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Adaptive co-evolution of strategies and network leading to optimal cooperation level in spatial prisoner's dilemma game*

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We study evolutionary prisoner's dilemma game on adaptive networks where a population of players co-evolves with their interaction networks. During the co-evolution process, interacted players with opposite strategies either rewire the link between them with probability p or update their strategies with probability $1 - p$ depending on their payoffs. Numerical simulation shows that the final network is either split into some disconnected communities whose players share the same strategy within each community or forms a single connected network in which all nodes are in the same strategy. Interestingly, the density of cooperators in the final state can be maximised in an intermediate range of p via the competition between time scale of the network dynamics and that of the node dynamics. Finally, the mean-field analysis helps to understand the results of numerical simulation. Our results may provide some insight into understanding the emergence of cooperation in the real situation where the individuals' behaviour and their relationship adaptively co-evolve.

Keywords: prisoner's dilemma game, adaptive network, co-evolution, cooperation

PACC: 0250, 0175, 0565

1. Introduction

Complex networks have received considerable attentions in recent years. They occur in a large variety of real-world systems ranging from ecology and epidemiology to neuroscience, socioeconomics and computer science.[1−4] So far, two major topics of network research can be explicitly discriminated: the first one is associated with the dynamics of networks. Here, the topology of the network itself is considered as a dynamical system, and it evolves in time according to some special rules. Studies in this area have shown that certain evolution rules give rise to peculiar network topologies. Notable examples include the formation of small-world network (SWN)[5] and scale-free networks (SFN).[6,7] The second topic of network research focuses on the dynamics of networks in which nodes represent dynamical units and links stand for the interactions between them. Many studies have shown that network topology plays a crucial role in the system's dynamics (see, for example, a recent review[4] and references therein).

Until recently, the two topics of network research were reported almost independently in the literature. In most real-world networks, however, network topology and the state of node are both dynamic entities rather than static ones, and the evolution of network topology is often correlated to the state of node and vice versa. Some notable examples are: in acquaintance networks, people are more likely to maintain a social connection if their views and values are similar. In an artificial neuronal network, the network topology can be improved in the course of training depending on the state of the nodes and then modified topology determines the dynamics of the state in the next trial. In transportation networks, the network topology has a direct influence on traffic flow. For an usually congested road, some new roads will be likely to build so as to alleviate congested circumstance. Studies on such adaptive co-evolution mainly include synchronisation,[8] opinion dynamics,[9−13] game theory[14−27] and epi-

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demic spread[28,29] on adaptive networks. These studies have indicated adaptive co-evolution of network and individual dynamics could bring some nontrivial results.[30]

Game theory provides a useful framework for describing the evolution of systems consisting of selfish individuals.[31−33] One simple game, the prisoner's dilemma game (PDG), has attracted most attention in theoretical and experimental studies.[34,35] Since Nowak and May showed that the PDG on a simple spatial structure could lead to the emergence and persistence of cooperation,[36] much attention has been paid to evolutionary game on different graphs.[37,38] Especially, the advance of graph theory and achievement of complex networks have provided a natural and meaningful framework to describe the population structure on which the evolution of cooperation is studied. The nodes represent players, while the edges denote links between players. Extensive studies indicated that games on many real networks (e.g. SWN and SFN) were far different from that on regular networks.[39−48] Most notably Ref. [45] has recognised SFN as extremely potent promoters of cooperative behaviour in the PDG as well as another famous game, the snowdrift game. For a comprehensive review of this field of research see Ref. [49]. On the other hand, the significance of investigating the PDG on adaptive networks has been addressed recently.[14−27] In such adaptive networks, according to some rewarding and punishment rules, cognitive players can cut their partnerships or rebuild new social links so as to improve their competitive position. For example, Zimmermann and Eguíluz[16] have demonstrated a co-evolution model leads to more robust cooperation than fixed networked games, and shed some light on what network type is adaptively appropriate for cooperation to emerge. When node and network topology both adaptively evolve, two time scales are identified: one is associated with node dynamics, and another is associated with network dynamics.

In the present work, we consider a co-evolution model described as follows. At first a pair of connected players with opposite strategies is randomly selected. Then the underlying network updates its topology via rewiring their link with probability p; and with complementary probability $1-p$ one player at the end of this link will change its strategy if its payoff is smaller than the other. Note that p and $1-p$ are time scales of the network dynamics and node dynamics, respectively. An interesting question is therefore how cooperation level of the whole network would depend on the relative time scale of the network dynamics to that of the node dynamics. Interestingly, we find that the density of cooperators in the final state, characterising the overall gain of the game, can be maximised at an intermediate value of the relative ratio of the two time scales. Mean-field analysis is used to help to understand the simulation results.

This paper is organised as follows. In Section 2, we propose the co-evolutionary model. Numerical simulation results of the adaptive network structure and cooperation behaviours are presented in Section 3. Mean-field analysis is put forward in Section 4 and main conclusions are addressed in Section 5.

2. Model description

In the standard PDG, the players can adopt either cooperation (C) or defection (D); two interacting players are offered a certain payoff, the reward R, for mutual cooperation and a lower payoff, the punishment P, for mutual defection. If one player cooperates while the other defects, then the cooperator gets the lowest sucker's payoff S, while the defector gains the highest payoff, the temptation to defect T. Thus, we have $T > R > P > S$. It is easy to see that defection is the better choice irrespective of the opponent's selection. For this reason, defection is the only evolutionary stable strategy in well-mixed populations.[33] We describe the state of a player, C or D, by a state vector $\boldsymbol{s} = (1, 0)$ or $\boldsymbol{s} = (0, 1)$, respectively. Each player interacts with its neighbours and collects payoff depending on the payoff parameters. The total payoff of a certain player i can be expressed as

$$P_i = \sum\nolimits_{j\in\Omega_i} \boldsymbol{s}_i \boldsymbol{J} \boldsymbol{s}_j^{\mathrm{T}}, \tag{1}$$

where $\boldsymbol{s}_i$ and $\boldsymbol{s}_j$ denote the strategy of node i and j, the sum runs over all the neighbouring sites of i (this set is indicated by Ω_i) and the payoff matrix is given by

$$\boldsymbol{J} = \begin{pmatrix} R & S \\ T & P \end{pmatrix}. \tag{2}$$

Following common studies,[15,16,20,36,50,51] we also start by rescaling the game such that it depends on a single parameter, i.e., we can choose $R = 1$, $P = S = 0$, and $T = 1+r$ $(0 \le r \le 1)$ representing the advantage of defectors over cooperators (or the temptation to defect), without any loss of generality of the game.

There are three possible pairwise interactions in the network. Two C players connected by a link (C–C link) will try to maintain the interaction because both players benefit from the trade process. Though two D players connected by a link (D–D link) receive the minimum payoff, either of them may expect another player to change the present strategy such that he will receive the highest payoff. When a C player and a D player are connected by a link (C–D link), the C player's payoff is much lower than the D player's. In this case, it is reasonable that this link is broken by the C player and then rewired, or they update their strategies. Therefore, the evolutionary updating only occurs at C–D link. Due to lack of prior knowledge of the new neighbour's strategy, the target node a new rewired link points to is selected at random.

According to the above reasonable hypothesis, the evolution runs as follows. At each step, each individual plays the PDG with the same current state with all its neighbours, and collects an aggregate payoff. Then, a pair of connected players (i, j) is chosen at random from the whole population. If both players have the same strategy, nothing happens. Otherwise, an attempt to rewire the link is made with probability p (for the sake of convenience we suppose i is a cooperator, and its neighbour j is a defector): a new player k is then chosen at random and the link (i, j) is rewired to (i, k) with probability q or to (j, k) with probability $1-q$. With complementary probability $1-p$, strategy updating takes place: the two players compare their payoff, and the player with less payoff imitates the strategy of another player. The evolutionary process ends up till all C–D links die out.

3. Monte carlo simulation

Initially, N individuals locate on the nodes of the Erdös–Rényi (ER) random graph with average connectivity $\langle k\rangle$, and they are randomly assigned to be either strategy C or D with equal probability 1/2. We perform Monte Carlo (MC) simulation for the course of co-evolution of the network and strategy updating described in detail in the above section.

It is clear that the model moves toward to decrease the number of C–D links such that no C–D link survives in the ultimate network. Thus, the network may be separated to a set of disconnected communities, each of which all individuals share the same strategy. The primary interest in our model is how cooperative phenomenon arises via the process of the adaptive co-evolution.

We first consider the case for $q = 1$. We depict the density of cooperators $\rho_{\rm C}$ as a function of p for several different values of r, as shown in Fig. 1, in which each dot corresponds to an average over 100 realisations of initial network and initial condition. For a given value of r, $\rho_{\rm C}$ increases rapidly when p exceeds a certain value and then decreases as p is increased again, thus indicating the existence of maximal $\rho_{\rm C}$ for an intermediate range of p in the studied networked PDG. In a word, the co-evolution of individual strategy and interacted network can promote cooperation among selfish individuals, and there exists a optimal p for which the cooperation level is maximised.

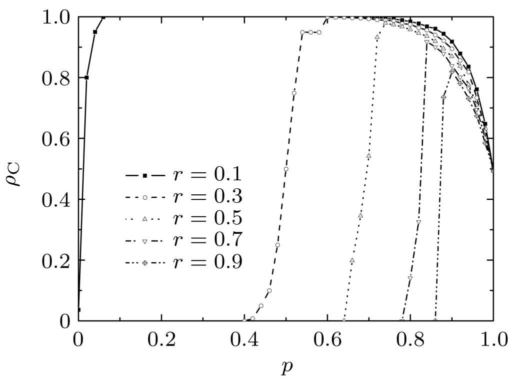

Fig. 1. The density of cooperators $\rho_{\rm C}$ as a function of p for several different values of r. Other parameters are $N = 500$, $\langle k\rangle = 20$, and $q = 1$.

For small p, i.e. the probability of rewiring C–D links is little, few C–D links become C–C ones and thus the average degree of C players is approximately equal to that of D players. In this case the defector's payoff is generally higher than cooperator's as a result of the prevalence of defector. For very large p, almost all C–D links quickly convert to C–C ones in the process of rewiring, which results in little chance of updating strategy. Thus, in this case the density of cooperators in the final state is evidently close to initial level. Most importantly, an intermediate range of p can ensure that the balance between rewiring events and strategy updating. This balance leads to the formation of hub (node with high connectivity), which plays a dominant role in promoting cooperation in the underlying network.[16,45,52]

In order to confirm this point, we use the normalised degree variance to measure the heterogeneity of the networks defined as $\sigma_k^2 = \left(\langle k^2\rangle - \langle k\rangle^2\right)/\langle k\rangle$. A larger σ_k^2 implies the more heterogeneity of the network. From Fig. 2, one can see that, when the value of p is relatively small the degree distribution of the resulting network approaches the Poisson distribution so as to $\sigma_k^2 \simeq 1$, similar to the initial one. When p increases, σ_k^2 rapidly increases to a maximum and then decreases. The inset of Fig. 2 shows degree distribution $P(k)$ of the maximal community of the final network for three typical values of p. There is a significant difference from the other two cases in the case $p = 0.75$ corresponding to a broad degree distribution. Interestingly, we find the degree distribution appears to follow a power law $P(k) \sim k^{-\gamma}$ over a significant part of its range. Therefore, a optimal p should exist for maximal cooperation level. Furthermore, this optimal p is dependent on the temptation to defect r; in detail, it becomes larger for a larger r. With increasing r, the rewiring event must be sufficiently frequent to guarantee the survival of cooperation. In particular, for relatively large r, the probability that an individual chooses for rewiring social ties should exceeds that of strategy updating in order to ensure the sustainability of cooperators.

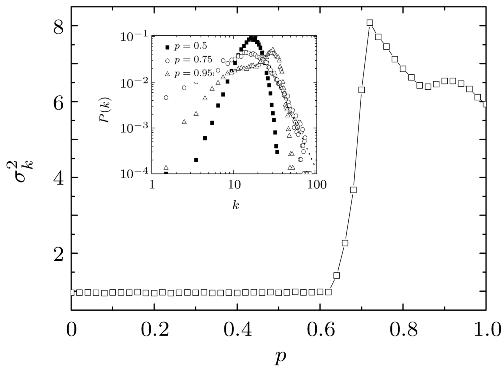

Fig. 2. Normalised degree variance σ_k^2 as a function of p. The inset depicts degree distribution of the giant component of the ultimate network for different p with a log–log scale. For $p = 0.75$ the distribution appears to follow a power law for part of its range, with exponent 4.0 ± 0.1, as indicated by the dotted line. Other parameters are $N = 500$, $\langle k \rangle = 20$, $r = 0.5$, and $q = 1$.

One recalls that the ultimate network may be split into some disconnected communities, each of which all agents share the same strategy. In real-world biological or social systems, segregation phenomena play a crucial role in sustaining diversity at many levels-cellular, functional, organisational, ecological, cultural.[53] For example, in human societies many sub-populations or communities may exhibit mutually conflicting cultural traits such that interactions between these communities are scarce, and thus can be effectively considered as disconnected from each other.[54] It is concerned how the number of communities and the size of the maximal community dependent on p. Figure 3 shows normalised size of the maximal community S/N and the number of communities $N_{\rm C}$ as a function of p, for some various values of r. For each value of r, we find that, beyond a critical value p_c, S/N continuously decreases from one, as shown in Fig. 3(a), and $N_{\rm C}$ increases from one and then decreases, as shown in Fig. 3(b). That is, the network begins to split when $p > p_c$. Contrasting with Fig. 1, one can notice that the location that the density of cooperators begins to increase from zero as p is varied just happens at $p = p_c$.

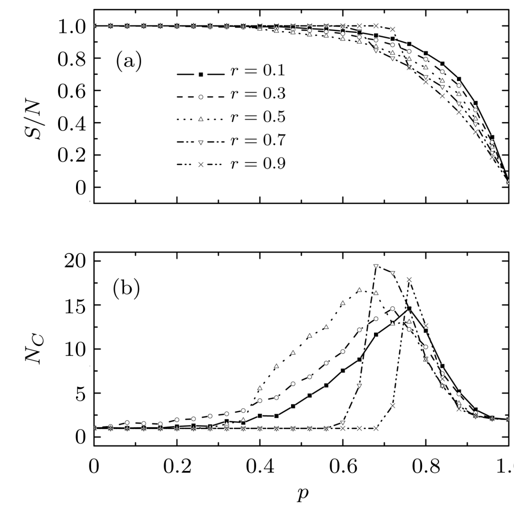

Fig. 3. Normalised size of the maximal community S/N and the number of communities $N_{\rm C}$ as a function of p, for several different r. Other parameters are $N = 500$, $\langle k \rangle = 20$, and $q = 1$.

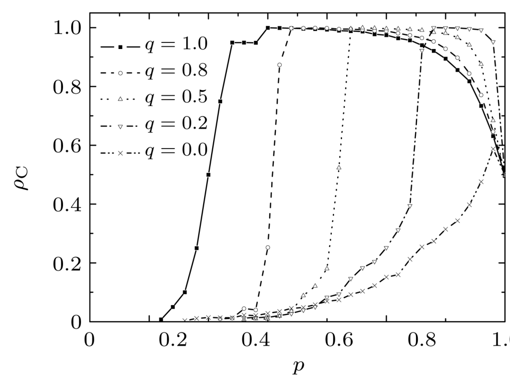

Fig. 4. The density of cooperators $\rho_{\rm C}$ as a function of p for several different values of q. Other parameters are $r = 0.3$, $N = 500$, $\langle k \rangle = 20$.

We now consider the case for $q \neq 1$. Figure 4 shows the density of cooperators $\rho_{\rm C}$ as a function of p for several different values of q, with other relevant parameters $r = 0.3$, $N = 500$, $\langle k \rangle = 20$. For any q, there also exists an intermediate range of p for which the cooperation level is maximised. As q is decreased, the location for the maximal $\rho_{\rm C}$ shifts to right. We also calculate the cases for some other values of r, and find that there is no essential difference in the main results.

4. Mean-field analysis

In order to understand above-mentioned results on adaptive network, we here provide some analytical illustration by using mean-field (MF) approximation. First of all, we introduce some quantities to characterise the state of system: (i) the magnetisation $m = \rho_{\rm C} - \rho_{\rm D}$, where $\rho_{\rm D}$ is the density of defectors; (ii) the number of links joining agents in the strategy C, $Nl_{\rm CC}$; (iii) the number of links joining agents of opposite strategies, i.e., of active links $Nl_{\rm CD} = Nl_{\rm DC}$. Since conservation rules $\rho_{\rm C} + \rho_{\rm D} = 1$, $l_{\rm CC} + l_{\rm CD} + l_{\rm DD} = \langle k \rangle /2$, the system's dynamical state can be expressed by the vector $\boldsymbol{X} = (m, l_{\rm CD}, l_{\rm CC})^{\rm T}$. According to the model's definition, the vector $\boldsymbol{X}$ can evolve via four routes at each elementary update: $\boldsymbol{X} \to \boldsymbol{X} + \nu^{\alpha}, \alpha = 1, 2, 3, 4$ with respective transition probabilities w^{α}. The displacement vectors and the associated probabilities are readily written: $N\nu^1 = (0, -1, 1)$, $w^1 = 2l_{\rm CD} \langle k \rangle^{-1} pq\rho_{\rm C}$; $N\nu^2 = (0, -1, 0)$, $w^2 = 2l_{\rm CD} \langle k \rangle^{-1} p(1-q)\rho_{\rm D}$; $N\nu^3 = (2, k_{\rm DD} - k_{\rm DC}, k_{\rm DC})$, $w^3 = 2l_{\rm CD} \langle k \rangle^{-1} (1-p) \Pr(P_{\rm C} > P_{\rm D})$; $N\nu^4 = (-2, k_{\rm CC} - k_{\rm CD}, -k_{\rm CC})$, $w^4 = 2l_{\rm CD} \langle k \rangle^{-1} (1-p) \Pr(P_{\rm C} < P_{\rm D})$. Here $k_{\rm CD} = l_{\rm CD}/\rho_{\rm C}$ is the average number of D neighbours of a C agent. Likewise, $k_{\rm CC} = 2l_{\rm CC}/\rho_{\rm C}$, $k_{\rm DC} = l_{\rm DC}/\rho_{\rm D}$, $k_{\rm DD} = 2l_{\rm DD}/\rho_{\rm D}$. $\Pr(\cdot)$ denotes probability, and $P_{\rm C}$ and $P_{\rm D}$ are the average payoffs of a cooperator and a defector, respectively, which can be calculated by MF approximation as

$$P_{\rm C} = k_{\rm CC}R + k_{\rm CD}S = k_{\rm CC},$$
$$P_{\rm D} = k_{\rm DC}T + k_{\rm DD}P = k_{\rm DC}(1 + r). \qquad (3)$$

The first two routes correspond to a rewiring event. The first route occurs whenever a C–D link is rewired to a C–C link. The probability that this route happens is proportional to the density of C–D links, $2l_{\rm CD}/\langle k \rangle$, times the probability that a C–D link is rewired to C–C link, $pq\rho_{\rm C}$. The second route takes place whenever a C–D link is rewired to a D–D link. The other two routes correspond to strategy updating, for which the change in magnetisation ($\pm 2/N$) is associated with changes in the densities of all types of links. For example, for the fourth route, when a C agent changes its strategy to D, its C–C links become D-C and its C–D links become D–D ones, and thus the changes of m, $l_{\rm CD}$, $l_{\rm CC}$ are $-2/N$, $(k_{\rm CC} - k_{\rm CD})/N$, $-k_{\rm CC}/N$, respectively. Furthermore, the probability of the fourth route is proportional to the density of C–D links, $2l_{\rm CD}/\langle k \rangle$, times the probability that a C agent changes its strategy, $(1-p)\Pr(P_{\rm C} < P_{\rm D})$. Thus, dynamical equations ruling the evolution of $\boldsymbol{X}$ can be written as $\mathrm{d}\boldsymbol{X}/\mathrm{d}t = \sum_{\alpha} \nu^{\alpha} w^{\alpha}$. Figure 5 shows the result obtained by numerical integration of the evolution of $\boldsymbol{X}$, compared with the results of simulations presented in the former section. It is clear that mean-filed approximation analysis qualitatively predicts the trends that $\rho_{\rm C}$ changes with p, r and q.

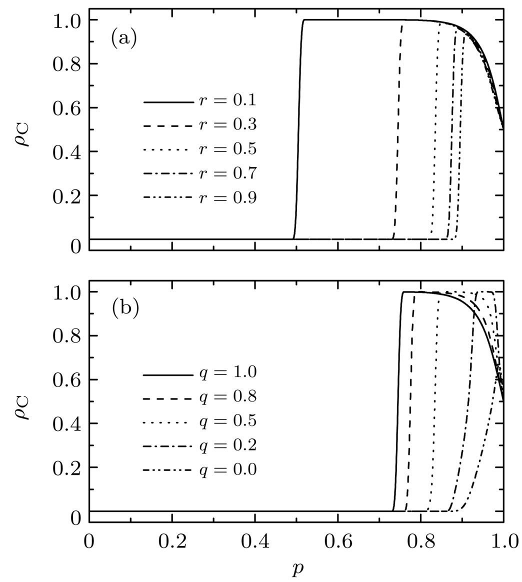

Fig. 5. Mean-field results: (a) the density of cooperators $\rho_{\rm C}$ as a function of p for several different values of r with $q = 1$ fixed; (b) the density of cooperators $\rho_{\rm C}$ as a function of p for several different values of q with $r = 0.3$ fixed. Other parameters are $N = 500$, $\langle k \rangle = 20$.

5. Conclusions

In conclusion, we use MC simulations and MF analysis to investigate evolutionary PDG on adaptive network topologies where a population of players coevolve with their interaction network. Our model describes that interacted players with opposite strategies either rewire their link or update their strategies depending on the payoff during the co-evolutionary process. We select a quantity p to regulate the time scales of network evolution and node dynamics. Interestingly, for a moderate range of p, the density of cooperators in the final state can be maximised, which attributes to competitive balance of the two time scales such that the emergence of so-called hub providing sufficient condition for cooperation to dominate. Since real social network itself is dynamic instead of static, our results may provide some insight into understanding the occurrence of cooperative behaviours in the real world.

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