Transfer of Noise into Signal through One-Way Coupled Chemical Oscillators

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The constructive effects of noise and disorder in nonlinear systems, especially stochastic resonance (SR),[1] have drawn great research interests. Recently, the frontier of this interest has shifted to spatially extended systems. A variety of interesting effects of noise and disorder have been reported, including, for example, array-enhanced stochastic resonance,[2] array-enhanced coherent resonance,[3] spatio-temporal stochastic resonance,[4] noise-sustained chemical waves,[5] noise-induced pattern transition,[6] taming spatio-temporal chaos by disorder.[7] In one-dimensional systems, specifically, an interesting phenomenon is noise-enhanced propagation.[8–15] It is found that noise can help a regular signal to propagate along a coupled array of bistable elements,[8–12] excitable systems,[13–14] or even monostable elements.[15] It often happens that an array of coupled dynamic elements is only subjected to external signal at only one end, such that the noise-enhanced propagation behavior is of great importance, especially for signaling processes in living systems.

However, we are still far from deeply understanding the effects of noise in coupled nonlinear systems. Actually, this is a rather "complex" problem, of which the complexity arises, for example, from the diversity of the node type, the coupling strength as well as coupling ways, the state of each site, the noise intensity and property. Therefore, it is still on the way to find new phenomena and new laws. Herein, we have investigated how a chain of one-way coupled chemical oscillators, all tuned near the supercritical Hopf bifurcation (HB), responds to local parameter noise at the first site. Interestingly, we find that the noise-induced oscillation (NIO) triggered at the first oscillator can propagate along the chain with considerable enhancement and noise suppression, such that at the end of the chain a rather regular signal is obtained. The effective signal-to-noise ratio (SNR) obtained at the end site is several orders larger than that at the first one, indicating that the system has transferred noise input into signal output.

The model we used herein was proposed to describe a first order, exothermic, irreversible reaction A→B in a continuously stirred tank reactor (CSTR).[16] The total mass flow (with rate r) carries components A and B continuously into and out of the reaction vessel. A reacts to form B and heat is released, which is removed through a cooling coil maintained at a constant temperature T. The mass and energy balance for component A can be described by the following deterministic Equations (1a) and (1b):

$$dx/dt = 1-x-\mu x E(y) = f(x,y;r) \tag{1a}$$

$$dy/dt = (1+\varepsilon)^{-1}[-\alpha y + \nu\mu x E(y)] = g(x,y;r) \tag{1b}$$

where $E(y)=\exp[y/(1+\eta y)]$, x and y are dimensionless concentration and temperature, respectively. The parameters α, η, μ, ν are related to r and T via $\alpha=1+4.08/r$, $\eta=T^*/8827$, $T^*=(885.8r+11.02T)(2.7r+11.02)$, $\mu=8.2365\times10^{11}e^{\frac{-1}{\eta}}/r$, and $\nu=271.46/(\eta T^*)$. The constant ε is 0.65. Herein, we choose r as the control parameter and keep $T=292$ K. Then a supercritical HB occurs at $r_h\approx0.998$ and below this value the oscillation region starts.

The one-way coupled CSTR system is then described by Equations (2a) and (2b):

$$dx_j/dt = f(x_j,y_j;r_j) + C(x_{j-1}-x_j) \tag{2a}$$

$$dy_j/dt = g(x_j,y_j;r_j) \tag{2b}$$

where j running from 2 to M is the index of the oscillator (M is the number of oscillators), and C is the coupling strength.

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Note that the behavior of the first site ($j=1$) is still described by Equations (1a) and (1b), and the coupling is considered via linear mass diffusion of *A*. For our purposes, we keep all the control parameters $r_{j=1,..,N}$ identically to be $r_0=1.01$, which is slightly higher than the HB, such that no oscillation exists in the absence of noise. Then we perturb the mass flow rate of the first site by noise, that is, $r_1=r_0[1+D\xi(t)]$, where $\xi(t)$ is Gaussian white noise with zero mean and unit variance, and *D* denotes the noise intensity. Since the behavior of the first site is not affected by the coupling, one would expect to find NIO and internal signal stochastic resonance, as already been reported before.[17] NIO is distinct from random noise because of the existence of a clear peak in its power spectrum, and internal signal stochastic resonance is characterized by the occurrence of a maximum SNR of the NIO with the variation of the noise intensity *D*.

When we investigate the dynamic behavior of the oscillators one by one, we find that the NIO triggered at the first site has propagated along the chain with considerable amplification and noise suppression. At the end of the chain, a rather "regular" signal with large amplitude and clear periodicity is obtained. The results obtained for $M=20$, $C=0.4$ and $D=0.025$ are displayed in Figure 1 a to 1 c. In Figure 1 a, the time series of x_1 and x_{20} are shown. It is apparent that x_{20} is much more regular and stronger than x_1. The corresponding power spectrum density (PSD) of x_1 and x_{20} are shown in Figure 1 b. For x_1, only one principal peak is observed. But for x_{20}, we find clearly multiple peaks at the base frequency and a few higher harmonics; and compared to the first site, the principal peak becomes much higher and narrower, the noise background becomes lower, and the base frequency shifts slightly to a smaller value. In Figure 1 c, the variations of the signal level $S=P(A)$, noise background $N=P(B)$ and the SNR along the chain are displayed, where $P(\cdot)$ denotes the PSD value of a given point. Note that the SNR is not simply calculated as *S/N*, but as $\text{SNR}=(S/N)\cdot\omega_A/(\omega_C-\omega_A)$. This equation takes into account the relative width of the peak. The points A, B, and C are shown in Figure 1 b, where A points to the peak, B is the minimum point of the PSD curve inside $(0, \omega_A)$, and C is determined via $P(C)=P(A)/e$. Note that the definition here is in spirit identical to that used in the literature.[14,17,18] When the NIO propagates along the chain, the signal level increases monotonically to be $\approx 10^2$ times larger, the noise background decreases sharply to $\approx 10^2$ times smaller, and consequently the SNR of $j=20$ is about 10^4 times larger than that of $j=1$. Therefore, a noise input at one end of the chain has been effectively transferred to a rather "regular" signal output at the other end.

In Figure 1 d, the contour plot of the SNR (in logarithmic scale) for different site and noise intensity is shown. For small and large *D* values, the SNR increases monotonically along the chain, while for intermediate values of *D*, the SNR shows a maximum in the middle. But for any noise intensity considered, the SNR of site 20 is always much larger than that of site 1. The dashed line in Figure 1 d corresponds to $D=0.025$ used in Figures 1 a to 1 c, where the SNR on *j* has a maximum at $j\approx 14$.

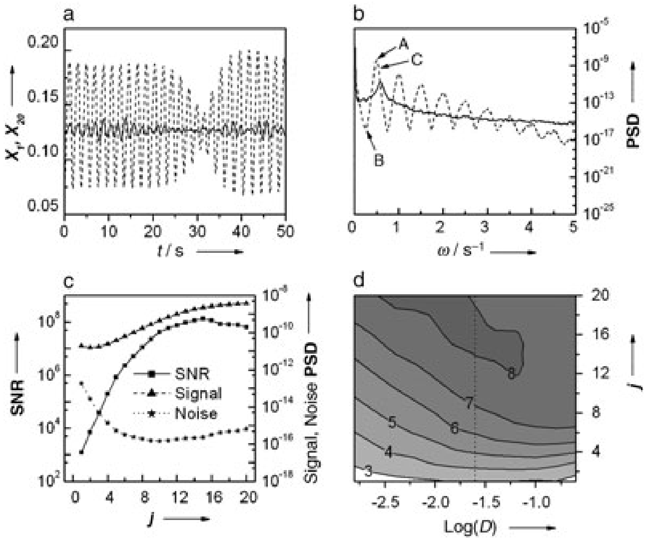

Figure 1. a) Stochastic oscillations for $j=1$ and $j=20$. The same scales are used. b) Corresponding power spectra for x_1 and x_{20}. Points A, B, and C demonstrate how to get the signal level S, noise background N, and the effective SNR for x_{20}: $S=P(A)$, $N=P(B)$, and $\text{SNR}=(S/N)\omega_A/(\omega_C-\omega_A)$. Note that the PSD are calculated from time series containing 16384 points, and the smoothed curves shown here are obtained via nearest-averaging over 50 points. Arbitrary units are used for the PSD. c) Variations of the SNR (■, left axis), signal level (▲, right axis) and noise background (⋆, right axis) along the chain. d) Contour plot of the SNR for different oscillator and noise intensity; the values on the contour lines denote the logarithm of the SNR. The dashed line corresponds to $D=0.025$ used in Figure 1 a to 1 c. For all diagrams a) to d), the coupling strength is $C=0.4$.

In short, we find that local noise stimulation can induce remote signal output in a chain of one-way coupled chemical oscillators tuned close to the HB. To check how this phenomenon depends on the setup of the system, we have also studied other cases. Firstly, we find that "global noise" is destructive. If all the sites in the chain are globally subjected to parameter noises (whether correlated or not), the SNR of site 20 will be much smaller, as displayed in Figure 2 a. One should

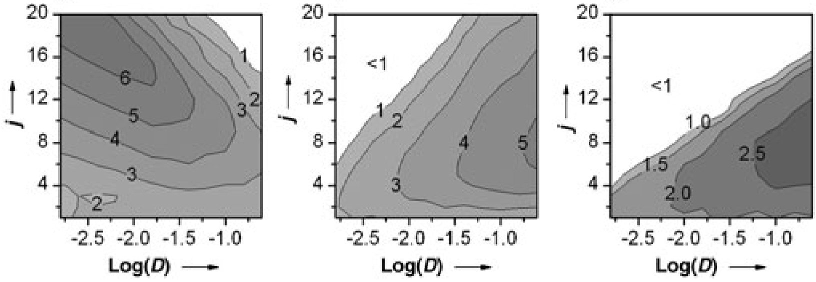

Figure 2. SNR plots for different cases from Figure 1. a) Global noise is used. The results shown here are for uncorrelated noises, but same qualitative results are obtained for correlated ones; b) $r_0=1.03$; c) two-way coupling. In all three diagrams, the values on the contour lines denote the logarithm of the SNR.

note that "global noise" was once used to denote fully spatial-correlated noises in coupled systems,[2,3,8] while it simply means that all the sites are noisy herein. Secondly, if the control parameter is moved away from the HB, the NIO may only propagate for a short length. Figure 2b shows the SNR plot for $r_0 = 1.03$, where no signal output is available at the end of chain for small noise intensities. For further larger r_0, the maximal SNR decreases, and its location in the D–j plane moves towards the direction of larger D and smaller j, indicating that propagation of the NIO becomes harder. Finally, if the oscillators are two-way coupled, the NIO cannot reach the end site at all, as shown in Figure 2c. Therefore, "local noise", "being close to the HB", and "one-way coupling" are necessary for the amplified propagation of the NIO.

It would be instructive to compare our results with previous work on "noise-enhanced propagation", which mainly considered how noise can help the propagation of a periodic signal along coupled bistable elements.[8–12] In contrast to those studies, our system works in the region close to a HB and no external periodic signal is needed: local noise input can induce remote signal output. In addition, it was demonstrated previously that global noise was necessary for the undamped propagation of the external signal,[12] while in the present work, we find that global noise is destructive to the propagation of the NIO. One also notes that most studies before used additive noises, while the present results are obtained for multiplicative parameter noises (nevertheless, very similar results have been found when additive noises are used).

In real systems, coupling strength is also an important parameter. Therefore, it is helpful to investigate the role of the coupling constant C. The results are shown in Figure 3 for C running from 0.2 to 0.7. For small and large coupling strengths, the SNR undergoes a clear maximum along the chain for a given D, and the NIO may damp out before reaching the end site, as shown in Figures 3a and 3f. If the coupling is weak, a larger noise tends to favor the signal propagation, and for a strong coupling, a smaller noise is better. But for an intermediate coupling strength, for example, $C = 0.3$ as shown in Figure 3b, the SNR always increases along the chain no matter what the noise intensity is. Therefore, an optimal coupling strength exists for the amplified propagation of the NIO. One notes that this is also different from previous studies, where a stronger coupling was always more favorable for noise-enhanced propagation of external signals.[8–10,12]

Although we have adopted a specific chemical model in the present study, it is likely that such a phenomenon may also exist in other dynamic systems. By use of a model for calcium signaling, we were able to obtain very similar results: noise-induced calcium oscillation can propagate along a cascade of cells with considerable enhancement, when the cells have a parameter value close to the deterministic Hopf bifurcation (the results are not shown here). Recently, is was demonstrated that sensory systems in vivo may work in the very vicinity of supercritical Hopf bifurcations through self-tuned mechanisms;[19,20] therefore, it could happen that such systems respond to ambient noisy stimulus with regular electrical signals using the mechanism described herein.

The use of a CSTR model makes it possible for a relevant experimental setup to testify the main results of the present work. On one hand, the mass flow rate into the first CSTR can be controlled via a syringe pump connected to a computer, where one can easily generate parameter noise. In a recent experimental work, by adopting parameter noise in such a way, noise-induced oscillations and internal signal stochastic resonance were successfully demonstrated in a single Belousov–Zhabotinsky CSTR system.[21] One notes here that parameter noise is easier to handle experimentally than an additive noise. On the other hand, CSTR reactors can be linearly coupled directly through mass diffusion[22–26] or indirectly via computer control,[27] and the coupling strength of either direction can be adjusted via outflows attached to the reactor[25] or pumps.[26] We look forward to such an experimental study in the near future.

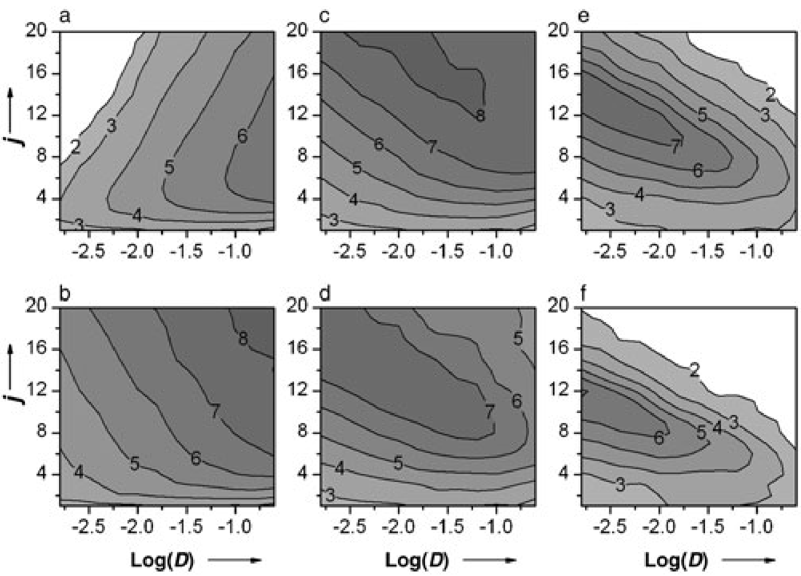

Figure 3. SNR plot for different coupling strengths. From a) to f), the coupling constant C is 0.2, 0.3, 0.4, 0.5, 0.6, and 0.7, respectively. It is clear that C = 0.3 is optimal for the amplified propagation of the NIO.

To conclude, we have studied how local noise could induce remote order in one-way coupled CSTR oscillators, all tuned near the supercritical Hopf bifurcation. When the first oscillator is subjected to parameter noise, stochastic oscillation is induced and propagates along the chain with considerable amplification and noise suppression. As a consequence, a rather regular signal output is obtained at the other end of the chain. By detailed numerical simulations, we find that local noise, being close to the Hopf

bifurcation, one-way coupling, and optimal coupling strength are important for this phenomenon. Since coupled oscillatory networks are often encountered in real systems, we hope our results could find
interesting applications.

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