Combinatorics, 2016 Fall, USTC
Homework 11

- The due is on **Tuesday**, Dec. 13, at beginning of the class.
- Solve all problems.

1. Using the corollary proved in lecture on Dec 6, prove the approximate version of Turán's Theorem: For any n-vertex K_{r+1}-free graph G, we have $e(G) \leq \frac{r-1}{2r} n^2$.

2. Prove the exact version of Turán's Theorem: For any n-vertex K_{r+1}-free graph G,

$$e(G) \leq e(T_r(n))$$

with equality if and only if G is the Turán graph $T_r(n)$. (Hint: by induction on r.)

3. Let $\mathcal{F}$ be an independent system of $[n]$. Let $\pi \in S_n$ be a random permutation of $[n]$. By considering the set

$$X = \{i : \{\pi(1), \pi(2), ..., \pi(i)\} \in \mathcal{F}\},$$

give a new proof of that

$$|\mathcal{F}| \leq \binom{n}{\lfloor n/2 \rfloor}.$$

4. Given a graph G with $\chi(G) = n$ and a proper coloring $f : V(G) \to [n]$, prove that for any color $i \in [n]$, there exists a vertex of color i adjacent to a vertex of every other color.

5. Let H be the graph obtained from K_4 by deleting one edge. Prove that for infinity many integers n, there exists an n-vertex H-free graphs with at least $c \cdot n^{6/5}$ edges for some absolute constant $c > 0$.