Traffic flow characteristics in a mixed traffic system consisting of ACC vehicles and manual vehicles: A hybrid modelling approach

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\section*{A B S T R A C T}

In this paper, we have investigated traffic flow characteristics in a traffic system consisting of a mixture of adaptive cruise control (ACC) vehicles and manual-controlled (manual) vehicles, by using a hybrid modelling approach. In the hybrid approach, (i) the manual vehicles are described by a cellular automaton (CA) model, which can reproduce different traffic states (i.e., free flow, synchronised flow, and jam) as well as probabilistic traffic breakdown phenomena; (ii) the ACC vehicles are simulated by using a car-following model, which removes artificial velocity fluctuations due to intrinsic randomisation in the CA model. We have studied the traffic breakdown probability from free flow to congested flow, the phase transition probability from synchronised flow to jam in the mixed traffic system. The results are compared with that, where both ACC vehicles and manual vehicles are simulated by CA models. The qualitative and quantitative differences are indicated.

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1. Introduction

Although it is still not feasible for fully automated vehicles to operate autonomously in a highway environment, recent advances in technology have made it a more near-term objective for partially or semiautomated vehicles designed to operate with manually driven vehicles in highway traffic. Adaptive cruise control (ACC) vehicles are the first step toward that direction. ACC is a driver assistance system designed for driver's comfort and safety. Through equipping with a forward-looking sensor, an ACC vehicle can detect the range (distance) as well as the range rate (relative speed) to its preceding vehicle accurately and instantaneously. Thus in an ACC vehicle, the delay due to driver reaction time is eliminated and a control system automatically adjusts the vehicle speed to keep the vehicle at the desired headway to the preceding vehicle. Due to their different behavior with manual vehicles, the introduction of ACC vehicles will influence the characteristics of traffic flow, including traffic safety, highway capacity, traffic stability and so on. Therefore, before ACC vehicles are deployed on a large scale, their impact on the characteristics of traffic flow need to be investigated carefully.

Many works have been done to assess the effects of ACC vehicles on highway traffic [1–30]. The headway policy of ACC vehicles has been investigated in Ref. [2–4]. The effect of ACC vehicles on highway safety has been studied in Ref. [5–7]. Vanderwerf et al. reported the impact of ACC vehicles on highway capacity [8]. Environmental benefits have been found by Zhang and Ioannou [9] due to reduced exhaust emissions. Moreover, the effects of ACC vehicles on traffic flow stability have been widely studied [6,7,10–13]. Treiber et al. [14,15] reported that if 10% of vehicles were equipped with ACC, the additional travel time due to traffic jams could be reduced by more than 80%. Kerner [16] found that ACC vehicles can suppress a wide moving jam and thus promote stability. However, he also pointed out that in some cases ACC vehicles

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could induce congestion at bottlenecks. Ioannou and Stefanovic also reported unfavorable effects of ACC vehicles [17]. They found unwanted cut-ins in mixed traffic due to larger gaps in front of ACC vehicles. Davis [18] showed that ACC vehicles can suppress jams as well by making the flow string stable. He also proposed a cooperative merging for ACC vehicles to improve throughput and reduce travel time [19].

As is well known, traffic flow can be classified into free flow and congested flow. Moreover, the probabilistic traffic breakdown phenomenon from free flow to congested flow has been observed at different locations around the world. Recently, Kerner [31] found out that congested flow can be further classified into synchronised flow and jam. A probabilistic phase transition phenomenon from synchronised flow to jam is also observed. However, these phenomena could not be well reproduced by car-following models.

On the other hand, the three traffic states (i.e., free flow, synchronised flow, and jam) as well as probabilistic phase transition among them could be well reproduced in several cellular automaton (CA) models. Therefore, the authors have investigated traffic flow characteristic in a mixed traffic system, in which both ACC vehicles and manual vehicles are simulated by CA models [20,21]. Nevertheless, due to the intrinsic randomisation in the CA model, it is found that artificial velocity fluctuations exist in traffic system composed of 100% ACC vehicles. This is not consistent with real traffic features of ACC vehicles.

Based on these facts, we propose a hybrid modelling approach to study traffic flow characteristics in a mixture of ACC vehicles and manual vehicles. In the hybrid approach, the manual vehicles are simulated by a CA model, and the ACC vehicles are simulated by the car-following model. As a result, the deficiencies in previous works can be removed.

The paper is organised as follows. In the next section, the CA model used to simulate manual vehicles and the car-following model used to simulate ACC vehicles are briefly reviewed. Traffic flow characteristics of the mixed traffic system are presented and discussed in Section 3. Finally, conclusions are given in Section 4.

2. Hybrid modelling approach

In this section, we briefly review the CA model for manual vehicles and the car-following model for ACC vehicles. Here, the CA model used to simulate manual vehicles is the modified comfortable driving (MCD) model [22], which can reproduce the three traffic states as well as the probabilistic phase transition among them. The updating rules of the model are as follows.

1. Determination of the randomisation parameter $p_n(t + 1)$:

$$ p_n(t + 1) = p(v_n(t), b_{n+1}(t), t_{h,n}, t_{s,n}) $$

2. Acceleration:

- if $(b_{n+1}(t) = 0$ or $t_{h,n} ≥ t_{s,n}$ and $(v_n(t) > 0))$ then: $v_n(t + 1) = \min(v_n(t) + 2, v_{\text{max}})$
- else if $(v_n(t) = 0)$ then: $v_n(t + 1) = \min(v_n(t) + 1, v_{\text{max}})$
- else: $v_n(t + 1) = v_n(t)$

3. Braking rule:

$$ v_n(t + 1) = \min(d_n^{\text{eff}}, v_n(t + 1)) $$

4. Randomisation and braking: if $(\text{rand}() < p_n(t + 1))$ then: $v_n(t + 1) = \max(v_n(t + 1) - 1, 0)$

5. The determination of $b_{n+1}(t + 1)$:

- if $(v_n(t + 1) > v_n(t))$ or $(v_n(t + 1) ≥ v_c$ and $t_{f,n} > t_{c})$ then: $b_{n+1}(t + 1) = 0$
- else if $(v_n(t + 1) < v_n(t))$ then: $b_{n+1}(t + 1) = 1$
- else if $(v_n(t + 1) = v_n(t))$ then: $b_{n+1}(t + 1) = b_n(t)$

6. The determination of $t_{st,n}$:

- if $v_n(t + 1) = 0$ then: $t_{st,n} = t_{st,n} + 1$
- else: $t_{st,n} = 0$

7. The determination of $t_{f,n}$:

- if $v_n(t + 1) ≥ v_c$ then: $t_{f,n} = t_{f,n} + 1$
- else: $t_{f,n} = 0$

8. Vehicle motion: $x_n(t + 1) = x_n(t) + v_n(t + 1)$.

Here $d_n$ is the gap between vehicle $n$ and the preceding vehicle $n + 1$, $b_n$ is the status of the brake light (on/off $→ b_n = 1(0)$). The two times $t_{h,n} = d_n/v_n(t)$ and $t_{s,n} = \min(v_n(t), h)$, where $h$ determines the range of interaction with the brake light, are introduced to compare the time $t_{h,n}$ needed to reach the position of the preceding vehicle with a velocity dependent interaction horizon $t_{s,n} = d_n/v_n(t) = \max(v_{\text{anti}} - \text{gap}_{\text{safety}}, 0)$ is the effective distance, where $v_{\text{anti}} = \min(d_{n+1}, v_{n+1})$ is the anticipated velocity of the preceding vehicle in the next time step and gap_{safety} adjusts the effectiveness of the anticipation. rand() is a random number between 0 and 1, $t_{st,n}$ denotes the time that the vehicle $n$ stops and $t_{f,n}$ denotes the time that the vehicle $n$ in the state $v_n ≥ v_c$. The randomisation parameter $p$ is defined as follows:

$$ p(v_n(t), b_{n+1}(t), t_{h,n}, t_{s,n}) = \begin{cases} p_b : & \text{if } b_{n+1}(t) = 1 \text{ and } t_{h,n} < t_{s,n} \\ p_0 : & \text{if } v_n(t) = 0 \text{ and } t_{st,n} ≥ t_c \\ p_d : & \text{in all other case.} \end{cases} $$
In the car-following model for ACC vehicles, the constant time headway (CTH) policy is adopted, and the dynamics of the ACC vehicle \( n \) (vehicle \( n \) follows vehicle \( n + 1 \)) can be described by the following equation:

\[
\frac{dv_n(t)}{dt} + v_n(t) = V(\Delta x_n(t), \Delta v_n(t))
\]

where \( \Delta x_n(t) = x_{n+1}(t) - x_n(t) \) denotes the distance between vehicle \( n \) and its preceding vehicle. This quantity is the range (including vehicle length) and the range rate is \( \Delta v_n(t) = v_{n+1}(t) - v_n(t) \). Vehicle response is modelled by first-order dynamics with a time constant \( \tau \). The desired velocity is specified as \( V = \min\left\{ \frac{1}{H_d} (\Delta x_n(t) - D) + \beta \Delta v_n(t), v_{\text{max}} \right\} \), which is taken from the desired headway according to the CTH policy \( \Delta_n^{\text{desired}}(t) = H_d v_n(t) + D \), where \( H_d \) is the headway time preferred by ACC vehicles and \( D \) is a constant length which is equal to or slightly larger than the length of a vehicle. Note that the acceleration and deceleration in the model for ACC vehicles (car-following model) is determined by Eq. (1), which might result in large value that exceeds the physical limits. However, if we impose the physical limits of acceleration and deceleration on the ACC vehicles, a crash may occur because large decelerations also exist in the CA model for manual vehicles. Therefore, a hybrid model that considers limited acceleration and deceleration in both the car-following model and the CA model needs to be investigated in our future work.

In the hybrid modelling approach, one time step in CA model corresponds to 1 s and the time step used in car-following model is set to 0.1 s. From time step \( t \rightarrow t + 1 \), the updating process of the hybrid modelling approach is as follows. First, the velocities and positions of manual vehicles are updated. Then the ACC vehicles are updated according to the dynamics described in Eq. (1). The update of ACC vehicles will be executed ten times. If the preceding vehicle of an ACC vehicle \( n \) is a manual vehicle, then the velocity of the manual vehicle is assumed to be a constant \( v_{n+1}(t) \) from \( t \) to \( t + 1 \), and the positions of the manual vehicle between \( t \) and \( t + 1 \) are obtained by linear interpolation. Finally, it also should be stressed that the status of the brake lights of ACC vehicles are determined to be the same as manual vehicles as follows in order to guarantee the update of a manual vehicle following an ACC vehicle. If the time that the ACC vehicle in the state \( v_n \) exceeds \( v_{c1} \) or the vehicle accelerates in the updating time step (the time step for ACC vehicles), its brake light is set on. Otherwise if it decelerates in the updating time step, its brake light is set off. Or if it does not change velocity, its brake light does not change either.

### 3. Simulation results

In this section, the simulation results are presented and discussed. In the simulations, we assume the vehicles are driving on a circular road with length \( L = 15 \) km, which corresponds to 10000 cells. The length of a vehicle is 7.5 m, which corresponds to 5 cells. The parameters in the car-following model are as follows: \( \tau = 0.5 \) s, \( v_{\text{max}} = 30 \) m/s, \( D = 7.5 \) m. We set \( \beta = \tau / H_d \) to guarantee the traffic stability as pointed out in Ref. [18]. The parameters in MCD model are as follows: \( t_c = 9 \) s, \( t_{c1} = 30 \) s, \( v_c = 18 \) cells/s, \( v_{\text{max}} = 20 \) cells/s, \( p_4 = 0.25 \), \( p_5 = 0.94 \), \( p_0 = 0.5 \), \( h = 6 \) and \( \text{gap}_{\text{safety}} = 7 \) cells.

First, we discuss traffic flow characteristics of 100% manual vehicles and that of 100% ACC vehicles. Then we present traffic flow characteristics of a mixed traffic system. For a convenience of comparison, some results obtained from the situation that both ACC vehicles and manual vehicles are simulated by CA models (referred to as CA modelling approach) are also presented.

Fig. 1 shows the fundamental diagram of 100% manual vehicles. The curves are divided into three branches by four critical densities: free flow branch \( (0 < \rho < \rho_{c2}) \), synchronised flow branch \( (\rho_{c1} < \rho < \rho_{c4}) \) and jam branch \( (\rho_{c3} < \rho < \rho_{\text{jam}} = 133.33 \) veh/km). We denote the flow rate corresponding to \( \rho_{c2} \) as \( q_{\text{max}} \), which is the maximum flow rate that could be reached in free flow.

Fig. 2 shows the probabilistic traffic breakdown curve as well as the probabilistic transition curve from synchronised flow to jam. The probability is obtained as follows. Initially, the vehicles are homogeneously distributed on the road. Then a large number of runs of the same duration \( T_0 = 6000 \) s are carried out for a given density. For each run it is checked whether traffic breakdown (or transition from synchronised flow to jam) occurred within the given time interval \( T_0 \) or not. We record the number of realisations \( N_p \) where traffic breakdown (or transition from synchronised flow to jam) has occurred. Thus \( P = N_p / N_0 \) is the approximate probability that traffic breakdown (or transition from synchronised flow to jam) occurs during the time interval \( T_0 \) at the given density. In this paper, \( N_0 = 10000 \) is adopted.

The traffic breakdown probability is zero when the density is smaller than \( \rho_{c1} \) and reaches one when the density is larger than \( \rho_{c2} \). When \( \rho_{c1} < \rho < \rho_{c2} \), the traffic breakdown probability increases with the density. Similarly, the transition probability from synchronised flow to jam is zero when \( \rho < \rho_{c3} \) and reaches one when \( \rho > \rho_{c4} \). When \( \rho_{c3} < \rho < \rho_{c4} \), the transition probability also increases with the density. Fig. 3 shows the typical spatiotemporal patterns corresponding to the traffic breakdown and the transition from synchronised flow to jam.

The fundamental diagrams of 100% ACC vehicles (i.e., \( R = 1 \)) at different values of \( H_d \) are also shown in Fig. 1, where \( R \) denotes the ratio of ACC vehicles. The triangular curves of flow rate versus density are obtained as the CTH policy is adopted. It can be seen that both the maximum flow rate and the corresponding density decrease with an increase of \( H_d \).

Comparing the fundamental diagram of \( R = 1 \) and \( R = 0 \), one can see that the maximum flow rate for \( H_d = 1 \) is larger than \( q_{\text{max}} \), while that for \( H_d = 1.5 \) and 2.2 are smaller than \( q_{\text{max}} \). Moreover, the congested branches for \( H_d = 1 \) and \( H_d = 1.5 \)
Fig. 1. Fundamental diagram of 100% ACC vehicles at different values of $H_d$ (black lines) and of 100% manual vehicles (red lines). F, S, J mean the free flow branch, synchronised flow branch and jam branch of 100% manual vehicles, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 2. Probability of traffic breakdown (left curve) and transition probability from synchronised flow to jams (right curve) for 100% manual vehicles.

are above the jam branch of 100% manual vehicles while the congested branch for $H_d = 2.2$ is below the jam branch of 100% manual vehicles.

Now we study the mixed traffic system. First, we consider the traffic breakdown probability, which is shown in Fig. 4. One can see that:

(i) For $H_d = 1$, the traffic breakdown curve moves to the right with increasing $R$. This means that traffic breakdown occurs with more difficulty, i.e., ACC vehicles can enhance the stability of free flow. This result is qualitatively similar to that of the CA modelling approach (see Fig. 5(a)). Nevertheless, one can see that quantitatively, traffic breakdown occurs with more difficulty in our hybrid modelling approach (the traffic breakdown curves are to the right of those in Fig. 5(a) at a given $R$). This, might be due to artificial velocity fluctuations of ACC vehicles that are removed by using car-following model.

(ii) For $H_d = 1.5$, the traffic breakdown curve first moves to the right with increasing $R$. When $R \geq 0.6$, the curve begins to move to the left. This result is opposite to that of the CA modelling approach (see Fig. 5(b)), in which the traffic breakdown curve first moves to the left and then moves to the right. Although we believe the result from the hybrid modelling approach should be correct because there are no artifical velocity fluctuations of ACC vehicles, more investigations are needed for verification.

(iii) For $H_d = 2.2$, the traffic breakdown curve moves to the left with increasing $R$. This means that traffic breakdown occurs more easily, i.e., ACC vehicles weaken the stability of free flow. This result is also different from that of the CA modelling approach (see Fig. 5(c)). In that case, the traffic breakdown curve is essentially independent of $R$ when $R \geq 0.1$. 
Fig. 3. Typical spatiotemporal patterns of (a) traffic breakdown and (b) transition from synchronised flow to jams for 100% manual vehicles. In (a) $\rho = 22$ veh/km; (b) $\rho = 40$ veh/km. The vehicles are moving from left to right and the time is increasing in the up direction.

Fig. 4. Traffic breakdown probability for a mixture of ACC vehicles and manual vehicles. (a) $H_d = 1$; (b) $H_d = 1.5$; (c) $H_d = 2.2$.

(iv) The incorrect effect caused by artificial velocity fluctuations in the CA modelling approach is not observed in our hybrid modelling approach. This can be clearly seen by comparing Fig. 6(a) and Fig. 6(b).

(v) For each $R$, the probability curve for $H_d = 1$ is always rightmost and that for $H_d = 2.2$ is always leftmost (see Fig. 7). It means that with increasing $H_d$, the stability of free flow in mixed traffic is gradually weakened. This is because with increasing $H_d$, the average time headway of manual vehicles decreases for a given average density. As a result, traffic breakdown becomes easier.

Next, we consider the transition probability from synchronised flow to jam, which is shown in Fig. 8. One can see that

(i) For each $H_d$, it is found that a jam will not spontaneously appear from synchronised flow when $R$ exceeds a critical value $R_c$. In this case, even if a jam is induced (e.g., due to a car accident), the jam will soon dissolve (see Fig. 9). With increasing $H_d$, the value of $R_c$ decreases. For $H_d = 1, 1.5, 2.2$, the corresponding values of $R_c$ are about 0.3, 0.2 and 0.1 respectively.

(ii) With a fixed $R$, we found that the probability curve for $H_d = 1$ is leftmost and the probability curve for $H_d = 2.2$ is rightmost (see Fig. 8(c)). It means that with increasing $H_d$, the transition from synchronised flow to jam is more difficult to occur. This effect is opposite to that corresponding to traffic breakdown curve (c.f. Fig. 7).
Fig. 5. Traffic breakdown probability for a mixture of ACC vehicles and manual vehicles using the CA modelling approach. (a) $H_d = 1$; (b) $H_d = 1.5$; (c) $H_d = 2.2$.

Fig. 6. Comparison of traffic breakdown probability in (a) hybrid modelling approach and (b) CA modelling approach. One can see in (b), a large shift of the curve happens even if $H_d$ slightly changes. This is obviously not correct.

(iii) These results are qualitatively similar to that of the CA modelling approach (see Fig. 10). Quantitatively, traffic breakdown occurs with more difficulty in the hybrid modelling approach (the traffic breakdown curves are to the right of those in Fig. 10 at a given $R$, and $R_c$ is smaller in our hybrid modelling approach).

4. Conclusion

In this paper, we have investigated the traffic flow characteristics of a heterogeneous traffic system consisting of a mixture of ACC vehicles and manual vehicles, by using a hybrid modelling approach. In the hybrid approach, the manual vehicles are described by a CA model and the ACC vehicles are simulated by using a car-following model. As a result, the hybrid modelling approach has the advantages of both the CA model and the car-following model. On one hand, the three traffic states and
the probabilistic traffic breakdown of manual vehicles could be reproduced. On the other hand, there is no artificial velocity fluctuations with ACC vehicles.

We have studied the traffic breakdown probability from free flow to congested flow, and the transition probability from synchronised flow to jams. Some main conclusions are: (i) The introduction of ACC vehicles enhances the traffic stability of synchronised flow, and traffic stability will be furthermore enhanced with an increase of $H_d$. At a given $H_d$, there exists a critical value $R_c$. When $R > R_c$, jam will not spontaneously appear from a synchronised flow. (ii) The introduction of ACC vehicles may either enhance or weaken the stability of free flow, depending on $R$ and $H_d$. At a given $R$, free flow stability will be enhanced with a decrease of $H_d$.

We have compared the results of the hybrid modelling approach with that of the CA modelling approach. It is found that quantitatively, traffic flow is more stable in the hybrid modelling approach. This might be due to artificial velocity
Fig. 9. Dissolving of a traffic jam, which appears due to reasons such as car accidents. \( H_d = 1, \rho = 60 \text{ vehicles/km}, R = 0.4 \).

Fig. 10. Transition probability from synchronized flow to jams for a mixture of ACC vehicles and manual vehicles using the CA modelling approach. (a) \( H_d = 1 \); (b) \( H_d = 1.5 \); (c) \( H_d = 2.2 \).

fluctuations of ACC vehicles that are removed in the hybrid modelling approach. Some qualitative differences between the two approaches are also indicated. Although we believe the results from the hybrid modelling approach are correct, more investigations are needed for verification.

In future work, empirical calibration and verification of the hybrid modelling approach are needed. Then the hybrid modelling approach could be applied to study traffic flow in various road conditions, for example, multilane traffic and traffic system with bottlenecks (e.g., on-ramp, off-ramp, lane closures).

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References