beta(1 + p)}. \quad (14) \]

The (HD, HD) phase should be satisfied with \( \alpha > \beta_{\text{eff}} \), \( \alpha_{\text{eff}} > \beta \).

Similarly, one reads
\[ \frac{\beta_{\text{eff}}}{1 + \beta_{\text{eff}}} = \frac{\beta}{1 + \beta} = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{4p\alpha}{\beta(1 + \alpha)\alpha_{\text{eff}}(1 + \alpha)} + \frac{\beta}{1 + \beta} } \right). \quad (16) \]

which is equivalent to \( \beta_{\text{eff}} = \beta \) and \( \alpha_{\text{eff}} = p \). According to Eq. (15), the system is in the (HD, HD) phase when \( \alpha > \beta, \quad \beta < p \).

Thus, the current and density profiles in this phase correspond to
\[ J = 1 - p, \quad \rho = \frac{1}{1 + p}, \quad \rho_1 = 1 - \frac{\beta}{\alpha(1 + \beta)}, \quad \rho_0 = \frac{1}{1 + p}, \]
\[ \rho_{k+1} = \frac{1}{1 + p}, \quad \rho_N = \frac{\beta}{1 + \beta}. \quad (18) \]

The above analysis suggests that there are four possible steady-state phases ((LD, LD), (LD, HD), (HD, LD), and (HD, HD)) existing in this system (see Fig. 2). Theoretical predictions of phase boundaries are almost the same as computer simulations. The (LD, HD) phase is a coexistence line of first-order phase transitions between the (LD, LD) and (HD, HD) phases. The current in the (LD, LD) and (HD, HD) phases is determined by the values of \( \alpha \) or \( \beta \) (see Fig. 2). In other words, the current depends on the boundary conditions. In contrast, the (HD, LD) phase is similar to a maximal current phase. The current through this phase is \( J = p/(1 + p) \). One can see that this current is constant and has the maximal possible value compared to the (LD, LD) and (HD, HD) phases. Also, the current in the (HD, HD) phase is boundary conditions-independent.

The (HD, LD) phase region is specified by \( 0 < \alpha < 1 \) and \( p \leq \beta < 1 \). When \( p = 0 \), we have \( J = 0 \). This is obviously correct as \( p = 0 \) means that no particles can pass the defective link. Thus, the current in the system \( J = 0 \). When \( 0 < p < 1 \), one can see that the (HD, LD) phase region and the (HD, HD) phase region expand with the increase of \( p \), while the (LD, LD) phase region shrinks with the increase of \( p \) (see Figs. 2(a) and (b)). Finally, in the case of \( p = 1 \), the system reduces to the usual synchronous TASEP without local defect (see Fig. 2(c)).

We note that the phase diagram (see Fig. 2) of the synchronous TASEP with a local defect is related to understand the general property of traffic flow. Local defects in a system can be hindrances (e.g., road reductions or road works) on roads. Although these hindrances just cover very short road segments, they can cause congested traffic. In the (LD, LD) and (HD, HD) phases, only local deviations of density profiles can be observed when \( p \) changes. However, for example, when the system changes from the (LD, LD) phase to the (HD, LD) phase, a phase separation between high and low densities occurs at the local defect.

4. Domain wall dynamics

Although the mean-field method introduced in Section 3 predicts the existence of four phases, it fails for the calculation of the density in the (LD, HD) phase. This gives a direct justification to apply domain wall theory instead. The domain wall approach has been employed successfully to explain the phase behavior of a TASEP with a junction in random update in Ref. [39]. We believe that this approach could be applied to a TASEP with a local defect in parallel update. In this Letter, the domain wall approach is used to only calculate the density profile of the line \( \alpha = \beta < p \), i.e., the (LD, HD) phase. To determine a position of the domain wall in the system, we define \( x \) as \( x = i/N \), where \( i \) is the site index and...
$N$ is the number of sites in the one-dimensional lattice. The case of $0 < x \leq 0.5$, thus, corresponds to the domain wall moving at rate $v_L$ in the left subsystem, and $0.5 < x \leq 1$ for the domain wall moving at rate $v_R$ in the right subsystem (see Fig. 3). $v_L$ and $v_R$ can be given by

$$v_L = \frac{J_L}{\rho^L_+ - \rho^L_-,} \quad v_R = \frac{J_R}{\rho^R_+ - \rho^R_-},$$

where

$$J_L = \frac{\alpha}{1 + \alpha}, \quad \rho^L_+ = \frac{1}{1 + \alpha}, \quad \rho^L_- = \frac{\alpha}{1 + \alpha},$$

and

$$J_R = \frac{\beta}{1 + \beta}, \quad \rho^R_+ = \frac{1}{1 + \beta}, \quad \rho^R_- = \frac{\beta}{1 + \beta}. \quad (20)$$

As a result, $v_L$ and $v_R$ are rewritten as:

$$v_L = \frac{\alpha}{1 + \alpha}, \quad v_R = \frac{\beta}{1 + \beta}. \quad (22)$$

Similarly to Ref. [39], $q_L$ ($q_R$) is denoted as a probability to find the domain wall at any position in the left (right) subsystem. For a special site $i$ in the left (right) subsystem, the probability is obviously equal to $2q_L/N$ ($2q_R/N$). Then, at the local defect, we have

$$\frac{2v_L q_L}{N} = \frac{2v_R q_R}{N}. \quad (23)$$

In addition, normalized $q_L$ and $q_R$ are satisfied with:

$$q_L + q_R = 1. \quad (24)$$

Instituting Eq. (24) into Eq. (23), we obtain

$$q_L = \frac{v_R}{v_L + v_R}, \quad q_R = \frac{v_L}{v_L + v_R}. \quad (25)$$

As $\alpha = \beta < p$ corresponds to the transition line, we have $v_L = v_R$ and $q_L = q_R = 1/2$. These expressions reflect the fact that the domain wall can travel the left and right subsystems at the same probability.

Accordingly, the probability of the domain wall falling in a certain zone in the left subsystem is given by

$$\text{Prob}(x_{DW} < x) = 2q_L x, \quad 0 < x \leq 0.5, \quad (26)$$

and in the right subsystem

$$\text{Prob}(x_{DW} < x) = q_L + 2q_R (x - 0.5), \quad 0.5 < x \leq 1. \quad (27)$$

Thus, the density at any position in the system becomes

$$\rho(x)_m = \rho^m_+ \text{Prob}(x_{DW} > x) + \rho^m_- \text{Prob}(x_{DW} < x), \quad m = L, R. \quad (28)$$

Finally, from Eqs. (22)–(28), one can obtain

$$\rho(x)_L = \frac{\alpha}{1 + \alpha} + \frac{1 - \alpha}{1 + \alpha} x, \quad 0 < x \leq 0.5, \quad (29)$$

and

$$\rho(x)_R = \frac{1}{2} + \frac{1 - \alpha}{1 + \alpha} (x - 0.5), \quad 0.5 < x \leq 1. \quad (30)$$

Densities in the boundary conditions can be calculated as $\rho(x = 0)_L = \alpha(1 + \alpha)$ and $\rho(x = 1)_R = 1/(1 + \alpha).$ At the locally defective site $k (k = N/2)$, the densities are equal to $\rho(x = 0.5)_L = \rho(x = 0.5)_R = 1/2.$ These results are completely identical with the theoretical analysis in Refs. [32,38] without defect.

5. Theoretical calculations and computer simulations

In this section, computer simulations are carried out to validate our approximate theoretical analysis. The length of the system size is assumed to be $N = 1000$ sites. For larger size $N$, our simulations show almost identical results with the ones presented here.

The simulation results for the density profiles are shown in Fig. 4. In the (LD, LD), (HD, HD) and (HD, LD) phases, density profiles are still quantitatively and qualitatively the same as theoretical calculations (see Figs. 4(a)–(c)). As mentioned, the (LD, HD) phase corresponds to the phase coexistence line between the (LD, LD) and (HD, HD) phases. Fig. 4(d) shows the density profiles in this phase obtained from computer simulations and theoretical predictions. It is found that the density profiles derived from mean-field approximation are close to that obtained in computer simulations except around the local defect. One possible explanation of these deviations is that the interaction between the particles near the local defect is stronger than our predictions.

The effects of hopping probability $p$ on density profiles are examined when $\alpha < \beta$. The influence of $p$ with $\alpha > \beta$ can be obtained using particle–hole symmetry [16] and the phase diag
shown in Fig. 2. In Fig. 5(a), \( \alpha = 0.2, \beta = 0.6, \) and \( p = 0.1 \) lead to the (HD, LD) phase which is determined by \( \alpha_{\text{eff}} = \beta_{\text{eff}} = 0.1 \) (see Eq. (11)). When \( p > 0.2 \), a phase transition from the (HD, LD) phase to the (LD, LD) phase is observed. In the (LD, LD) phase, the bulk density \( \rho = \alpha / (1 + \alpha) \), which is independent of the value of \( p \). The details of densities near the locally defective site are shown in Fig. 5(b). It can be seen that the amplitude of density increment near the local defect in the left subsystem decreases with the increase of \( p \), while densities remain unchanged in the right subsystem when \( p > 0.2 \).

Currents with different hopping probability \( p \) are also investigated. For simplicity, we assume that exit rate \( \beta \) is constant in each figure, i.e., \( \beta = 0.5 \) in Fig. 6(a) and \( \beta = 1 \) in Fig. 6(b). Entrance rate \( \alpha \) changes within \([0, 1]\). From the phase diagram (see Fig. 2(a)) we observe that the system is in the (LD, LD) phase when \( p = 0.2, \beta = 0.5 \) and \( \alpha < 0.2 \). In these situations, \( J = \alpha / (1 + \alpha) \).

Upon increasing \( \alpha \) to 0.2, a saturation point \( S \) is reached with \( S = p = 0.2 \) and \( J = p / (1 + p) \approx 0.16667 \). Further increasing \( \alpha \), the system transits from the (LD, LD) phase to the (HD, LD) phase, while the current is constant (see Fig. 6(a)) since \( p \) dictates the dynamics of the system, in particular, the right subsystem. When \( p = 0.8 \) and \( \beta = 0.5 \), a phase transition from the (LD, LD) phase to the (HD, LD) phase can be observed with \( \alpha \) increasing to 0.5 (see Fig. 2(b)). Clearly, saturation point \( S \) is at \( S = \beta = 0.5 \) with \( J = \beta / (1 + \beta) = 0.3333 \). Similarly, we can obtain theoretically currents with \( \beta = 1 \) and \( p = 0.2 \) and 0.8 (see Fig. 6(b)). It can be seen that our theoretical calculations are in perfect agreement with computer simulations. In this Letter, saturation point \( S \) and corresponding current \( I_{\text{sat}} \) can be expressed as follows (see Eq. (31)). However, we note that Eq. (31) is not general since this equation gives the saturation point and current only for fixed \( p, \beta \) and upon varying \( \alpha \).

\[
S = \min(p, \beta), \quad I_{\text{sat}} = \min \left( \frac{p}{1 + p}, \frac{\beta}{1 + \beta} \right). \tag{31}
\]

In order to investigate the system behavior close to the phase boundaries, e.g., in Fig. 2(a), a verification with Monte Carlo simulations is carried out. Fig. 7 shows simulated results of density profiles near the theoretical phase boundaries (see Fig. 2(a)) with hopping probability \( p = 0.2 \). In Fig. 7(a), the system is in the (LD, LD) phase when \( \alpha = 0.195 \) and \( \beta = 0.5 \). When \( \alpha \) increases to 0.2, the phase boundary is reached. In other words, a saturation point (see Fig. 6) appears. Further increasing the value of \( \alpha \) (e.g., \( \alpha = 0.205 \)), the system transfers into the (HD, LD) phase. We can observe that the transition from the (HD, LD) phase to the (HD, LD) phase is discontinuous in the left subsystem, while it is continuous in the right subsystem. Also, the transition from the (HD, HD) phase to the (HD, LD) phase is continuous in the left subsystem, while it is discontinuous in the right subsystem (see Fig. 7(b)). Our simulation results show that there is little deviation of phase boundaries between theoretical calculations and computer simulations, compared to Ref. [16] in which the random update is used. The reasons for this are, probably, the different update procedures as well as different treatment with the local defect, e.g., see Eq. (4) in our model and Eqs. (6) and (7) in Ref. [16].

6. Summary

We have investigated a one-dimensional TASEP with a single local defect in parallel update via mean-field approximation and extensive computer simulations. The approximate calculation is accomplished via separation into two subsystems connected at the defect by current conservation. There are four possible stationary phases ((LD, LD), (LD, HD), (HD, LD) and (HD, HD)) in the system for \( p < 1 \). The (HD, LD) phase can be seen as a maximal current phase in which the current is constant and dictated by the local defect. With the increase of \( p \), the (HD, LD) phase region shrinks, while the (LD, LD) and (HD, HD) phases regions expand. A phenomenological domain wall approach is developed to predict density profiles of the (LD, HD) phase. Density profiles and currents are simulated, and show very good agreement with theoretical calculations.

Our approach can be extended to investigate effects of a local defect in synchronous TASEPs with long-range hopping or extended objects. It is also interesting to study the dynamics of synchronous multiple TASEPs with junctions where junction points can be viewed as local defects.

Acknowledgements

Authors gratefully acknowledge the thoughtful comments and suggestions of anonymous reviewers for the improvement of the manuscript. R. Wang acknowledges the support of Massey University International Visitor Research Fund (2007) and Massey University Research Fund (2007). R. Jiang acknowledges the support of National Basic Research Program of China (2006CB 705500),
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