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Enhancement of stochastic resonance by noise delay

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Abstract

Noise induced coherent oscillations (NICO) are investigated near a Hopf bifurcation point in the NO + CO catalytic reaction system, while the control parameter is perturbed by Gaussian noise, each random number of which has a duration time τ. With the variation of the noise intensity, the NICO-strength shows a maximum characteristic of the occurrence of stochastic resonance (SR). It's rather interesting that when τ increases for constant noise intensity, the NICO-strength goes through multi-maxima at $\tau_n \simeq (n+1)/2T_0$ (n is an positive integer and T_0 the period of of the NICO), indicating the possibility to enhance the strength of SR by suitable duration of noise. In addition, the maximum values at τ_n decays exponentially as $\exp[-(\tau_n - T_0/2)/\gamma$ with $\gamma \simeq 3.65$. © 1999 Published by Elsevier Science B.V. All rights reserved.

The phenomena of stochastic resonance (SR) has been extensively studied in the past decades (for a review of stochastic resonance, see Refs. [1–5]). Since it was originally proposed to account for the periodic oscillation of the Earth's ice ages [6], more and more scientists, whose majority vary from biology, physics to chemistry [7–9], have paid considerable attention to this rather 'counterintuitive' phenomena that noise with suitable intensity can increase the detectability of a weak input signal to a nonlinear system: the signal-to-noise ratio (SNR) goes through a maximum with the variation of the noise intensity.

The original model of SR includes a symmetric bistable system, a weak input periodic signal and additive external Gaussian white noise [10]. With the development of its application, however, the concept of SR has been widely extended. For example, the nonlinear system can be monostable [11,12], excitable [13–16], threshold-free [17–19], or spatial-extended [20–23], etc.; the input signal can be aperiodic or even chaotic [24–27]; and the noise can be colored, multiplicative or even be the intrinsic randomness of a deterministic chaotic system [28,29]. Very recently, Hu Gang et al. [30,31] proposed that the external signal can also be replaced by an 'internal signal', such as periodic oscillation in a deterministic system. When the control parameter locates near the bifurcation point between a stable node and a limit cycle, where the deterministic steady state is the stable node, noise induced coherent oscillation (NICO) is found and the NICO-strength goes through a maximum with the increment of the noise intensity, showing the characteristic of SR. Similar phenomena known as 'noisy precursors' or 'coherent resonance' [37] have also been reported. Here we would name

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this behavior *internal signal stochastic resonance* (ISSR) to underline the fact that external signal is absent.

Now another interesting and important tendency in studying SR has been to enhance the strength of it. For example, some scientists have studied the behavior called array enhanced stochastic resonance (AESR) [37], i.e., the maximum SNR in the SR curve can be enhanced if a nonlinear dynamic element is coupled with other elements in a one-dimensional array. Another clue to enhance the strength of SR is to change the quality of noise. According to the study of Daichi Nozaki et al., for $1/f^{\beta}$ colored noise, the case $\beta=1$ is the most favorable for the enhancement of SR in the FHN neuron model [38]. Alexander Neiman et al. [39] have investigated the influence of the correlation time of the colored noise and they found, as stated by them, memory effects of SR. In the threshold-free model [17–19], the authors also found that the bandwidth of the noise can considerably influence the output SNR. Chow et al. reported that the aperiodic stochastic resonance (ASR) can be enhanced by modulation of the noise intensity by the input signal or the unit's output rate signal (for AESR, see [40] and references therein).

In this Letter, we have adopted a chemical catalytic reaction system, the NO + CO catalytic reduction system, to study the NICO and henceforth ISSR behavior near a Hopf bifurcation. We prefer to a chemical system rather than a mathematical dynamical model [30,31] proposed in textbooks due not only to its realistic application but also to its abundant bifurcation character which may lead to more interesting behavior. On the other hand, Rappel et al. posed in Ref. [30,31] that the occurrence of ISSR was due to nonuniform motion on the deterministic limit cycle such that this phenomena might not exhibit near a Hopf bifurcation. In the present work, however, we found that this assertion may not be universal to other systems since we have also observed ISSR near a Hopf bifurcation.

Here we depict the specific character of noise $\xi(t)$ used in the present work, which is a type of shot noise composed of a periodic repetition of δ-spikes with Gaussian distributed strengths, i.e.,

$$\xi(t)=\sum_{j=0}^{\infty}\xi_j\Gamma(t-j\tau),$$

where j is an integer, and $\Gamma(x)=1$ for $0\le x<\tau$ and 0 otherwise; ξ_i is Gaussian distributed random value with zero mean and unit variance: $\langle\xi_i\rangle=0,\quad\langle\xi_i\xi_j\rangle=\delta_{ij}$.

In the numerical scheme, we choose $\tau=N\Delta t$, here N is a positive integer and Δt is the simulation time step. If $N=1$, the noise changes each time step and has no time-correlation so that one recovers a Gaussian white noise. In fact, $1/\tau$ measures the frequency bandwidth of the noise. Since the experimental realization of noise must be limited by a bandwidth, thus it should be rather important to investigate the influence of τ on the observed noisy dynamic behavior. It's rather interesting that, according to our present work, the NICO-strength goes through multi-maxima and the maximum strength show exponential-decay when τ increases.

The catalytic reduction of NO with CO on Pt(100) surface has been extensively studied both experimentally and theoretically. A lot of interesting phenomena have been observed, including kinetic oscillation, chaos and pattern formation [41–45]. The reaction is assumed to progress via the following steps:

$$CO+*\rightleftharpoons CO_a \tag{1a}$$
$$NO+*\rightleftharpoons NO_a \tag{1b}$$
$$NO_a+*\rightarrow N_a+O_a \tag{1c}$$
$$2N_a\rightarrow N_2+2* \tag{1d}$$
$$CO_a+O_a\rightarrow CO_2+2* \tag{1e}$$

where $*$ and 'a' denote an free adsorption site and an adsorbed species, respectively. Following Ref. [45], the differential equations for the three adsorbate coverages: θ_{CO},θ_{NO} and θ_O read:

$$\frac{d\theta_{CO}}{dt}=k_1p_{CO}(1-\theta_{CO}-\theta_{NO})-k_2\theta_{CO}-k_3\theta_{CO}\theta_O, \tag{2a}$$

$$\frac{d\theta_{NO}}{dt}=k_1p_{NO}(1-\theta_{CO}-\theta_{NO})-k_4\theta_{NO}-k_5\theta_{NO}\theta_{empty}, \tag{2b}$$

$$\frac{d\theta_O}{dt}=k_5\theta_{NO}\theta_{empty}-k_3\theta_{CO}\theta_O \tag{2c}$$

with

$$\theta_{empty}=\max\left[\left(1-\frac{\theta_{CO}+\theta_{NO}}{\theta^{inh}_{CO,NO}}-\frac{\theta_O}{\theta^{inh}_O},0\right)\right].$$

These equations describe the adsorption and desorption of NO and CO(k_1,k_2,k_4), the dissociation of NO(k_5), and the surface reaction between O_a and CO_a(k_3). The number of empty sites, θ_{empty}, depends on the NO, CO, and oxygen coverages. If they exceed certain values, $\theta_{NO,CO}^{\mathrm{inh}}$ and $\theta_{\mathrm{O}}^{\mathrm{inh}}$, no free sites are available and NO dissociation is inhibited. The various constants, which depend on temperature via Arrhenius law $k_i = \nu_i \exp(-E_i/RT)$, are all taken from experiments. Notice that the activation energy for NO and CO desorption, $E_{\mathrm{act}}^{\mathrm{NO,CO}}$, decrease with increasing coverage due to repulsive interactions:

$$E_{\mathrm{act}}^{\mathrm{NO,CO}}(\theta) = E_{\mathrm{act}}^{NO,CO}(0) - k_6 * (\theta_{\mathrm{NO}} + \theta_{\mathrm{CO}})^2,$$

which proves to be necessary for the occurrence of kinetic oscillation.

Fig. 1a displays a bifurcation diagram for this model, for $p_{\mathrm{CO}} = 3 \times 10^{-7}$ mbar and $T = 420$ K, where one observes two Hopf bifurcation points at $p_{\mathrm{NO}} \simeq 3.226 \times 10^{-7}$ mbar and 4.25×10^{-7} mbar between which is the oscillation region (see also Ref. [45]). We now choose $p_{\mathrm{NO}}^0 = 3.22 \times 10^{-7}$ mbar such that the deterministic steady state is the stable node, and perturbed it by noise, $p_{\mathrm{NO}} = p_{\mathrm{NO}}^0[1 + 2D\Gamma(t)]$, here D is the noise intensity, and $\Gamma(t)$ random noise obeying Gaussian distribution with zero mean value and unit variance. Firstly, we will study the white-noise case, i.e., $\tau = \Delta t$. For $D \neq 0$, an obvious peak appears in the power spectrum of the time series $\theta_{\mathrm{NO}}(t)$, which indicates the occurrence of NICO. With the increment of D, the peak becomes higher and wider and if D is too large, it's annihilated into the noise background. This behavior is depicted in Fig. 1b, where the power spectrum curves are obtained from averaging over 100 independent runs for each D. Interestingly, it seems that the variation tendency of the NICO-strength with D shows the characteristic of SR. The relative NICO-strength can be measured by h or $\beta = h/\Delta\omega$, here h is the peak-height and $\Delta\omega$ the width of the peak at the height $h/2$. The variation of h and β with D are shown in Fig. 1c, where both values go through a maximum, demonstrating the occurrence of ISSR.

Now one can further study the the influence of τ on the strength of NICO. When τ increases, a rather interesting behavior is found as shown in Fig. 2, where the noise intensity is fixed at $D = 10^{-4}$.

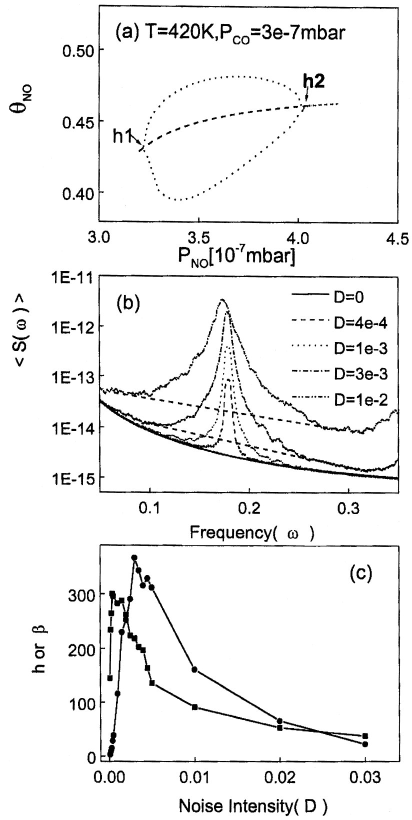

Fig. 1. (a) A typical bifurcation diagram for the NO+CO reduction system. Solid line: stable node; dashed line: unstable node or foci; dotted line: the amplitude of the oscillation. $h1$ and $h2$ are two supercritical Hopf bifurcation points (b). The average power spectrum of the noise induced oscillation for different noise intensities. The dashed lines denote the baseline for large noise. (c) Dependence of the peak height h (circles) and β (squares) on the noise intensity near the Hopf bifurcation point $h1$. Stochastic resonance behavior is observed.

Multi-maxima appear in the $h \sim \tau$ curve nearly periodically with period $T_\tau \simeq 550$. For $\tau \simeq (n+1)/2T_\tau$, here n is a positive integer, h reaches the maximum and for $\tau \simeq nT_\tau$, h is nearly zero. In addition, the maximum value at $\tau \simeq (n+1)/2T_\tau$ decay exponentially. Fitting with the function $h \sim \exp[-(\tau - \tau_0)/\gamma]$, we obtain $\tau_0 \simeq T_\tau/2$ and $\gamma \simeq 3.65$. Another interesting and rather surprising phenomena is that T_τ nearly equals to the period of the NICO T_0 (for $D = 10^{-4}$, the frequency of the NICO reads approximately 0.0018, see Fig. 1b).

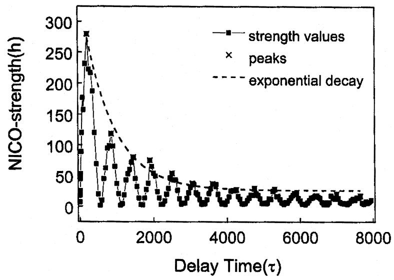

Fig. 2. The dependence of the NICO-strength on the delay time r. Multi-maxima appear nearly periodically. The dashed line fits the exponential decay of the peak.

The occurrence of NICO is not so surprising. Due to the perturbation of the control parameter, the system enters the oscillation region now and then and therefore the deterministic limit cycle is occasionally random-modulated resulting a Gaussian-expanded peak in the power spectrum. However, the occurrence of ISSR is rather counterintuitive and it can not be explained by previous reported theoretical models. Firstly, a potential model applicable to bistable or excitable systems is not suitable here, for there is no potential barrier to hinder the hopping between the stable node and the limit cycle. On the other hand, the threshold-free model proposed by Bezrukov et al. [17–19] does not work either, because an exponential relationship between input and output rates was needed there. In addition, the Hopf bifurcation here is supercritical and the motion on the deterministic limit cycle is quite uniform such that the 'nonuniform-motion' condition proposed by Rappel et al. [32–36] is yet not satisfied. So a new mechanism must be responsible for the ISSR observed in the present work. In fact, one has to account for the increment of the NICO-strength in the small D range. In the present work, we think that two facts might be responsible for it. Firstly, there exists a distance between p_{NO}^0 and the correct Hopf bifurcation point p^*. Secondly, with the increment of the control parameter p_{NO}^0, the strength of the deterministic oscillation also remarkably increases. So with the increment of D from zero, the chance for the system to pass across the distance $p^* - p_{\mathrm{NO}}^0$ to enter the stronger oscillation region would increase such that the NICO-strength could be enhanced. Of course, if D is too large, the coherent oscillation would be annihilated by the noise. In the present work, we have also perform numerical simulation work to investigate the influence of p_{NO}^0 on the ISSR. We find that with the approach of p_{NO}^0 to p^*, the ISSR peak becomes left-shifted and higher which implies that a smaller D can lead to the maximum NICO-strength. Notice that these results are in consistent with above discussions. However, one expects that a more complete and convincing explanation is necessary and a proper theoretical model would be great helpful in the future work.

The nontrivial dependence of the NICO-strength with τ indicates the important role of the relationship between the characteristic time scale of noise (τ) and the system (here is T_0). If they are of nearly the same magnitude, the effects of the noise are nearly annihilated; while if $\tau \simeq (n+1)/2T_0$ is satisfied, the effects of noise are the most significant and the NICO-strength attains the maximum; but if n is larger than one, the effects of noise are annihilated during each T_0 and consequently decays with the increment of n, which can be well approximated by an exponential-decay function. Here we would like to stress two main points. On one hand, if comparing to the white-noise case, one expects that suitable choice of noise-duration-time τ can considerably enhance the strength of ISSR. On the other hand, the match of T_τ and T_0 as well as the exponential decay of the maximum h at $\tau \simeq (n+1)/2T_0$ are surprising, which may open further perspective in the study on the interacting mechanism of noise with nonlinear system near Hopf bifurcation points.

Acknowledgements

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