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Best Spatiotemporal Performance Sustained by Optimal Number of Random Shortcuts on Complex Networks *

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The spatial synchronization and temporal coherence of FitzHugh–Nagumo (FHN) neurons on complex networks are numerically investigated. When an optimal number of random shortcuts are added to a regular neural chain, the system can reach a state which is nearly periodic in time and almost synchronized in space. More shortcuts do not increase the spatial synchronization too much, but will obviously destroy the temporal regularity.

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Complex networks have gained considerable attention^[1–3] in the past few years due to their importance in mimicking real systems of the world, such as social,^[4] biological,^[5,6] and electronic communication^[7–9] networks. In fact, any coupled system can be viewed as a network consisting of a number of sites connected with a certain topology graph, where vertices represent the dynamic elements of the system and the edges represent the interactions or couplings between them. Two distinct types of complex networks, the small-world network (SWN)^[10] and the scale-free network (SFN),^[11] have been studied extensively due to their important relevance to many real-world structures. Studies on complex networks can be divided into two main categories. While currently the dominant one is to study the topological properties of complex networks and various mechanisms to determine the topology, the other one, which is considered to be more important, is to study how the interplay between the intrinsic dynamic of the constituent elements and their complex connectivity can influence the system's dynamical features. It is now accepted that the network topology can considerably influence the collective behaviour of the system, although a thorough understanding is still needed. For instance, it was found that any spreading rate can lead to the whole infection of disease in a scale-free small-world network,^[12,13] stochastic resonance^[14] and synchronization^[15,16] can be considerably enhanced on SWN, and small-world connections can improve the probability of spiral wave formation in excitable media,^[17] tame spatiotemporal chaos,^[18] and eliminate oscillation death,^[19] to list just a few examples.

Neural networks have always been the research subject in this area. The elementary processing units

in the central nervous system are neurons which are connected to each other in an intricate pattern. It is known that a single neuron in vertebrate cortex connects to more than 10^4 postsynaptic neurons via synapses, forming complex networks.^[20] Therefore, adding a number of long-range connections among them is reasonable and feasible. It has been shown that the topology of the FitzHugh–Nagumo (FHN) neuron network has an important impact on the effective frequency range of the system,^[21] and small world chaotic Hindmarsh–Rose neural networks can achieve collective phase synchronization,^[22] the dynamic neuronal system with small world connectivity can be adjusted to an optimal sensitive state for signal processing in the presence of additive noise.^[23] Thus, the studies of the influence of complex network topology on the dynamics of real neural systems are of great significance.

In this Letter, we investigate the synchronization and coherence of noisy FHN neurons on complex networks, constructed by randomly adding shortcuts to a regular one-dimensional lattice. On the original regular chain, the ensemble shows a rather disorder state due to the presence of noise and heterogeneity. Being consistent with the literature, we find that the neurons can be more synchronized with the increase of the number of random shortcuts. When investigating the collective temporal behaviour of the neuron network, however, we find that for an optimal number of shortcuts, the temporal regularity of the collective spike train reaches a maximum, corresponding to a nearly-periodic state in time. We also notice that the neurons are already nearly-synchronized in space at this state. Therefore, the distinct finding of our work is the existence of an optimal amount of random shortcuts sustaining a most spatiotemporal-ordered state.

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The system we consider here is N coupled FHN neurons on a complex network, which is constructed as follows. We start from a one-dimensional lattice under a periodic boundary condition, composed of $N = 60$ elements and each site is connected to its two nearest neighbours. Then we randomly add links between non-nearest sites, and the number of random shortcuts is denoted by M . In the limit case when the neurons are globally-coupled to each other, the network contains $N(N-1)/2$ edges. Hence the fraction of random shortcuts is given by $p = M/[N(N-1)/2]$. Since the structural properties of complex networks, e.g. characteristic length $L(p)$ and clustering coefficient $C(p)$ in SWN, vary together with randomness p , it is important to study the effect of p on the dynamics of complex networks. One may use the parameter p to measure the randomness of the network. However, we should note that for a given p , there could be a lot of network realizations.

The dynamics of the noisy FHN^[24] neuron network can be described by the following equations:

$$\begin{aligned} \varepsilon \frac{dx_i}{dt} &= x_i - \frac{x_i^3}{3} - y_i + \sum_j g_{ij}(x_j - x_i), \\ \frac{dy_i}{dt} &= x_i + a_i + D\xi_i(t), \end{aligned} \quad (1)$$

where $i, j = 1, 2, \dots, N$. The variables x in Eq. (1) are the membrane potential of the neuron and y is related to the time-dependent conductance of the potassium channels in the membrane; $\varepsilon = 0.01$ is a small parameter allowing one to separate the motions of the fast and slow variables.^[25] For a single FHN neuron, the system has an excitable stable fixed point for $|a| > 1$, and a stable limit cycle for $|a| < 1$. We assume that all the neurons stay at the excitable state in the absence of noise such that all a_i are larger than 1. Heterogeneity is also introduced by assuming a uniform distribution of a_i inside $(1, 1.1)$. $D\xi_i(t)$ denotes the noise of the i th neuron, where D is the noise strength (we consider a homogeneous noise level along the network) and $\xi_i(t)$ is Gaussian white noise with zero mean value and unit variance, i.e. $\langle \xi_i(t)\xi_j(t') \rangle = \delta_{ij}\delta(t-t')$. It is well-known that noise will induce firing activities of the FHN neuron, a general property of excitable systems. Due to the heterogeneity of a_i , the neurons will have different average firing frequencies^[26] if they are not coupled, although the noise levels are the same. Here g_{ij} is the coupling strength between neuron i and j , which is determined by the coupling pattern of the system. If these two neurons are connected to each other, we have $g_{ij} = 0.03$, and otherwise $g_{ij} = 0$. Numerical integration of Eq. (1) is carried out by explicit Euler method with a time step 0.001 s.

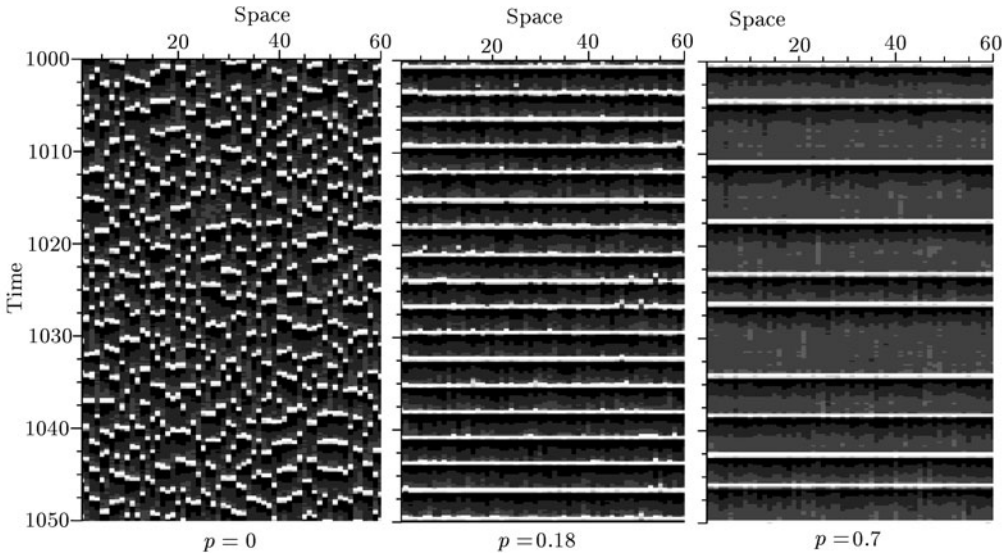


Fig. 1. Spatiotemporal evolution of $x_i(t)$ for (a) $p = 0.0$, (b) $p = 0.18$, (c) $p = 0.70$. Time increases from top to bottom.

To focus on the effect of the network topology, we fix the noise strength $D = 0.2$, which is able to trigger spikes for the above-mentioned parameters, and vary p . The spatiotemporal evolution of all the 60-neuron membrane potential $x_i(t)$ is displayed in Fig. 1. Time passes from top to bottom. Figure 1(a) corresponds to the regular lattice case, i.e. $p = 0$. In the bright regions, the neurons fire, while in the dark ones they

are in quiescent states. It shows that without random shortcuts, the network shows rather irregular spiking activity in spacetime. However, when a certain number of random shortcuts are present, we find that the system can show a rather regular spatiotemporal pattern, which is almost synchronized in space and nearly periodic in time (see Fig. 1(b) for $p = 0.18$). If the number of random shortcuts is further increased (see

Fig. 1(c) for $p = 0.7$, the firing activities of the neurons are still synchronized in space, but the periodicity in time is clearly lost. Therefore, the existence of an optimal number of random shortcuts sustaining a most spatiotemporal ordered state is demonstrated.

To further quantitatively characterize this behaviour, we introduce standard deviation σ and reciprocal coefficient of variance R to measure the spatial synchronization and temporal regularity of the network collective firing activity, respectively. Here σ is defined as

$$\sigma = [\langle \sigma(t) \rangle]$$

with

$$\sigma(t) = \sqrt{\frac{\left[\frac{1}{N} \sum_{i=1}^N x_i(t)^2 - \left\{ \frac{1}{N} \sum_{i=1}^N x_i(t) \right\}^2 \right]}{(N-1)}}, \quad (2)$$

where $\langle \bullet \rangle$ represents the averaging over time and $[\bullet]$ denotes the averaging over 50 different network realizations for each p . Clearly from the definition, a smaller value of $\langle \sigma(t) \rangle$ denotes better synchronization, and $\langle \sigma(t) \rangle = 0$ corresponds to the complete synchronization. Here σ is a time-average of $\sigma(t)$, which is a reasonable measure of the synchronization in space for a long time. One should note that σ contains no information of the system temporal regularity. A state completely synchronized in space may correspond to many temporal patterns, periodic in time, totally random, or even chaotic. Hence, we further introduce R to characterize the system temporal regularity, which is defined as in Eq. (3):

$$R = \left[\frac{\langle T \rangle}{\sqrt{\langle T^2 \rangle - \langle T \rangle^2}} \right], \quad (3)$$

where $[\bullet]$ has the same meaning as above. $\langle T \rangle$ and $\langle T^2 \rangle$ are the mean and mean-squared interspike intervals (ISI), respectively. To describe the network collective behaviour, we have introduced the average

membrane potential as $x_{\text{out}}(t) = \frac{1}{N} \sum_{i=1}^N x_i(t)$. A spike

is defined to occur each time x_{out} across 0.5 mV. Note that R represents a measurement of the spike coherence, in the way that a larger R represents better periodicity in time, and for a completely periodic spike train R is infinity. Such a measure as R is of biological significance because it is related to the spike timing precision of the information processing in real neural systems.

Figure 2(a) depicts the dependence of σ on p . We can see that the value of σ decreases monotonically when p increases, approaching zero as p is large enough. This enhancement of synchronization by adding connections is consistent with many previous

studies and the mechanism has been well-understood. One only needs to note that σ decreases very sharply in a narrow range around $p = 0.1$, and after that, σ undergoes a rather even change. There is a turning point at $p \sim 0.15$. Correspondingly, the curve of R versus p is displayed in Fig. 2(b). Interestingly, a clear maximum is shown at $p \approx 0.18$, where the neurons reaches the best performance in their collective temporal dynamics. On the other hand, it is important to emphasize that at such an optimal case, σ already reaches the even-varying stage and has a rather low value (see Fig. 2(a)), indicating that the neurons are already almost-synchronized in space. Therefore, the system is both nearly-periodic in time and almost-synchronized in space, i.e. it is in a most spatiotemporal ordered state.

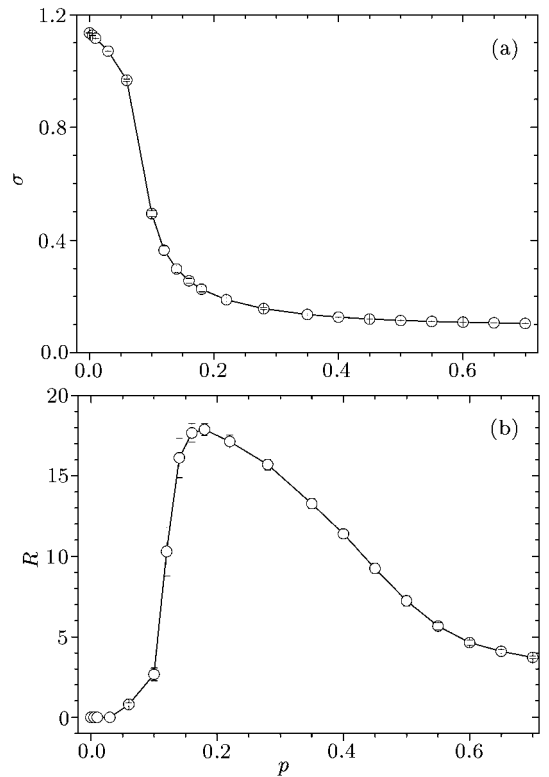


Fig. 2. Dependences of (a) σ and (b) R on p .

The study of synchronization of coupled oscillatory elements is an intensively developing branch of nonlinear science^[27–29] for its relevance to many problems of physics, chemistry, and life science, in particular to neuroscience. Synchronization of coupled neurons has been suggested as a mechanism for binding spatially distributed features into a coherent object and may play an important role in revealing communication pathways in neurons.^[30] Moreover, it is known that neural information is mainly transmitted using a code based on the time intervals between neural firings^[31], and the reciprocal coefficient of variance R

is of biological significance because it is related to the spike timing precision of the information processing in neural systems. Hence, it is necessary to study the synchronization and the coherence of the neurons. In real neural systems, neurons cannot interact by regular connection, but are connected to each other in an intricate pattern forming a complex network. Thus our findings of the dynamics of FHN neurons on complex networks may find its applications for a better understanding of the behaviour of real neural systems.

In conclusion, we have studied the synchronization and coherence of FHN neurons on complex networks. It is found that complex network connectivity can lead synchronization which is absent in the regular lattice, and this spatial synchronization of FHN neurons is enhanced as p increases. The temporal coherence of the network collective dynamics is also calculated. Interestingly, a novel phenomenon is found such that there is an optimal fraction of random connections which sustains a state of the maximal spatiotemporal order, which is nearly-periodic in time and almost-synchronized in space. Since real neural networks are often complex, we hope that our findings will find interesting applications.

References

- [1] Dorogovtsev S N and Mendes J F F 2002 *Adv. Phys.* **51** 1079
- [2] Albert R and Barabasi A L 2002 *Rev. Mod. Phys.* **74** 47
- [3] Strogatz S H 2001 *Nature* **410** 268
- [4] Watts D J and Strogatz S H 1998 *Nature* (London) **393** 440
- [5] Barabasi A L and Albert R 1999 *Science* **286** 509
- [6] Newman M E J 2001 *Proc. Natl. Acad. Sci. USA* **98** 404
- [7] Jeong H, Mason S P, Barabasi A L and Oltvai Z N 2001 *Nature* (London) **441** 41
- [8] Camacho J, Guimerá R and Amaral L N 2002 *Phys. Rev. Lett.* **88** 228102
- [9] Lawrence S and Giles C L 1998 *Science* **280** 98
- [10] Lawrence S and Giles C L 1999 *Nature* **400** 107
- [11] Albert R, Jeong H and Barabasi A L 1999 *Nature* **401** 130
- [12] Pastor-Satorras R and Vespignani A 2001 *Phys. Rev. Lett.* **86** 3200
- [13] Pastor-Satorras R 2002 *Phys. Rev. E* **65** 035108
- [14] Gao Z, Hu B and Hu G 2002 *Phys. Rev. E* **65** 016209
- [15] Barahona M and Pecora L M 2002 *Phys. Rev. Lett.* **89** 054101
- [16] Hong H, Choi M Y and Kim B J 2002 *Phys. Rev. E* **65** 026139
- [17] He D, Hu G, Zhan M, Ren W and Gao Z 2002 *Phys. Rev. E* **65** 055204
- [18] Qi F, Hou ZH and Xin H W 2003 *Phys. Rev. Lett.* **91** 064102
- [19] Hou Z H and Xin H W 2003 *Phys. Rev. E* **68** 055103
- [20] Gerstner W and Kistler W M 2002 *Spiking Neuron Models* (Cambridge: Cambridge University Press)
- [21] Li S H, Wang H L and Ouyang Q 2004 *Chin. Phys.* **13** 448
- [22] Wang Q Y and Lu Q S 2005 *Chin. Phys. Lett.* **22** 1329
- [23] Xu X J, Wang S J, Tang W and Wang Y H 2005 *Chin. Phys. Lett.* **22** 507
- [24] FitzHugh R A 1966 *Biophys. J.* **1** 445
- [25] Pikovsky A S and Kurths J 1997 *Phys. Rev. Lett.* **78** 775
- [26] Hu B and Zhou C 2000 *Phys. Rev. E* **61** R1001
- [27] Kuramoto Y 1984 *Chemical Oscillations, Waves and Turbulence* (Berlin: Springer)
- [28] Haken H 1993 *Advanced Synergetics: Instability Hierarchies of Self-Organizing Systems* (Berlin: Springer)
- [29] Pikovsky A, Rosenblum M and Kurths J 2001 *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge: Cambridge University Press)
- [30] Shuai J W and Durand D M 1999 *Phys. Lett. A* **264** 289
- [31] Eccles J C 1973 *The Understanding of the Brain* (New York: McGraw-Hill)