Discovering Classes of Strongly Equivalent Logic Programs with Negation as Failure in the Head

Jianmin Ji

School of Computer Science and Technology
University of Science and Technology of China
Hefei, China
jianmin@ustc.edu.cn

Abstract. In this paper, we apply Fangzhen Lin's methodology of computer aided theorem discovery to discover classes of strongly equivalent logic programs with negation as failure in the head. Specifically, with the help of computers, we discover exact conditions that capture the strong equivalence between small sets of rules, which have potential applications in the theory and practice of logic programming. In the experiment, we extend the previous approach to semi-automatically generate plausible conjectures. We also show that it is possible to divide the original problem in simpler cases and combine their solutions in order to obtain the solution of the original problem.

1 Introduction

Fangzhen Lin introduced a methodology, called computer-aided theorem discovery [2], to discover some theorems using computers in a given theory. The methodology has been successfully applied to discover classes of strongly equivalent logic programs in the theory of logic programming [3].

In this paper, we report on another successful experiment of the methodology for logic programs with negation as failure in the head [1] and make three contributions. First, we extend Lin and Chen's approach [3] to semi-automatically generate candidates of theorems that need to be discovered in the experiment. Second, we show that when the methodology cannot be directly applied, since it would be computationally unfeasible, it is possible to divide the original problem in simpler cases and combine their solutions in order to obtain the solution of the original problem. Third, we discover the new and non-trivial conditions that capture certain classes of strongly equivalent logic programs, which contribute to the theory and practice of logic programming.

2 Logic programs with negation as failure in the head

Logic programming with answer set semantics has been considered as one of the most popular nonmonotonic rule-based formalisms [1]. In this paper, we consider only fully grounded finite logic programs.

Let L be a propositional language, *i.e.*, a set of atoms. An *extended logic program* (ELP) is a finite set of (*extended*) *rules* of the form

$$a_1 \vee \cdots \vee a_k \vee not\, a_{k+1} \vee \cdots \vee not\, a_h \leftarrow a_{h+1}, \ldots, a_m, not\, a_{m+1}, \ldots, not\, a_n, \quad (1)$$

where $n \geq m \geq h \geq k \geq 0$, $n \geq 1$ and $a_1, \ldots, a_n$ are atoms in L. If $h = k$, it is a *disjunctive rule*; if $h = k$ and $m = n$, it is a *positive rule*. In particular, a *disjunctive logic program* (DLP) is a finite set of disjunctive rules and a positive program is a finite set of positive rules. An ELP is also called a *logic program with negation as failure in the head* [1]. Note that, generally it is impossible to translate an ELP to a DLP without adding new atoms.

We will also write rule r of form (1) as $head(r) \leftarrow body(r)$, where $head(r) = head^+(r) \vee head^-(r)$, $body(r) = body^+(r) \wedge body^-(r)$, $head^+(r)$ is $a_1 \vee \cdots \vee a_k$, $head^-(r)$ is $\neg a_{k+1} \vee \cdots \vee \neg a_h$, $body^+(r)$ is $a_{h+1} \wedge \cdots \wedge a_m$, and $body^-(r)$ is $\neg a_{m+1} \wedge \cdots \wedge \neg a_n$. In the following, we identify $head^+(r)$, $head^-(r)$, $body^+(r)$, $body^-(r)$ with their corresponding sets of atoms.

Two ELPs P_1 and P_2 are *strongly equivalent*, if for any ELP P', programs $P_1 \cup P'$ and $P_2 \cup P'$ have the same set of answer sets. In general, checking if two ELPs or DLPs are strongly equivalent is coNP-complete.There is a mapping from logic programs to propositional theories and showed that two logic programs are strongly equivalent iff their corresponding theories in propositional logic are equivalent. This result provides the basis for applying Lin's computer-aided theory discovery.

3 Discovering Classes of Strongly Equivalent ELPs

In this paper, we extend Lin and Chen's approach to discovering classes of strongly equivalent ELPs. We focus on discovering necessary and sufficient conditions for answering the *k-m-n* problem for ELPs, *i.e.*, is an ELP $\{r_1, \ldots, r_k, u_1, \ldots, u_m\}$ strongly equivalent to an ELP $\{r_1, \ldots, r_k, v_1, \ldots, v_n\}$?

Following Lin's computer-aided theory discovery, we first construct a first-order language F_L based on the propositional language L of ELPs. In specific, F_L has equality, two unary predicates H_1 and H_2, and four unary predicates PH_r, NH_r, PB_r, and NB_r for each rule r in L. An *intended model* of F_l is one whose domain is L, and for each rule $r \in L$, the unary predicates PH_r, NH_r, PB_r, and NB_r are interpreted by the sets of atoms $head^+(r)$, $head^-(r)$, $body^+(r)$, and $body^-(r)$, respectively.

Theorem 1. *P_1 and P_2 are strongly equivalent in L iff the following sentence*

$$\forall x(H_1(x) \supset H_2(x)) \supset \left(\bigwedge_{r \in P_1} \gamma(r) \equiv \bigwedge_{r \in P_2} \gamma(r) \right) \tag{2}$$

is true in all intended models of F_L, where $\gamma(r)$ is the conjunction of the following two sentences:

$$[\forall x(PB_r(x) \supset H_1(x)) \wedge \forall x(NB_r(x) \supset \neg H_2(x))] \supset [\exists x(PH_r(x) \wedge H_1(x)) \vee \exists x(NB_r(x) \wedge \neg H_2(x))]\,,$$
$$[\forall x(PB_r(x) \supset H_2(x)) \wedge \forall x(NB_r(x) \supset \neg H_2(x))] \supset [\exists x(PH_r(x) \wedge H_2(x)) \vee \exists x(NB_r(x) \wedge \neg H_2(x))]\,.$$

Given a *k-m-n* problem, *i.e.*, $P_1 = \{r_1, \ldots, r_k, u_1, \ldots, u_m\}$ and $P_2 = \{r_1, \ldots, r_k, v_1, \ldots, v_n\}$, if a conjecture for answering the *k-m-n* problem is represented by the formula $\exists \boldsymbol{x} \forall \boldsymbol{y}\, \Phi$ in F_L, then verifying the conjecture is equivalent to verifying the formula $\exists \boldsymbol{x} \forall \boldsymbol{y}\, \Phi \supset (2)$. Now we have the following theorem.

Theorem 2. *Given ELPs $P_1 = \{r_1, \ldots, r_k, u_1, \ldots, u_m\}$ and $P_2 = \{r_1, \ldots, r_k, v_1, \ldots, v_n\}$ in the propositional language L, if $\exists \boldsymbol{x} \forall \boldsymbol{y}\, \Phi$ is a property about P_1 and P_2 in F_L, where $\boldsymbol{x}$ is a tuple of w variables, and Φ a quantifier-free, function-free, and constant-free formula, then the following two assertions are equivalent:*

1. *If $\exists \boldsymbol{x} \forall \boldsymbol{y}\, \Phi$ is true in F_L, then P_1 is strongly equivalent to P_2.*
2. *For any ELPs P_1 and P_2 with at most $w + 2(k + \max\{m, n\})$ atoms when $\min\{m, n\} > 0$ and $\max\{w + 2k, 1\}$ atoms when $\min\{m, n\} = 0$, if $\exists \boldsymbol{x} \forall \boldsymbol{y}\, \Phi$ is true in an intended model of F_L, then P_1 is strongly equivalent to P_2.*

Then the correctness of the conjecture for the *k*-*m*-*n* problem can be verified by considering corresponding ELPs with a small size of atoms.

4 The Theorems

4.1 The 0-1-0 problem

This problem asks if a rule can always be deleted from any program. With the help of computers[1], we get the following experimental result:

Lemma 1. *If a rule r mentions three distinct atoms, then $\{r\}$ is strongly equivalent to $\emptyset$ iff $(head^+(r) \cup body^-(r)) \cap body^+(r) \neq \emptyset$ or $head^-(r) \cap body^-(r) \neq \emptyset$.*

Lemma 2. *If there is a rule r such that $\{r\}$ and $\emptyset$ are strongly equivalent, but the condition in Lemma 1 does not hold, then there is a such rule that mention at most three atoms.*

Theorem 3 (The 0-1-0 problem). *Lemma 1 holds in the general case, without any restriction on the number of atoms in r.*

4.2 The 1-1-0 and the 0-1-1 problems

With the help of computers, we get the following result for the 1-1-0 problem:

Lemma 3. *For any two rules r_1 and r_2 that mention four atoms, $\{r_1, r_2\}$ and $\{r_1\}$ are strongly equivalent iff one of the following three conditions is true:*

1. *$\{r_2\}$ is strongly equivalent to $\emptyset$.*
2. *$head^+(r_1) \subseteq head^+(r_2) \cup body^-(r_2)$, $head^-(r_1) \subseteq head^-(r_2) \cup body^+(r_2)$, $body^+(r_1) \subseteq body^+(r_2)$, and $body^-(r_1) \subseteq body^-(r_2)$.*
3. *$head^+(r_1) \subseteq body^-(r_2)$, $head^-(r_1) \subseteq head^-(r_2) \cup body^+(r_2)$, $body^+(r_1) \subseteq head^-(r_2) \cup body^+(r_2)$, and $body^-(r_1) \subseteq body^-(r_2)$.*

Lemma 4. *If there are two rules r_1 and r_2 such that $\{r_1, r_2\}$ and $\{r_1\}$ are strongly equivalent, but none of the three conditions in Lemma 3 hold, then there are two such rules that mention at most four atoms.*

Theorem 4 (The 1-1-0 problem). *Lemma 3 holds in the general case, without any restriction on the number of atoms in r_1 and r_2.*

[1] Source codes of computer programs for verifying conjectures can be downloaded from http://staff.ustc.edu.cn/%7ejianmin/discover/code.zip.

Theorem 5 (The 0-1-1 problem). *For any two rules r_1 and r_2, $\{r_1\}$ and $\{r_2\}$ are strongly equivalent iff one of the following three conditions is true:*

1. *$\{r_1\}$ and $\{r_2\}$ are both strongly equivalent to $\emptyset$.*
2. *$body^+(r_1) = body^+(r_2)$, $body^-(r_1) = body^-(r_2)$, $head^-(r_1) \cup body^+(r_1) = head^-(r_2) \cup body^+(r_2)$, and $head^+(r_1) \cup body^-(r_1) = head^+(r_2) \cup body^-(r_2)$.*
3. *$head^+(r_1) \subseteq body^-(r_1)$, $head^+(r_2) \subseteq body^-(r_2)$, $body^-(r_1) = body^-(r_2)$, and $head^-(r_1) \cup body^+(r_1) = head^-(r_2) \cup body^+(r_2)$.*

4.3 The 2-1-0, 0-2-1, and 0-2-2 problems

As the 2-1-0 problem is too hard to be solved directly, we need to first divide the problem into simpler cases.

Property 1. For any rules r_i and r_3, $\{r_i, r_3\}$ and $\{r_i\}$ are not strongly equivalent iff $\{r_3\}$ is not strongly equivalent to $\emptyset$ and one of the five conditions is true:

1. There is an atom p such that: $p \in body^-(r_i)$ and $p \notin body^-(r_3)$.
2. There is an atom p such that: $p \in head^-(r_i)$ and $p \notin head^-(r_3) \cup body^+(r_3)$.
3. There is an atom p such that: $p \in body^+(r_i)$ and $p \notin head^-(r_3) \cup body^+(r_3)$.
4. There is an atom p such that: $p \in head^+(r_i)$ and $p \notin head^+(r_3) \cup body^-(r_3)$.
5. There are two atoms p, q such that: $p \in body^+(r_i)$, $p \notin body^+(r_3)$, $p \in head^-(r_3)$, $q \in head^+(r_i)$, $q \notin body^-(r_3)$ and $q \in head^+(r_3)$.

Property 2. For any rules r_1, r_2 and r_3, one of the four conditions is true:

1. $\{r_3\}$ is strongly equivalent to $\emptyset$.
2. $\{r_i, r_3\}$ is strongly equivalent to $\{r_i\}$, for $i = 1, 2$.
3. $\{r_3\}$ is not strongly equivalent to $\emptyset$, one of the conditions from (1) - (4) of Property 1 is true, and the condition (5) of Property 1 is not true, where $i = 1$ or 2.
4. $\{r_3\}$ is not strongly equivalent to $\emptyset$, $\{r_1, r_3\}$ is not strongly equivalent to $\{r_1\}$, $\{r_2, r_3\}$ is not strongly equivalent to $\{r_2\}$, and the condition (5) of Property 1 is true, where $i = 1$ or 2.

Lemma 5. *For any three rules r_1, r_2 and r_3 that make the condition (3) of Property 2 true and mention at most five atoms, $\{r_1, r_2, r_3\}$ and $\{r_1, r_2\}$ are strongly equivalent if there is an atom p such that:*

1. *$p \in (head^-(r_1) \cup body^+(r_1)) \cap (body^-(r_2) \cup head^+(r_2))$,*
2. *$\{r_i^*, r_3\}$ is strongly equivalent to $\{r_i^*\}$, for $i = 1, 2$,*
3. *If $p \in body^+(r_1) \cap body^-(r_2)$, then $head^+(r_1) \subseteq body^-(r_3)$,*
4. *If $p \in body^+(r_1) \cap head^+(r_2)$, then $head^+(r_1) \subseteq body^-(r_3)$ or $body^+(r_2) \subseteq body^+(r_3)$,*

where r_1^* is a new rule obtained from r_1 by deleting p from $head^-(r_1)$ and $body^+(r_1)$, and r_2^* is obtained from r_2 by deleting p from $body^-(r_2)$ and $head^+(r_2)$.

Lemma 6. *For any three rules r_1, r_2 and r_3 that make the condition (3) of Property 2 true and mention at most five atoms, $\{r_1, r_2, r_3\}$ and $\{r_1, r_2\}$ are strongly equivalent if there is an atom p such that:*

1. $p \in (body^-(r_1) \cup head^+(r_1)) \cap (head^-(r_2) \cup body^+(r_2))$,
2. $\{r_i^*, r_3\}$ *is strongly equivalent to* $\{r_i^*\}$, *for* $i = 1, 2$,
3. *If* $p \in body^-(r_1) \cap body^+(r_2)$, *then* $head^+(r_2) \subseteq body^-(r_3)$,
4. *If* $p \in head^+(r_1) \cap body^+(r_2)$, *then* $head^+(r_2) \subseteq body^-(r_3)$ *or* $body^+(r_1) \subseteq body^+(r_3)$,

where r_1^* *is a new rule obtained from* r_1 *by deleting* p *from* $body^-(r_1)$ *and* $head^+(r_1)$, *and* r_2^* *is obtained from* r_2 *by deleting* p *from* $head^-(r_2)$ *and* $body^+(r_2)$.

Lemma 7. *For any three rules* r_1, r_2 *and* r_3 *that make the condition (3) of Property 2 true and mention at most five atoms,* $\{r_1, r_2, r_3\}$ *and* $\{r_1, r_2\}$ *are strongly equivalent iff the condition in Lemma 5 or Lemma 6 is true.*

Lemma 8. *If there are three rules* r_1, r_2 *and* r_3 *such that the condition (3) of Property 2 is true,* $\{r_1, r_2, r_3\}$ *and* $\{r_1, r_2\}$ *are strongly equivalent, but the condition in Lemma 7 does not hold, then there are three such rules that mention at most five atoms.*

Theorem 6. *Lemma 7 holds in the general case, without any restriction on the number of atoms in* r_1, r_2 *and* r_3.

Lemma 9. *For any three rules* r_1, r_2 *and* r_3 *that make the condition (4) of Property 2 true and mention at most six atoms,* $\{r_1, r_2, r_3\}$ *and* $\{r_1, r_2\}$ *are strongly equivalent if there are two atoms* p *and* q *such that:*

1. $p \in head^-(r_1) \cap head^+(r_2) \cap head^+(r_3)$, $p \notin body^+(r_1)$ *and* $p \notin body^-(r_2)$,
2. $q \in head^+(r_1)$ *and* $q \in body^+(r_2)$,
3. $\{r_i^*, r_3\}$ *is strongly equivalent to* $\{r_i^*\}$, *for* $i = 1, 2$,
4. $body^+(r_2) \setminus \{q\} \subseteq body^+(r_3)$, *and* $body^+(r_1) \subseteq body^+(r_3)$,

where r_1^* *is a new rule obtained from* r_1 *by deleting* p *from* $head^-(r_1)$ *and deleting* q *from* $head^+(r_1)$, *and* r_2^* *is obtained from* r_2 *by deleting* p *from* $head^+(r_2)$.

Lemma 10. *For any three rules* r_1, r_2 *and* r_3 *that make the condition (4) of Property 2 true and mention at most six atoms,* $\{r_1, r_2, r_3\}$ *and* $\{r_1, r_2\}$ *are strongly equivalent if there are two atoms* p *and* q *such that:*

1. $p \in head^+(r_1) \cap head^-(r_2) \cap head^+(r_3)$, $p \notin body^-(r_1)$ *and* $p \notin body^+(r_2)$,
2. $q \in body^+(r_1)$ *and* $q \in head^+(r_2)$,
3. $\{r_i^*, r_3\}$ *is strongly equivalent to* $\{r_i^*\}$, *for* $i = 1, 2$,
4. $body^+(r_1) \setminus \{q\} \subseteq body^+(r_3)$, *and* $body^+(r_2) \subseteq body^+(r_3)$,

where r_1^* *is a new rule obtained from* r_1 *by deleting* p *from* $head^+(r_1)$, *and* r_2^* *is obtained from* r_2 *by deleting* p *from* $head^-(r_2)$ *and deleting* q *from* $head^+(r_2)$.

Lemma 11. *For any three rules* r_1, r_2 *and* r_3 *that make the condition (4) of Property 2 true and mention at most six atoms,* $\{r_1, r_2, r_3\}$ *and* $\{r_1, r_2\}$ *are strongly equivalent iff the condition in Lemma 7, Lemma 9 or Lemma 10 is true.*

Lemma 12. *If there are three rules r_1, r_2 and r_3 such that the condition (4) of Property 2 is true, $\{r_1, r_2, r_3\}$ and $\{r_1, r_2\}$ are strongly equivalent, but the condition in Lemma 11 does not hold, then there are three such rules that mention at most six atoms.*

Theorem 7. *Lemma 11 holds in the general case, without any restriction on the number of atoms in r_1, r_2 and r_3.*

Theorem 8. *For any three rules r_1, r_2 and r_3, $\{r_1, r_2, r_3\}$ and $\{r_1, r_2\}$ are strongly equivalent iff one of the following three conditions is true:*

1. *$\{r_3\}$ is strongly equivalent to $\emptyset$.*
2. *$\{r_i, r_3\}$ is strongly equivalent to $\{r_i\}$, where $i = 1$ or 2.*
3. *the condition in Lemma 5, 6, 6, or 10 is true.*

Theorem 9 (The 0-2-1 problem). *For any three rules r_1, r_2 and r_3, $\{r_1, r_2\}$ and $\{r_3\}$ are strongly equivalent iff the following two conditions are true:*

1. *$\{r_1, r_2, r_3\}$ and $\{r_1, r_2\}$ are strongly equivalent, and*
2. *$\{r_i, r_3\}$ and $\{r_3\}$ are strongly equivalent, for $i = 1, 2$.*

Theorem 10 (The 0-2-2 problem). *For any rules r_1, r_2, r_3 and r_4, $\{r_1, r_2\}$ and $\{r_3, r_4\}$ are strongly equivalent iff the following two conditions are true:*

1. *$\{r_1, r_2, r_i\}$ and $\{r_1, r_2\}$ are strongly equivalent, for $i = 3, 4$, and*
2. *$\{r_3, r_4, r_i\}$ and $\{r_3, r_4\}$ are strongly equivalent, for $i = 1, 2$.*

5 Conclusion

In this paper, we report on another successful experiment of Lin's computer-aided theory discovery for discovering classes of strongly equivalent extended logic programs. The paper makes three contributions. First, we extend Lin and Chen's approach to semi-automatically generate plausible conjectures. Second, we show that when the methodology cannot be directly applied, since it would be computationally unfeasible, it is possible to divide the original problem in simpler cases and combine their solutions in order to obtain the solution of the original problem. Third, we discover the new and non-trivial conditions that capture certain classes of strongly equivalent extended logic programs, which contribute to the theory and practice of logic programming.

Acknowledgments. We thank Xiaoping Chen for helpful discussions. The work was supported by NSFC under grant 61175057, NSFC for the Youth under grant 61403359, as well as the USTC Key Direction Project and the USTC 985 Project.

References

1. Inoue, K., Sakama, C.: Negation as failure in the head. The Journal of Logic Programming 35(1), 39–78 (1998)
2. Lin, F.: Finitely-verifiable classes of sentences. In: Proceedings of the 8th International Symposium on Logical Formalizations of Commonsense Reasoning (Commonsense'07). pp. 89–94 (2007)
3. Lin, F., Chen, Y.: Discovering classes of strongly equivalent logic programs. In: Proceedings of the 19th International Joint Conference on Artificial Intelligence (IJCAI-05). pp. 516–521 (2005)