Pedestrian flow dynamics in a lattice gas model coupled with an evolutionary game

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This paper studies unidirectional pedestrian flow by using a lattice gas model with parallel update rules. Game theory is introduced to deal with conflicts that two or three pedestrians want to move into the same site. Pedestrians are either cooperators or defectors. The cooperators are gentle and the defectors are aggressive. Moreover, pedestrians could change their strategy. The fundamental diagram and the cooperator fraction at different system width W have been investigated in detail. It is found that a two-lane system exhibits a first-order phase transition while a multilane system does not. A microscopic mechanism behind the transition has been provided. Mean-field analysis is carried out to calculate the critical density of the transition as well as the probability of games at large value of W. The spatial distribution of pedestrians is investigated, which is found to be dependent (independent) on the initial cooperator fraction when W is small (large). Finally, the influence of the evolutionary game rule has been discussed.

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I. INTRODUCTION

In cities, there are large crowds of people in the pedestrian subway, passageway of the stadium, train station, and pedestrian street. The congestion of pedestrian flow influences critically people’s activities and capacity of these infrastructure facilities. Especially, it might cause serious trampling accidents. Therefore pedestrian flow attracts more and more attention in recent years [1,2]. Some interesting pedestrian dynamics have been observed, such as jamming and clogging, lane formation, “faster-is-slower” effect, etc. [1,2].

Main research methods of pedestrian flow include empirical or experimental study by means of video analysis [3–6] and simulations. The approach of simulation is based on pedestrian flow models. Most of the current research focuses on microscopic modeling since microscopic models are more flexible and efficient to simulate individual pedestrian motion and complicated interactions. The microscopic pedestrian flow models can be classified into social force model [1,7–9] and cellular automaton (CA) models [10–22]. While the social force model is continuous in space and time, CA models are discrete.

In a previous paper [23], we have studied pedestrian flow in a lattice gas model, which is a type of CA model. In that work, the parallel update rules have been adopted and the conflict that several pedestrians intend to move to the same site is solved simply by introducing probabilities. Specifically, when two or three pedestrians wish to move to the same site, one of them is randomly chosen with probability of 1/2 or 1/3 to move into the site; other pedestrians does not move.

In the real world, an evolutionary game usually occurs when conflict exists. In particular, the coupling between an evolutionary game with traffic dynamics has also been investigated in some previous literature. For instance, Perc has introduced the evolutionary game between neighboring agents in the Biham-Middleton-Levine (BML) model and he found that a traffic flow seizure is induced [24]. The authors have introduced the evolutionary game into random walk to study immigration behavior [25]. Ji et al. have studied conflict between pedestrians and vehicles in a signalized intersection [26] as well as conflict in pedestrian flow [27] by using a majority-based game. Recently, Tanimoto et al. [28] and Zheng and Cheng [29] have introduced game theory to study the evacuation process.

Inspired by the previous studies, we introduce the evolutionary game into a lattice gas model and study the coupling effect between evolutionary game and pedestrian flow dynamics. We have studied the fundamental diagram and the cooperator fraction at different system width W. An interesting behavior is observed: while the two lane system exhibits a first-order phase transition, the multilane system does not. We have presented mean-field analysis to calculate the probability of games. We also find that the spatial distribution of pedestrians depends on the initial cooperator fraction $f_{c0}$ when W is small, whereas it becomes independent of $f_{c0}$ when W is large.

The paper is organized as follows. The model is described in Sec. II. Simulation results and mean-field analysis are presented and discussed in Sec. III. Finally, we give conclusions in Sec. IV.

II. MODEL

The model is considered in a periodic unidirectional pedestrian flow system. It is defined on a square lattice of width W and length L. There are n pedestrians in the channel. The density $\rho$ is defined as $\rho = n/(W \cdot L)$. When a pedestrian arrives at the wall of the channel, it is reflected by the wall and never goes out through the wall.

For simplicity, the drift strength $D_f$ is set to 1 in our model (Here, $D_f$ represents a strength of bias to the right direction [12,23]). The situation $D_f < 1$ will be studied in future work. As a result, when the site ahead is empty, a pedestrian will try to move forward. Otherwise, (i) if both left and right sites are empty, the pedestrian will try to move to the left or right with probability of games at large value of W. The spatial distribution of pedestrians is investigated, which is found to be dependent (independent) on the initial cooperator fraction when W is small (large). Finally, the influence of the evolutionary game rule has been discussed.

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TABLE I. The payoff matrix of two-person game.

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>½</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>q</td>
</tr>
</tbody>
</table>

equal probability 0.5; (ii) if the left or the right site is empty, the pedestrian will try to move to the site; (iii) the pedestrian does not move.

If there is no conflict, the pedestrian could successfully move to the target site. However, when two or three pedestrians try to move to the same site, the conflict is solved by introducing a two-person and three-person game similar to the prisoner’s dilemma game [30–32] as follows.

Pedestrians in the channel are either cooperators (C) or defectors (D). Cooperators are gentle and defectors are aggressive. When two or three pedestrians try to move to the same site, a two-person or three-person game occurs. Tables I and II show the corresponding payoff matrix, which gives the moving probabilities of the pedestrians.

When two pedestrians intend to move to the same site, there are three cases: (i) if both are cooperators, one of them can move to the goal site with equal probability $\frac{1}{2}$; (ii) if one is a cooperator and the other is a defector, the defector moves and the cooperator does not move; (iii) if both are defectors, one of them can move with equal probability $q$, where $0 \leq q < \frac{1}{2}$.

When three pedestrians try to move to the same site, there are four cases: (i) if they are all cooperators, one of them can move to the goal site with equal probability $\frac{1}{3}$; (ii) if one is a defector, the other two are cooperators, the defector moves and the cooperators do not move; (iii) if one is a cooperator, the other two are defectors, the cooperator does not move and one of the defectors moves with equal probability $q$; (iv) if they are all defectors, one of them moves with equal probability $p$, where $0 \leq p < \frac{1}{3}$ and $p < q$.

Pedestrians changing their strategy in the evolution process is taken into account in our model as shown in Fig. 1. In two-person games, when a defector meets another defector, both change into cooperators. When a defector meets a cooperator, the defector remains unchanged, but the cooperator changes into a defector. Similarly, in three-person games, when three defectors meet, all of them change into cooperators. When a cooperator meets two defectors or a defector meets two cooperators, the defectors remain unchanged and the cooperators change into defectors.

III. RESULTS AND DISCUSSIONS

This section carries out the Monte Carlo simulations for pedestrian flow using the above model. Periodic boundary condition and random initial distribution are employed. The parameter $q$ is set to 0.3, $p$ is set to 0.2. The length of the channel $L$ is set to 2000.

Figure 2 shows the fundamental diagrams (FDs) of the system at different values of system width $W$, in which the initial fraction of cooperators $f_{c0} < 1$. We would like to mention that the FDs are independent of $f_{c0}$ provided $f_{c0} < 1$.

In the FDs, the flow rate is defined as the average number of pedestrians passing through the location $x = L/2$ in one time step divided by $W$.

As expected, the FDs consist of two branches. When the density is smaller than a critical density $\rho_{cr}$, the system is in free flow and every pedestrian can move forward with velocity 1. When the density exceeds $\rho_{cr}$, the system is in congested flow. On the congested branch, the flow rate decreases continuously with the increase of density when system width $W > 2$. Moreover, with the increase of $W$, the congested branch gradually lowers until $W$ reaches a large value ($W \approx 100$). This can be understood as follows. Since on the wall side, no pedestrian competes for the space ahead of a pedestrian, pedestrians could move faster along the wall. With the increase of $W$, the ratio of pedestrians that move along the wall decreases. This lowers the average velocity and consequently lowers flow rate.

FIG. 1. (Color online) Schematic illustration of how pedestrians change strategy. A blue full circle labeled C indicates a cooperator and a red full circle labeled D indicates a defector. (a) and (b) show the situation for the two-person game and three-person game, respectively.

FIG. 2. (Color online) Plots of flow rate against the density for different system width $W$. Here, the initial cooperators fraction $f_{c0} < 1$.
However, when $W = 2$, a qualitatively different phenomenon is observed on the congested branch: a sudden drop of flow rate occurs at $\rho'_c \approx 0.674$. An intuitive explanation of the difference is that there is no three-person game when $W = 2$.

Now we investigate the fraction of cooperators $f_c$, which is the order parameter in the first-order phase transition as shown below, in the stationary state. When the density $\rho < \rho_c$, $f_c$ depends on $f_{c0}$. Nevertheless, the flow rate is independent of $f_c$ because every pedestrian moves with velocity 1. Therefore next we focus on the situation $\rho > \rho_c$ and the results are shown in Fig. 3, which are independent of $f_{c0}$ provided $f_{c0} < 1$. It can be seen that while $f_c$ changes continuously with the system density $\rho$ when $W > 2$, a first-order phase transition of $f_c$ exists at $\rho'_c$ when $W = 2$. This clarifies why the drop of flow rate occurs when $W = 2$.

For a better understanding of the role of a three-person game in the first-order phase transition, we design a special four-lane model as shown in Fig. 4. In this model, a fence with some “holes” is placed at the center of the four-lane channel. The fence is like a wall. The pedestrians cannot cross the fence. In contrast, the pedestrians can cross through the holes. The hole fraction (number of holes divided by system length $L$) is denoted as $\alpha$. When $\alpha = 0$, the special four-lane channel reduces to two separated two-lane channels. When $\alpha = 1$, the special four-lane model becomes an ordinary four-lane model defined above. Figure 5 shows that the first-order phase transition vanishes gradually with the increase of $\alpha$, which demonstrates that increasing the three-person games does lead to disappearance of first-order phase transition.

To explore the microscopic mechanism behind this transition, we study a small system $W \times L = 2 \times 2$. Considering the periodic boundary and symmetry, there are eight configurations when $\rho = \frac{3}{4}$, as shown in Fig. 6. In the case $f_{c0} < 1$, only configuration ⑦ evolves into $f_c = 1$. The other six configurations constitute a closed cycle, and the evolution among each other is shown in Fig. 6. Thus when starting from the six configurations, $f_c = 1$ could not be reached. When $L$ is large, although it will become much more complex, one can imagine that a similar evolution process still could be observed. When $\rho > \rho'_c$, the initial random configurations constitute a closed cycle, except that an infinite small ratio of initial configurations evolve into $f_c = 1$. Therefore $f_c = 1$ could not be reached. In contrast, if $\rho < \rho'_c$, such a closed cycle is broken, and $f_c = 1$ becomes a unique attractor. We have tested the case $L = 3$. It is found that a closed cycle
exists when $\rho = 5/6$ and it is broken when $\rho = 4/6$.\textsuperscript{1} We also would like to mention that on the other hand, the closed cycle will never be broken when $W > 2$. Therefore there is no first-order phase transition.

We make an approximation analysis of the critical density $\rho_{cr}$, through calculating the probability of a two-person game. We would like to mention that at $\rho = \rho_{cr}$, although there is a sudden drop of $f_{cr}$, the probability of a two-person game changes almost continuously (our simulation shows that there is only a slight drop, which might be due to finite-size effect). In the density range $\rho < \rho_{cr}$ and $\rho > \rho_{cr}$, the probability is calculated in a different way. The critical density $\rho_{cr}$ is thus determined by matching the probability in the two ranges. When $\rho < \rho_{cr}$, there are $(1 - \rho)WL$ empty sites and $\rho WL$ pedestrians; on average there are $\rho WL - (1 - \rho)WL = (\rho - 0.5)WL$ conflicts in each time step.

When the density $\rho > \rho_{cr}$, the conflicts can be approximated by $\rho^2(1 - \rho)WL$ if ignoring the correlation. Here $1 - \rho$ means that site $S_1$ is empty, $\rho^2$ means that sites $S_2, S_3, S_4$ are occupied, with site $S_2$ upstream of site $S_1$, site $S_3$ neighboring site $S_1$ (on the other lane), site $S_4$ downstream of site $S_1$. Therefore the critical density $\rho_{cr}$ is calculated by $\rho - 0.5 = \rho^3(1 - \rho)$, which yields $\rho_{cr} \approx 0.583$. This underestimates the critical density due to neglecting the correlation. More efforts are needed to calculate $\rho_{cr}$ more accurately in future work.

To investigate the probabilities of two-person and three-person games at large values of $W$, we carry out a simple mean-field analysis. Figure 7 shows the sites to be considered in mean-field analysis. Part (I) corresponds to the probability of a two-person game that two lateral pedestrians attempt to move to the empty site 2. In this case, site 1 is empty. In part (I), term $a$ means that sites 1, 4, and 6 are occupied and site 2 is empty; term $b$ indicates that site 8 is occupied and site 3 is empty, the coefficient 2 corresponds to the symmetry of sites; term $c$ indicates that site 8 and site 3 are empty; term $d$ means that sites 1, 4, and 6 are occupied and site 2 is empty; term $e$ indicates that site 8 and site 3 are occupied and site 5 is empty; term $f$ corresponds to that site 8 and site 5 are empty and site 7 is empty; term $g$ indicates that site 8 and site 7 are empty and site 3 and site 5 are occupied. In part (II), term $h$ means that sites 1 and 2 are empty and sites 3, 4, 5, 6 are occupied; term $i$ corresponds to that site 8 and site 7 are empty; term $j$ corresponds to that site 8 and site 7 are occupied; term $k$ corresponds to that one of site 8 and site 7 is occupied and the

\[
P_{2g} = \frac{\rho^3(1 - \rho)}{\rho(1 - \rho) + \rho(1 - \rho) + 2(1 - \rho)(1 - \rho)} \\
+ 2 \frac{1}{2} \rho \rho(1 - \rho) + 2 \frac{1}{2} (1 - \rho)(1 - \rho) \\
+ \frac{1}{2} \cdot \frac{1}{2} (1 - \rho)(1 - \rho) \\
+ \frac{\rho^2(1 - \rho)}{\rho(1 - \rho) + 2(1 - \rho)(1 - \rho)} \\
+ \rho \rho + 2 \frac{1}{2} \rho(1 - \rho) \\
\]

Here, part (I) corresponds to the probability of a two-person game that a forward moving pedestrian and a lateral moving pedestrian try to move to the empty site 2. In this case, site 1 is occupied. Part (II) corresponds to the probability of a two-person game that two lateral pedestrians attempt to move to the empty site 2. In this case, site 1 is empty. In part (I), term $a$ means that sites 1, 4, and 6 are occupied and site 2 is empty; term $b$ indicates that site 8 is occupied and site 3 is empty, the coefficient 2 corresponds to the symmetry of sites; term $c$ indicates that site 8 and site 3 are empty; term $d$ means that sites 1, 4, and 6 are occupied and site 2 is empty; term $e$ indicates that site 8 and site 3 are occupied and site 5 is empty; term $f$ corresponds to that site 8 and site 5 are empty and site 7 is empty; term $g$ indicates that site 8 and site 7 are empty and site 3 and site 5 are occupied. In part (II), term $h$ means that sites 1 and 2 are empty and sites 3, 4, 5, 6 are occupied; term $i$ corresponds to that site 8 and site 7 are empty; term $j$ corresponds to that site 8 and site 7 are occupied; term $k$ corresponds to that one of site 8 and site 7 is occupied and the

\[
\text{FIG. 7. Sketch of the sites considered in mean-field analysis. The above arrow shows the moving direction of pedestrians.}
\]

\[\text{FIG. 8. (Color online) Probability of two-person game and three-person game versus density. Here the system width } W = 100.\]
other is empty. Similarly, we can also obtain the probability of a three-person game,

\[
P_{3g} = \rho^5 (1 - \rho)^3 + 2 \frac{1}{2} \rho^3 (1 - \rho)^2 + \frac{1}{2} \cdot \frac{1}{2} (1 - \rho)^2 (1 - \rho)\]  

(2)

Here, term \(l\) means that sites 1, 3, 4, 5, and 6 are occupied and site 2 is empty; term \(m\) indicates that sites 7 and 8 are both occupied; term \(n\) corresponds to that one of site 8 and site 7 is occupied and the other is empty; term \(o\) indicates that sites 7 and 8 are both empty. Figure 8 shows the probability of a two-person game and a three-person game versus density. It can be seen that the mean-field result of \(P_{3g}\) is qualitatively consistent with the simulation result, but underestimates the maximum value. The mean-field result of \(P_{2g}\) also underestimates the maximum value. Moreover, it overestimates when the density \(\rho < 0.5\). This is because the correlation is strong in this density range (for instance, there is no interaction among pedestrians when \(\rho < \rho_{cr}\)).

It can be seen from Fig. 3 that the cooperator frequency \(f_c\) is close to 0.5 at large values of \(W\). Therefore by ignoring the correlation we can estimate probabilities of different kinds of games as follows: \(\frac{1}{4} P_{2g}\) for the \(CC\) game, \(\frac{1}{4} P_{2g}\) for the \(CD\) game, \(\frac{1}{4} P_{2g}\) for the \(DD\) game, \(\frac{1}{4} P_{3g}\) for the \(CCC\) game, \(\frac{1}{3} P_{3g}\) for the \(CCD\) game, \(\frac{1}{3} P_{3g}\) for the \(CDD\) game, \(\frac{1}{3} P_{3g}\) for the \(DDD\) game. These approximation results are plotted in Figs. 9 and 10. It can be seen that the approximation results are in good agreement with simulations in the cases of \(CD\), \(CCD\), and \(CDD\) games (it overestimates when \(\rho < 0.5\) in the case of the \(CD\) game as mentioned before), but underestimate the maximum value of the \(CC\), \(DD\), \(CCC\), and \(DDD\) games.

FIG. 9. (Color online) Probability of three kinds of two-person games versus density. Here the system width \(W = 100\).

FIG. 10. (Color online) Probability of four kinds of three-person games versus density. Here the system width \(W = 100\).
Figure 3 also shows that (i) when $W > 2$, $f_c$ decreases with the increase of $W$. Moreover, (ii) when $W$ is small (e.g., $W = 3$), $f_c$ increases monotonically with the density. However, (iii) when $W$ is large, $f_c$ reaches minimum at intermediate density. These are believed to be related to the times of three-person games, denoted by $r$. Generally speaking, $f_c$ decreases with the increase of $r$, which is explained as follows. If ignoring the correlation, while two-person games turn three cooperators into four cooperators [Fig. 1(a)], three-person games keep the cooperators number, which is 6, unchanged [Fig. 1(b)]. Namely, two-person games increase $f_c$ and three-person games do not influence $f_c$. By considering the correlation, it is logical to believe that two-person games tend to increase $f_c$, and three-person games tend to decrease $f_c$.

At a given density, it can be easily understood that $r$ increases with the increase of $W$, which explains item (i). When $W$ is large, $r$ is quite large at intermediate density (see Fig. 8), which lowers $f_c$. When $W$ is small, $r$ is relatively small even at intermediate density and is not able to lower $f_c$. This explains items (ii) and (iii).

We study the evolution behavior of defector ratio $f_s = 1 - f_c$ in the vicinity of $\rho'_{cr}$. As shown in Fig. 11, when $\rho > \rho'_{cr}$, $f_d$ will finally remain a constant value after a transient time. However, when $\rho < \rho'_{cr}$, $f_d$ decays exponentially after a transient time. This system behavior is exactly the same as observed in a random walk process coupled with an evolutionary game, which thus provides another evidence for the argument that "while the second-order phase transition produces power-law decay, the first-order phase transition does not."\(^2\)

Next we consider the case $f_o = 1$. Figure 12 shows the difference of flow rate between $f_o = 1$ and $f_o < 1$. It can be seen that the difference is nontrivial. When $W$ is small, the difference quickly decreases with $W$ provided $\rho$ is not close to 1. However, with the increase of $W$, it becomes slowly decreasing with $W$. When $W \gtrsim 100$, the difference remains unchanged with $W$.

We argue that the flow rate difference has two contributions: (1) the first contribution is obvious, i.e., the defectors and the cooperators have different moving probabilities; (2) the other contribution is that spatial distribution of the pedestrians changes between $f_o = 1$ and $f_o < 1$. To demonstrate this, we study the spatial distribution of the pedestrians by investigating the distribution of the number of neighbors of a pedestrian. Here the neighbors are defined as Moore neighbors. Figure 13 compares such distributions between $f_o = 1$ and $f_o < 1$. While there is remarkable difference between the distributions when $W$ is small [see, e.g., Figs. 13(a) and 13(b)], there is no notable difference when $W$ is large [see, e.g., Fig. 13(c)]. As a result, when $W$ is small, contribution (2) exists. Thus the flow rate difference is quite large. Since contribution (2) quickly decreases with the increase of $W$, the flow rate difference

\(^2\)For more details, please refer to Refs. [25,33]. We would like to mention that presently the argument is a conjecture, which needs rigorous proof.

FIG. 11. (Color online) Semilog plot of $f_d$ against the time for different values of $\Delta = \rho - \rho'_{cr}$. The critical density $\rho'_{cr} = 0.674$. From top to bottom, $\Delta$ is 0.002, 0.000, $-0.002$, and $-0.004$.

FIG. 12. (Color online) Plots of difference of flow rate between $f_o = 1$ and $f_o < 1$ for different system width $W$.

FIG. 13. (Color online) Probabilities histogram of the number of Moore neighbors between $f_o = 1$ and $f_o < 1$ for system width (a) $W = 2$, (b) $W = 3$, (c) $W = 100$. The density (a) $\rho = 0.7$; (b),(c) $\rho = 0.5$. In (b) and (c), the histogram is based on the statistic of pedestrians away from the wall.
also quickly decreases. When $W$ is large, contribution (2) disappears and the flow rate difference is completely due to contribution (1), thus it becomes independent of $W$.

Finally, we discuss the rule in the evolutionary game presented in Fig. 1. Our rule is very simple, in which $DD$ simply changes into $CC$ and $DDD$ also suddenly changes to $CCC$. We have also studied the situation that when two or three $D$ meet, each changes into $C$ with probability $\beta < 1$. The results at $W = 2$ are shown in Fig. 14. One can see that $\rho_c$ decreases with the decrease of $\beta$, and the first-order phase transition finally disappears. When $W > 2$, the results are qualitatively unchanged with the decrease of $\beta$. In future work, one needs to take into account more realistic evolutionary game rules (e.g., in Refs. [34–36]) and investigate their influence on the pedestrian flow dynamics.

IV. CONCLUSION

This paper has studied the unidirectional pedestrian flow using a lattice gas model with parallel update rules. To solve the conflict that several pedestrians try to move to the same site, we introduce an evolutionary game. An interesting first-order phase transition is observed in the two-lane system. Before the transition, the defector fraction $f_d$ decays exponentially while it remains constant above the transition. This transition behavior is exactly the same as observed previously in a different system [25]. However, this phase transition does not exist in multilane system. By considering a small system, we have provided a microscopic mechanism of the transition. Moreover, we have carried out a simple mean-field analysis to investigate the probability of the games, which are consistent with the simulations to some extent.

We have also studied the spatial distribution of pedestrians. It is found that when the system width $W$ is small, the spatial distribution depends on the initial defector fraction $f_{c0}$: there is a remarkable difference between $f_{c0} = 1$ and $f_{c0} < 1$. However, this difference disappears when $W$ is large. As a result, the flow rate difference between $f_{c0} = 1$ and $f_{c0} < 1$ is completely due to that the defectors and the cooperators have different moving probabilities and the difference becomes independent of $W$.

Finally, we have briefly discussed the influence of the evolutionary game rules. It is shown that changing the rules might lead to qualitatively different results (e.g., decreasing $\beta$ leads to disappearance of the first-order phase transition in the case $W = 2$). Therefore in our future work a more realistic evolutionary game will be introduced to study unidirectional pedestrian flow and counterflow. Moreover, empirical data need to be collected for verification.

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