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Stochastic resonance in liquid membrane oscillator

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The oil/water liquid membrane oscillator which has been intensively investigated in recent years is generally considered a promising model for excitable biomembranes. The present work deals with the stochastic resonance (SR) phenomena liquid membrane oscillators with the model proposed by Yoshikawa, and the mechanism of SR and some interesting behaviors are also discussed. When SR occurs, the input signal is greatly amplified with the cooperation of noise and the output signal-to-noise ratio is also dramatically enhanced. We consider SR as a possible functional mechanism of some sensory cells.
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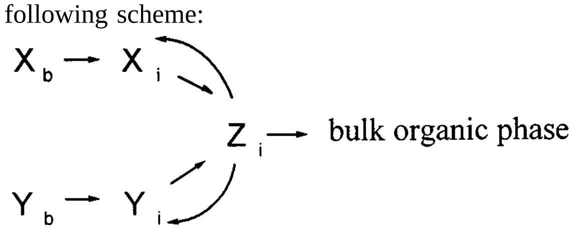
I. INTRODUCTION

The process of biomembranes is a very important one in living organisms for maintaining life. Of them, the dynamic process of excitable cell membranes has attracted much attention and been a focus of research in life science. In recent years, various excitable artificial membranes have been extensively investigated and considered as simple effective models for excitable biomembranes.[1] Among them, the oil/water liquid membrane bears the most promise for this purpose. In 1978, Dupeyrat and Nakche reported the quasiperiodic variations of interfacial tension and electrical potential at an oil/water interface.[2] Thereafter, Yoshikawa and coworkers and other authors investigated this kind of phenomena extensively and also greatly improved the experiments.[3–9] Meanwhile, the o/w liquid membrane is considered to model the membranes of some sensory cells, such as olfactory and gustatory cells because of their similar dynamic behaviors.[1,4,7]

Stochastic resonance (SR) is a rather new research subject which was first brought out in the 1980s. In general, noise is almost omnipresent and is often thought of as a negative element which usually induces disorder. Therefore people usually spare no effects to eliminate it. However, this is not always the case for nonlinear systems, that is, under certain conditions it will help establish a new order by modifying the existing states or inducing transitions between coexisting states. Imposing an input which contains noise and signal on a nonlinear system, people will find that increasing the noise intensity will help to enhance the signal-to-noise ratio (SNR) of the output under some conditions. And the value of the SNR passes its maximum over the noise with various intensities, which is called the stochastic resonance phenomenon, a kind of nonlinear cooperation effect among the noise, signal, and nonlinear system.[10] People have already found that some sensory cells show not only high selectivity but also unusually high sensitivity on some excitants, which is still an open question. Recent works[11–13] show that SR is a potential mechanism for the detection of weak signals by sensory cells. On the liquid membrane oscillator, researchers have mainly concentrated their interest on its oscillatory behavior in the past two decades; in this paper, we investigate another interesting dynamic behavior—the SR phenomenon in the system based on the model proposed by Yoshikawa and co-workers.

II. DYNAMIC MODEL OF LIQUID MEMBRANE OSCILLATOR

Many possible interpretations and related models have been proposed in the last two decades. Until now, the model by Yoshikawa and co-workers[6,8] has been the most complete and successful, in which the whole process can be shown in the following scheme:



(1)

where X, Y, and Z stand for the surfactant, the cooperative spice (usually alcohol), and the complex of X and Y, respectively; the subscripts i,b represent the sites at the o/w interface and the bulk water phase, respectively. In a succinct description, the spices X and Y arrive at the oil/water interface through diffusion from bulk aqueous solution phase and aggregate at the interface. When complex Z gathers up to a critical coverage, it is abruptly transferred across the interface into the organic phase. The periodic formation and disruption of the monolayer of the complex Z are responsible for the oscillations. The related governing dynamic equations read

$$\frac{dX_i}{dt}=D_X(X_b-X_i)-K_1Z_i\,, \tag{2}$$

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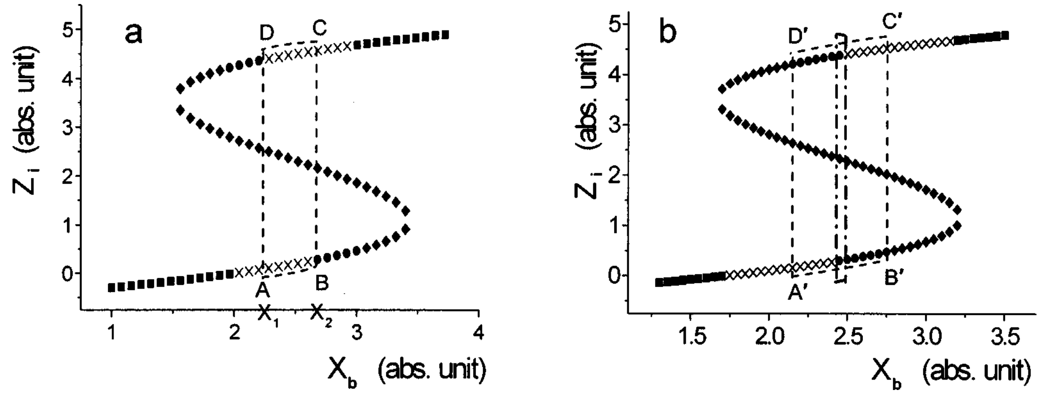

FIG. 1. Bifurcation diagram of the system solution Z_i vs X_b. (■) stable nodes, (◆) saddle nodes, (●) unstable foci, (×) stable foci. Parameters: $D_1=2.0$, $D_2=1.0$, $K_1=1.4$, $K_2=1.4$, $K_4=5.0$, $K_5=-7.0$, $K_6=3.30$, $K_7=20.0$, $\omega=0.02$, $Y_0=3.00$, (a) $K_3=20.0$, (b) $K_3=21.1$. (These parameters remain unchanged except for special setting.)

$$\frac{dY_i}{dt}=D_Y(Y_b-Y_i)-K_2Z_i, \tag{3}$$

$$\frac{dZ_i}{dt}=K_3(X_i+Y_i)-K_4G(Z_i). \tag{4}$$

In Eqs. (2) and (3), the first term denotes the diffusion and the second denotes the feedback from the decomposition of the complex *Z*. In Eq. (4), with the assumption that the phase transfer of *Z* relates to the interface tension, the first term denotes the production rate of *Z* and the second term concerns the phase transfer rate of *Z*. $G(z)$ is a nonlinear function with ''*N*'' shape which enables the system to be excitable. In this paper we make

$$G(Z)=Z^3+K_5Z^2+K_6Z+K_7. \tag{5}$$

III. RESULTS AND DISCUSSIONS

A. Bifurcation diagram

Figure 1 shows the dependency of the system solutions' stability on the parameter X_b. In the bifurcation diagrams, the stable foci and nodes are stable steady states while the unstable foci and saddle nodes are unstable. There is a bistable region in Fig. 1(a), namely, region *ABCD*, while there are none in Fig. 1(b). In Fig. 1(b), limit cycles encompass three unstable steady states in the region environed by the dash-dotted line, where extensive studies have been done in the past two decades not only in theory but in experiment[3–9] because oscillations occur in this kind of region. In the following, we shall focus our attention on another interesting nonlinear behavior—the SR phenomenon in the liquid membrane oscillator system.

B. Modulation of the parameter X_b

To modulate the parameter X_b, let

$$X_b'=X_b+\alpha\cos(2\pi\omega t)+\beta\Gamma(t) \tag{6}$$

and substitute X_b with X_b' in Eq. (2). In Eq. (6), the second term is a modulating signal, where α is the amplitude of the cosine signal and ω is the signal frequency; the third term is the noise term, where β is the noise intensity, $\Gamma(t)$ stands for a zero-mean Gaussian noise, namely, $\langle\Gamma(t)\rangle=0$, $\langle\Gamma(t)\Gamma(t')\rangle=2\delta(t-t')$. While numerically solving the kinetic equations [including Eq. (5)], $\Gamma(t)$ is isochronously constructed by a random number generator (e.g., 100 numbers in a signal period in this paper) and each number with Gaussian distribution is kept for a while (in this paper, 1/100 of the signal period) until another is generated. Supposing the noise is a train of spikes in which each number holds for a shorter time span, the results appear very similar [Figs. 2(b) and 2(c)].

C. Stochastic resonance phenomenon

From the relative amplitude of the signal (the system's input) and the value of variable Z_i (the output), it is easy to see that the signal is dramatically amplified. The amplifying grade depends on the location of X_b, the width of the bistable region, and the gap between the two stable steady states. The signal can be effectively amplified 10–30 times with the parameters given in this paper.

With the proper parameters, the SNR of the system output passes through a maximum over the variation of the noise intensity, which suggests the occurrence of stochastic resonance in this system. The peak position of SR curves and the maximal SNR value rely on the signal amplitude α. As we know, the SR phenomenon can take place in a bistable system, however, we find that SR can also occur in a nonbistable system, e.g., the system shown in Fig. 1(b) [Figs. 2(b) and 2(c)], which will be discussed later. For the sake of simplicity, we mainly investigate the bistable systems, e.g., ones in region *ABCD*.

With the bifurcation diagram [Fig. 1(a)], we will give a schematic description of the SR mechanism. Supposing the system on the lower stable branch at the beginning and $X_b+\alpha<X_2$, when $\beta=0$, driven by the input signal the system only oscillates with small amplitude on the lower branch; when $\beta\neq 0$, if X_b' is always smaller than X_2, the system will still stay at the lower steady state [Fig. 3(a)]. With the increase of the noise amplitude β, at a certain occasion when $X_b'>X_2$, the system transits from the lower state to the upper state [Fig. 3(b)]. If β becomes large enough, the transition and the signal are almost completely synchronous and the

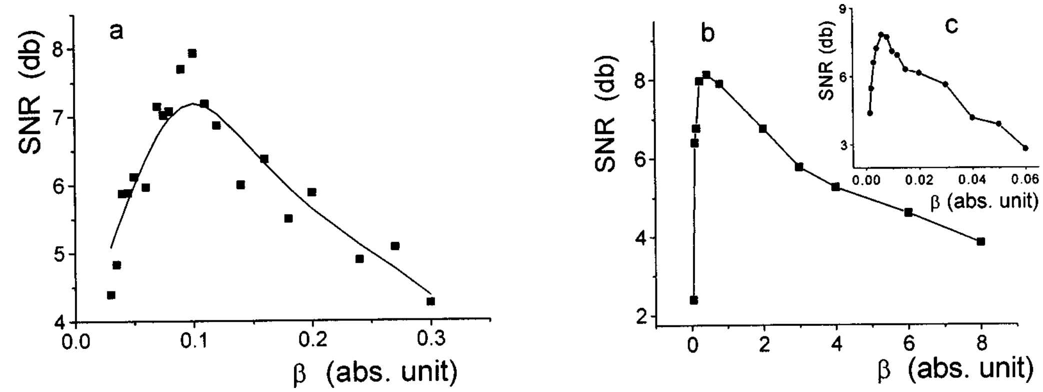

FIG. 2. The dependency of the signal-to-noise ratio (SNR) on the noise intensity β. (a) Systems in region *ABCD*, $X_b=2.475$, $\alpha=0.20$; (b), (c) SR diagrams of systems in the region $A'B'C'D'$ $\alpha=0.02$, $X_b=2.485$, (b) the random number is not kept, (c) the number is kept until another is generated.

SNR at the signal frequency reaches its maximum [Fig. 3(c)]. But with the further increase of β, they become less synchronous and the SNR will decrease, to [Fig. 3(d)].

For a further study on this system without noise in the input, at a certain frequency we divide the range of α into three domains (Fig. 4). The domains relate to the signal frequency, for instance, when $\omega=0.02$, the width of domain II is approximately 0.023 while it is 2.0×10^{-6} if $\omega=0.04$. In domain III, the system transits synchronously with the signal because the signal is strong enough to reign the system. In domain II, the solution becomes very susceptible to the calculating error and transits irregularly. We think the calculating error plays a role as does noise in triggering the transition at a proper occasion. In domain I, the system oscillates on the lower branch with a small amplitude (supposing it stays there initially). There are two interesting phenomena in this domain. One is that the system does not transit from the lower stable state to the upper one immediately when $a+X_b>X_2$ ($X_b'>X_2$ when $\beta\neq0$) and it even can stay there without transition. We consider it as a kind of transient behavior. From another viewpoint, the bifurcation diagrams based on the dynamic equations (2)–(4) are ''stationary'' ones, however, when the input (including signal and noise) is imposed on the system, its ''potential surface'' (if it could be called so) is modified by the input. Consequently, the stability of the states and even the topological structure of the bifurcation diagram changes from the ''stationary'' bifurcation diagrams. Briefly, the stability of the unstable states (without input) may change owing to the presence of the input, which is a possible reason why the system can stay at the unstable states of the ''stationary'' bifurcation diagram. As mentioned before, SR is a rather common phenomenon in a bistable system. However, it can also occur in the system without coexisting stable states shown in Fig. 1(b) [Figs. 2(b) and 2(c)]. We regard that perhaps the lower and upper unstable states change their stability and a bistable region appears for a short period, which may be responsible for the similar behavior between the two kinds systems with and without a bistable region. Another interesting phenomenon in this domain is that if α is large enough, a very tiny noise can trigger the transition. Figure 5 shows the dependency of the minimal noise which can trigger a transition (in the first 200 signal periods) on the signal amplitude and frequency. It is very surprising that simply increasing the signal amplitude will not induce a transition before a critical value, but a very tiny noise will. We think that it results from the long-time accumulation of the tiny noise's action. In our numerical solving of the equations, the smaller the frequency, the larger

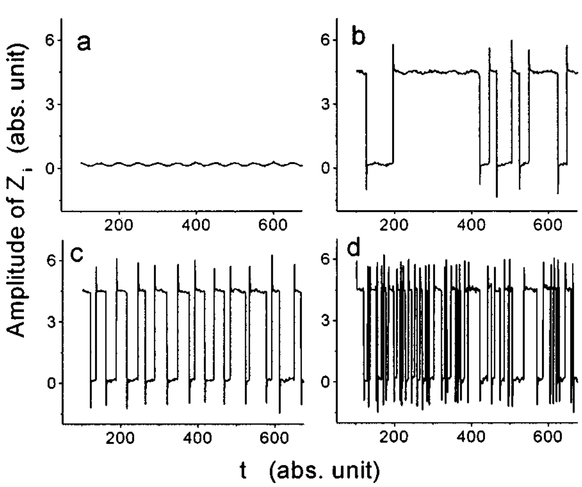

FIG. 3. The transitions between two stable states at different noises ($\alpha=0.150$, $X_b=2.475$). (a) $\beta=0.03$, (b) $\beta=0.07$, (c) $\beta=0.15$, (d) $\beta=0.25$.

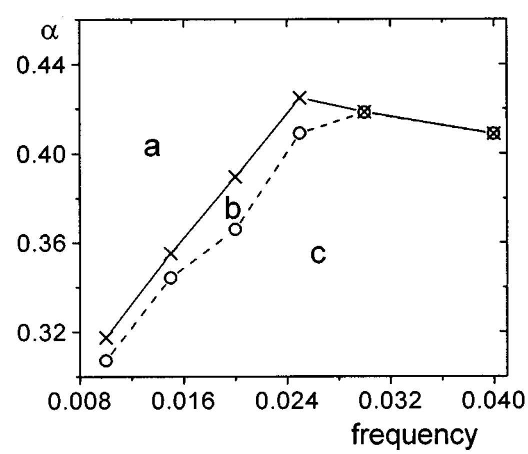

FIG. 4. The diagram of domains of signal intensity (α) at different signal frequency (ω). (a) domain I, (b) domain II, (c) domain III (described in the context).

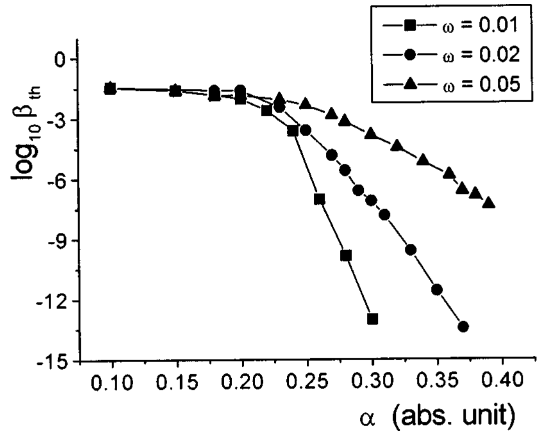

FIG. 5. The dependency of minimal noise intensity β_{th} on the signal frequency (ω) and amplitude (α) ($X_b = 2.475$).

the accumulating times in a signal period. Therefore, in Fig. 5, at a given signal amplitude α, the smaller the frequency, the smaller the minimal noise intensity β_{th}. Obviously, the effect of the accumulation is not simply a linear one.

D. Discussion

From some results and discussions above, when SR occurs, with the assistance of noise the weak input signal is effectively amplified, that is, at the same time the output also has very high SNR value. In other words, the weak signal embedded in a noisy environment can become clearer and easier to detect. SR phenomena widely exist in nonlinear systems with[10,11] (e.g., bistable, multistable systems) or without a threshold,[14] not only for the simple sinusoidal signal but also for the nonperiodic signals.[15–17]

Generally speaking, the dynamic process of excitable cell membrane involves many nonlinear subprocesses which provide the system with rich nonlinear dynamic behaviors such as bistability and multistability. In addition, the signal transfer in or among cells is inevitably disturbed by the noise induced by internal fluctuation and external environment. Therefore, it is possible that SR is a very common phenomenon in this kind of system and might also be a potential functional mechanism in which a weak signal arising from some certain excitants can be effectively detected by sensory cells.

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