Network operation reliability in a Manhattan-like urban system with adaptive traffic lights

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ABSTRACT

Traffic breakdown to global gridlock occurring in congested traffic network makes the serious traffic congestion even much worse. This paper has proposed to use Network Operation Reliability (NOR) to quantitatively depict the probabilistic feature of traffic breakdown to global gridlock. The Nagel–Schreckenberg cellular automaton model has been used to simulate the traffic flow in a Manhattan-like urban network. A simple adaptive traffic light strategy has been proposed. It has been shown that if vehicles choose to use geometric shortest path, the adaptive traffic signals are able to remarkably enhance the NOR and sometimes the average velocity and the arrival rate as well. The vehicle distribution has been investigated, which has heuristically explained the enhancement of the NOR. A simple perimeter control strategy has been shown to fail to enhance the NOR. Finally, we show that if the time shortest path information could be provided and updated timely, then the NOR can be remarkably enhanced but the adaptive traffic signals have only trivial effect on NOR.

1. Introduction

Traffic congestion in transportation network is serious almost all over the world. A series of proposals to lessen congestion have been presented, such as congestion pricing (Yang and Bell, 1997; Verhoef, 2002) that has been adopted in Singapore and London, the travel restriction based on license plate tail number as adopted in Beijing, advanced traveller information system (ATIS) like variable message sign (VMS) board (Gan and Ye, 2013; Jindahra and Choocharukul, 2013), the variable speed limit, the ramp metering and so on (Papageorgiou et al., 2003; Frejo and Camacho, 2012; Carlson et al., 2011).

In network traffic flow, recently it has been found empirically that the average flow and average density are related by a unique, reproducible curve, which is named as Macroscopic Fundamental Diagram (MFD) (Geroliminis and Daganzo, 2008). Utilizing the MFD, optimal control methodologies such as perimeter and boundary flow control strategies, hierarchical control that considers network heterogeneity as well as cooperative traffic control have been studied (Aboudolas and Geroliminis, 2013; Geroliminis et al., 2013; Haddad et al., 2013; Haddad and Shraiber, 2014; Haddad, 2015; Keyvan-Ekbatani et al., 2015; Yildirimoglu et al., 2015; Yildirimoglu and Geroliminis, 2014; Ramezani et al., 2015).
In urban traffic networks, the traffic signals control the flows of vehicles at signalized intersections. Therefore, the optimization of traffic signal plans is one major target of intelligent transportation system research. The traffic signal systems can be classified into two types: static systems and dynamic ones. In static systems, the pre-timed signal might have been optimized via off-line optimization approaches, using historically measured data. However, the systems are problematic since the historical data do not accurately reflect the current traffic conditions.

In the dynamic systems, the traffic signal timing reacts adaptively to real-time traffic conditions based on a responsive technique in which real-time measured data is used. As a result, it is expected that the adaptive traffic signals are able to enhance the performance of traffic networks.

Researchers have proposed many adaptive traffic signal strategies and tested them at isolated intersection and traffic networks (see e.g., El-Tantawy et al., 2013; Zheng and Recker, 2013; Zhang et al., 2013; Brilon and Wietholt, 2013; Maslekar et al., 2013; Mckenney and White, 2013; Xin et al., 2013; Mirchandani and Head, 2001; Lo, 1999; Trabia et al., 1999; Abdulhai et al., 2003; Spall and Chin, 1997; Balaji and Srinivasan, 2010; Yu and Recker, 2006; Cai et al., 2009; Gartner and Stamatidis, 1998; Mirchandani and Zou, 2007; Fang and Elefteriadou, 2008; Kouvelas et al., 2014; Varaiya, 2013; Diaoaki et al., 2002, 2003; Gayah et al., 2014). It has been shown that adaptive traffic signals could improve the traffic network performance (e.g., increasing average velocity, reducing average delay and achieving higher capacity).

This paper studies the effect of adaptive traffic signal on the performance of traffic network from a different perspective. It has been shown that when the network density is large, traffic breakdown to global gridlock with zero flow will occur (Daganzo et al., 2011; Mazloumian et al., 2010; Gayah et al., 2014). Since the traffic flow has a probabilistic nature to breakdown into global gridlock, we propose an index “network operation reliability” (NOR) to quantitatively describe this kind of performance of traffic network. Our studies show that the adaptive traffic signal is able to significantly enhance the NOR under certain circumstance.

The paper is organized as follows. In the next section, the simulation model is introduced. In Section 3, we have presented the simulation results to show the probabilistic nature of traffic network’s breakdown into global gridlock. The effect of adaptive traffic signal on the NOR is discussed in Section 4. Section 5 makes discussion on the effect of various factors on the NOR. Finally, conclusion and future work are given in Section 6.

2. Model

We study traffic flow in urban traffic network in which all the intersections are signalized. We use the Nagel–Schreckenberg cellular automaton model (Nagel and Schreckenberg, 1992) to simulate the motion of vehicles. For simplicity, we consider a Manhattan-like urban system as shown in Fig. 1(a), which has also been previously studied in the literature, see e.g., Gayah et al. (2014), Ortigosa et al. (2015), and Zhang et al. (2013). The network consists of \( N \times N \) square lattice. Any two successive intersections are connected by two lanes, one for each direction, which are both divided into cells. Vehicle driving is restricted to the right lane. In each traffic phase, the traffic lights stay green for one ingoing street and red for the other ingoing streets (yellow light is not considered). When the green light is on, vehicles on the corresponding ingoing street can go straight ahead, turn left or right, or make a U-turn.

Vehicles, except for the leading ones in each lane, travel in the system following the Nagel–Schreckenberg rules as follows:

\[ v_i \rightarrow \min(v_{\text{max}}, v_i + 1); \]
\[ v_i \rightarrow \min(d_i, v_i); \]
\[ v_i \rightarrow \max(v_i - 1, 0) \text{ with a braking probability } p; \]
\[ x_i = x_i + v_i. \]

Here \( v_{\text{max}} \) is the maximum velocity of vehicles, \( x_i \) is the position of the \( i \)th vehicle and \( d_i \) is the number of empty cells ahead of the \( i \)th vehicle.

The update rules for the leading vehicles on each street are:

a. Traffic light is green
   * If the desired outgoing street is in jam or the intersection is occupied at the moment, \( d \) is the number of empty cells ahead to the intersection.
   * Otherwise, \( d \) is the number of empty cells ahead to the last vehicle of the desired outgoing street.

b. Traffic light is red
   * \( d \) is the number of empty cells ahead to the intersection.

Initially, the vehicles are distributed in the network randomly. Each vehicle is given with a randomly selected destination. When a vehicle reaches its destination, a new destination is chosen randomly from the system. This is just like a taxi. When it arrives at the destination, the passengers get off and new passengers get on and go to new destination (assume that there are always new passengers getting on once passengers get off).
In some previous simulations, to mimic the origin–destination behavior, it was assumed that each vehicle makes a random choice about which link it wants to turn into at the intersection (Zhang et al., 2013; Mazloumian et al., 2010; de Gier et al., 2011; Gayah et al., 2014). This paper considers the route choice behavior of drivers. However, for simplicity, it assumes that vehicles move along the geometric shortest path. If there is only one geometric shortest path to the destination, then the driver moves along this path. However, since the network is a grid network, there is usually more than one geometric shortest path between an origin and destination pair.\footnote{We have made simplification of the intersections in our model and assume that the intersections occupy only one cell. All intersection movements are treated the same when calculating the geometric shortest path. This should have trivial effect when the links are long and the intersections are small. However, when the links are not so long and the intersections are large, the turning track length may have a nontrivial effect on the geometric shortest path. This simplification needs to be improved in our future work.} For example, from A to B as shown in Fig. 1(a), there are three geometric

Fig. 1. (a) The sketch of the Manhattan-like urban network. (b) The global gridlock induced by the loop jam structure, which is shown by dashed line. To enable the visualization, a small size is used in which \( N = 5 \) and \( L = 17 \). Here red means left-turning vehicles, blue means right-turning vehicles, yellow means straight-going vehicles. The global density \( \rho = 0.6 \). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
shortest paths, in which vehicles can go (i) West-West-North-North; (ii) West-North-West-North; (iii) North-West-West-North. In this case, we assume that at the intersections where the geometric shortest paths bifurcate, there is ATIS that provides information of the mean velocity of the two related outgoing streets to the drivers, and the drivers would choose the outgoing street with a larger mean velocity. The mean velocity of each street is updated from simulation step to step. As shown in Li et al. (2011), the ATIS could improve the performance of traffic network.

Now we present the adaptive traffic light strategy. The traffic light period at all intersections is set as \( T \), and the traffic lights are switched in a clockwise manner. For each ingoing street, green light period consists of a static period and a dynamic one.

1. Static green light time \( T_{\text{static}} \), which denotes the minimum green time, is the same for all ingoing streets.
2. Dynamic green light time \( T_{\text{dynamic}} \) is set to be proportional to the vehicle density on the street

\[
T_{\text{dynamic}} = \frac{\rho_i^2}{\sum_{i=1}^{m} \rho_i^2} \times (T - mT_{\text{static}})
\]

Here \( m \) is the number of ingoing streets at the intersections. \( m = 4 \) for most intersections that are not on the boundary. For the intersections on the boundary, \( m = 3 \). \( \rho_i \) is instantaneous vehicle density on the ingoing street \( i \) of the intersection, obtained at time step \( t = 0 \). \( T = 2T \ldots \) This strategy on the one hand, guarantees the minimum green time for the ingoing streets \( (T_{\text{static}}) \). On the other hand, it gives more green time to the ingoing streets with higher density. \( \alpha \) is a tunable parameter, which reflects the strategy’s preference over ingoing streets with large densities. In the special case \( \alpha = 1 \), the strategy reduces to that studied in Gayah et al. (2014). Also note that in the special case \( T_{\text{static}} = T/m \), one has \( T_{\text{dynamic}} = 0 \). When \( \alpha = 0 \), \( T_{\text{dynamic}} = \frac{T_{\text{static}}}{m} \) is fixed. In the latter two cases, the adaptive traffic light strategy reduces to the fixed strategy as used in Li et al. (2011). Since the green time should be an integer, we choose \( m - 1 \) \( T_{\text{dynamic}} \) randomly at each intersection and roundoff their non-integer part, and let the last \( T_{\text{dynamic}} \) equal to the difference between \( T - mT_{\text{static}} \) and the sum of the \( m - 1 \) \( T_{\text{dynamic}} \).

The dynamic traffic light formula in Eq. (1) is similar to Webster’s formula (Webster, 1958). The difference is that Eq. (1) is based on densities while Webster’s formula is based on saturation levels. However, Webster’s formula is "optimal" in some sense only for isolated junctions and free flow conditions. Consideration for Webster’s formula is questionable for saturated networks.

3. Network operation reliability

This section presents and discusses the evolution of the average velocity of the system. In the simulation, the system size is set as \( N \times N = 24 \times 24 \) and the length of the street is set as \( L = 100 \) cells. Assume each cell corresponds to 6 m, the length of the streets is 600 m. The intersections are assumed to occupy only one cell. The maximum velocity \( v_{\text{max}} = 3 \) and the randomization probability \( p = 0.1 \). Assume each time step corresponds to 1 s, the maximum velocity corresponds to 64.8 km/h (18 m/s). The traffic light period is set as \( T = 80 \) unless otherwise mentioned. The Supplemental demo Video S1 shows the movement of vehicles in a system with small size. In the video, the red symbol denotes an example vehicle and the blue symbol denotes its destination.

Fig. 2(a) shows a typical evolution of the average velocity of the system at low global density. One can see that the average velocity becomes stationary after a transient time. Moreover, the evolution process is almost the same under different runs, in which different random seeds are used and other configurations are the same. Note that Knoop et al. (2013) has also studied a Manhattan-like grid network with square blocks and periodic boundary conditions. The intersections are un-signalized ones, which are treated as merges and diverges. The cell transmission model is used in their simulations. They show that the network flow is not constant for a fixed number of vehicle, but decreases due to local queuing and spill back processes around the congestion “nuclei”, which corresponds to the transient state shown in our simulations. It would be interesting to study the NOR in Knoop’s model and compare with the results in the present paper in the future work.

However, with the increase of the density, the situation becomes different, see Fig. 2(b). The average velocity firstly becomes stationary after a transient time as before. Then it begins to drop. This is because the entrance sections of some streets have been jammed. As a result, vehicles cannot enter these streets and thus congestion spills back and gradually propagates to the whole network. If the congestion propagation has led to global gridlock, in which a loop jam structure has formed as shown in Fig. 1(b), then the average velocity will finally decrease to zero. See Supplemental Video S2, which shows the evolution process of an example.

Fig. 2(c) shows the evolution of the average velocity at the same global density under several different runs. One can see that the density of the stationary state is different. This means that the occurrence of congestion propagation has a
probabilistic nature. Nevertheless, even if the average velocity drops due to the congestion propagation, this does not necessarily mean that global gridlock will occur. Fig. 2(d) shows two typical examples that the average velocity recovers after it drops to a very small value, in which the global gridlock has not formed. See Supplemental Video S3, which shows the evolution process of one example. Finally, when the density is large enough, the traffic breakdown occurs so quickly that the stationary state cannot be observed, see Fig. 2(e).

To quantitatively depict this probabilistic nature of global gridlock formation, we define the network operation reliability (NOR) as follows. At given time interval $T_{\text{inter}}$, the NOR is determined by $\text{NOR} = \frac{1}{C_0} P$. Here $P$ is the probability that global gridlock has formed, which is calculated by $P = \frac{n_p}{n}$. Here $n$ is the total number of runs, and $n_p$ is the number of runs in which the average velocity becomes zero, which definitely implies that the global gridlock has occurred.

4. Effect of adaptive traffic signal

In the adaptive traffic signal strategy (1), there are three parameters $T_{\text{static}}, \alpha$ and $T$. This section studies the effect of the three parameters on the NOR.

4.1. Effect of $T_{\text{static}}$

Fig. 3 shows the dependence of NOR on the global density under adaptive traffic light strategy with different static green light time $T_{\text{static}}$. The parameter $\alpha = 1$ and the simulation time interval is set as $T_{\text{inter}} = 10^5$. For each density, the total number of runs $n = 300$. Different density levels are simulated at different simulation runs. In each run, the vehicle number is fixed. One can see that at a given $T_{\text{static}}$, when the global density is low, the network could well operate and no gridlock happens. With the increase of global density, the NOR gradually decreases and gridlock could happen. When the global density is large enough, NOR decreases to zero and gridlock will always happen.
Fig. 3 also shows that compared with the fixed strategy ($T_{\text{static}} = 20$), the adaptive traffic signal remarkably enhances the NOR. Moreover, the smaller $T_{\text{static}}$ is (which corresponds to more flexible traffic light strategy), the more the NOR is enhanced.

To understand the change of NOR under adaptive traffic signal, we investigate the vehicle distribution. Fig. 4 shows the distribution of density on each street in the stationary state under different values of $T_{\text{static}}$. In the fixed strategy (Fig. 4(a)), one can see that surprisingly, the density in the center area is not large. The large densities appear on the four corners. With the decrease of $T_{\text{static}}$, on the one hand, the vehicles gradually tend to assemble in the center area. On the other hand, the maximum value of local density decreases. Let $\rho_{\text{max},k}$ denote the maximum value of the density on the streets in the direct $k \in \{\text{eastbound, westbound, northbound, southbound}\}$. Fig. 5(a) shows $q_{\text{max}}$, the average value of $\rho_{\text{max},k}$ over four directions, which decreases as $T_{\text{static}}$ decreases. This means that decrease of the maximum value of local density leads to the enhancement of NOR, which is because the initialization of congestion spillover and propagation become difficult when the maximum local density decreases.

Figs. 6(a) and 7(a) show the effect of $T_{\text{static}}$ on the average velocity $\bar{v}$ and the average arrival rate $\bar{q}$ in the stationary state. The latter is defined as $\bar{q} = N_{\text{arr}}/N_{\text{total}}$. Here $N_{\text{arr}}$ denotes the number of vehicles that arrive their destinations per time step and $N_{\text{total}}$ denotes the total number of vehicles. One can see that $\bar{v}$ and $\bar{q}$ behave very similarly. At low global densities, decreasing $T_{\text{static}}$ is able to improve $\bar{v}$ and $\bar{q}$. However, with the increase of global density, decreasing $T_{\text{static}}$ has essentially not improved them.

4.2. Effect of $\alpha$

Next we investigate the effect of parameter $\alpha$. Fig. 8 shows the dependence of NOR on the parameter $\alpha$. One can see that the NOR firstly increases with the increase of $\alpha$. The maximum NOR could be achieved at intermediates value of $\alpha$. Then the NOR begins to decrease with the further increase of $\alpha$. Figs. 6(b) and 7(b) shows that the average velocity and the average arrival rate could also be enhanced at intermediate values of $\alpha$. 

Fig. 3. NOR versus the global density under different values of $T_{\text{static}}$. The parameters $\alpha = 1$, $T_{\text{interval}} = 10^3$.

Fig. 4. The density distribution of vehicles in the stationary state under different values of $T_{\text{static}}$. The parameter (a) $T_{\text{static}} = 20$, (b) $T_{\text{static}} = 15$, (c) $T_{\text{static}} = 10$, (d) $T_{\text{static}} = 5$. Other parameters $\alpha = 1$, $\rho = 0.22$. 

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Comparing with distributing the green time equally among the four ingoing streets, it is easy to understand that giving more green time to ingoing streets with large densities can benefit the network. However, giving too much green time to ingoing streets with large densities is a waste because the streets do not need so long green time, in particular when the outgoing streets are in jam and do not allow entrance of vehicles. Therefore, an intermediate value of $\alpha$ is preferred.

Fig. 9 shows the density distribution of vehicles under different values of $\alpha$. One can see that when $\alpha$ increases from small value, the density distribution also shrinks toward the center area and $\rho_{\text{max}}$ also decreases, see Fig. 5(b). Therefore, the NOR increases. However, when $\alpha$ further increases, $\rho_{\text{max}}$ increases. Thus, the NOR decreases.
4.3. Effect of \( T \)

Now we study the effect of traffic light period \( T \). As shown in Fig. 10(a), at the specific parameter values, the NOR is very sensitive to \( T \) and exhibits a complicated changing behavior. Two maximum values have been observed at \( T/T_{\text{static}} = 77 \) and \( 95 \), and one minimum value has been observed at \( T/T_{\text{static}} = 85 \). Nevertheless, Figs. 6(c) and 7(c) show that the average velocity and the average arrival rate are almost independent of \( T \).

Fig. 11 shows the density distribution of vehicles under different values of \( T \), which essentially neither shrinks nor expands. The high density region at the four corners expand with the increase of \( T \). Fig. 5(c) shows that there is a maximum value of \( \rho_{\text{max}} \) at \( T/T_{\text{static}} = 85 \), which leads to the minimum value of NOR.

To explore the origin of the two maximum value of NOR, we examine the average density \( \bar{\rho} \) in the center area with size \( 5 \times 5 \). Fig. 5(d) shows that there is a minimum value of \( \bar{\rho} \) at \( T/T_{\text{static}} = 77 \), which leads to one maximum value of NOR. Moreover, when \( T \) is in the range \( 85 < T < 95 \), \( \bar{\rho} \) is almost a constant but \( \rho_{\text{max}} \) decreases. Thus, NOR increases in this range and reaches another maximum value of NOR at \( T/T_{\text{static}} = 95 \).
Nevertheless, NOR is not always so sensitive to $T$. As shown in Fig. 10(b), when the density is small, NOR equals to 1 in the studied range of $T$. When $q$ increases to 0.38, NOR always equals to zero in the studied range of $T$. With the gradual increase of density from small value to large value, Fig. 10(b) shows how the two local maximum and one local minimum emerge and disappear.

We have changed the parameter $T_{\text{static}}$, and similar results are observed, see e.g., Fig. 10(c), in which $T_{\text{static}} = 15$.

5. Discussion

5.1. Effect of $p$

For the Nagel–Schreckenberg model, the randomization parameter $p$ determines the outflow from traffic lights. This subsection studies the effect of randomization $p$ on the NOR. The simulation results show that NOR depends on $p$ similar to $T$, see Fig. 12. One can see that when the global density $\rho = 0.1$, NOR always equal to 1 in the studied parameter range of $p$. With the increase of $\rho$, two local maxima and a local minimum gradually appear, see e.g., curve of $\rho = 0.29$. When $\rho$ increases to 0.45, NOR becomes zero in the studied parameter range of $p$. This might be because the free speed of a vehicle equals to $v_{\text{max}} - p$. Thus, with the increase of $p$, the free speed decreases and the time $L/(v_{\text{max}} - p)$ to transverse a link increases. More studies concerning $v_{\text{max}}$ and $L$ are needed to investigate this issue in the future work.

5.2. Effect of $T_{\text{interval}}$

In the calculation of NOR, there is another parameter $T_{\text{interval}}$. This subsection investigates the effect of $T_{\text{interval}}$ on the NOR. Fig. 13 shows that the NOR is independent of $T_{\text{interval}}$ when the global density is low (e.g., $\rho = 0.26$). When the global density
increases (e.g. $\rho = 0.31$ and 0.36), the NOR equals to 1 when $T_{\text{interval}}$ is small. It gradually decreases to zero with the increase of $T_{\text{interval}}$. Nevertheless, the basic conclusion that the adaptive traffic signal could enhance the NOR does not change, as can be seen from Fig. 14, which shows the NOR with $T_{\text{interval}} = 8 \times 10^5$, shorter than that shown in Fig. 3.

We would like to mention that it would be better if we could obtain the distribution of the network traffic breakdown time. However, since the traffic breakdown takes very long time, especially when NOR is close to or equals 1, to obtain the distribution is very time consuming. Therefore, we propose to use NOR instead of simulating the distribution of traffic breakdown time.

Fig. 10. Dependence of NOR on the traffic light cycle $T$. The parameter $\alpha = 1$. In (a), the density $\rho = 0.297$. In (a) and (b), $T_{\text{static}} = 5$. In (c), $T_{\text{static}} = 15$. 

(a) 

(b) 

(c)
In this subsection, we relax the constraint that the global density is constant to mimic the loading and unloading processes. We assume that in each time step, \( n_{\text{car}} \) vehicles will be added into the system until the global density reaches

![Graphs showing density distribution](image)

**Fig. 11.** The density distribution of vehicles in the stationary state under different values of \( T \). The parameter (a) \( T = 74 \), (b) \( T = 76 \), (c) \( T = 86 \), (d) \( T = 92 \), (e) \( T = 98 \), (f) \( T = 104 \). Other parameters \( T_{\text{static}} = 5 \), \( \alpha = 1.0 \), \( \rho = 0.297 \).

**5.3. Time-changing density**

In this subsection, we relax the constraint that the global density is constant to mimic the loading and unloading processes. We assume that in each time step, \( n_{\text{car}} \) vehicles will be added into the system until the global density reaches
Then vehicles will be removed from the system with probability \( p_{\text{dis}} \) when they arrive at the destinations, until the global density reaches \( \rho_{\text{ma}} \). As a result, the average global density \( \rho_{\text{ave}} = \frac{\rho_{\text{ma}} + \rho_{\text{mi}}}{2} \), and the fluctuation amplitude \( A = \rho_{\text{ma}} - \rho_{\text{mi}} \).

Firstly, we present the evolution process of average velocity and system density. Fig. 15(a) shows an example in which traffic breakdown does not happen. One can clearly see the fluctuation of system density and average velocity. In contrast, Fig. 15(b) and (c) shows two examples in which traffic breakdown has occurred and the system finally evolves into gridlock. One can see that the traffic breakdown initiates when system density is close to \( \rho_{\text{ma}} \). We have examined the traffic breakdown in many other runs, and same feature has been observed in most cases. At given average density \( \rho_{\text{ave}} \); \( \rho_{\text{ma}} \) increases with \( A \). This factor makes network traffic unstable. However, on the other hand, the appearance of \( \rho_{\text{ma}} \) decreases because the density changing period increases, which should benefit the network traffic stability. As a combined effect of the two factors, the NOR curve changes with \( A \) in a non-monotonic way. It firstly moves right when \( A \) increases from zero to 0.1, which means that density fluctuation makes the network traffic flow more stable, see Fig. 15(d). When \( A \) increases from 0.1 to 0.2, the NOR curve essentially does not move. However, further increase of \( A \) destabilizes the network traffic, so that the NOR curve moves left.
Fig. 14. NOR versus the global density under different values of $T_{\text{static}}$. The parameters $\alpha = 1$, $T_{\text{interval}} = 8 \times 10^4$. 

Fig. 15. (a) An example of the time-changing density in which traffic breakdown does not happen. (b and c) Two examples in which traffic breakdown has occurred. (d) The NOR under different values of fluctuation amplitude $A$. The parameters $n_{\text{car}} = 2$, $p_{\text{dis}} = 0.3$, $T_{\text{static}} = 5$, $\alpha = 1$, $T = 80$. 
5.4. Effect of OD pattern

Now we study the effect of OD pattern on the NOR. We suppose that when vehicles arrive at their destinations, they choose a new destination in the inner $16 \times 16$ grid with probability $p_{in}$ and in the outer grid with probability $1 - p_{in}$. The cell number equals to 108800 in the inner $16 \times 16$ grid, and 240000 in the whole network. Thus, in the special case $p_{in} = \frac{34}{75}$, the situation reduces to that a new destination is chosen randomly in the whole network.

Fig. 16(a) shows the NOR under different values of $p_{in}$. One can see that the NOR curve moves left with the increase of $p_{in}$. This means that when more vehicles go to city center, network traffic flow becomes more unstable. Note that similar studies performed by Mazloumian et al. (2010) also showed that gridlock occurs within a short time when a small fraction of vehicles are routed to a destination area. Fig. 16(b)–(d) shows the density distribution of vehicles in the stationary state under different values of $p_{in}$. One can see that with the increase of $p_{in}$, the center area becomes more congested, which leads to decrease of network stability.

![Figure 16](image-url)

**Fig. 16.** (a) The NOR under different values of $p_{in}$. (b–d) The density distribution of vehicles in the stationary state under different values of $p_{in}$. The parameter (b) $p_{in} = 0.3$, (c) $p_{in} = \frac{34}{75}$, (d) $p_{in} = 0.6$. Other parameters $T_{static} = 5$, $x = 1.0$, $T = 80$, $\rho = 0.23$. 

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**References:**
Mazloumian et al. (2010)
5.5. Macroscopic fundamental diagram

Fig. 17(a)–(c) presents macroscopic fundamental diagram of the whole network, the inner $16 \times 16$ grid, and the outer grid, respectively. Fig. 17(a) shows that under fixed traffic signals, the flow rate of the whole network firstly increases very quickly. When $q$ exceeds 0.07, the flow rate becomes increasing very slowly. The flow rate reaches the maximum at $q \approx 0.24$. Then it begins to decrease with the further increase of $q$. When $q > 0.26$, the NOR becomes zero. The network is very unstable and the stationary state essentially cannot be maintained. With the implement of adaptive traffic signals, the maximum flow rate is reached much earlier at $q \approx 0.09$. Then the flow rate slowly decreases. Nevertheless, the flow rate is always larger than that under fixed signals. Moreover, since NOR is enhanced, the stationary state can be observed at larger densities until $q \approx 0.33$.

Fig. 17(b) shows that macroscopic fundamental diagram of the inner $16 \times 16$ grid is very similar to that of the whole network. However, as center area of the network, the inner $16 \times 16$ grid can operate at much larger densities, since the density distribution is rather homogeneous in the inner $16 \times 16$ grid than in the whole network.

Fig. 17(c) shows that macroscopic fundamental diagram of the outer grid is quite different. Under fixed signals, the maximum flow rate is reached at $q \approx 0.075$. With the implement of adaptive traffic signals, the maximum flow rate is reached at $q \approx 0.043$. Then the flow rate decreases quite fast.

5.6. Perimeter control

Perimeter control manipulates the flows across the region border to regulate the critical densities in the region, aiming to enhance network performance. However, the network might still lead to gridlock if the control is applied in a heavily congested network. Haddad and Geroliminis (2012) has conducted stability analysis of the traffic perimeter control problem in two-region urban cities and derived the boundaries of the stable and unstable regions. Later Haddad and Shraiber (2014) has proposed a robust perimeter control design for an urban region.

![Fig. 17. Macroscopic fundamental diagram of (a) the whole network, (b) the inner $16 \times 16$ grid, (c) the outer grid. The parameters are $T = 80$, $x = 1$.](image-url)
In this subsection, we study a simple perimeter control strategy. In the beginning of each period, when the density in the inner $16 \times 16$ grid exceeds a threshold $\rho_{cr}$, then no vehicle is allowed to enter the inner region in the period.

Fig. 18(a) shows the NOR under different values of $\rho_{cr}$. When $\rho_{cr}$ is large, the perimeter control strategy has no effect on NOR since the density in the inner $16 \times 16$ grid seldom or never exceeds $\rho_{cr}$. When $\rho_{cr}$ is below $\rho_{cr} \approx 0.6$, the NOR curve begins to move left, which means network traffic flow becomes more unstable. Fig. 18(b) shows an example of time series of the average velocity in which traffic breakdown has occurred. Fig. 18(c)–(h) shows the typical density distribution in the evolution process of gridlock. One can see that traffic congestions occur on the region border. As a result, vehicles inside the inner grid sometimes are hindered by the border congestion and cannot go out of the inner region. This finally leads to gridlock of the whole network.

Fig. 18. (a) The NOR under different values of $\rho_{cr}$. (b) An example of time series of the average velocity in which traffic breakdown has occurred. (c–h) Six typical density distributions in the evolution process of gridlock. (c) $t = 30,000$, (d) $t = 31,000$, (e) $t = 32,000$, (f) $t = 33,000$, (g) $t = 34,000$, (h) $t = 36,000$. The parameters are $\rho_{cr} = 0.4$, $\rho = 0.25$, $T = 80$, $\alpha = 1$, $T_{static} = 5$. 
Our simulation results show that the simple perimeter control strategy fails to enhance NOR. This might be also related to the situation that drivers use the geometric shortest path. In the future work, other perimeter control strategies as well as adaptive route choice behavior need to be tested.

5.7. Links with unequal length

In the previous studies, the length of the links in the grid network equal to each other. As a result, in most cases, there is more than one geometric shortest path between an origin and a destination. This subsection relaxes the constraint. We suppose the length of the links is a uniformly distributed variable in the range between 50 cells and 100 cells. Consequently, there is usually only one geometric shortest path between a OD pair. Fig. 19(a) compares the NOR under fixed traffic signals and adaptive traffic signals. One can see that (i) the network traffic is very unstable. The NOR has decreased to zero even though the global density is very small. (ii) The adaptive traffic signals are still able to enhance the NOR. Fig. 19(b) and (c) shows the density distribution under fixed traffic signals and adaptive traffic signals. It can be seen that the distribution is very inhomogeneous. Vehicles tend to accumulate on a small portion of links.

5.8. Time shortest path

Route choice behavior is an important factor in the network traffic flow and has been widely studied in the literature (see e.g., Knoop et al., 2012; Yildirimoglu et al., 2015; Mahmassani et al., 2013; Leclercq et al., 2015). This subsection considers

![Fig. 19. (a) The NOR in the network in which links have unequal length. (b and c) The density distribution in the stationary state. (b) Corresponds to fixed traffic signal with T_{static} = 20, (c) corresponds to adaptive traffic signals with T_{static} = 5. The parameters are T = 80, \alpha = 1, \rho = 0.013. For better visualization, the networks in (b and c) are still plotted as before, in which links' different length was not distinguished.](image-url)
the situation that the vehicles move along the time shortest path in the grid network. We suppose the ATIS is able to collect real-time travel time on the links and provides the information to the drivers. Specifically, when a vehicle enters/leaves a link, it sends the information to the ATIS so that ATIS can calculate the travel time of the vehicle. Then the ATIS calculates the average travel time of the vehicles on all links and thus the time shortest path between any two intersections.\(^4\) It sends the information to the vehicles. The information is supposed to be updated every \(T_{tu}\) time steps. The drivers are supposed to drive along the time shortest path provided by the ATIS.

Fig. 20(a) shows the NOR under the time shortest path situation. One can see that the NOR is remarkably enhanced compared with that under the geometric shortest path situation. However, the adaptive traffic signals fails to enhance the NOR. Fig. 20(b) and (c) show the density distribution of the vehicles. One can see that the density distribution is already quite homogeneous under the time shortest path situation and fixed traffic signals, see Fig. 20(b). As a result, the adaptive traffic lights have only trivial effect, see Fig. 20(c).

Now we study the situation that the information of time shortest path is not updated. We suppose that the information is updated for 100 periods. Then due to some system problems, the information of time shortest path ceases to update. Fig. 21(a) shows that the network traffic is very unstable. The NOR has decreased to zero even though the density is very small. However, the adaptive traffic signals are still able to enhance the NOR. Fig. 21(b) and (c) shows that the density distribution is also inhomogeneous. This is similar to the situation as studied in Section 5.7, which is because there is usually only one fixed path between an origin and a destination under both situations.

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\(^4\) In calculating the time shortest path, we consider the moving time of the vehicles in the links and ignore the moving time of the vehicles in the intersections. As pointed out in Footnote 1, this simplification needs to be improved if the links are not so long and the intersections are large.
6. Conclusion and future work

Nowadays, traffic congestion in transportation network is serious and widely observed almost in all large cities. When traffic breakdown to global gridlock occurs in congested traffic network, the network completely collapses. This makes the serious traffic congestion even much worse.

To quantitatively depict the probabilistic nature of traffic breakdown to global gridlock in traffic network, this paper has proposed the index of NOR. We have used the Nagel–Schreckenberg cellular automaton model to simulate the traffic flow in a Manhattan-like urban network. A simple adaptive traffic signal strategy has been proposed. It has been shown that when the vehicles choose to use geometric shortest path, the adaptive traffic signal is able to remarkably enhance the NOR and sometimes the average velocity and the arrival rate as well. The vehicle distribution has been investigated and it is found that the enhancement of NOR is due to a decrease of the maximum value of local density and sometimes depends on the average density in the center area. We have also studied a simple perimeter control strategy, and found that the strategy fails to enhance the NOR, since the vehicles going out of the inner region sometimes are hindered by border congestion. Finally, we found that if the time shortest path information could be provided and updated timely, then the NOR can be remarkably enhanced but adaptive traffic signals have only trivial effect on NOR.

Although our model is very simple, it has captured the most important issues in network traffic, i.e., the route choice behavior, the spillover and propagation of traffic congestion, and the formation of gridlock. The simple model in this paper can be extended in several directions in future work. For instance, (i) To model the real route choice behavior of drivers. (ii) Heterogeneous traffic signal setup at different intersections. The traffic light period $T$, the static time $T_{\text{static}}$, and the parameter $\rho = 0.01$. 

Fig. 21. (a) The NOR under the time shortest path situation but the information update is ceased after 100 periods. (b and c) The density distribution. (b) corresponds to fixed traffic signal with $T_{\text{static}} = 20$, (c) corresponds to adaptive traffic signals with $T_{\text{static}} = 5$. The parameters are $T = 80$, $T_{a} = 80$, $\alpha = 1$, $\rho = 0.01$. 

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z could be different among the intersections. (iii) Traffic light strategy. We need to study other adaptive traffic signal strategies and check whether they are better than the one proposed in the present paper, or whether better strategy can be designed. (iv) Network structure: a more realistic road network and multilane road sections need to be considered. (v) More realistic origin–destination data need to be deployed. (vi) To consider how to classify the system into multi-reservoirs, and to investigate advanced perimeter and boundary flow control strategy (Aboudolas and Geroliminis, 2013; Geroliminis et al., 2013; Daganzo, 2007; Ji and Geroliminis, 2012; Ramezani et al., 2015).

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Appendix A. Supplementary material

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