Phase transition and bistable phenomenon of granular flows down a chute with successive turnings

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ABSTRACT

This paper studies the granular flow down a chute with two successive turnings, which play the role of bottlenecks for the granular flow system and determine the granular flow state in main section between them. With the increase of main section width $D$, phase transition from dilute to dense granular flow is observed: When the main section width $D$ is small (large), the granular flow at upper (lower) bottleneck is dense and the granular flow is dilute (dense) in the main section. More interestingly, a bistable region is exhibited, in which either dilute flow or dense flow may occur and continue for the entire run. In this region, the packing in the reservoir will affect initial flow rate and then affect the flow pattern. This study can be viewed as a paradigm for the jamming and unjamming transitions under shear due to gravity.

1. Introduction

Granular system exhibits solid-like and fluid-like properties, but has distinct features when compared with classical solids and fluids. Its peculiar properties have attracted much attention from physical society. Granular system is intrinsically far from thermal equilibrium and therefore exhibits nonlinear instabilities such as force chains [1], solitons [2], surface waves [3–5], segregation [6–8], jamming [9–16], granular flow [17–20], granular clock [21,22], etc. Dynamical behavior of granular flow is one of the most challenging subjects in this field [23–25]. The granular flow can be normally classified into three states: dilute flow, dense flow and jammed, where the grains behave like gas, liquid and solid, respectively. The phase transitions among the three states have attracted intensive interest. For example, To et al. studied the jamming phenomenon of granular flow in a hopper [10].

In real transport of granular material in industrial, mining and agricultural processing, sometimes there are successive bottlenecks along the transport chute or pipe, such as turning, confined exit, reduction of chute width or pipe diameter, variation of chute or pipe inclination angle, and so on. Nevertheless, there are very few studies on the effect of successive bottlenecks in granular flow system. Hajra et al. studied granular mixing and segregation in zigzag chute flow [30]. Alonso-Marroquin et al. reported the flow rate across a bottleneck increases if an obstacle is placed before it [31]. A comprehensive study on the effects of successive bottlenecks is missing. Moreover, the jamming transition problem in shearing granular system is important and have attract much attention recently [15]. Our study can be a typical paradigm for the jamming and unjamming transitions with shear under gravity.

This paper focuses on the phase transition features in the granular chute flow system with successive bottlenecks. Varying the main section width $D$, the granular flow shows dilute to dense transition. In particular, a bistable phenomenon is observed that the flow can be either dilute or dense for the entire run. We find that the granular packing fraction in the hopper before flowing can be varying from 0.7 to 0.8. This behavior really affects the initial flow rate, and then affects the flowing behavior in the bistable region.
2. Experimental setup and discrete element simulation method

The experiments are carried out in a quasi-two-dimensional channel with an inclination angle of 30° (See Fig. 1). The channel is established with specially shaped glass spacers between a metal base and a glass cover plate. The gap between the base and the cover plate is kept at 3.0 mm to ensure a single-layer flow of stainless steel beads of diameter \( d_0 = 2.5 \pm 0.001 \) mm. The mass of a single steel bead is about 0.06 g. At the top of the channel, there is a hopper reservoir that stores steel beads. The hopper is connected to an entrance section with width \( D_1 = 50 \) mm and length \( L_1 = 500 \) mm. The entrance section connects with the main section via a turning (point A). The main section width \( D \) is adjustable, and its length \( L = 500 \) mm. The main section connects with an exit section with width \( D_2 = 20 \) mm and length \( L_2 = 200 \) mm via another turning (point B).

A thin plate is inserted at the exit of the hopper and the steel beads are poured into the hopper. Then the plate is pulled out very quickly and granular flows are initiated by allowing the steel beads to fall by gravity. The total mass \( M \) of the beads falling out of the exit is measured as a function of time \( t \) by an electronic balance with sensitivity of 0.1 g and a weighing period of 0.2 s. The flow rate \( j(t) \) is obtained by the slope of the recorded \( M(t) \) curve.

We carry out discrete element (DE) simulation on this problem. Since the system involves dense flow state, we adopt soft particle simulation with a spring-dashpot contact model. In this model, the normal forces \( F^n \) and tangential forces \( F^t \) of particle-particle and particle-wall collisions are formulated as:

\[
F^n = -k^n \Delta U^n + b^n \Delta V^n, \tag{1}
\]

\[
F^t = \min \{ k^t \Delta V^t \delta t, \mu F^n \}, \tag{2}
\]

where \( k^n \) and \( k^t \) are the normal and tangential spring stiffness coefficients, \( b^n \) is the normal viscous damping coefficient, \( \mu \) is the Coulomb friction coefficient, \( \Delta U^n \) is the overlap between the bodies in contact, \( \Delta V^n \) and \( \Delta V^t \) represent the relative normal and tangential velocities, \( \delta t \) is the simulation step time. The parameters for the discrete element simulation and the material properties are shown in Table 1.

For the sake of simplicity, we carry out two-dimensional simulation and treat the particles as disks. The collisions of particles with the base and the cover plate are not taken into consideration. Nevertheless, we select a much higher damping coefficient than reality in simulation (note that in simulation all parameters are normalized by particle diameter, density and gravity), so as to account for the energy dissipation.

3. Experiment observations and simulation

Fig. 2 shows typical flow patterns at two different main section widths \( D \). One can see that in the main section, the flow is dilute at \( D = 20 \) mm, while it becomes dense at \( D = 50 \) mm. When \( D = 20 \) mm, the granular flow at upper turning A is dense, but the granular flow at turning B is dilute. On the other hand, when \( D = 50 \) mm, the granular flow at lower turning B is dense. Thus the increment of \( D \) from 20 mm to 50 mm actually decreases the overall flow rate. This is different from the intuition that increasing the channel width will increase flow rate.

Fig. 3 shows typical time series of granular flow rate with different main section width \( D \). When \( D \) is small (Fig. 3(a)), the system is in a dilute flow state. When \( D \) is large (Fig. 3(b)), the granular flow is dense. In both cases, the flow rate fluctuates with time due to the forming and breaking of dynamic arches at the turning points. The higher fluxes in the time series correspond to the breaking of dynamic arches. An interesting bistable phenomenon emerges at \( D = 28 \) mm. As shown in Fig. 3(c) and (d), the granular flow can be either dilute or dense at \( D = 28 \) mm. Once dilute flow begins, the flow will remain dilute and there is a much higher flux. One question is that the dilute flow might be simply a transient state. To check the stability of the dilute flow state, we do the experiment for more than 30 minutes. The granular flow remains dilute. Thus we conclude that the dilute flow is stable.

We find that the bistable phenomenon actually depends on the initial flow rate. One can see that a dilute flow state is developed when the initial flow rate is high (Fig. 3(c)), while a dense flow state is developed if the initial flow rate is low (Fig. 3(d)). When the initial flow rate is high, the granular flow at upper turning A becomes dense first, with a discharge flux lower than the dilute-to-dense transition flux of B. As a result, B will not become dense, and the main section remains in dilute flow for the rest of the flowing. On the other hand, when the initial flow rate is low (but still larger than the dilute-to-dense transition flux of B), point A is not dense by the initial flow. When the initial flow reaches point B, point B becomes dense first. The densely flowing front propa-
Close observations show that the initial flow rate depends on the initial granular packing in the hopper. Although the particles are poured to the hopper randomly before flowing, there are minor differences in the microstructure of granular packing of the hopper. The packing near the exit can be either loose packing with a lower quasi-2D density (Fig. 4(a)) or crystal-alike packing with a higher quasi-2D density (Fig. 4(b)). In experiment, we find that the packing fraction can vary from 0.7 to 0.8. The packing density will affect the flow rate of granular flow [20]. In our experiment, when the packing is loose (crystal-alike), the initial flux will be high (low). The differences in the packing structure will affect the flow rate at the first moment of flowing, and then affect the flow pattern.

Fig. 5 shows the variation of mean flow rate $\langle J \rangle$ vs main section width $D$, from experiment and simulation. With the increase of $D$, the system’s behavior shows two curves: one for dilute flow state and the other for dense flow state. For the dilute flow, the system’s flow rate increase monotonically with $D$. Bistable phenomenon emerges in the region of $28 \text{ mm} \leq D \leq 30 \text{ mm}$ (experiment) and $27 \text{ mm} \leq D \leq 29 \text{ mm}$ (simulation). One can see that the simulation results are in qualitative agreement with experiment.

An interesting phenomenon is that the flow rate increases with $D$ in the dense flow state in the bistable region (see the lower curve of the bistable region). This behavior can be explained as follows. In the dense flow state, bottleneck B plays the main role. At this point, although the exit section width $D_2$ remains 20 mm, the effective flowing width of the turning point B increases with $D$, since the spheres are flowing from the upper-right direction of point B. This effect can lead to a lower probability of dynamic gates upwards to point A. Thus the main section is finally in dense flow state.
Fig. 6. (Color online.) Time serials of dense flow rate at $D = 28$ mm and $D = 30$ mm.

Fig. 7. (Color online.) Stable heap and free flowing layer. (a) Experiment snapshot ($D = 50$ mm) taken with a regular video camera with 1/30 s exposure time. The blurry part (upside of red dashed line) corresponds to flowing particles and the sharp part is static. (b) Illustration of the channel when $D = 40$ mm.

Fig. 8. (Color online.) Simulation results of mean flow rate $\langle J \rangle$ vs main section width $D$. The blue dashed lines are eye guidances for the bistable region. The data are obtained by averaging with 20 individual simulations.
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References