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A two variable model, which has been proposed to describe a first-order, exothermic, irreversible reaction $A \rightarrow B$ carried out in a continuous stirred tank reactor (CSTR), is investigated when the control parameter is modulated by random and/or periodic forces. Within the bistable region where a limit cycle and a stable node coexist, stochastic resonance (SR) is observed when both random and periodic modulations are present. In the absence of periodic external signal noise induced coherent oscillations (NICO) appear when the control parameter is randomly modulated near the supercritical Hopf bifurcation point. In addition, the NICO-strength goes through a maximum with the increment of the noise intensity, characteristic for the occurrence of *internal signal stochastic resonance* (ISSR). [S0021-9606(99)51525-0]

I. INTRODUCTION

The phenomenon of stochastic resonance (SR) has been extensively studied in the last decade.[1] The main result of SR, which is in some ways considered counterintuitive, shows a constructive role of noise, since the response of the system to external periodic signal may be enhanced with the addition of an optimized amount of noise. In this sense, the main fingerprint of SR is the appearance of a maximum in the output signal-to-noise ratio (SNR) at a nonzero noise level.

In spite of the fact that the original SR model has been concerned with a bistable system subjected to external periodic signal and additive Gaussian white noise,[2] it is now known that there are different situations where SR appears. For example, the nonlinear system can be monostable,[3] excitable,[4] spatially extended,[5] or nondynamic;[6] the signal can be aperiodic or chaotic;[7] the noise can be colored, multiplicative,[8] or even substituted by the intrinsic randomness of a deterministic chaotic system, which leads to the so-called noise-free SR.[9] Very recently, it was reported that the external signal can be replaced by ''internal signal,'' as, for instance, the deterministic oscillation of the system.[10] When the control parameter is randomly modulated near the bifurcation point between a stable node and a stable limit cycle, noise induced coherent oscillation (NICO) is observed, the strength of which goes through a maximum with the increment of the noise intensity, showing the occurrence of SR, which may be named *internal signal stochastic resonance* (ISSR).

Although SR has found important applications in different fields of science, there are so far only a few reports concerning the study of SR in chemical reaction systems.[11,12] However, chemical reactions often go far from equilibrium and show complex nonlinear spatiotemporal behavior, such as multistability, oscillation, deterministic chaos, chemical waves, and so on. Therefore, one expects that SR, as well as ISSR or even noise-free SR, should be rather common here. In our previous work we have reported SR in the $CO+O_2$ oxidation and NO+CO reduction systems.[12] In the present work we will study the SR and ISSR behavior of a two-variable model, which has been proposed to describe a first order, exothermic, irreversible reaction $A \rightarrow B$ in continuous stirred tank reactor (CSTR). This study might provide instructions for future experimental work.

II. THE MODEL AND RESULTS

According to the irreversible $A \rightarrow B$ reaction carrying out in CSTR, the total mass flow (with rate j) carries heat and component A and B continuously into and out of the reaction vessel; heat is removed through a cooling coil which is maintained at a temperature T_c; component A reacts to form B and heat is released. The mass balance and the energy balance for component A can be described by two coupled first-order differential equations as following:[13]

$$\dot{x} = 1 - x - xDE(y), \tag{1.1}$$

$$\dot{y} = -\beta y + xBDE(y), \tag{1.2}$$

where $E(y) = \exp[y/(1+\eta y)]$, x and y are dimensionless concentration and temperature, respectively; for certain choices of the physical quantities, the parameters B, D, β, and η are related to the control parameters j and T_c via the following equations:

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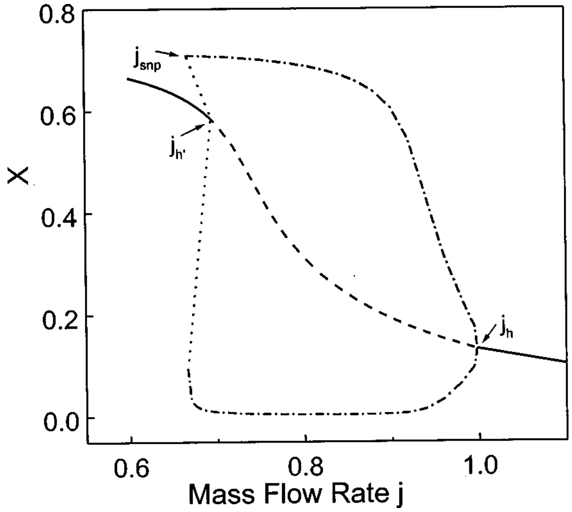

FIG. 1. The bifurcation diagram of the CSTR model. The stable (completely unstable) stationary states are denoted with a solid (dashed) line; the stable (unstable) limit cycles are denoted with a dot–dashed (dotted) line. The parameter path contains a *snp* bifurcation value j_{snp} and a supercritical and subcritical Hopf bifurcation value: j_h and $j_{h'}$.

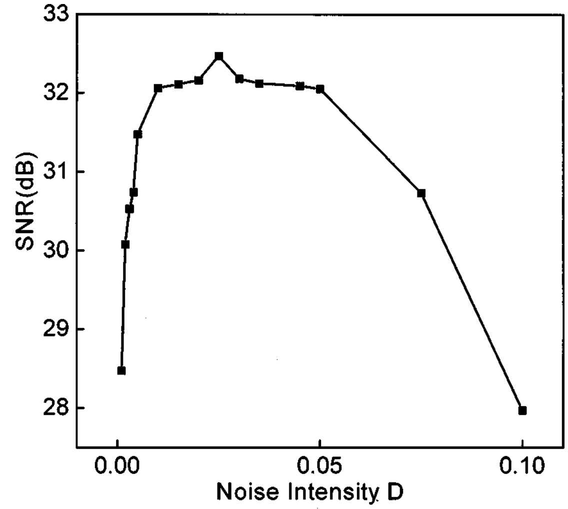

FIG. 2. The dependence of the output SNR on the noise intensity, when the control parameter is modulated by periodic force as well as noise near the bistable region. Stochastic resonance is observed.

$$\eta=\frac{T^*}{8827},\quad D=\frac{8.2365\times 10^{10}e^{-1/\eta}}{j},$$

$$B=\frac{271.46}{\eta T^*},\quad \beta=1+\frac{4.08}{j},\quad T^*=\frac{885.8j+11.02T_c}{2.7j+11.02}.$$

For more details of the CSTR equations and the physical quantities, one can turn to Ref. 13 and references there. Figure 1 shows the bifurcation diagram for Eq. (1) with the variation of j, keeping $T_c=292\,\mathrm{K}$. The parameter path contains a saddle-node bifurcation value of periodic orbits (j_{snp}) and a supercritical and subcritical Hopf bifurcation value: j_h and $j_{h'}$. Within the bistable region between j_{snp} and $j_{h'}$, the stable node is surrounded by a smaller unstable limit cycle (the dot line) and a larger stable limit cycle (the dash–dot line). Notice that this diagram is similar to that in Ref. 13 and we redraw it here just for convenience.

A. Stochastic resonance

To study the SR behavior, we choose the control parameter j to be located near the bistable region, where the deterministic attractor is the stable node, and modulate it by both periodic and random forces, as

$$j=j_0[1+\alpha\sin 2\pi f_s t+2D\dot{\xi}(t)],\tag{2}$$

where $j_0=0.6661$, α and f_s are the amplitude and frequency of the periodic signal, respectively; $\xi(t)$ denotes Gaussian white noise with zero mean and unit variance: $\langle\xi(t)\rangle=0$ and $\langle\xi(t)\xi(t')\rangle=\delta(t-t')$; D is the noise intensity. Notice that the periodic signal is subthreshold ($\alpha=0.073$), i.e., it cannot lead to deterministic transitions between the stable node and the limit cycle, and $f_s(0.02)$ is much smaller than that of the deterministic oscillation $f_0(\simeq 0.24)$ near j_{snp}. Equations (1) and (2) are solved by standard explicit Eular method. With the increment of noise intensity, the system shows standard behavior like that observed in classical SR model: A small amount of noise leads occasionally to transition from the stable node state to the oscillation state; this transition nearly synchronizes the periodic signal at an optimized noise level (SR occurs); and for too large noise, the transition is totally dominated by randomness. We choose $x(t)$ as the output signal and calculate its power spectrum $S\langle\omega\rangle$ by Fourier transformation of the correlation function $\langle x(t)x(t')\rangle$. The SNR is defined as usual at $\omega=f_s$ and its dependence on D is depicted in Fig. 2, where a maximum is present, characteristic for the occurrence of SR.

B. Internal signal stochastic resonance

To study the ISSR behavior, we choose the control parameter j to be located in the right vicinity of the supercritical Hopf bifurcation point j_h and perturb it by Gaussian white noise

$$j=j_0[1+2D\xi(t)],\tag{3}$$

with $j_0=0.998$. For $D=0$, the deterministic oscillation is absent. If D exceeds a certain level, however, an obvious peak shows in the output power spectrum, indicating the occurrence of NICO. With the increment of noise level, the NICO-strength is first considerably enhanced and finally annihilated into the noise background. In addition, for suitable noise level, higher harmonics up to $7\omega_0$ (here ω_0 is the fundamental frequency of the NICO) appear in the power spectrum and their strength also goes through nonmonotonical variation. Obviously, the dependence of the NICO-strength on noise intensity shows the characteristic of SR. The power spectra for different noise level are presented in Fig. 3. Notice that since the oscillation is induced by noise, the power spectrum of $x(t)$ of a separate run is not smooth and the SNR should be calculated by statistical averaging over enough independent runs to decrease the random effects, e.g., in Fig. 3 each curve is obtained by averaging over 50 independent runs. The ISSR behavior are depicted in Fig. 4, where the NICO-strength is measured by SNR/$\Delta\omega$, here $\Delta\omega$ is the width of the NICO-peak at the half of its height. The higher

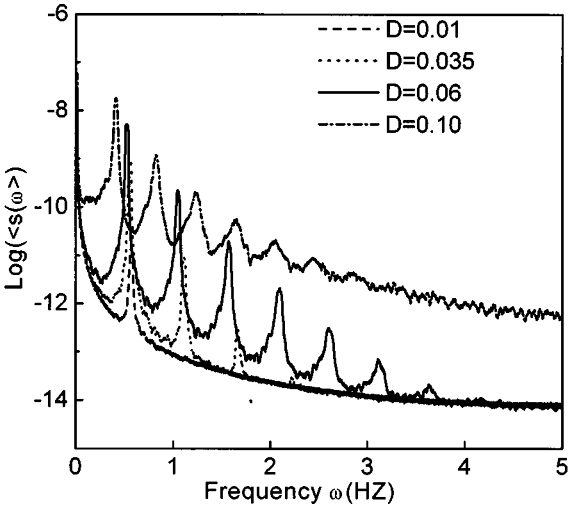

FIG. 3. The average power spectra for different noise level, when the control parameter is random modulated near the supercritical Hopf bifurcation value j_h. For $D=0.06$, the NICO-strength reaches the maximum and even $7\omega_0$ component is observed.

harmonics also show ISSR behavior, as, for instance, the ISSR-curve for $2\omega_0$-harmonic and $3\omega_0$-harmonic is shown in Fig. 4. It is found that all the harmonic oscillations reach the maximum strength at a same noise level $D\simeq0.06$.

III. CONCLUSION

In this paper, we have studied the noisy dynamic behavior of a two variable model, which has been proposed to describe an exothermic irreversible $A\rightarrow B$ reaction carried in a CSTR. When the control parameter is located near the bistable region and modulated by periodic force as well as noise, the transition between the stable node and the limit cycle can nearly synchronize the periodic signal at an optimized noise level, i.e., stochastic resonance (SR) occurs. If the control parameter is randomly modulated near a supercritical Hopf bifurcation point, noise induced coherent oscillations, including higher harmonics up to even seventh order, are observed. The strength of the fundamental component and all the harmonics go through a maximum with the increment of noise intensity, showing the occurrence of internal signal stochastic resonance (ISSR). We expect that our work might help find experimental SR in CSTR systems and open further perspectives in the study of ISSR.

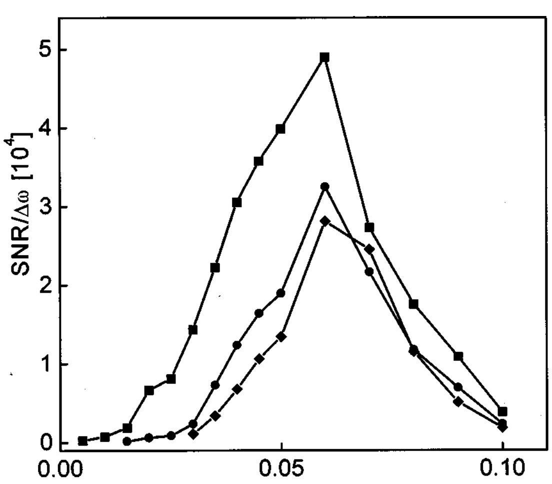

FIG. 4. The dependence of the NICO-strength of ω_0 (squares), $2\omega_0$ (circles), and $3\omega_0$ (diamonds) components on noise intensity. All the harmonics reaches the maximum at $D\simeq0.12$. Notice the strength of the $2\omega_0(3\omega_0)$ components are multiplied by 5(20), respectively.

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